

Chapter 6

Future Worth Analysis

Systematic Economic Analysis Technique

- 1. Identify the investment alternatives**
- 2. Define the planning horizon**
- 3. Specify the discount rate**
- 4. Estimate the cash flows**
- 5. Compare the alternatives**
- 6. Perform supplementary analyses**
- 7. Select the preferred investment**

Future Worth Analysis

Single Alternative

Future Worth Method

- converts all cash flows to a single sum equivalent at the end of the planning horizon using $i = MARR$
- used mostly for financial planning
- not a popular corporate DCF method

$$FW (i\%) = \sum_{t=0}^n A_t (1+i)^{n-t}$$

(take all cash flows to “time n ” and add them up!)

Example 6.1

A \$500,000 investment in a surface mount placement machine is being considered. Over a 10-year planning horizon, it is estimated the SMP machine will produce net annual savings of \$92,500. At the end of 10 years, it is estimated the SMP machine will have a \$50,000 salvage value. Based on a 10% MARR and future worth analysis, should the investment be made?

$$\begin{aligned}FW &= -\$500K(F|P 10\%,10) + \$92.5K(F|A 10\%,10) + \$50K \\ &= \$227,341.40 \\ &= FV(10\%,10,-92500,500000)+50000 \\ &= \$227,340.55\end{aligned}$$

Example 6.2

How does future worth change over the life of the investment? How does future worth change when the salvage value decreases geometrically and as a gradient series?

Chapter 6 tables and figures (11-14-08) [Compatibility Mode] - Microsoft Excel

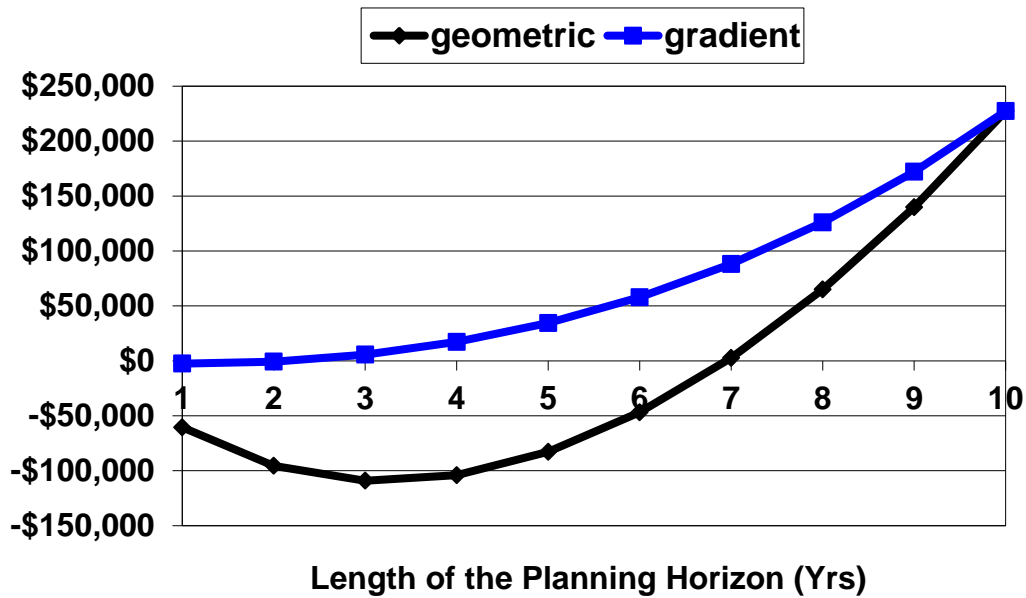
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D13 $=FV(\$B\$1,A13,-B13+C13,-\$B\$3)+\$C13$

	A	B	C	D	E	F	G	H	I	J
1	MARR =	10%								
	Planning Horizon	CF	SV_{geometric}	FW_{geometric}	SV_{gradient}	FW_{gradient}				
2										
3	0	-\$500,000	\$500,000	\$0.00	\$500,000	\$0.00	=B3+E3			
4	1	\$92,500	\$397,000	-\$60,500.00	\$455,000	-\$2,500.00	=FV(\$B\$1,A4,-B4,-\$B\$3)+\$E4			
5	2	\$92,500	\$315,218	-\$95,532.00	\$410,000	-\$750.00	=FV(\$B\$1,A5,-B5,-\$B\$3)+\$E5			
6	3	\$92,500	\$250,283	-\$109,041.91	\$365,000	\$5,675.00	=FV(\$B\$1,A6,-B6,-\$B\$3)+\$E6			
7	4	\$92,500	\$198,725	-\$104,032.72	\$320,000	\$17,242.50	=FV(\$B\$1,A7,-B7,-\$B\$3)+\$E7			
8	5	\$92,500	\$157,787	-\$82,745.78	\$275,000	\$34,466.75	=FV(\$B\$1,A8,-B8,-\$B\$3)+\$E8			
9	6	\$92,500	\$125,283	-\$46,803.32	\$230,000	\$57,913.43	=FV(\$B\$1,A9,-B9,-\$B\$3)+\$E9			
10	7	\$92,500	\$99,475	\$2,679.67	\$185,000	\$88,204.77	=FV(\$B\$1,A10,-B10,-\$B\$3)+\$E10			
11	8	\$92,500	\$78,983	\$65,008.32	\$140,000	\$126,025.24	=FV(\$B\$1,A11,-B11,-\$B\$3)+\$E11			
12	9	\$92,500	\$62,713	\$139,840.33	\$95,000	\$172,127.77	=FV(\$B\$1,A12,-B12,-\$B\$3)+\$E12			
13	10	\$142,500	\$50,000	\$227,340.55	\$50,000	\$227,340.55	=FV(\$B\$1,A13,-B13+C13,-\$B\$3)+\$E13			
14	FW =	\$227,340.55	=FV(B1,A13,,NPV(B1,B4:B13)-B3)							
15										
16										
17										
18										
19										
20										
21										
22										
23										
24										

Figure 6.1(a) Figure 6.1(b) Figure 6.2 Figure 6.3 Figure 6.4 Figure 6.5 Figure 6.6

Future Worth of the SMP Machine Investment



	A	B	C	D	E	F	G	H	I	J
1	MARR =	10%								
	Planning Horizon	CF	SV _{geometric}	FW _{geometric}	SV _{gradient}	FW _{gradient}				
2										
3	0	-\$500,000	\$500,000	\$0.00	\$500,000	\$0.00	=B3+E3			
4	1	\$92,500	\$397,000	-\$60,500.00	\$455,000	-\$2,500.00	=FV(\$B\$1,A4,-B4,-\$B\$3)+\$E4			
5	2	\$92,500	\$315,218	-\$95,532.00	\$410,000	-\$750.00	=FV(\$B\$1,A5,-B5,-\$B\$3)+\$E5			
6	3	\$92,500	\$250,283	-\$109,041.91	\$365,000	\$5,675.00	=FV(\$B\$1,A6,-B6,-\$B\$3)+\$E6			
7	4	\$92,500	\$198,725	-\$104,032.72	\$320,000	\$17,242.50	=FV(\$B\$1,A7,-B7,-\$B\$3)+\$E7			
8	5	\$92,500	\$157,787	-\$82,745.78	\$275,000	\$34,466.75	=FV(\$B\$1,A8,-B8,-\$B\$3)+\$E8			
9	6	\$92,500	\$125,283	-\$46,803.32	\$230,000	\$57,913.43	=FV(\$B\$1,A9,-B9,-\$B\$3)+\$E9			
10	7	\$92,500	\$99,475	\$2,679.67	\$185,000	\$88,204.77	=FV(\$B\$1,A10,-B10,-\$B\$3)+\$E10			
11	8	\$92,500	\$78,983	\$65,008.32	\$140,000	\$126,025.24	=FV(\$B\$1,A11,-B11,-\$B\$3)+\$E11			
12	9	\$92,500	\$62,713	\$139,840.33	\$95,000	\$172,127.77	=FV(\$B\$1,A12,-B12,-\$B\$3)+\$E12			
13	10	\$142,500	\$50,000	\$227,340.55	\$50,000	\$227,340.55	=FV(\$B\$1,A13,-B13+ C13,-\$B\$3)+\$E13			
14	FW =	\$227,340.55	=FV(B1,A13,, -NPV(B1,B4:B13)-B3)							

Notice, a negative FW occurs until the 3rd year with gradient decreases and until the 7th year with geometric decreases; also, FW achieves a minimum during the 1st year with gradient decreases and during the 3rd year with geometric decreases.

Example 6.3

A recent engineering graduate began investing at age 23, with a goal of achieving a net worth of \$5 million by age 58. If the engineer obtains an annual return of 6.5% and makes a first investment of \$5000, what gradient increase is required?

$$G(A|G 6.5\%,36) + \$5000 = \$5,000,000(A|F 6.5\%,36)$$

$$G = [\$5,000,000(A|F 6.5\%,36) - \$5000]/(A|G 6.5\%,36)$$

$$(A|F 6.5\%,36) = 0.065/[(1.065)^{36} - 1] = 0.0075133$$

$$(A|G 6.5\%,36) = \{(1.065)^{36} - [1 + 36(0.065)]\}/\{0.065[(1.065)^{36} - 1]\}$$
$$= 11.22339$$

$$G = [\$5,000,000(0.0075133) - \$5000]/11.22339 = \$2901.66$$

Example 6.3 (Continued)

Suppose the return on the investment is quite uncertain. Specifically, suppose it can be between 4% and 10%. What will be the impact on the value of the investment portfolio when the engineer is 58?

Answer: it will have a value between \$3.41 million and \$9.13 million.

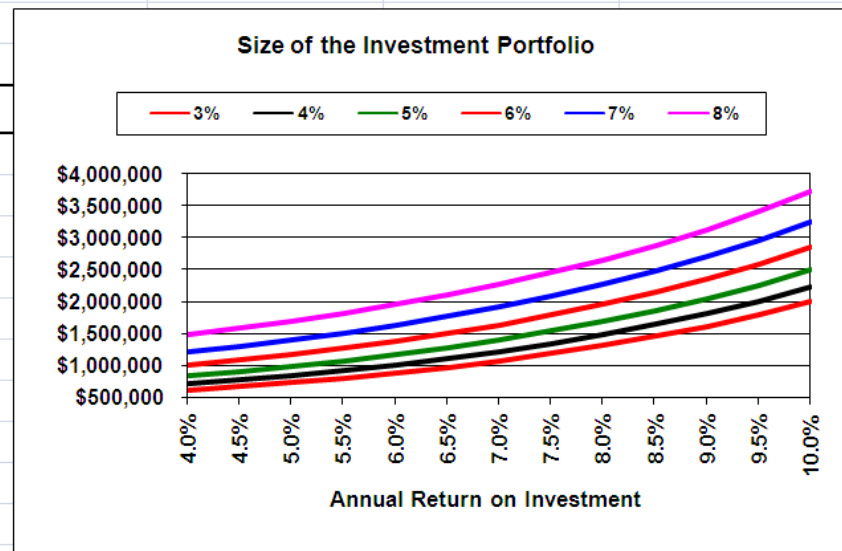
Example 6.3 (Continued)

Suppose the engineer makes geometric increases in annual investments. Specifically, suppose annual investments increase by 3%, 4%, 5%, 6%, 7%, or 8%. What will be the impact on the value of the investment portfolio when the engineer is 58?

Answer: it will have a value between \$0.6 million and \$3.7 million.

Return on Investment	Rate of Increase in Annual Investments					
	3%	4%	5%	6%	7%	8%
4.0%	\$602,827.11	\$710,296.02	\$843,941.79	\$1,010,829.86	\$1,220,001.61	\$1,483,029.91
4.5%	\$659,700.04	\$773,445.91	\$914,437.67	\$1,089,957.85	\$1,309,312.75	\$1,584,399.05
5.0%	\$723,384.45	\$843,941.79	\$992,882.77	\$1,177,717.93	\$1,408,031.51	\$1,696,059.28
5.5%	\$794,761.41	\$922,717.61	\$1,080,269.25	\$1,275,166.62	\$1,517,285.60	\$1,819,217.29
6.0%	\$874,828.95	\$1,010,829.86	\$1,177,717.93	\$1,383,495.62	\$1,638,345.09	\$1,955,229.96
6.5%	\$964,717.59	\$1,109,473.78	\$1,286,495.10	\$1,504,049.43	\$1,772,640.76	\$2,105,623.47
7.0%	\$1,065,707.98	\$1,220,001.61	\$1,408,031.51	\$1,638,345.09	\$1,921,784.67	\$2,272,114.82
7.5%	\$1,179,250.82	\$1,343,943.31	\$1,543,943.91	\$1,788,094.57	\$2,087,593.51	\$2,456,636.14
8.0%	\$1,306,989.35	\$1,483,029.91	\$1,696,059.28	\$1,955,229.96	\$2,272,114.82	\$2,661,361.97
8.5%	\$1,450,784.88	\$1,639,219.94	\$1,866,442.27	\$2,141,932.00	\$2,477,656.61	\$2,888,740.18
9.0%	\$1,612,745.56	\$1,814,729.25	\$2,057,426.11	\$2,350,662.17	\$2,706,820.71	\$3,141,526.60
9.5%	\$1,795,258.93	\$2,012,064.72	\$2,271,647.59	\$2,584,198.93	\$2,962,540.46	\$3,422,824.21
10.0%	\$2,001,028.73	\$2,234,062.33	\$2,512,086.44	\$2,845,678.57	\$3,248,123.06	\$3,736,127.17

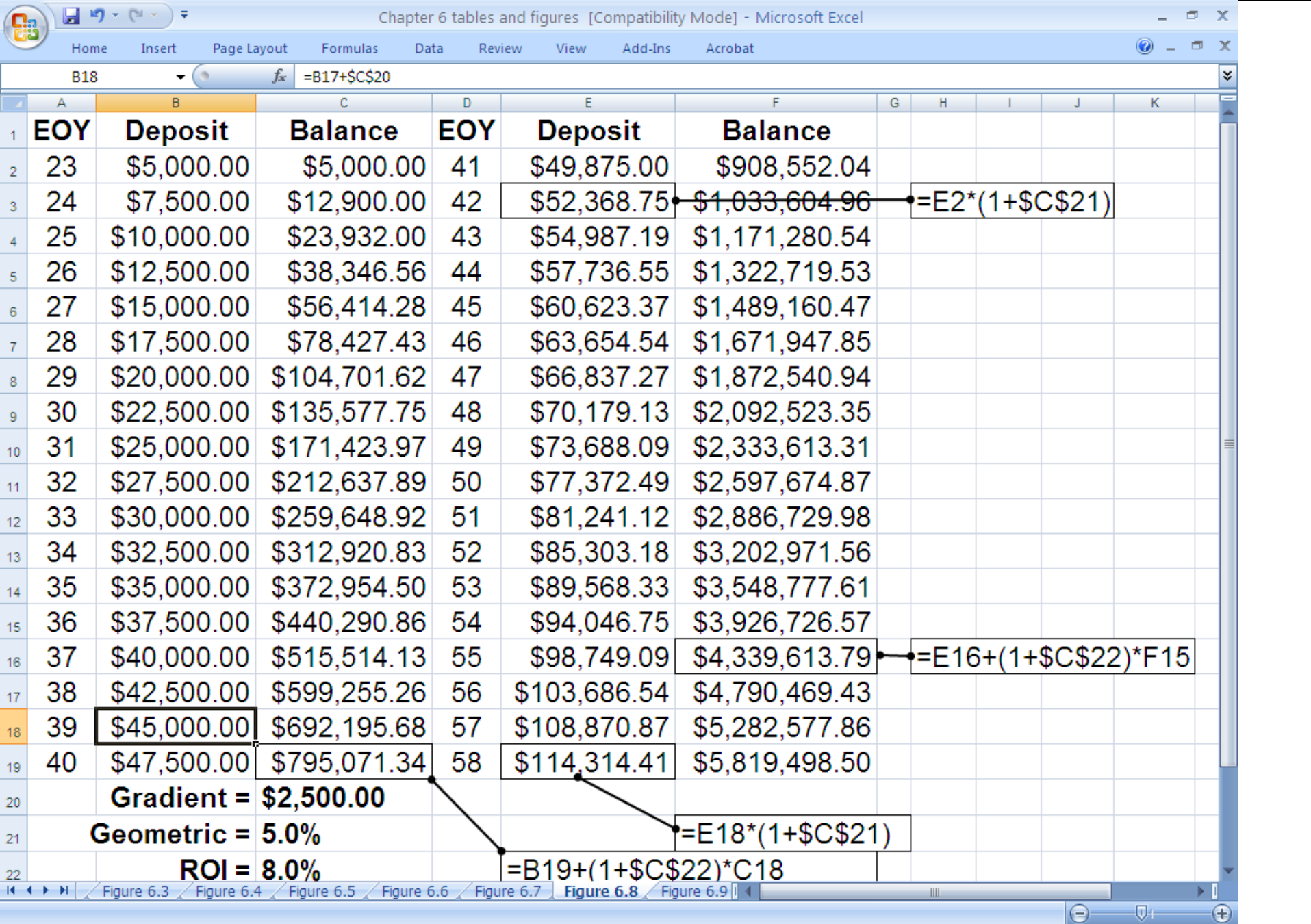
$$=5000*((1+\$A15)^{36}-(1+B\$2)^{36})/(\$A15-B\$2)$$



Example 6.3 (Continued)

Based on the results of the analysis, the engineer decides to increase by \$2500 the annual investment until age 40; the next 18 annual investments are 5% greater than the previous investment. What will be the impact on the value of the investment portfolio when the engineer is 58?

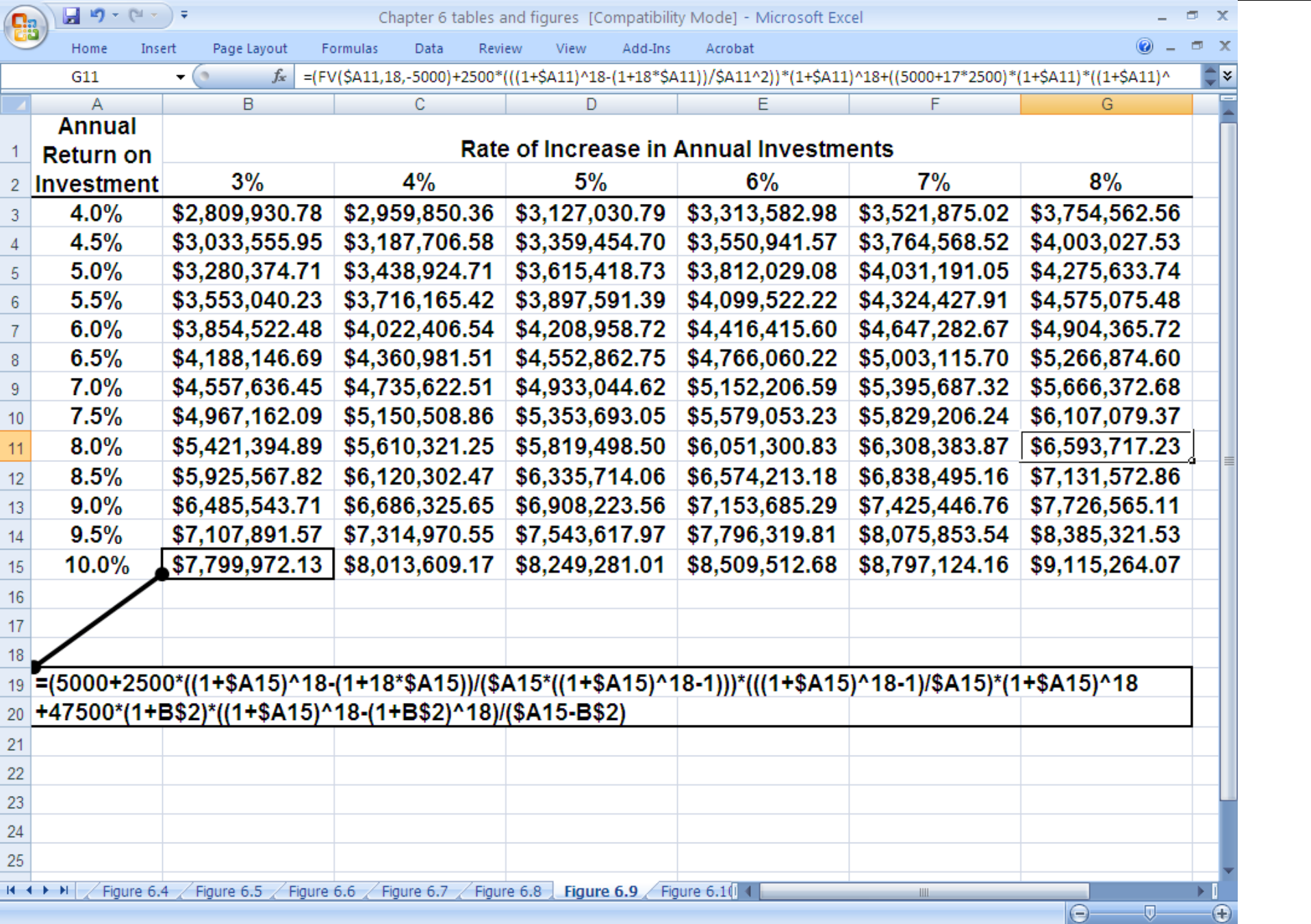
Answer: The investment portfolio will equal \$5,819,498.50.



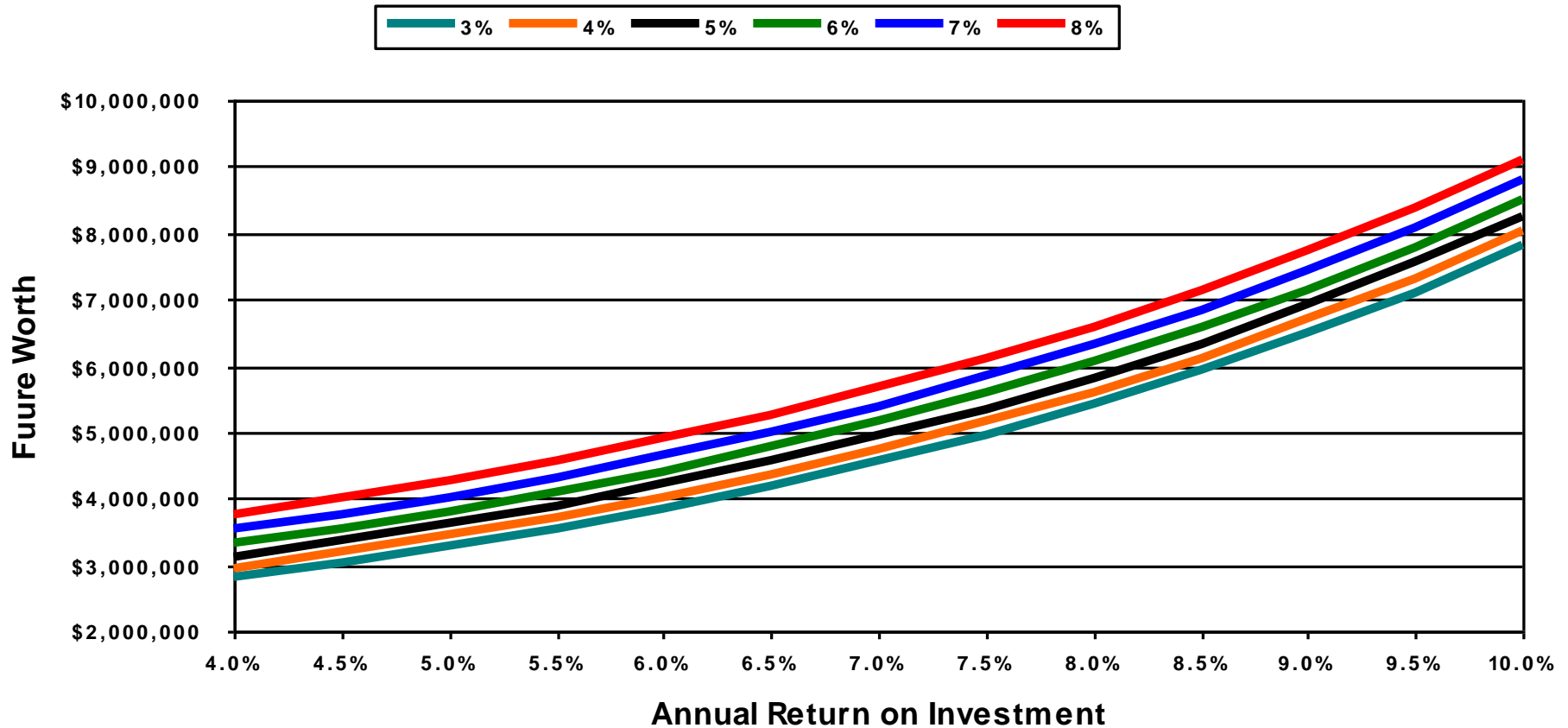
Example 6.3 (Continued)

Based on the previous results with a combination of gradient and geometric increases in annual investments, what will be the effect on the investment portfolio at age 58 if the geometric increases are 3%, 4%, 5%, 6%, 7%, and 8%, and the annual return on investment in the portfolio ranges from 4% to 10% in half percent increments?

Answer: The investment portfolio will range from \$2.8 million to \$9.1 million.



Size of the Investment Portfolio for Various Geometric Rates of Increase in Annual Investments after Age 40



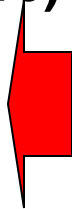
Future Worth Analysis

Multiple Alternatives

Example 6.4

Recall the example involving two design alternatives (A & B) for a new ride (The Scream Machine) in a theme park. A costs \$300,000, has revenue of \$55,000/yr, and has a negligible salvage value at the end of the 10-year planning horizon; B costs \$450,000, has revenue of \$80,000/yr, and has a negligible salvage value. Based on a FW analysis and a 10% MARR, which is preferred?

$$\begin{aligned}FW_A(10\%) &= -\$300,000(F/P\ 10\%,10) + \$55,000(F/A\ 10\%,10) \\ &= \$98,436.10 \\ &= \mathbf{FV(10\%,10,-55000,300000) = \$98,435.62}\end{aligned}$$

$$\begin{aligned}FW_B(10\%) &= -\$450,000(F/P\ 10\%,10) + \$80,000(F/A\ 10\%,10) \\ &= \$107,810.60 \\ &= \mathbf{FV(10\%,10,-80000,450000) = \$107,809.86}\end{aligned}$$


Example 6.4

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Analyze the impact on FW based on salvage values decreasing geometrically to 1¢ after 10 years; and analyze the impact of changes in the *MARR* on the recommendation.



Example 6.5

A recent 22-year old engineering graduate is choosing between 2 retirement plans: with plan 1, up to 4% of salary is matched by employer and, in the past, has earned 6% annual returns; with plan 2, a 1.5% fee is paid, matching up to 4% still occurs, and the investments being considered return between 2% and 12% annually. Her current salary is \$55,000; she assumes her salary will increase at an annual rate of 5%. Which should she choose?

$$FW_1(6\%) = 2(0.04)(\$55,000)(F|A_1 \ 6\%,5\%,40) = \$1,428,120.90$$

$$FW_2(2\%) = 2(0.04)(\$55,000)(0.985)(F|A_1 \ 2\%,5\%,40) = \$698,055.57$$

$$FW_2(12\%) = 2(0.04)(\$55,000)(0.985)(F|A_1 \ 12\%,5\%,40) = \$5,325,308.50$$

She chose the 2nd plan; which would you choose?

Example 6.6

Recall the example with two design alternatives for The Scream Machine: A costs \$300,000, has revenue of \$55,000/yr, and has a negligible salvage value at the end of the 10-year planning horizon; and B costs \$450,000, has revenue of \$80,000/yr, and has a negligible salvage value. Based on an incremental FW analysis and a 10% MARR, which is preferred?

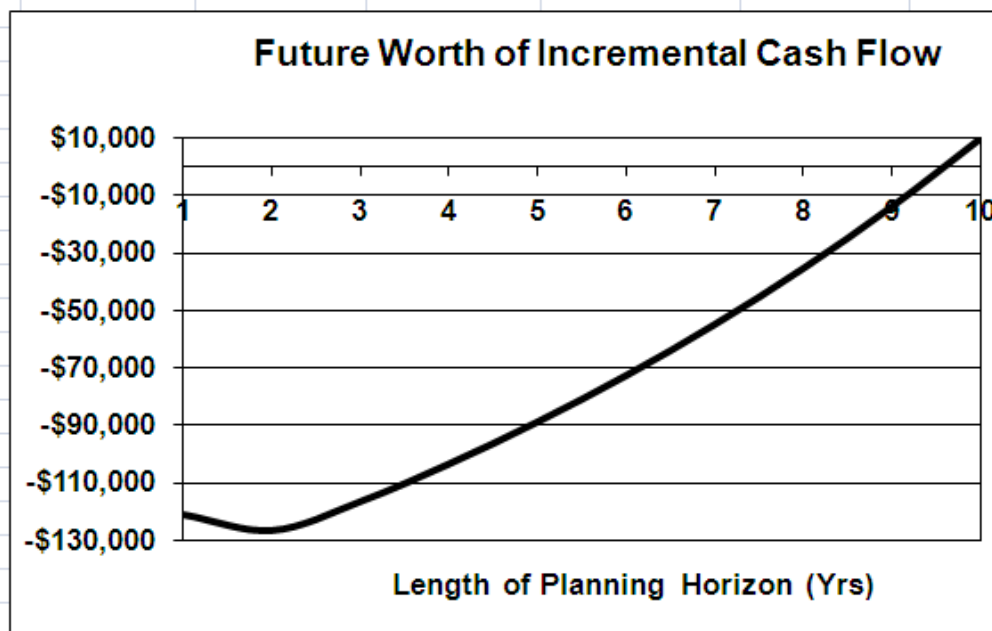
$$\begin{aligned}FW_A(10\%) &= -\$300,000(F/P\ 10\%,10) + \$55,000(F/A\ 10\%,10) \\ &= \$98,436.10 > \$0 \\ &= FV(10\%,10,-55000,300000) = \$98,435.62 > \$0 \\ &\text{(A is better than "do nothing")}\end{aligned}$$

$$\begin{aligned}FW_{B-A}(10\%) &= -\$150,000(F/P\ 10\%,10) + \$25,000(F/A\ 10\%,10) \\ &= \$9374.50 > \$0 \\ &= FV(10\%,10,-25000,150000) \\ &= \$9374.25 > \$0 \\ &\text{(B is better than A)}\end{aligned}$$

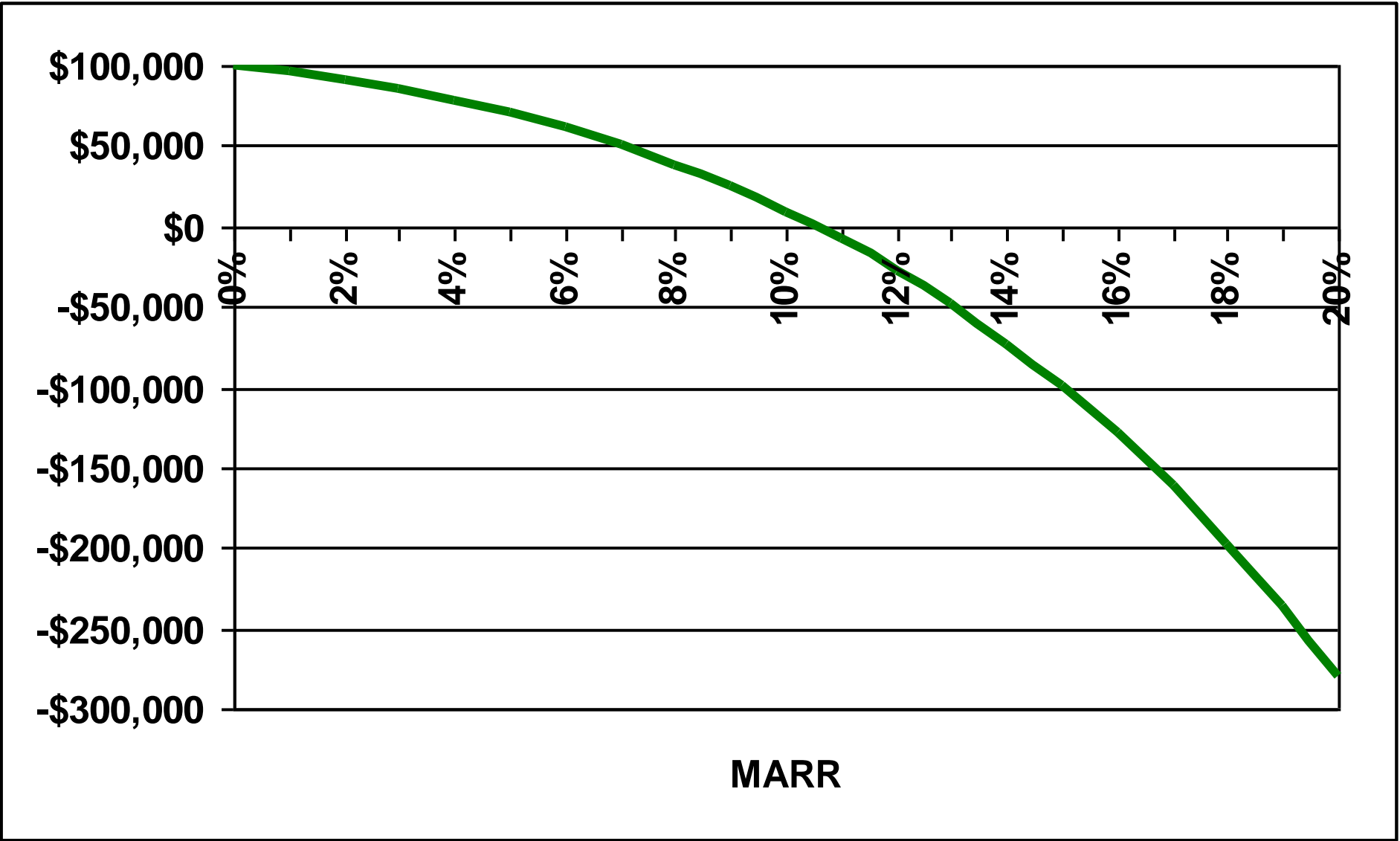
Prefer B

G13 fx =FV(10%,A13,-E13,-\$E\$3)+F13

	A	B	C	D	E	F	G	H	I	J
1	Planning	Geometric Series Rate (A) = -85.800%			Geometric Series Rate (B) = -86.364%					
2	Horizon	CF(A)	SV(A)	FW(A)	CF(B)	SV(B)	FW(B)	CF(B-A)	SV(B-A)	FW(B-A)
3	0	-\$300,000.00	\$300,000.00	\$0.00	-\$450,000.00	\$450,000.00	\$0.00	-\$150,000.00	\$150,000.00	\$0.00
4	1	\$55,000.00	\$42,599.95	-\$232,400.05	\$80,000.00	\$61,360.84	-\$353,639.16	\$25,000.00	\$18,760.88	-\$121,239.12
5	2	\$55,000.00	\$6,049.19	-\$241,450.81	\$80,000.00	\$8,367.00	-\$368,133.00	\$25,000.00	\$2,317.82	-\$126,682.18
6	3	\$55,000.00	\$858.98	-\$216,391.02	\$80,000.00	\$1,140.90	-\$333,009.10	\$25,000.00	\$281.92	-\$116,618.08
7	4	\$55,000.00	\$121.98	-\$183,853.02	\$80,000.00	\$155.57	-\$287,409.43	\$25,000.00	\$33.60	-\$103,556.40
8	5	\$55,000.00	\$17.32	-\$147,355.18	\$80,000.00	\$21.21	-\$236,300.29	\$25,000.00	\$3.89	-\$88,945.11
9	6	\$55,000.00	\$2.46	-\$107,107.29	\$80,000.00	\$2.89	-\$179,950.76	\$25,000.00	\$0.43	-\$72,843.47
10	7	\$55,000.00	\$0.35	-\$62,820.38	\$80,000.00	\$0.39	-\$117,948.62	\$25,000.00	\$0.05	-\$55,128.24
11	8	\$55,000.00	\$0.05	-\$14,102.75	\$80,000.00	\$0.05	-\$49,743.86	\$25,000.00	\$0.00	-\$35,641.11
12	9	\$55,000.00	\$0.01	\$39,486.93	\$80,000.00	\$0.01	\$25,281.70	\$25,000.00	\$0.00	-\$14,205.23
13	10	\$55,000.00	\$0.00	\$98,435.62	\$80,000.00	\$0.00	\$107,809.86	\$25,000.00	\$0.00	\$9,374.25



Incremental Future Worth as a Function of the *MARR*



MARR

Example 6.8

Perform an investment portfolio analysis for the investment involving two design alternatives for The Scream Machine.

$$\begin{aligned}FW_{DN}(10\%) &= \$450,000(F|P\ 10\%,10) = \$1,167,183.00 \\ &= FV(10\%,10,-450000) = \$1,167,184.11\end{aligned}$$

$$\begin{aligned}FW_B(10\%) &= \$80,000(F/A\ 10\%,10) = \$1,274,993.60 \\ &= FV(10\%,10,-80000) = \$1,274,993.97\end{aligned}$$

$$\begin{aligned}FW_A(10\%) &= \$55,000(F/A\ 10\%,10) + \$150,000(F/P\ 10\%,10) \\ &= \$1,265,619.10 \\ &= FV(10\%,10,-55000) + FV(10\%,10,-150000) \\ &= \$1,265,619.72\end{aligned}$$



More on Unequal Lives

Principle #8

**Compare investment alternatives
over a common period of time**

Example 6.9

If an investor's MARR is 12%, which mutually exclusive investment alternative maximizes the investor's future worth, given the parameters shown below?

EOY	CF(1)	CF(2)	CF(3)
0	-\$10,000	-\$15,000	-\$20,000
1	\$5,000	\$5,000	\$0
2	\$5,000	\$5,000	\$3,000
3	\$10,000	\$5,000	\$6,000
4		\$5,000	\$9,000
5		\$5,000	\$12,000
6		\$7,500	\$15,000

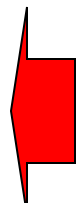
What planning horizon should be used? What assumptions are made regarding Alt. 1 for years 4, 5, and 6?

Example 6.9 (Continued)

If we use a 6-year planning horizon and assume no cash flows will occur in years 4, 5, and 6 for Alt. 1, the future worths will be as follows:

$$\begin{aligned}FW_1(12\%) &= -\$10,000(F|P\ 12\%,6) \\ &\quad + [\$5000(F|A\ 12\%,3) + \$5000](F|P\ 12\%,3) = \$10,990.43 \\ &= FV(12\%,6,-5000,10000) + FV(12\%,3,5000,-5000) = \$10,990.36\end{aligned}$$

$$\begin{aligned}FW_2(12\%) &= -\$14,500(F|P\ 12\%,6) + \$5000(F|A\ 12\%,6) = \$11,955.56 \\ &= FV(12\%,6,-5000,14500) = \$11,955.52\end{aligned}$$

$$\begin{aligned}FW_3(12\%) &= -\$20,000(F|P\ 12\%,6) + \$3000(A|G\ 12\%,6)(F|A\ 12\%,6) \\ &= \$13,403.40 \\ &= FV(12\%,6,-1000 * NPV(12\%,0,3,6,9,12,15) + 20000) = \$13,403.27\end{aligned}$$


Example 6.9 (Continued)

If we use a 6-year planning horizon and assume Alt. 1 repeats with identical cash flows for years 4, 5, and 6 for Alt. 1, the cash flow profiles will be as follows:

EOY	CF(1')	CF(2)	CF(3)
0	-\$10,000	-\$15,000	-\$20,000
1	\$5,000	\$5,000	\$0
2	\$5,000	\$5,000	\$3,000
3	\$0	\$5,000	\$6,000
4	\$5,000	\$5,000	\$9,000
5	\$5,000	\$5,000	\$12,000
6	\$10,000	\$7,500	\$15,000

Example 6.9 (Continued)

Under the assumption that Alt. 1 is repeated with identical cash flows for years 4, 5, and 6, the future worths will be as follows:

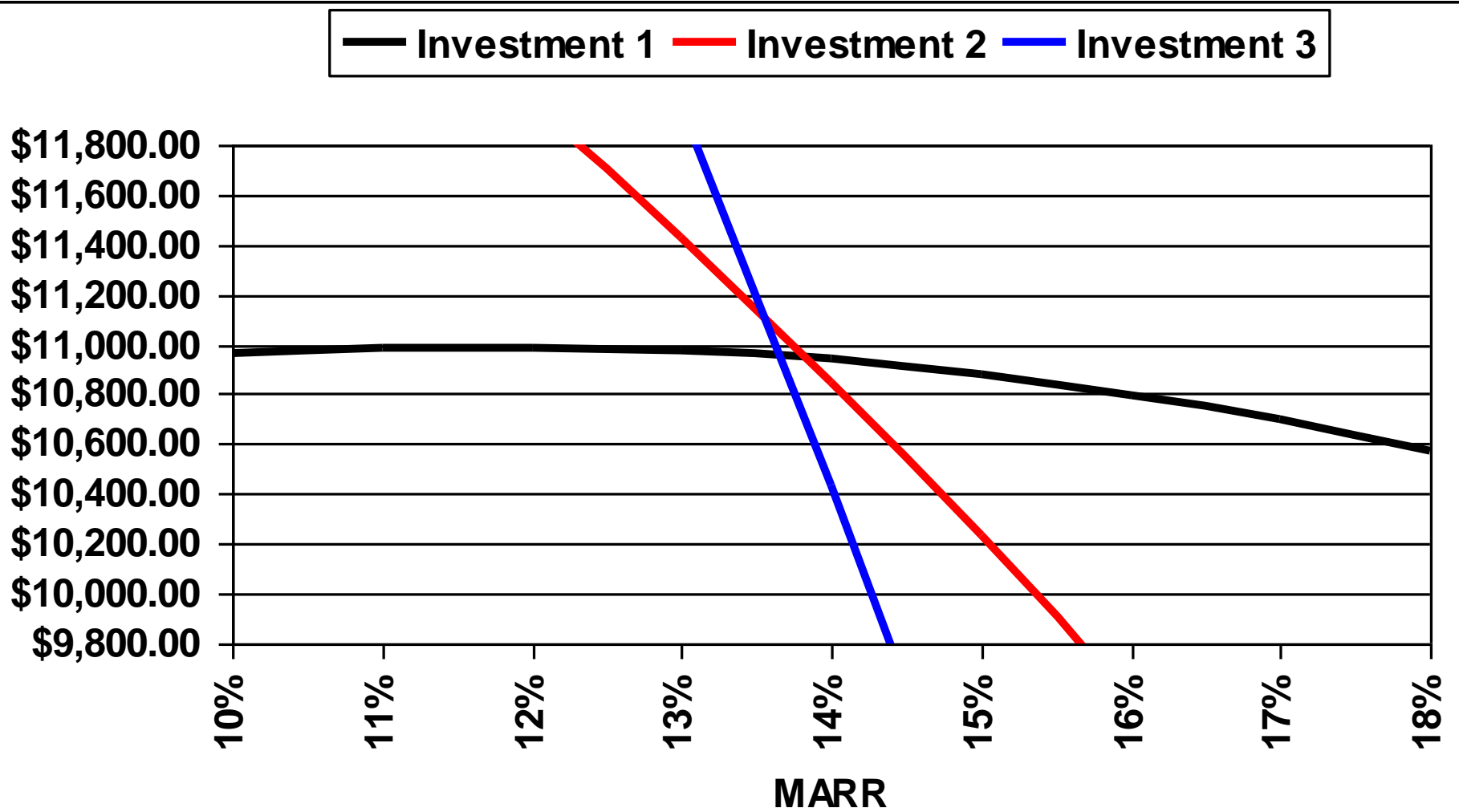
$$\begin{aligned}FW_1(12\%) &= -\$10,000(F|P\ 12\%,6) + \$5000(F|A\ 12\%,6) - \$5000(F|P\ 12\%,3) \\ &\quad + \$5000 \\ &= FV(12\%,6,-5000,10000) + FV(12\%,3,,5000) + 5000 \\ &= \$18,813.08\end{aligned}$$

$$\begin{aligned}FW_2(12\%) &= -\$14,500(F|P\ 12\%,6) + \$5000(F|A\ 12\%,6) \\ &= FV(12\%,6,-5000,14500) \\ &= \$11,955.52\end{aligned}$$

$$\begin{aligned}FW_3(12\%) &= -\$20,000(F|P\ 12\%,6) + \$3000(A|G\ 12\%,6)(F|A\ 12\%,6) \\ &= FV(12\%,6,-1000*NPV(12\%,0,3,6,9,12,15) + 20000) \\ &= \$13,403.27\end{aligned}$$

Is it reasonable to assume an investment alternative equivalent to Alt. 1 will be available in 3 years? If so, why was the MARR set equal to 12%?

Future Worths Assuming Investment 1 Is Not Repeated



Pit Stop #6—It's Time to Put the Peddle to the Metal!

1. True or False: Future worth analysis is the most popular *DCF* measure of economic worth.
2. True or False: Unless non-monetary considerations dictate otherwise, choose the mutually exclusive investment alternative that has the greatest future worth, regardless of the lives of the alternatives.
3. True or False: If $FW > 0$ when the $MARR = 20\%$, then $DPBP < 5$ years.
4. True or False: If $FW < 0$, then $PW < 0$.
5. True or False: If $FW(A) > FW(B)$, then $DPBP(A) < DPBP(B)$, and $PBP(A) < PBP(B)$.
6. True or False: When using future worth analysis with mutually exclusive alternatives having unequal lives, use a planning horizon equal to the least common multiple of lives.

Pit Stop #6—It's Time to Put the Peddle to the Metal!

1. True or False: Future worth analysis is the most popular *DCF* measure of economic worth. **FALSE**
2. True or False: Unless non-monetary considerations dictate otherwise, choose the mutually exclusive investment alternative that has the greatest future worth, regardless of the lives of the alternatives. **FALSE**
3. True or False: If $FW > 0$ when the $MARR = 20\%$, then $DPBP < 5$ years. **FALSE**
4. True or False: If $FW < 0$, then $PW < 0$. **TRUE**
5. True or False: If $FW(A) > FW(B)$, then $DPBP(A) < DPBP(B)$, and $PBP(A) < PBP(B)$. **FALSE**
6. True or False: When using future worth analysis with mutually exclusive alternatives having unequal lives, use a planning horizon equal to the least common multiple of lives. **FALSE (it is situation and circumstance dependent)**