Chapter 7

Annual Worth Analysis

Systematic Economic Analysis Technique

1. Identify the investment alternatives
2. Define the planning horizon
3. Specify the discount rate
4. Estimate the cash flows
5. Compare the alternatives
6. Perform supplementary analyses
7. Select the preferred investment

Annual Worth Analysis

Single Alternative

Annual Worth Method

- converts all cash flows to a uniform annual series over the planning horizon using i=MARR
- a popular DCF method

$$AW(i\%) = \left[\sum_{t=0}^{n} A_t (1+i)^{-t} \right] \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

$$AW(i\%) = PW(i\%)(A \mid P\ i\%, n)$$

SMP Investment

A $500,000 investment in a surface mount placement machine is being considered. Over a 10-year planning horizon, it is estimated the SMP machine will produce net annual savings of $92,500. At the end of 10 years, it is estimated the SMP machine will have a $50,000 salvage value. Based on a 10% MARR and annual worth analysis, should the investment be made?

AW(10%) = -$500K(A|P 10%,10) + $92.5K
+ $50K(A|F 10%,10)
= $14,262.50
=PMT(10%,10,500000,-50000)+92500
= $14,264.57

SMP Investment

How does annual worth change over the life of the investment? How does annual worth change when the salvage value decreases geometrically and as a gradient series?

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D12 =PMT(B1,A12,-B3,-C12)+B12

MARR =	**10%**					
Planning Horizon	**CF**	**$SV_{geometric}$**	**$AW_{geometric}$**	**$SV_{gradient}$**	**$AW_{gradient}$**	**Excel Formula for $AW_{gradient}$**
0	**-$500,000**	**$500,000**	**n/a**	**$500,000**	**n/a**	
1	**$92,500**	**$397,000**	**-$60,500.00**	**$455,000**	**-$2,500.00**	**=PMT(B1,A4,-B3,-E4)+B4**
2	**$92,500**	**$315,218**	**-$45,491.43**	**$410,000**	**-$357.14**	**=PMT(B1,A5,-B3,-E5)+B5**
3	**$92,500**	**$250,283**	**-$32,943.17**	**$365,000**	**$1,714.50**	**=PMT(B1,A6,-B3,-E6)+B6**
4	**$92,500**	**$198,725**	**-$22,416.01**	**$320,000**	**$3,715.26**	**=PMT(B1,A7,-B3,-E7)+B7**
5	**$92,500**	**$157,787**	**-$13,553.55**	**$275,000**	**$5,645.57**	**=PMT(B1,A8,-B3,-E8)+B8**
6	**$92,500**	**$125,283**	**-$6,066.06**	**$230,000**	**$7,506.01**	**=PMT(B1,A9,-B3,-E9)+B9**
7	**$92,500**	**$99,475**	**$282.45**	**$185,000**	**$9,297.27**	**=PMT(B1,A10,-B3,-E10)+B10**
8	**$92,500**	**$78,983**	**$5,684.59**	**$140,000**	**$11,020.15**	**=PMT(B1,A11,-B3,-E11)+B11**
9	**$92,500**	**$62,713**	**$10,297.92**	**$95,000**	**$12,675.58**	**=PMT(B1,A12,-B3,-E12)+B12**
10	**$137,500**	**$49,794**	**$14,251.63**	**$50,000**		
***AW* =**	**$14,264.57**	**=PMT(B1,10,500000,-50000)+92500**				

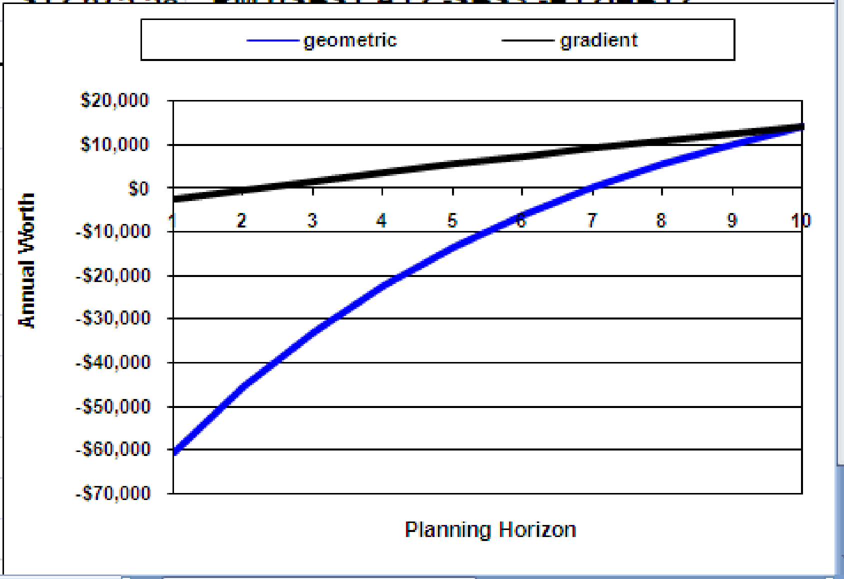


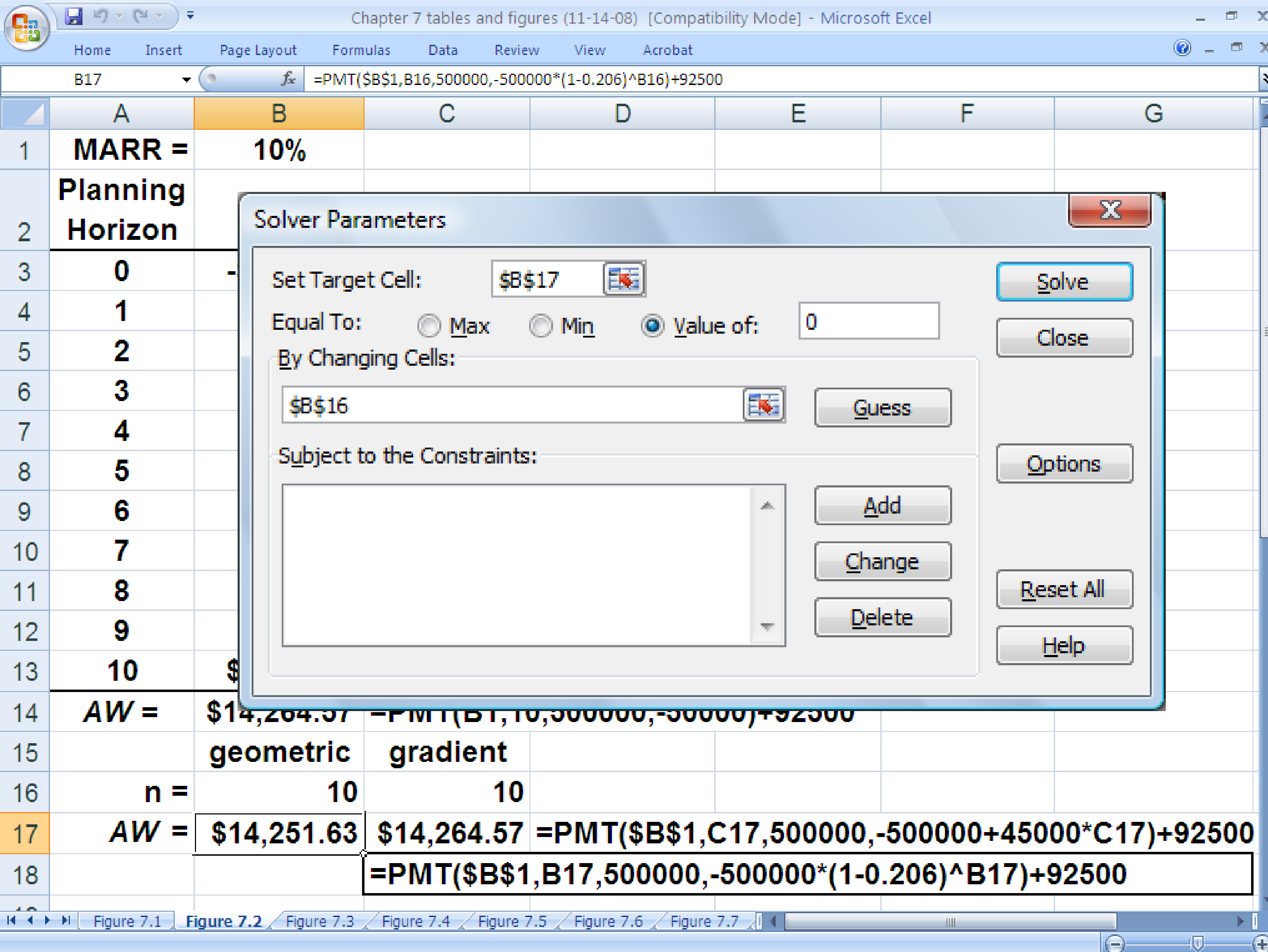
Figure 7.1 | Figure 7.2 | Figure 7.3 | Figure 7.4 | Figure 7.5 | Figure 7.6 | Figure 7.7

Let's use **SOLVER** to determine the *DPBP* using *AW* analysis.

B17 =PMT(B1,B16,500000,-500000*(1-0.206)^B16)+92500

	A	B	C	D	E	F
1	MARR =	10%				
2	Planning Horizon	CF	$SV_{geometric}$	$AW_{geometric}$	$SV_{gradient}$	$AW_{gradient}$
3	0	-$500,000	$500,000	n/a	$500,000	n/a
4	1	$92,500	$397,000	-$60,500.00	$455,000	-$2,500.00
5	2	$92,500	$315,218	-$45,491.43	$410,000	-$357.14
6	3	$92,500	$250,283	-$32,943.17	$365,000	$1,714.50
7	4	$92,500	$198,725	-$22,416.01	$320,000	$3,715.26
8	5	$92,500	$157,787	-$13,553.55	$275,000	$5,645.57
9	6	$92,500	$125,283	-$6,066.06	$230,000	$7,506.01
10	7	$92,500	$99,475	$282.45	$185,000	$9,297.27
11	8	$92,500	$78,983	$5,684.59	$140,000	$11,020.15
12	9	$92,500	$62,713	$10,297.92	$95,000	$12,675.58
13	10	$137,500	$49,794	$14,251.63	$50,000	$14,264.57
14	*AW* =	$14,264.57	=PMT(B1,10,500000,-50000)+92500			
15		geometric	gradient			
16	n =	10	10			
17	*AW* =	$14,251.63	$14,264.57	=PMT(B1,C17,500000,-500000+45000*C17)+92500		
18			=PMT(B1,B17,500000,-500000*(1-0.206)^B17)+92500			

Figure 7.1 | Figure 7.2 | Figure 7.3 | Figure 7.4 | Figure 7.5 | Figure 7.6 | Figure 7.7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B17
=PMT(B1,B16,500000,-500000*(1-0.206)^B16)+92500
MARR =
10%
Planning Horizon
0
1
2
3
4
5
6
7
8
9
10
AW =
geometric
gradient
n =
10
10
AW =
$14,251.63
$14,264.57
=PMT(B1,C17,500000,-500000+45000*C17)+92500
=PMT(B1,B17,500000,-500000*(1-0.206)^B17)+92500
Solver Parameters
Set Target Cell:
B17
Equal To:
Max
Min
Value of:
0
By Changing Cells:
B16
Guess
Subject to the Constraints:
Add
Change
Delete
Solve
Close
Options
Reset All
Help
Figure 7.1
Figure 7.2
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Figure 7.6
Figure 7.7

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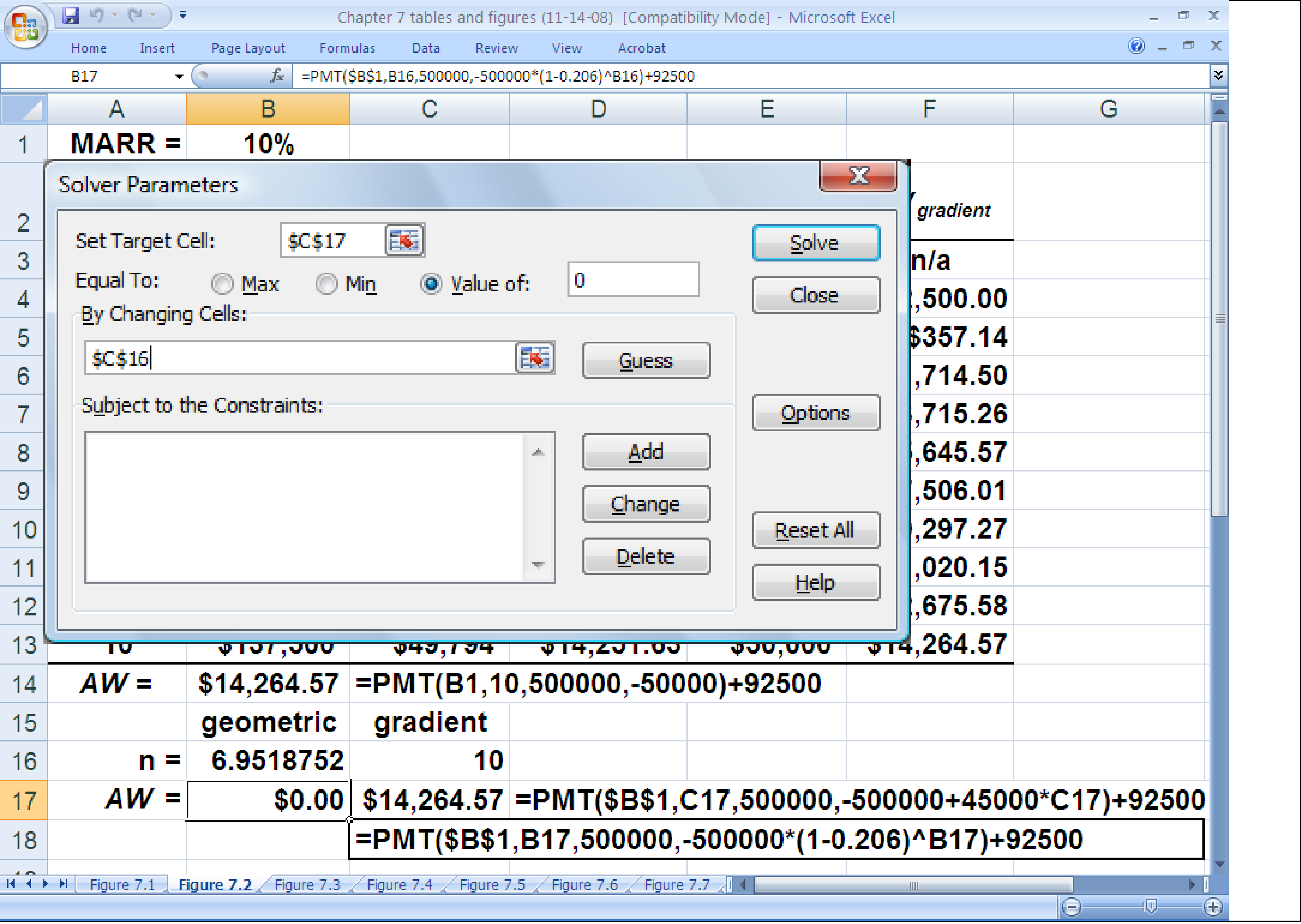
B17 =PMT(B1,B16,500000,-500000*(1-0.206)^B16)+92500

	A	B	C	D	E	F	G
1	MARR =	10%					
2	Planning Horizon	CF	$SV_{geometric}$	$AW_{geometric}$	$SV_{gradient}$	$AW_{gradient}$	
3	0	-$500,000	$500,000	n/a	$500,000	n/a	
4	1	$92,500	$397,000	-$60,500.00	$455,000	-$2,500.00	
5	2	$92,500	$315,218	-$45,491.43	$410,000	-$357.14	
6	3	$92,500	$250,283	-$32,943.17	$365,000	$1,714.50	
7	4	$92,500	$198,725	-$22,416.01	$320,000	$3,715.26	
8	5	$92,500	$157,787	-$13,553.55	$275,000	$5,645.57	
9	6	$92,500	$125,283	-$6,066.06	$230,000	$7,506.01	
10	7	$92,500	$99,475	$282.45	$185,000	$9,297.27	
11	8	$92,500	$78,983	$5,684.59	$140,000	$11,020.15	
12	9	$92,500	$62,713	$10,297.92	$95,000	$12,675.58	
13	10	$137,500	$49,794	$14,251.63	$50,000	$14,264.57	
14	*AW* =	$14,264.57	=PMT(B1,10,500000,-50000)+92500				
15		geometric	gradient				
16	n =	6.9518752	10				
17	*AW* =	$0.00	$14,264.57	=PMT(B1,C17,500000,-500000+45000*C17)+92500			
18			=PMT(B1,B17,500000,-500000*(1-0.206)^B17)+92500				

Figure 7.1 | Figure 7.2 | Figure 7.3 | Figure 7.4 | Figure 7.5 | Figure 7.6 | Figure 7.7



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B17
=PMT(B1,B16,500000,-500000*(1-0.206)^B16)+92500
MARR =
10%
Solver Parameters
Set Target Cell:
C17
Equal To:
Max
Min
Value of:
0
By Changing Cells:
C16
Guess
Subject to the Constraints:
Add
Change
Delete
Solve
Close
Options
Reset All
Help
gradient
n/a
,500.00
$357.14
,714.50
,715.26
,645.57
,506.01
,297.27
,020.15
,675.58
,264.57
AW =
$14,264.57
=PMT(B1,10,500000,-50000)+92500
geometric
gradient
n =
6.9518752
10
AW =
$0.00
$14,264.57
=PMT(B1,C17,500000,-500000+45000*C17)+92500
=PMT(B1,B17,500000,-500000*(1-0.206)^B17)+92500
Figure 7.1
Figure 7.2
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Figure 7.7

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C17 =PMT(B1,C16,500000,-500000+45000*C16)+92500

	A	B	C	D	E	F	G
1	MARR =	10%					
2	Planning Horizon	CF	$SV_{geometric}$	$AW_{geometric}$	$SV_{gradient}$	$AW_{gradient}$	
3	0	-$500,000	$500,000	n/a	$500,000	n/a	
4	1	$92,500	$397,000	-$60,500.00	$455,000	-$2,500.00	
5	2	$92,500	$315,218	-$45,491.43	$410,000	-$357.14	
6	3	$92,500	$250,283	-$32,943.17	$365,000	$1,714.50	
7	4	$92,500	$198,725	-$22,416.01	$320,000	$3,715.26	
8	5	$92,500	$157,787	-$13,553.55	$275,000	$5,645.57	
9	6	$92,500	$125,283	-$6,066.06	$230,000	$7,506.01	
10	7	$92,500	$99,475	$282.45	$185,000	$9,297.27	
11	8	$92,500	$78,983	$5,684.59	$140,000	$11,020.15	
12	9	$92,500	$62,713	$10,297.92	$95,000	$12,675.58	
13	10	$137,500	$49,794	$14,251.63	$50,000	$14,264.57	
14	*AW* =	$14,264.57	=PMT(B1,10,500000,-50000)+92500				
15		geometric	gradient				
16	n =	6.9518752	2.1699745				
17	*AW* =	$0.00	$0.00	=PMT(B1,C17,500000,-500000+45000*C17)+92500			
18			=PMT(B1,B17,500000,-500000*(1-0.206)^B17)+92500				

Figure 7.1 | Figure 7.2 | Figure 7.3 | Figure 7.4 | Figure 7.5 | Figure 7.6 | Figure 7.7

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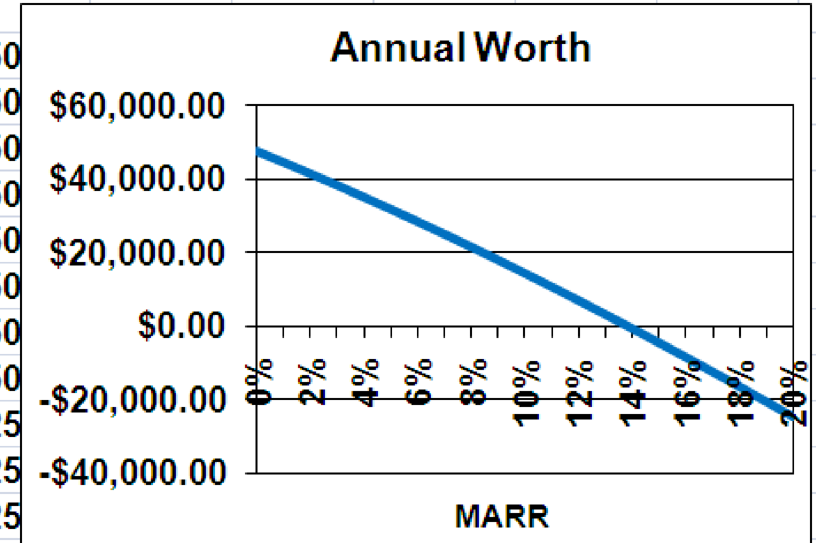
C24 =PMT(C23,10,500000,-50000)+92500

	A	B	C
1	*MARR*	*AW*	
2	0%	$47,500.00	=PMT(A2,10,500000,-50000)+9250
3	1%	$44,488.07	=PMT(A3,10,500000,-50000)+9250
4	2%	$41,403.06	=PMT(A4,10,500000,-50000)+9250
5	3%	$38,246.27	=PMT(A5,10,500000,-50000)+9250
6	4%	$35,019.08	=PMT(A6,10,500000,-50000)+9250
7	5%	$31,722.94	=PMT(A7,10,500000,-50000)+9250
8	6%	$28,359.42	=PMT(A8,10,500000,-50000)+9250
9	7%	$24,930.12	=PMT(A9,10,500000,-50000)+9250
10	8%	$21,436.73	=PMT(A10,10,500000,-50000)+925
11	9%	$17,880.96	=PMT(A11,10,500000,-50000)+925
12	10%	$14,264.57	=PMT(A12,10,500000,-50000)+925
13	11%	$10,589.36	=PMT(A13,10,500000,-50000)+92500
14	12%	$6,857.13	=PMT(A14,10,500000,-50000)+92500
15	13%	$3,069.70	=PMT(A15,10,500000,-50000)+92500
16	14%	-$771.09	=PMT(A16,10,500000,-50000)+92500
17	15%	-$4,663.43	=PMT(A17,10,500000,-50000)+92500
18	16%	-$8,605.49	=PMT(A18,10,500000,-50000)+92500
19	17%	-$12,595.47	=PMT(A19,10,500000,-50000)+92500
20	18%	-$16,631.59	=PMT(A20,10,500000,-50000)+92500
21	19%	-$20,712.09	=PMT(A21,10,500000,-50000)+92500
22	20%	-$24,835.24	=PMT(A22,10,500000,-50000)+92500
23		*MARR* =	10.000%
24		*AW* =	$14,264.57



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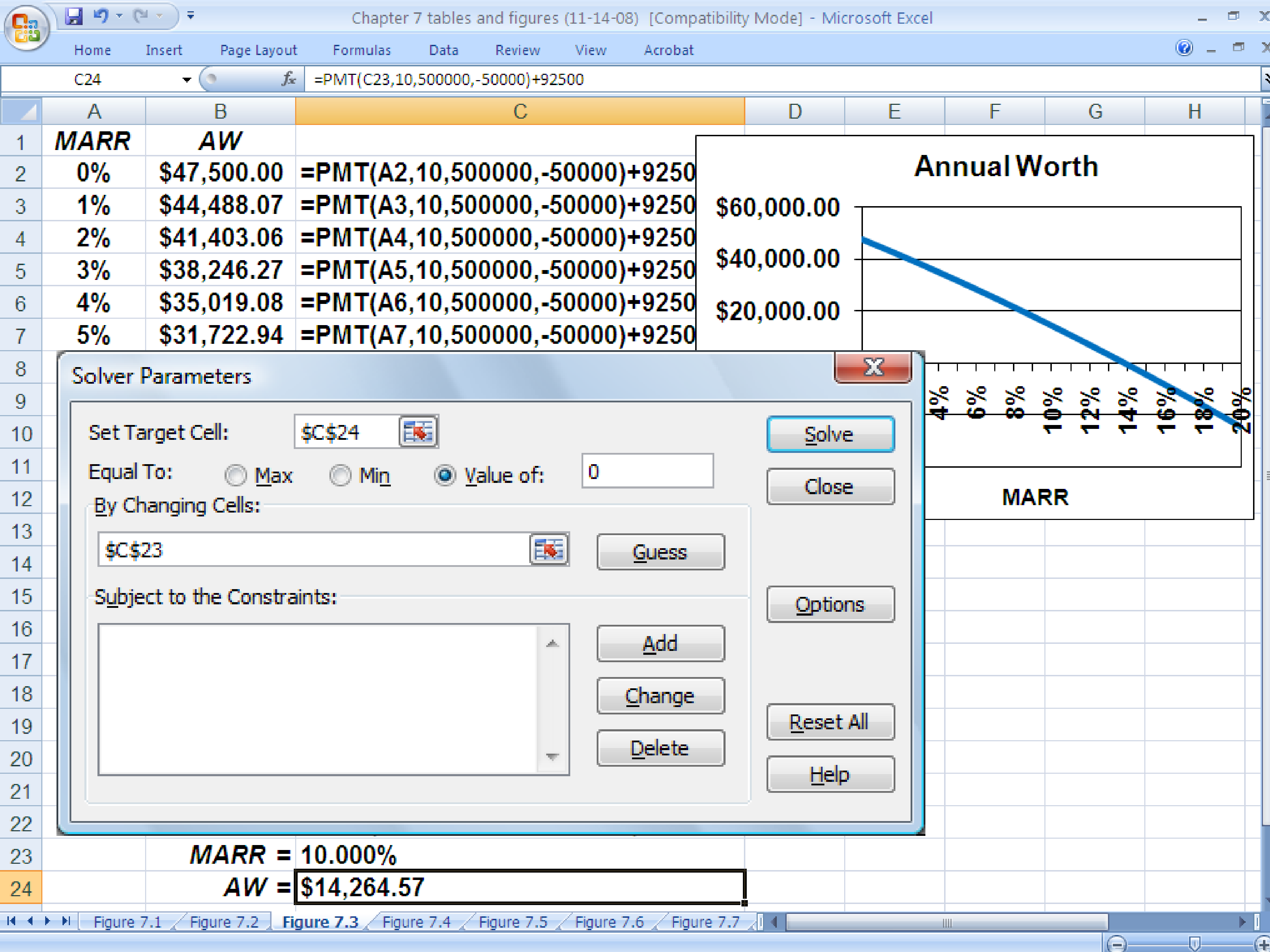

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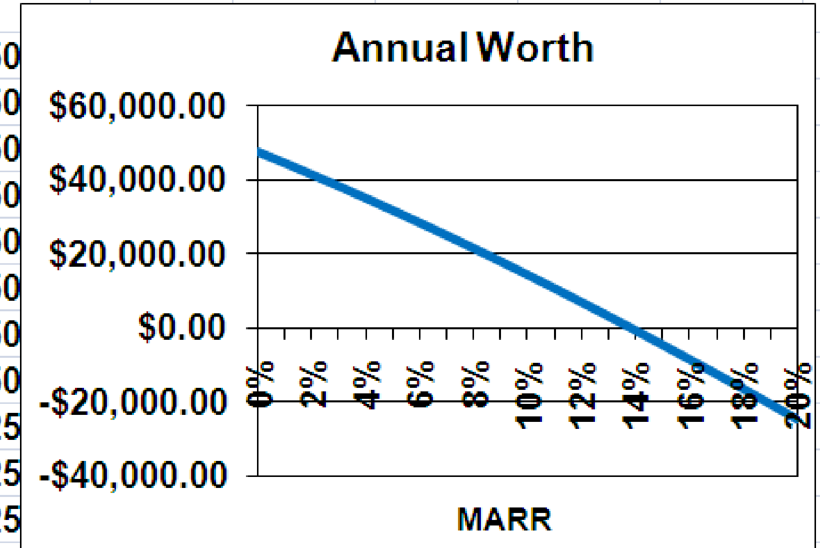
C24 =PMT(C23,10,500000,-50000)+92500

	A	B	C
1	*MARR*	*AW*	
2	0%	$47,500.00	=PMT(A2,10,500000,-50000)+9250
3	1%	$44,488.07	=PMT(A3,10,500000,-50000)+9250
4	2%	$41,403.06	=PMT(A4,10,500000,-50000)+9250
5	3%	$38,246.27	=PMT(A5,10,500000,-50000)+9250
6	4%	$35,019.08	=PMT(A6,10,500000,-50000)+9250
7	5%	$31,722.94	=PMT(A7,10,500000,-50000)+9250
8	6%	$28,359.42	=PMT(A8,10,500000,-50000)+9250
9	7%	$24,930.12	=PMT(A9,10,500000,-50000)+9250
10	8%	$21,436.73	=PMT(A10,10,500000,-50000)+925
11	9%	$17,880.96	=PMT(A11,10,500000,-50000)+925
12	10%	$14,264.57	=PMT(A12,10,500000,-50000)+925
13	11%	$10,589.36	=PMT(A13,10,500000,-50000)+92500
14	12%	$6,857.13	=PMT(A14,10,500000,-50000)+92500
15	13%	$3,069.70	=PMT(A15,10,500000,-50000)+92500
16	14%	-$771.09	=PMT(A16,10,500000,-50000)+92500
17	15%	-$4,663.43	=PMT(A17,10,500000,-50000)+92500
18	16%	-$8,605.49	=PMT(A18,10,500000,-50000)+92500
19	17%	-$12,595.47	=PMT(A19,10,500000,-50000)+92500
20	18%	-$16,631.59	=PMT(A20,10,500000,-50000)+92500
21	19%	-$20,712.09	=PMT(A21,10,500000,-50000)+92500
22	20%	-$24,835.24	=PMT(A22,10,500000,-50000)+92500
23		*MARR* =	13.800%
24		*AW* =	$0.00



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Annual Worth Analysis

Multiple Alternatives

Example 7.4

Recall the example involving two alternative designs for a new ride at a theme park: Alt. A costs $300,000, has net annual after-tax revenue of $55,000, and has a negligible salvage value at the end of the 10-year planning horizon; Alt. B costs $450,000, has revenue of $80,000/yr., and has a negligible salvage value. Based on an AW analysis and a 10% MARR, which is preferred?

$AW_A(10\%)$ = -$300,000(*A*/*P* 10%,10) + $55,000

= -$300,000(0.16275) + $55,000 = $6175.00

=PMT(10%,10,300000)+55000 = $6176.38

$AW_B(10\%)$ = -$450,000(*A*/*P* 10%,10) + $80,000

= -$450,000(0.16275) + $80,000 = $6762.50

=PMT(10%,10,450000)+80000 = $6764.57

Analyze the impact on AW based on salvage values decreasing geometrically to 1¢ after 10 years.

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B16 =PMT(10%,B15,300000,-300000*(1+D1)^B15)+55000

Planning Horizon	Geometric Series Rate (A) = -85.800% CF(A)	SV(A)	AW(A)	Geometric Series Rate (B) = -86.364% CF(B)	SV(B)	AW(B)
0	-$300,000.00	$300,000.00	n/a	-$450,000.00	$450,000.00	n/a
1	$55,000.00	$42,599.95	-$232,400.05	$80,000.00	$61,360.84	-$353,639.16
2	$55,000.00	$6,049.19	-$114,976.58	$80,000.00	$8,367.00	-$175,301.43
3	$55,000.00	$858.98	-$65,374.93	$80,000.00	$1,140.90	-$100,606.98
4	$55,000.00	$121.98	-$39,614.96	$80,000.00	$155.57	-$61,928.34
5	$55,000.00	$17.32	-$24,136.41	$80,000.00	$21.21	-$38,705.39
6	$55,000.00	$2.46	-$13,881.90	$80,000.00	$2.89	-$23,322.95
7	$55,000.00	$0.35	-$6,621.61	$80,000.00	$0.39	-$12,432.43
8	$55,000.00	$0.05	-$1,233.20	$80,000.00	$0.05	-$4,349.80
9	$55,000.00	$0.01	$2,907.84	$80,000.00	$0.01	$1,861.76
10	$55,000.00	$0.00	$6,176.38	$80,000.00	$0.00	$6,764.57

$DPBP_A$ = 10.000 $DPBP_B$ = 10.000

AW_A = $6,176.38 AW_B = $6,764.57

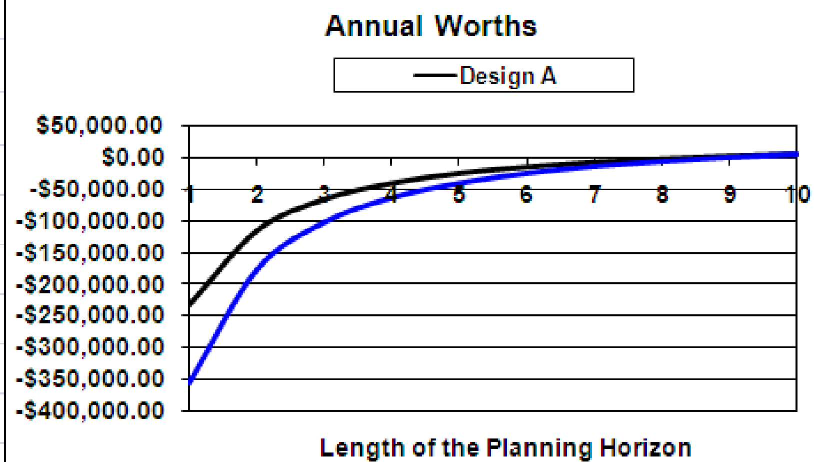
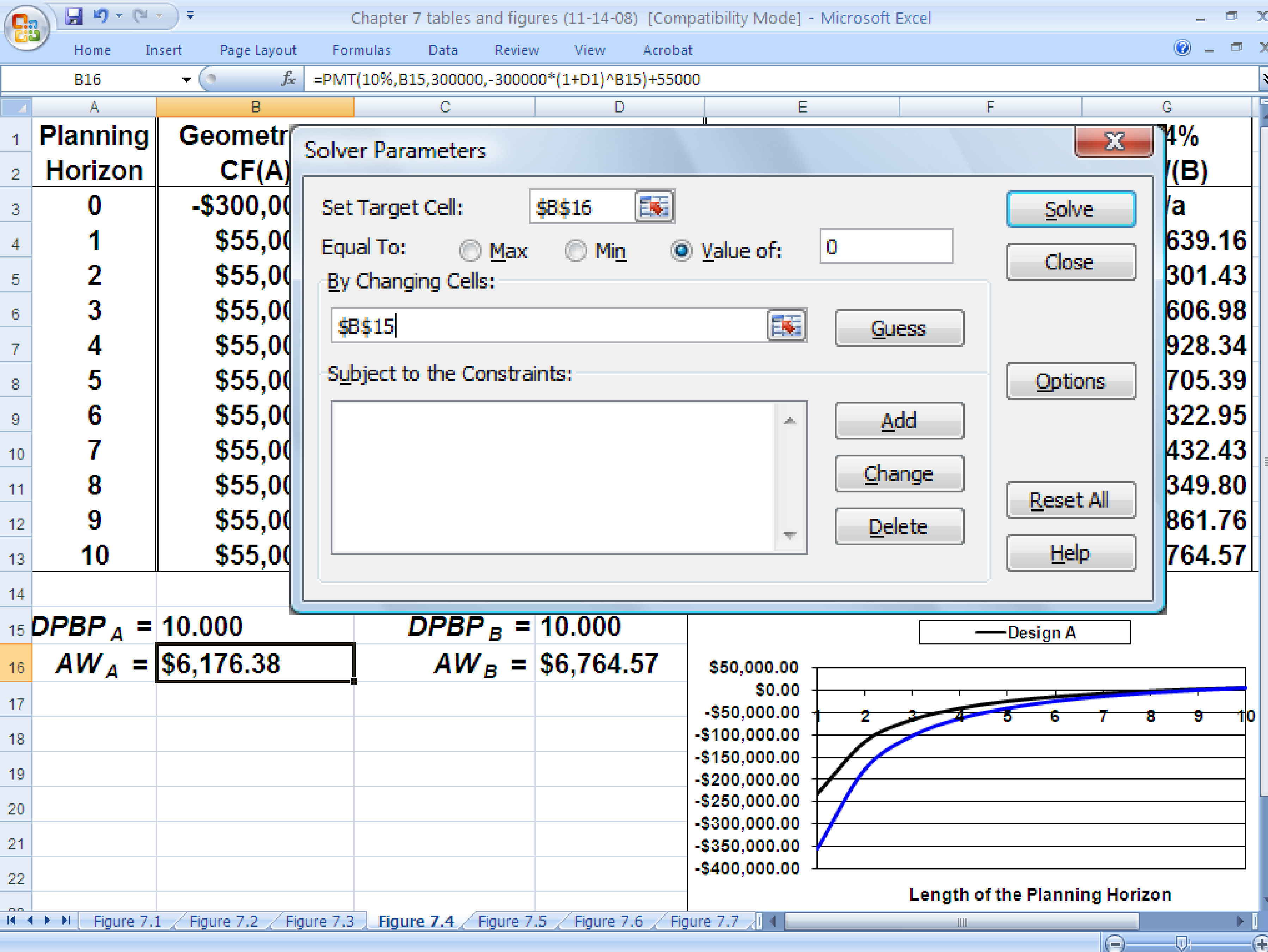


Figure 7.1 Figure 7.2 Figure 7.3 Figure 7.4 Figure 7.5 Figure 7.6 Figure 7.7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B16
=PMT(10%,B15,300000,-300000*(1+D1)^B15)+55000
Planning Horizon
Geometr
CF(A)
0
1
2
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10
-$300,00
$55,00
Solver Parameters
Set Target Cell:
B16
Equal To:
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4%
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/a
639.16
301.43
606.98
928.34
705.39
322.95
432.43
349.80
861.76
764.57
DPBP A = 10.000
AW A = $6,176.38
DPBP B = 10.000
AW B = $6,764.57
Design A
$50,000.00
$0.00
-$50,000.00
-$100,000.00
-$150,000.00
-$200,000.00
-$250,000.00
-$300,000.00
-$350,000.00
-$400,000.00
1 2 3 4 5 6 7 8 9 10
Length of the Planning Horizon
Figure 7.1
Figure 7.2
Figure 7.3
Figure 7.4
Figure 7.5
Figure 7.6
Figure 7.7

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B16 =PMT(10%,B15,300000,-300000*(1+D1)^B15)+55000

Planning Horizon	Geometric Series Rate (A) = -85.800% CF(A)	SV(A)	AW(A)	Geometric Series Rate (B) = -86.364% CF(B)	SV(B)	AW(B)
0	-$300,000.00	$300,000.00	n/a	-$450,000.00	$450,000.00	n/a
1	$55,000.00	$42,599.95	-$232,400.05	$80,000.00	$61,360.84	-$353,639.16
2	$55,000.00	$6,049.19	-$114,976.58	$80,000.00	$8,367.00	-$175,301.43
3	$55,000.00	$858.98	-$65,374.93	$80,000.00	$1,140.90	-$100,606.98
4	$55,000.00	$121.98	-$39,614.96	$80,000.00	$155.57	-$61,928.34
5	$55,000.00	$17.32	-$24,136.41	$80,000.00	$21.21	-$38,705.39
6	$55,000.00	$2.46	-$13,881.90	$80,000.00	$2.89	-$23,322.95
7	$55,000.00	$0.35	-$6,621.61	$80,000.00	$0.39	-$12,432.43
8	$55,000.00	$0.05	-$1,233.20	$80,000.00	$0.05	-$4,349.80
9	$55,000.00	$0.01	$2,907.84	$80,000.00	$0.01	$1,861.76
10	$55,000.00	$0.00	$6,176.38	$80,000.00	$0.00	$6,764.57

$DPBP_A$ = 8.273 $DPBP_B$ = 10.000

AW_A = $0.00 AW_B = $6,764.57

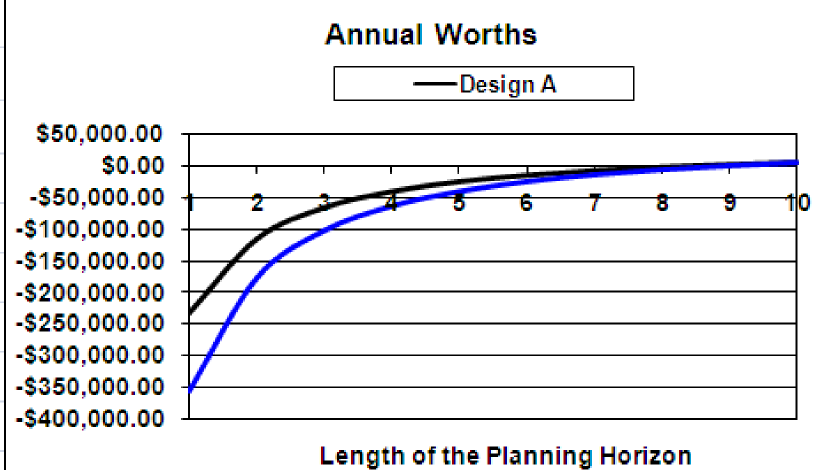
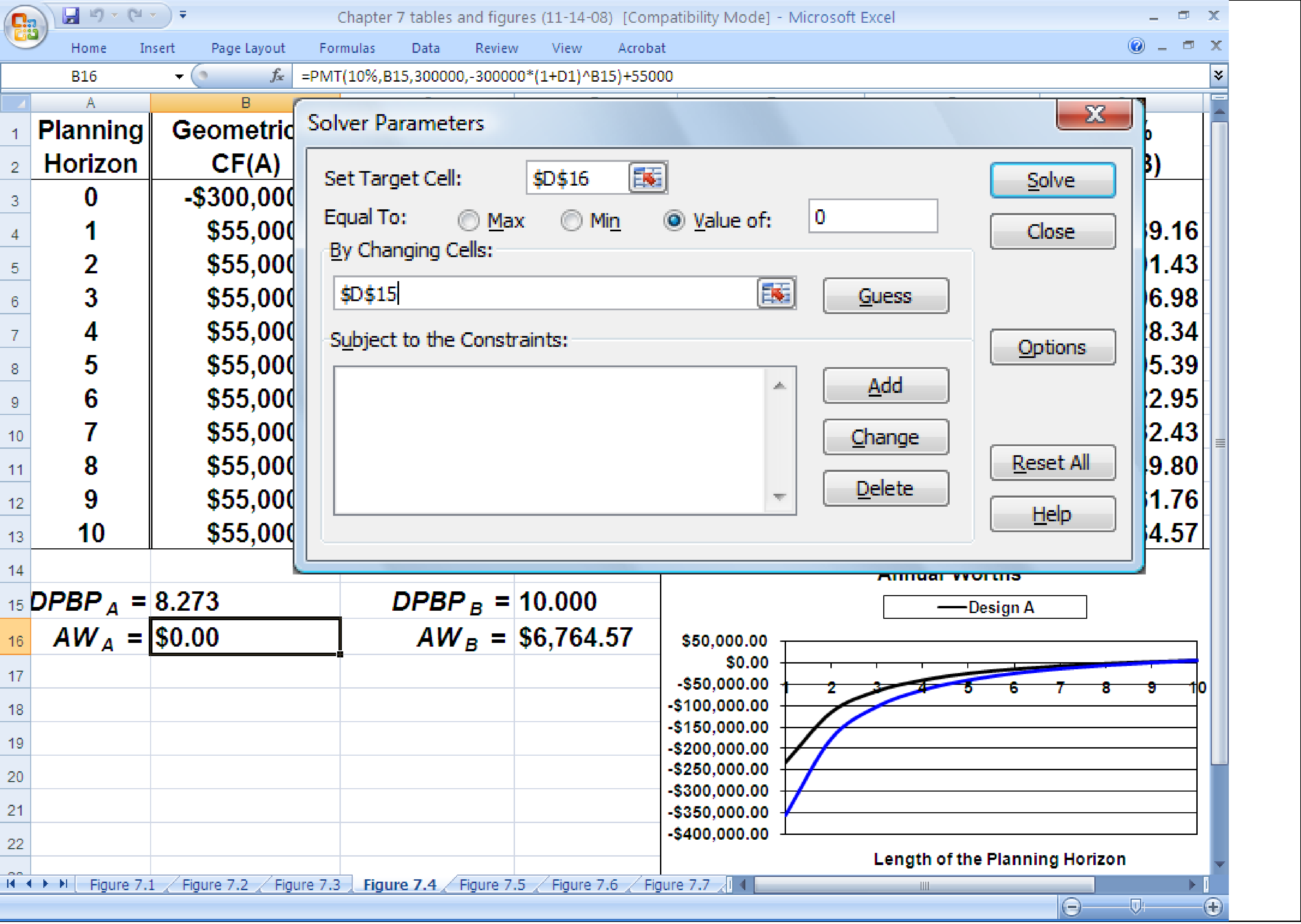


Figure 7.1 | Figure 7.2 | Figure 7.3 | Figure 7.4 | Figure 7.5 | Figure 7.6 | Figure 7.7


Chapter 7 tables and figures (11-14-08) [Compatibility Mode] - Microsoft Excel
Home
Insert
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B16
=PMT(10%,B15,300000,-300000*(1+D1)^B15)+55000
Planning Horizon
Geometric CF(A)
0
-$300,000
1
$55,000
2
$55,000
3
$55,000
4
$55,000
5
$55,000
6
$55,000
7
$55,000
8
$55,000
9
$55,000
10
$55,000
Solver Parameters
Set Target Cell:
D16
Equal To:
Max
Min
Value of:
0
By Changing Cells:
D15
Guess
Subject to the Constraints:
Add
Change
Delete
Solve
Close
Options
Reset All
Help
DPBP A = 8.273
DPBP B = 10.000
AW A = $0.00
AW B = $6,764.57
Design A
$50,000.00
$0.00
-$50,000.00
-$100,000.00
-$150,000.00
-$200,000.00
-$250,000.00
-$300,000.00
-$350,000.00
-$400,000.00
Length of the Planning Horizon
Figure 7.1
Figure 7.2
Figure 7.3
Figure 7.4
Figure 7.5
Figure 7.6
Figure 7.7

Chapter 7 tables and figures (11-14-08) [Compatibility Mode] - Microsoft Excel

D16 =PMT(10%,D15,450000,-450000*(1+G1)^D15)+80000

Planning Horizon	Geometric Series Rate (A) = -85.800% CF(A)	SV(A)	AW(A)	Geometric Series Rate (B) = -86.364% CF(B)	SV(B)	AW(B)
0	-$300,000.00	$300,000.00	n/a	-$450,000.00	$450,000.00	n/a
1	$55,000.00	$42,599.95	-$232,400.05	$80,000.00	$61,360.84	-$353,639.16
2	$55,000.00	$6,049.19	-$114,976.58	$80,000.00	$8,367.00	-$175,301.43
3	$55,000.00	$858.98	-$65,374.93	$80,000.00	$1,140.90	-$100,606.98
4	$55,000.00	$121.98	-$39,614.96	$80,000.00	$155.57	-$61,928.34
5	$55,000.00	$17.32	-$24,136.41	$80,000.00	$21.21	-$38,705.39
6	$55,000.00	$2.46	-$13,881.90	$80,000.00	$2.89	-$23,322.95
7	$55,000.00	$0.35	-$6,621.61	$80,000.00	$0.39	-$12,432.43
8	$55,000.00	$0.05	-$1,233.20	$80,000.00	$0.05	-$4,349.80
9	$55,000.00	$0.01	$2,907.84	$80,000.00	$0.01	$1,861.76
10	$55,000.00	$0.00	$6,176.38	$80,000.00	$0.00	$6,764.57

$DPBP_A$ = 8.273 $DPBP_B$ = 8.674

AW_A = $0.00 AW_B = $0.00

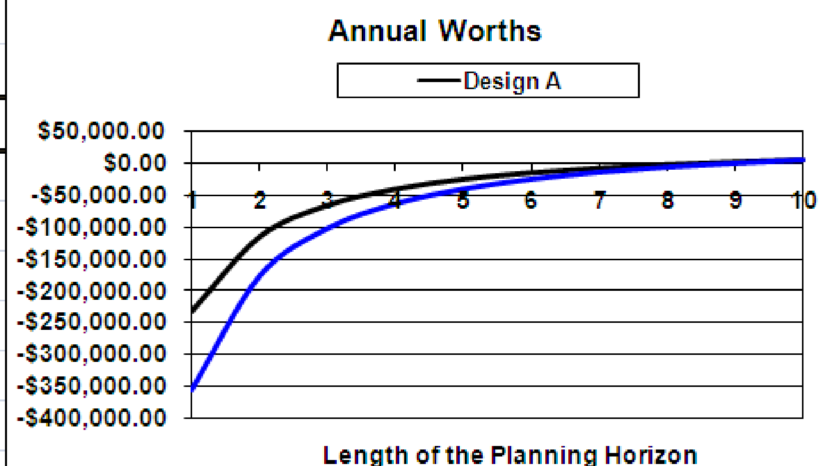
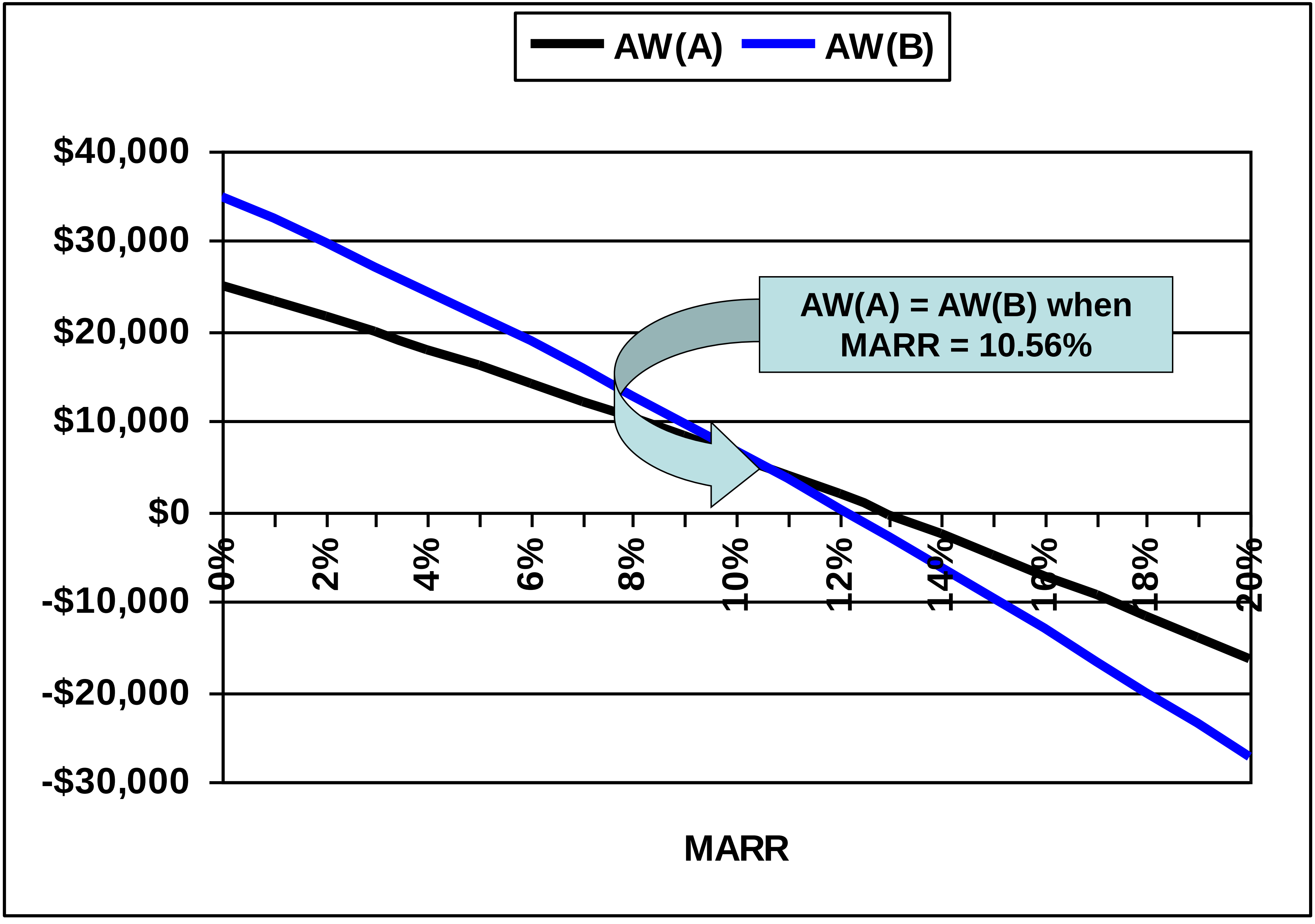


Figure 7.1 Figure 7.2 Figure 7.3 Figure 7.4 Figure 7.5 Figure 7.6 Figure 7.7


AW(A)
AW(B)
$40,000
$30,000
$20,000
$10,000
$0
-$10,000
-$20,000
-$30,000
AW(A) = AW(B) when MARR = 10.56%
0%
2%
4%
6%
8%
10%
12%
14%
16%
18%
20%
MARR

Example 7.5

For The Scream Machine alternatives (A costing \$300,000, saving \$55,000, and having a negligible salvage value at the end of the 10-year planning horizon; B costing \$450,000, saving \$80,000, and having a negligible salvage value), using an incremental AW analysis and a 10% MARR, which is preferred?

AW_A(10%) = -\$300,000(*A/P* 10%,10) + \$55,000
= -\$300,000(0.16275) + \$55,000 = \$6175.00
=PMT(10%,10,300000)+55000 = \$6176.38 > \$0
(A is better than "do nothing")

AW_{B-A}(10%) = -\$150,000(*A/P* 10%,10) + \$25,000
= -\$150,000(0.16275) + \$25,000 = \$587.50
=PMT(10%,10,150000)+25000 = \$588.19 > \$0
(B is better than A)

Prefer B

Example 7.6

If an investor's MARR is 12%, which mutually exclusive investment alternative maximizes the investor's future worth, given the parameters shown below?

EOY	CF(1)	CF(2)	CF(3)
0	-$10,000	-$15,000	-$20,000
1	$5,000	$5,000	$0
2	$5,000	$5,000	$3,000
3	$10,000	$5,000	$6,000
4		$5,000	$9,000
5		$5,000	$12,000
6		$7,500	$15,000

Consider 3 scenarios: individual life cycles; least common multiple of lives; and "one-shot" investments

Example 7.6 (Continued)

Scenario 1: individual life cycles

$AW_1(12\%) = -\$10{,}000(A|P\ 12\%,3) + \$5000 + \$5000(A|F\ 12\%,3)$

$= \$2318.25$

=PMT(12%,3,10000,-5000)+5000 = $2318.26

$AW_2(12\%) = -\$15{,}000(A|P\ 12\%,6) + \$5000 + \$2500(A|F\ 5\%,6)$

$= \$1659.63$

=PMT(12%,6,15000,-2500)+5000 = $1659.68

$AW_3(12\%) = -\$20{,}000(A|P\ 12\%,6) + \$3000(A|G\ 12\%,6)$

$= \$1651.55$

=PMT(12%,6,-1000*NPV(12%,0,3,6,9,12,15)+20000)

= $1651.63

Example 7.6 (Continued)

Scenario 2: least common multiple of lives

EOY	CF(1')	CF(2)	CF(3)
0	-$10,000	-$15,000	-$20,000
1	$5,000	$5,000	$0
2	$5,000	$5,000	$3,000
3	$0	$5,000	$6,000
4	$5,000	$5,000	$9,000
5	$5,000	$5,000	$12,000
6	$10,000	$7,500	$15,000

AW_1(12%) = -$10,000(A|P 12%,6) + $5000 + $5000(A|F 12%,6)

- $5000(A|P 12%,3)(A|P 12%,6)

= $2318.22

=PMT(12%,6,10000,-5000)+5000

+PMT(12%,6,PV(12%,3,,-5000))= $2318.26

AW_2(12%) = $1473.17 = $1473.23

AW_3(12%) = $1651.55 = $1651.63

Identical results!

Example 7.6 (Continued)

Scenario 3: "one-shot" investments

EOY	CF(1)	CF(2)	CF(3)
0	-$10,000	-$14,500	-$20,000
1	$5,000	$5,000	$0
2	$5,000	$5,000	$3,000
3	$10,000	$5,000	$6,000
4	$0	$5,000	$9,000
5	$0	$5,000	$12,000
6	$0	$5,000	$15,000

$AW_1(12\%)$ = {-$10,000 + [$5000(P|A 12%,3)

+ $5000(*P/F* 12%,3)]}(*A/P* 12%,6) = $1354.32

=PMT(12%,6,10000-PV(12%,3,-5000,-5000))

= $1354.29

$AW_2(12\%)$ = $1473.17 = $1473.23

$AW_3(12\%)$ = $1651.55 = $1651.63

Considering scenarios 1 and 2, is it reasonable to assume an investment alternative equivalent to Alt. 1 will be available in 3 years? If so, why was the MARR set equal to 12%?

Example 7.7

Three industrial mowers (Small, Medium, and Large) are being evaluated by a company that provides lawn care service. Determine the economic choice, based on the following cost and performance parameters:

	Small	Medium	Large
First Cost:	$1,500	$2,000	$5,000
Operating Cost/Hr	$35	$50	$76
Revenue/Hr	$55	$75	$100
Hrs/Yr	1,000	1,100	1,200
Useful Life (Yrs)	2	3	5

Use AW analysis to determine the preferred mower, based on a MARR of 12%.

Example 7.7 (Continued)

AW_{small} = -\$1500(*A*/*P* 12%,2) + \$20(1000) = \$19,112.45
=PMT(12%,2,1500)+20*1000 = \$19,112.45

AW_{med} = -\$2000(*A*/*P* 12%,3) + \$25(1100) = \$26,667.30
=PMT(12%,3,2000)+25*1100 = \$26,667.30

AW_{large} = -\$5000(*A*/*P* 12%,5) + \$24(1200) = \$27,412.95
=PMT(12%,5,5000)+24*1200 = \$27,412.95

What did we assume when solving the example?

Example 7.8

If a 5-year planning horizon were used, what salvage values are required to have the same AW as before? The small mower will be replaced at the end of year 4; the medium mower will be replaced at the end of year 3. One year of service of the small mower will have the following cash flows:

SV_{small} = \$19,112.45(*F/A* 12%,1) - \$20,000(*F/A* 12%,1) + \$1500(*F/P* 12%,1) = \$792.45

=FV(12%,1,-19112.45)-FV(12%,1,-20000,1500)

= \$792.45

SV_{med} = \$26,667.30(*F/A* 12%,2) - \$27,500(*F/A* 12%,2) + \$2000(F|P 12%,2) = \$743.48

=FV(12%,2,-26667.3)-FV(12%,2,-27500,2000)

= \$743.48

SV_{large} = \$0

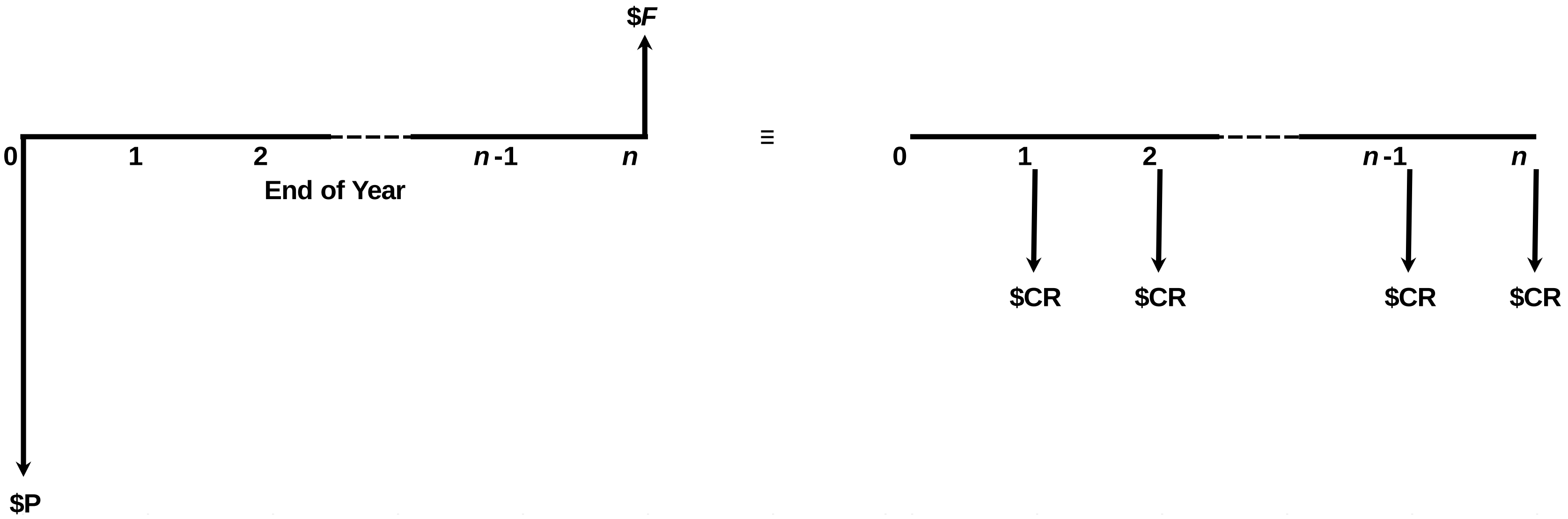
Principle #8

Compare investment alternatives over a common period of time

Fundamentals of Engineering Examination

Even though you might not encounter a situation in your professional practice that requires the least common multiple of lives assumption to be used, it is very likely you will have problems of this type on the FE Exam. Therefore, you need to be familiar with how to solve such problems. Specifically, on the FE Exam, *unless instructions are given to do otherwise*, <u>calculate the annual worth for a life cycle of each alternative and recommend the one that has the greatest annual worth</u>.

Capital Recovery Cost



CFD for Capital Recovery Cost (CR).

Capital Recovery Cost Formulas

CR = P(A|P i,n) – F(A|F i, n)

CR = (P-F)(A|F i, n) + Pi

CR = (P-F)(A|P i,n) + Fi

CR =PMT(*i,n,-P,F*)

Example

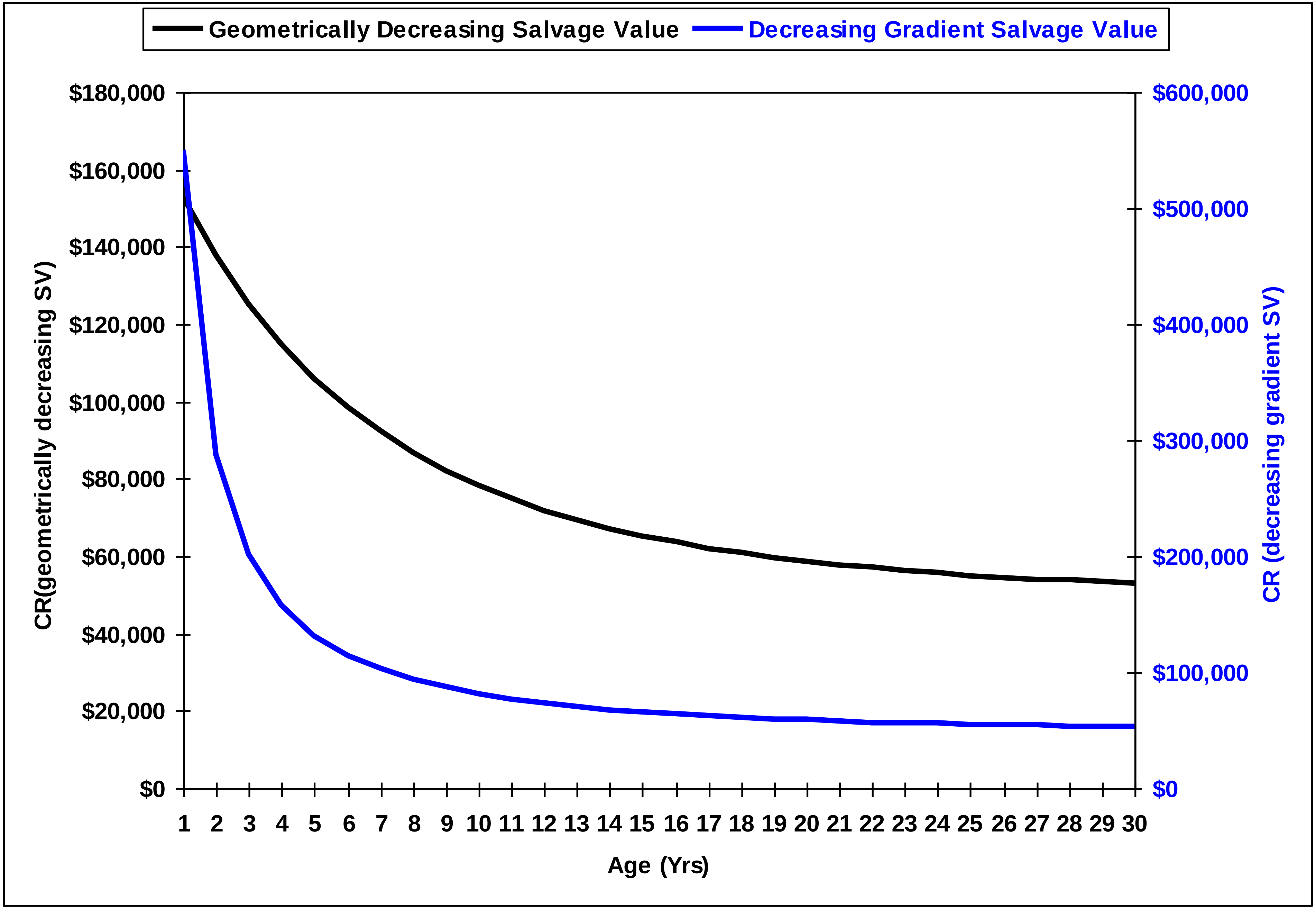
P = $500,000 F = $50,000 i = 10% n = 10 yrs

CR = $500,000(0.16275) - $50,000(0.06275) = $78,237.50

CR = $450,000(0.06275) + $500,000(0.10) = $78,237.50

CR = $450,000(0.16275) + $50,000(0.10) = $78,237.50

CR =PMT(10%,10,-500000,50000) = $78,235.43



Geometrically Decreasing Salvage Value
Decreasing Gradient Salvage Value
$180,000
$160,000
$140,000
$120,000
$100,000
$80,000
$60,000
$40,000
$20,000
$0
CR(geometrically decreasing SV)
$600,000
$500,000
$400,000
$300,000
$200,000
$100,000
$0
CR (decreasing gradient SV)
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
Age (Yrs)

Pit Stop #7—No Time to Coast

1. True or False: Annual worth analysis is the most popular *DCF* measure of economic worth.
2. True or False: Unless non-monetary considerations dictate otherwise, choose the mutually exclusive investment alternative that has the greatest annual worth over the planning horizon.
3. True or False: The capital recovery cost is the uniform annual cost of the investment less the uniform annual worth of the salvage value.
4. True or False: If *AW* > 0, then *PW*>0, and *FW*>0.
5. True or False: If *AW*(A) *>* *AW*(B), then *PW*(A) > *PW*(B).
6. True or False: If *AW* (A) < *AW*(B), then AW(B-A) > 0.
7. True or False: If *AW*(A) > *AW*(B), then *CW*(A) > *CW*(B) and *DPBP*(A) < *DPBP*(B).
8. True or False: *AW* can be applied as either a ranking method or as an incremental method.
9. True or False: To compute capital recovery cost using Excel, enter =PMT(i%,n,-P,F) in any cell in a spreadsheet.
10. True or False: When using annual worth analysis with mutually exclusive alternatives having unequal lives, always use a planning horizon equal to the least common multiple of lives.

Pit Stop #7—No Time to Coast

1. True or False: Annual worth analysis is the most popular *DCF* measure of economic worth. FALSE
2. True or False: Unless non-monetary considerations dictate otherwise, choose the mutually exclusive investment alternative that has the greatest annual worth over the planning horizon. TRUE
3. True or False: The capital recovery cost is the uniform annual cost of the investment less the uniform annual worth of the salvage value. TRUE
4. True or False: If *AW* > 0, then *PW* > 0, and *FW* > 0. TRUE
5. True or False: If *AW*(A) *>* *AW*(B), then *PW*(A) > *PW*(B). TRUE
6. True or False: If *AW* (A) < *AW*(B), then AW(B-A) > 0. TRUE
7. True or False: If *AW*(A) > *AW*(B), then *CW*(A) > *CW*(B) and *DPBP*(A) < *DPBP*(B). FALSE
8. True or False: *AW* can be applied as either a ranking method or as an incremental method. TRUE
9. True or False: To compute capital recovery cost using Excel, enter =PMT(i%,n,-P,F) in any cell in a spreadsheet. TRUE
10. True or False: When using annual worth analysis with mutually exclusive alternatives having unequal lives, always use a planning horizon equal to the least common multiple of lives. FALSE