Chapter 5

Present Worth Analysis

Systematic Economic Analysis Technique

- 1. Identify the investment alternatives
- 2. Define the planning horizon
- 3. Specify the discount rate
- 4. Estimate the cash flows
- 5. Compare the alternatives
- 6. Perform supplementary analyses
- 7. Select the preferred investment

Measures of Economic Worth

- Present Worth (≥ \$0)
- Future Worth (≥ \$0)
- Annual Worth (≥ \$0)
- Capitalized Worth (≥ \$0)
- Discounted Payback Period (< Value, e.g. 2 yrs)
- Payback Period (≤ Value)
- Internal Rate of Return (≥ MARR)
- External Rate of Return (≥ MARR)
- Modified Internal Rate of Return (> MARR)
- Benefit/Cost Ratio (≥ 1.0)

Measures of Economic Worth

- Ranking Methods or Incremental Methods
 - Present Worth
 - Future Worth
 - Annual Worth
 - Capitalized Worth
 - Discounted Payback Period
 - Payback Period
- Incremental Methods
 - Internal Rate of Return
 - External Rate of Return
 - Modified Internal Rate of Return
 - Benefit/Cost Ratio

Measures of Economic Worth

- The following are consistent measures of economic worth, i.e., yield the same recommendation (if performed correctly)
 - Present Worth
 - Future Worth
 - Annual Worth
 - Internal Rate of Return
 - External Rate of Return
 - Benefit/Cost Ratio
- Capitalized worth yields the same recommendation if the planning horizon is infinitely long or equal to a least common multiple of lives of the investment alternatives

Present Worth Analysis

- Ranking versus incremental analysis
- Before-tax versus after-tax analysis
- Equal versus unequal lives
- Single versus multiple alternatives
- Finite versus infinite planning horizon
- Discounted payback period and payback period analyses

Present Worth Analysis

Single Alternative

Present Worth Method

- converts all cash flows to a single sum equivalent at time zero using i = MARR over the planning horizon
- the most popular DCF method

$$PW (i\%) = \sum_{i=0}^{n} A_{i} (1+i)^{-t}$$

(bring all cash flows back to "time zero" and add them up!)

A \$500,000 investment in a surface mount placement machine is being considered. Over a 10-year planning horizon, it is estimated the SMP machine will produce net annual savings of \$92,500. At the end of 10 years, it is estimated the SMP machine will have a \$50,000 salvage value. Based on a 10% MARR and a present worth analysis, should the investment be made?

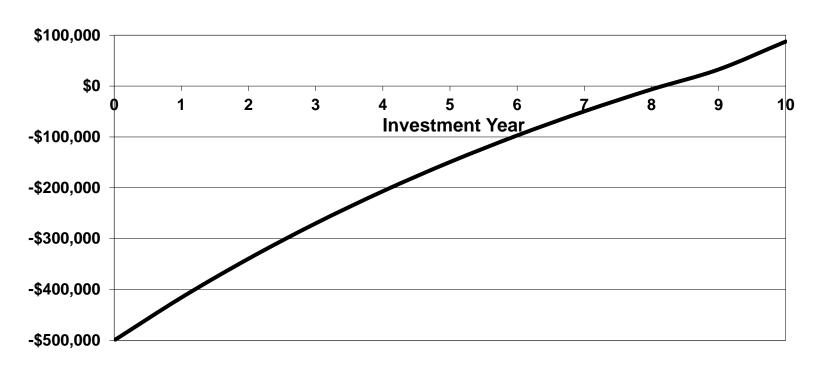
```
PW = -$500K + $92.5K(P|A 10%,10) + $50K(P|F 10%,10)
= $87,650.50
=PV(10%,10,-92500,-50000)-500000
= $87,649.62
```

Solving with Excel®,

```
(ignores salvage value until EOY = 10)
MARR =
           10%
 EOY
                      Cum(PW)
            CF
                     -$500,000.00 =B3
          -$500,000
           $92,500
                     -$415,909.09 =NPV($B$1,$B$4:B4)+$B$3
                     -$339,462.81 = NPV($B$1,$B$4:B5)+$B$3
   2
           $92,500
                     -$269,966.19 =NPV($B$1,$B$4:B6)+$B$3
   3
           $92,500
                     -$206,787.45 =NPV($B$1,$B$4:B7)+$B$3
           $92,500
                     -$149,352.22 =NPV($B$1,$B$4:B8)+$B$3
   5
           $92,500
                      -$97,138.39 =NPV($B$1,$B$4:B9)+$B$3
   6
           $92,500
                      -$49,671.26 =NPV($B$1,$B$4:B10)+$B$3
           $92,500
                       -\$6,519.33 = NPV(\$B\$1,\$B\$4:B11) + \$B\$3
           $92,500
                       $32,709.70 = NPV($B$1,$B$4:B12)+$B$3
   9
           $92,500
                       $87,649.62 = NPV($B$1,$B$4:B13)+$B$3
          $142,500
  10
        87,649.62 = NPV(B1,B4:B13)+B3
 PW =
```

Plotting Cumulative Present Worth,

Present Worth of the Investment (ignores salvage value until EOY = 10)



Present Worth Analysis

Multiple Alternatives

$$Maxim ize PW_{j}(i\%) = \sum_{i=0}^{n} A_{ji}(1+i)^{-t}$$

Choose the alternative with the greatest present worth

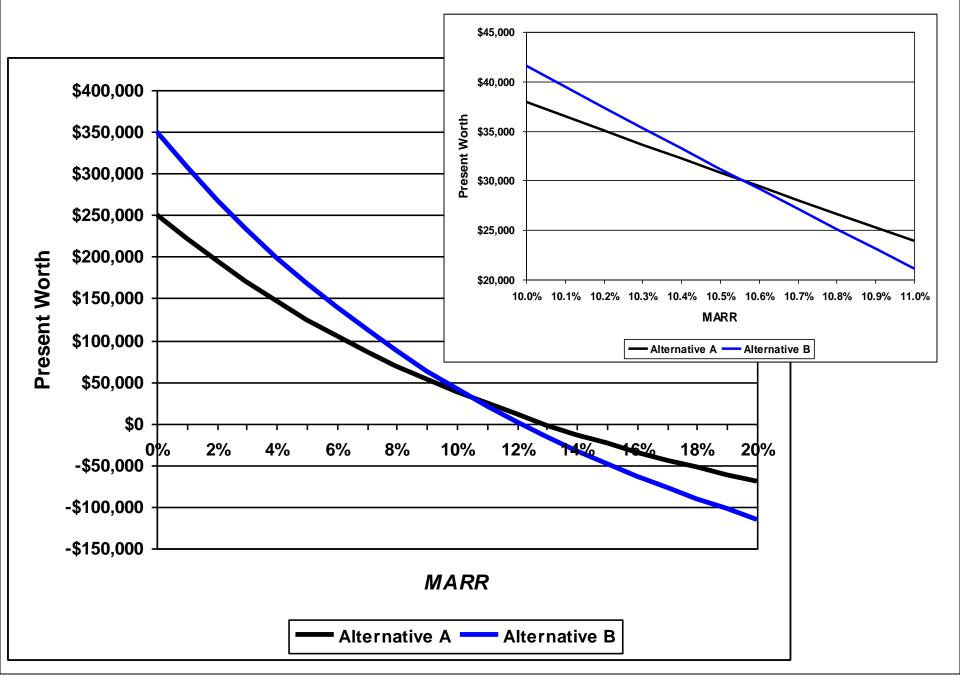
Two design alternatives (A & B) are being considered for a new ride (The Scream Machine) at a theme park in Florida. Alternative A requires a \$300,000 investment and will produce net annual revenue of \$55,000/yr. Alternative B requires a \$450,000 investment and will produce net annual revenue of \$80,000/yr. At the end of the 10-yr planning horizon, both designs will have negligible salvage values. Based on a 10% *MARR*, which should be chosen? (The "do nothing" alternative is feasible and assumed to have a PW of \$0.)

```
\begin{aligned} \text{PW}_{\text{A}}(10\%) &= -\$300,000 + \$55,000(\textit{P/A}\ 10\%,10) = \$37,951.35 \\ &= \text{PV}(10\%,10,-55000) - 300000 = \$37,951.19 > \$0 \\ &\text{(A is better than doing nothing)} \\ \text{PW}_{\text{B}}(10\%) &= -\$450,000 + \$80,000(\textit{P/A}\ 10\%,10) = \$41,565.60 \\ &= \text{PV}(10\%,10,-80000) - 450000 = \$41,565.37 > \text{PW}_{\text{A}} \\ &\text{(B is better than A)} \end{aligned}
```

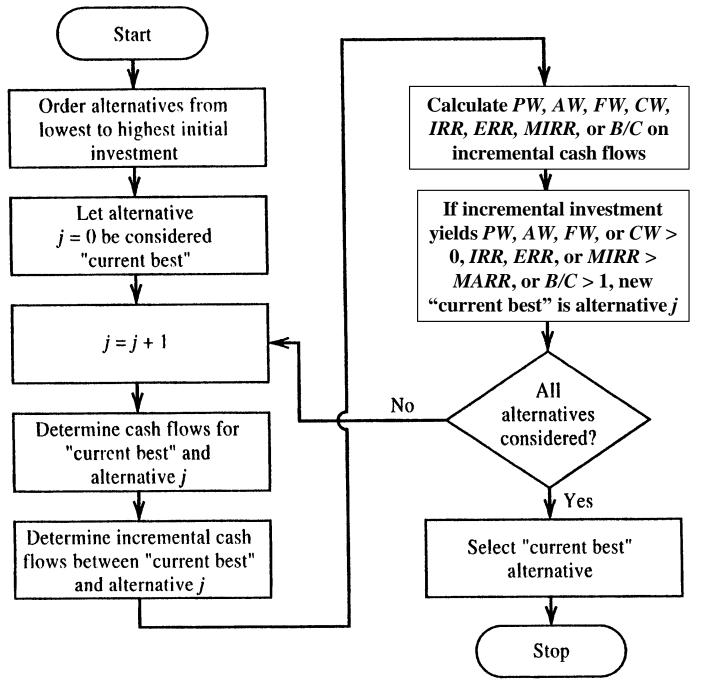
Two design alternatives (A & B) are being considered for a new ride (The Scream Machine) at a theme park in Florida. Alternative A requires a \$300,000 investment and will produce net annual revenue of \$55,000/yr. Alternative B requires a \$450,000 investment and will produce net annual revenue of \$80,000/yr. At the end of the 10-yr planning horizon, both designs will have negligible salvage values. Based on a 10% *MARR*, which should be chosen? (The "do nothing" alternative is feasible and assumed to have a PW

How does PW change with changing MARR?

```
PW_A(1070) = -$300,000 + $33,000(P/A 1070,10) = $37,931.33
=PV(10\%,10,-55000)-300000 = $37,951.19 > $0
(A is better than doing nothing)
PW_B(10\%) = -$450,000 + $80,000(P/A 10\%,10) = $41,565.60
=PV(10\%,10,-80000)-450000 = $41,565.37 > PW_A
(B is better than A)
```



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Flow Chart for the Incremental Comparison of Investment Alternatives

Let's use an incremental approach to evaluate the two design alternatives for a new ride at a theme park. Recall, Alternative A required a \$300,000 investment and produced annual revenue of \$55,000; Alternative B required a \$450,000 investment and produced annual revenue of \$80,000. At the end of the 10-yr planning horizon, both had negligible salvage values. Based on a 10% *MARR*, which should be chosen?

$$PW_{A}(10\%) = -\$300,000 + \$55,000(P/A 10\%,10) = \$37,951.35 \\ = PV(10\%,10,-55000) - 3000000 = \$37,951.19 > \$0 \\ \text{(A is better than doing nothing)} \\ PW_{B-A}(10\%) = -\$150,000 + \$25,000(P/A 10\%,10) = \$3,614.25 \\ = PV(10\%,10,-25000) - 1500000 = \$3,614.18 > \$0 \\ \text{(B is better than A)} \\$$

Present Worth Analysis

"One Shot" Investments

Two investment alternatives (1 & 2) are available, with the CFDs shown below. They are "one shot" investments. Using a 15% MARR, which should be chosen?

MARR = 15%

Two investment alternatives (1 & 2) are available, with the CFDs shown below. They are "one shot" investments. Using a 15% MARR, which should be chosen?

Discounted Payback Period Analysis

Single Alternative

Discounted Payback Period Method

- determines how long it takes to fully recover an investment while considering the time value of money
- increasing in popularity
- determine the smallest value of m such that

$$\sum_{t=0}^{m} A_t \left(1+i\right)^{-t} \geq 0$$

(determine the point in time when cumulative discounted cash flow \geq \$0)

Discounted Payback Period

- EASTMAN calls this the net present value payback year
- Let's use Excel's® SOLVER and/or GOAL SEEK to determine the DPBP for the SMP investment with salvage value decreasing as geometric and gradient series.

Based on a 10% MARR, how long does it take for the \$500,000 investment in a surface mount placement machine to be recovered, based on an annual savings of \$92,500 and a negligible salvage value, regardless of how long the machine is used?

```
# years = NPER(10%,92500,-500000)
= 8.16 years
```

Example 5.6 (Continued)

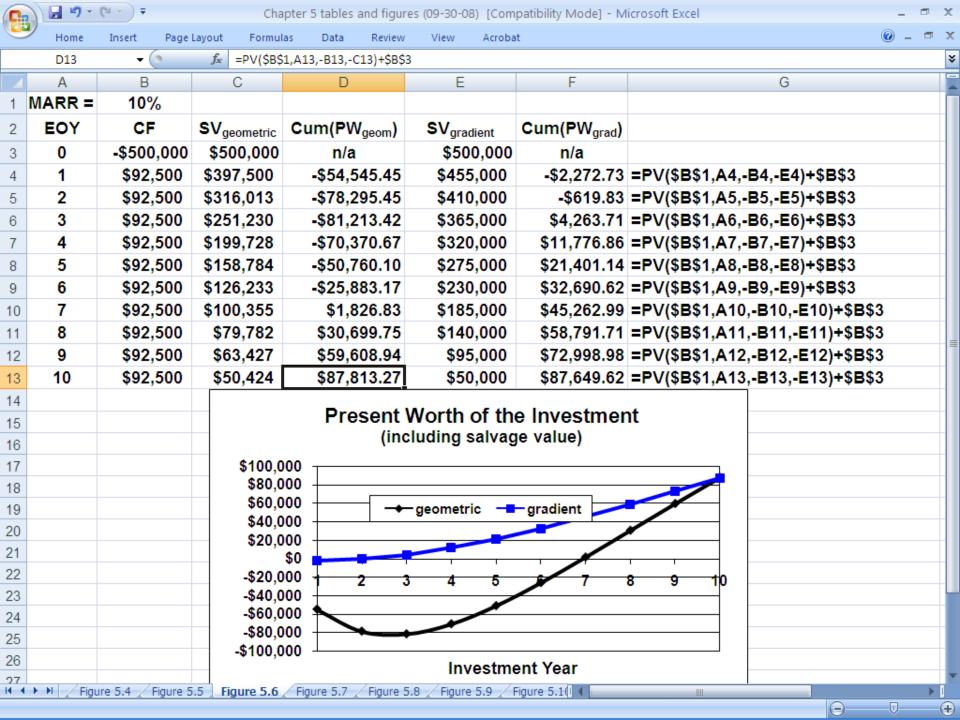
How is the PW of the investment in the SMP machine affected when salvage value decreases from \$500,000 to \$50,000 over the 10-year planning horizon? Consider both geometric and gradient decreases.

Example 5.6 (Continued)

How is the PW of the investment in the SMP machine affected when salvage value decreases from \$500,000 to \$50,000 over the 10-year planning horizon? Consider both geometric and gradient decreases.

$$G = (\$500,000 - \$50,000)/10 = \$45,000/yr$$

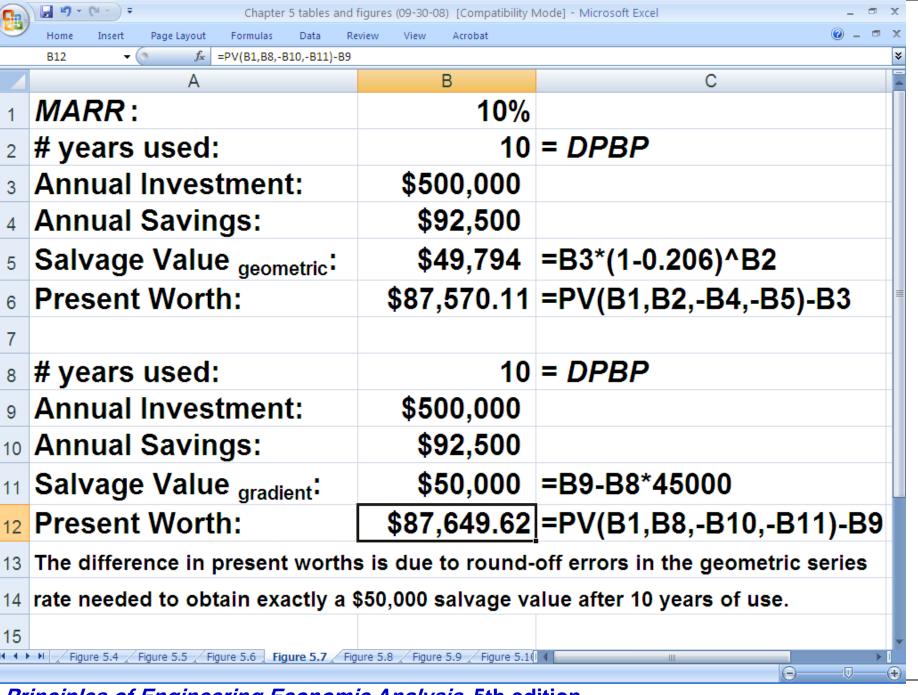
 $j = RATE(10,,-500000,50000) = -20.6\%/yr$

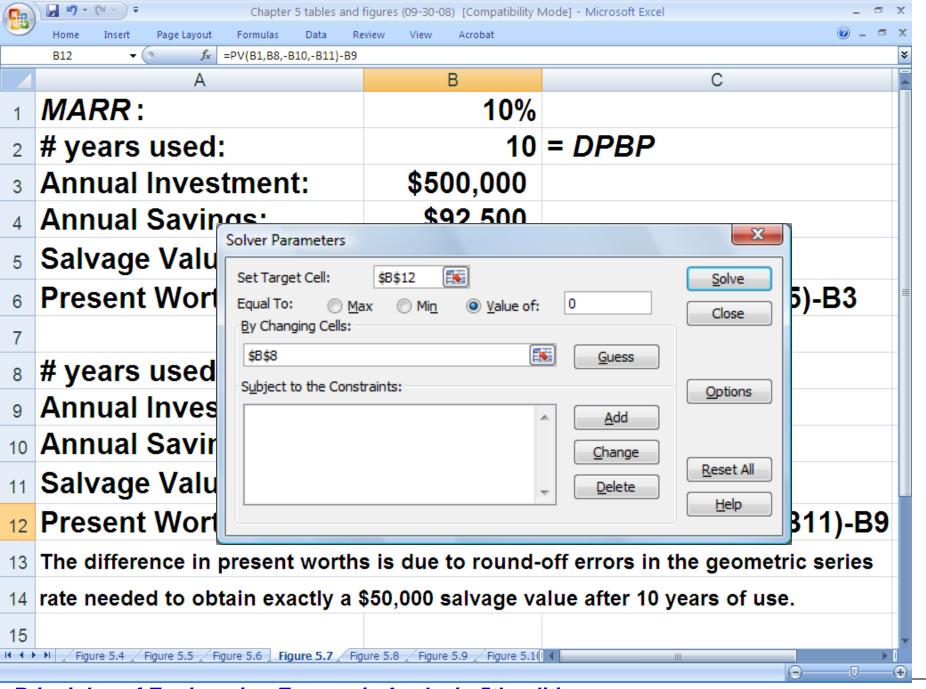


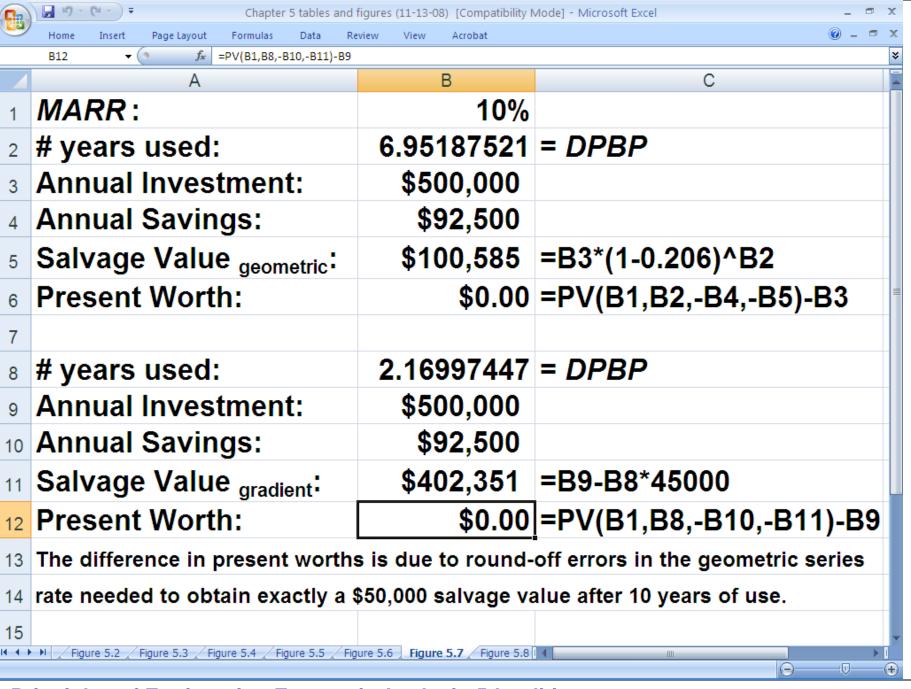
Based on a 10% MARR, how long does it take for the \$500,000 investment in a surface mount placement machine to be recovered, based on an annual savings of \$92,500 and a salvage value at the end of n years equal to

- a) $$500,000(1 0.206)^n$ and
- b) \$500,000 \$45,000n?

The Excel® SOLVER tool is used to solve the example.









Payback Period Method



- EASTMAN calls it the cash payback year
- determines the length of time required to recover the initial investment without considering the time value of money
- not equivalent to those already considered
- a popular method of valuing investments
- determine the smallest value of m such that

$$\sum_{t=0}^{m} A_{t} \ge 0$$

(ignores cash flows that occur after the payback period)

Why Use the Payback Period Method?

- does not require interest rate calculations
- does not require a decision concerning the MARR
- easily explained and understood
- reflects a manager's attitudes when capital is limited
- hedge against uncertainty of future cash flows
- provides a rough measure of the liquidity of an investment

What is the payback period for the \$500,000 SMP investment, given an annual savings of \$92,500?

```
PBP = $500,000/$92,500
= 5.4054 years
=NPER(0%,92500,-500000)
= 5.4054
```

Discounted Payback Period Analysis

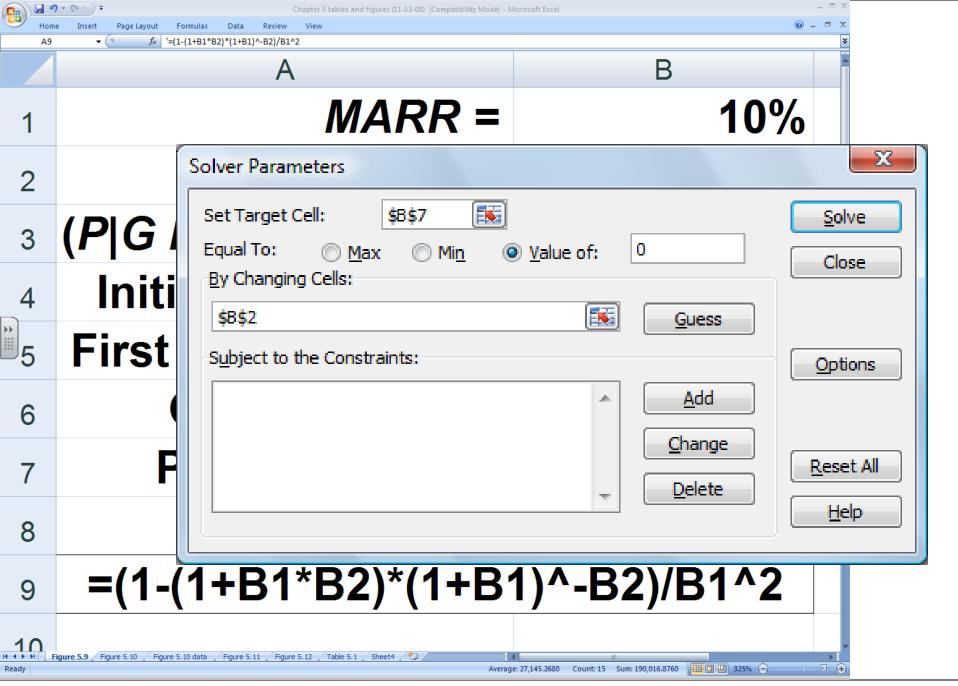
Multiple Alternatives

Now, suppose a third design (alternative C) is developed for The Scream Machine. As before, A requires a \$300,000 investment and produces revenue of \$55,000/yr; and B requires a \$450,000 investment and produces revenue of \$80,000/yr. The new design (C) requires a \$150,000 investment and produces 1st year revenue of \$45,000; thereafter, revenue decreases by \$5000/yr. Based on a 10% *MARR*, which design has the smallest DPBP?

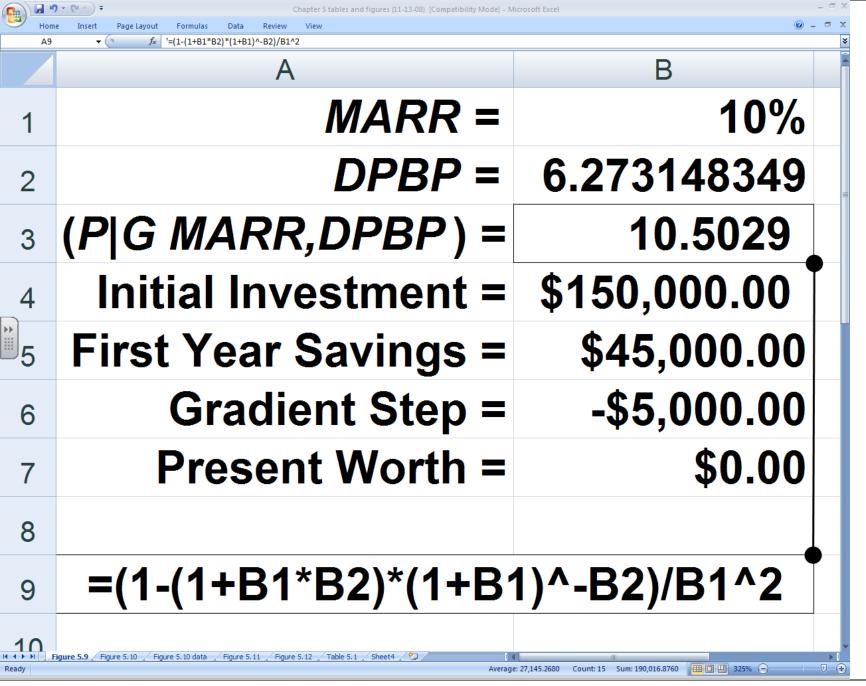
```
DPBP_A(10\%) = NPER(10\%, -55000, 300000) = 8.273 \text{ years}

DPBP_B(10\%) = NPER(10\%, -80000, 450000) = 8.674 \text{ years}

DPBP_C(10\%) = 6.273 \text{ years} (using the Excel® SOLVER tool)
```



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Now, suppose a third design (alternative C) is developed for The Scream Machine. As before, A requires a \$300,000 investment and produces revenue of \$55,000/yr; and B requires a \$450,000 investment and produces revenue of \$80,000/yr. The new design (C) requires a \$150,000 investment and produces 1st year revenue of \$45,000; thereafter, revenue decreases by \$5000/yr. Based on a 10% *MARR*, which design has the smallest DPBP?

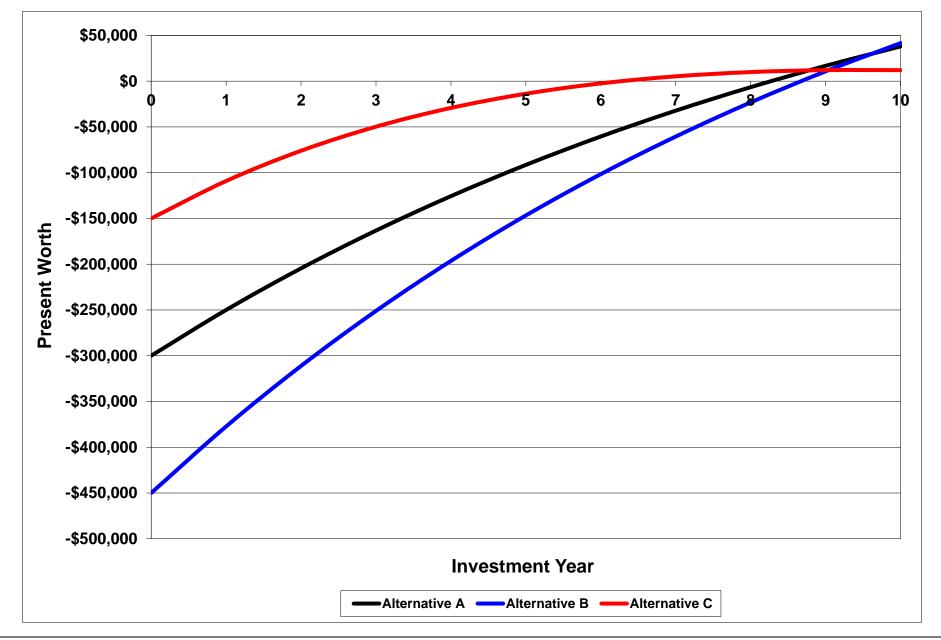
```
DPBP_A(10\%) = NPER(10\%, -55000, 300000) = 8.273 \text{ years}

DPBP_B(10\%) = NPER(10\%, -80000, 450000) = 8.674 \text{ years}

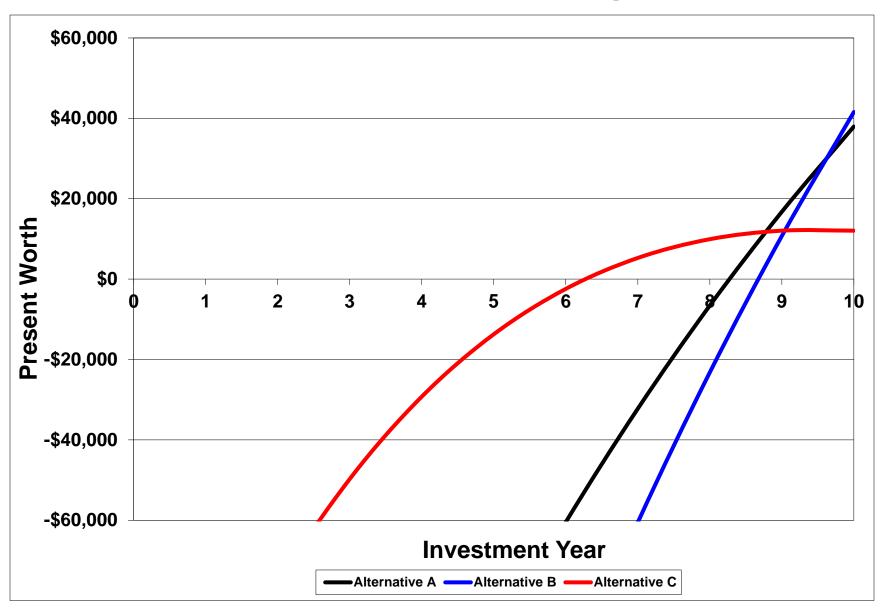
DPBP_C(10\%) = 6.273 \text{ years (using SOLVER)}
```

Note: $PW_C(10\%) = -\$150,000 + \$45,000(P|A 10\%,10) - \$5,000(P|G 10\%,10)$ $PW_C(10\%) = \$12,048.81 < PW_A(10\%) < PW_B(10\%)$ B is best, not C!!

Present Worth as a Function of Investment Duration



Close-Up of Critical Region



Three investments are available, but only one can be pursued: invest \$10,000 and obtain \$5,000/yr for 2 yrs, plus \$1,000 after 5 yrs; invest \$10,000 and receive \$5,000, \$4,000, \$3,000, \$2,000, and \$1,000 over the next 5 yrs; invest \$10,000 and receive \$2,500/yr for 5 yrs, plus \$10,000 after 5 yrs. Which is best using PBP? using PW and a MARR of 10%?

EOY	CF(1)	CumCF(1)	CF(2)	CumCF(2)	CF(3)	CumCF(3)
0	-\$10,000	-\$10,000	-\$10,000	-\$10,000	-\$10,000	-\$10,000
1	\$5,000	-\$5,000	\$5,000	-\$5,000	\$2,500	-\$7,500
2	\$5,000	\$0	\$4,000	-\$1,000	\$2,500	-\$5,000
3	\$0	\$0	\$3,000	\$2,000	\$2,500	-\$2,500
4	\$0	\$0	\$2,000	\$4,000	\$2,500	\$0
5	\$1,000	\$1,000	\$1,000	\$5,000	\$12,500	\$12,500
PBP =		2 yrs		2.33 yrs		4 yrs
<i>PW</i> (10%) =	-\$701.39		\$2,092.13	_	\$5,686.18	_

PBP ranking: 1, 2, 3

PW ranking: 3, 2, 1

Capitalized Worth Analysis

Single Alternative

Capitalized Worth Method

- a perpetuity is an investment that has an infinite life
- the capitalized worth is the present worth of a perpetuity
- the capitalized worth indicates the amount of money needed "up front" such that the interest earned will cover the cash flow requirements forever for the investment
- used mostly by government

$$CW(i) = AW(i)/i$$

Every 10 years the dome of the state capital building has to be cleaned, sand blasted, and re-touched. It costs \$750,000 to complete the work. Using a 5% MARR, what is the capitalized cost for the refurbishment of the capital dome?

```
CC = $750,000 + $750,000(P|F 5%,10) + $750,000(P|F 5%,10) + ... or CC = $750,000(A|P 5%,10)/0.05 = $750,000(0.1295)/0.05 = $1,942,500 CC = PMT(5%,10,-750000)/0.05 = $1,942,569 or CC = $750,000 + $750,000(A|F 5%,10)/0.05 = $750,000 + $750,000(0.0795)/0.05 = $1,942,500 CC = 750000+PMT(5%,10,,-750000)/0.05 = $1,942,569 Recall, (A|P i\%,n) = (A|F i\%,n) + i
```

A new highway is to be constructed. Asphalt paving will be used. The asphalt will cost \$150/ft, including the material and paving operation. Due to heavy usage, the asphalt is expected to last 5 yrs before requiring resurfacing. The cost of resurfacing will be the same/ft. Paved ditches must be installed on each side of the highway and will cost \$7.75/ft to install; ditches will have to be re-paved in 15 yrs at a cost equal to the initial cost. Four pipe culverts are required/mile; each costs \$8,000 and will last 10 yrs; replacements will cost \$10,000, each, forever. Annual maintenance of the highway will cost \$9,000/mi. Cleaning each culvert will cost \$1,250/yr. Cleaning and maintaining each ditch will cost \$3.75/ft every year. Using a 5% MARR, what is the capitalized cost (CC) per mile for the highway?

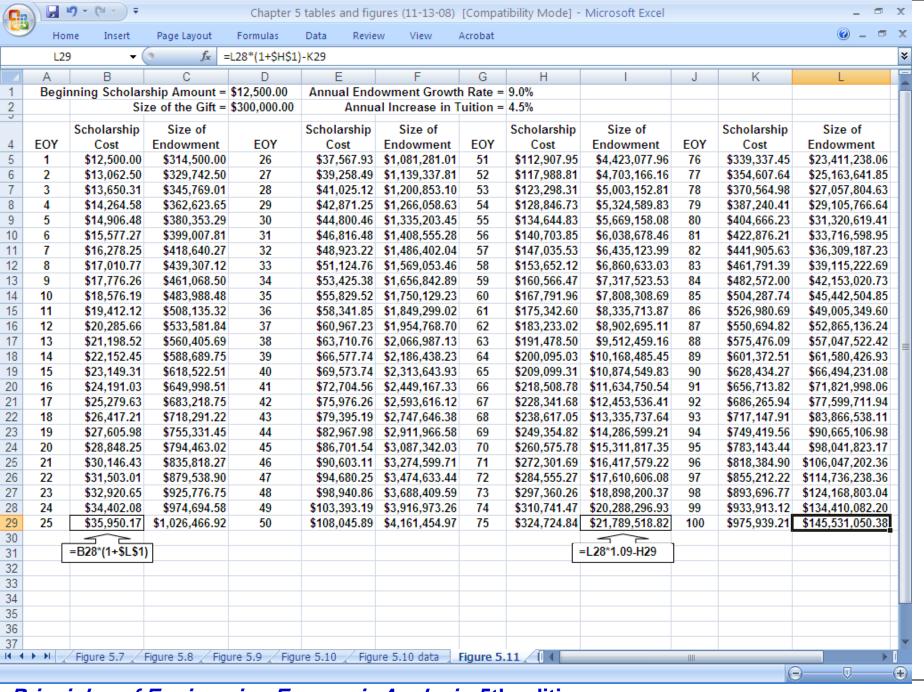
Example 5.12 (Solution)

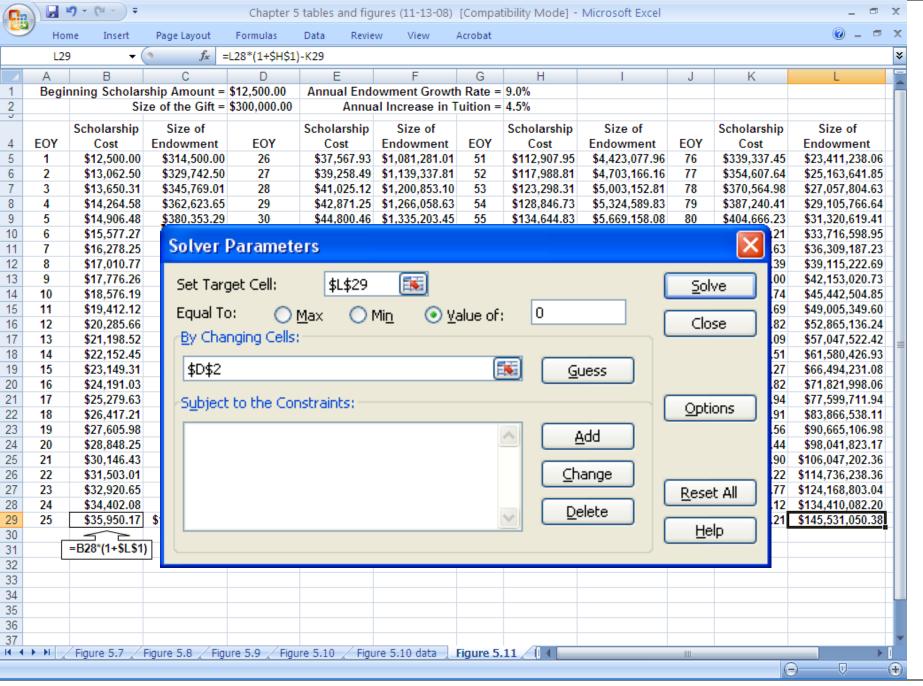
```
Paving Highway and Ditches/mile
   CC = 5,280 \text{ ft/mi}[\$150/\text{ft}(A|P 5\%,5) + \$7.75/\text{ft}(A|P 5\%,15)]/0.05
        = $3,737,409
        =5280*(PMT(5\%,5,-150)+PMT(5\%,15,-7.75))/0.05 = $3,737,487
Highway Maintenance/mile
   CC = \$9,000/0.05 = \$180,000
Ditch Maintenance/mile
   CC = 2(5,280 \text{ ft/mi})(\$3.75/\text{ft})/0.05 = \$792,000
Culverts/mile
   CC = 4[(\$8,000 + \$1,250/0.05 + \$10,000(A|F 5\%,10)/0.05]
        = $195,600
        =4*(8000+1250/0.05+PMT(5\%,10,,-10000)/0.05)
        = $195,604
Highway/mile
   CC = \$3,737,487 + \$180,000 + \$792,000 + \$195,604
        = $4,905,091
```

How much will it cost to endow a \$12,500 scholarship if the endowment earns 4.5% interest?

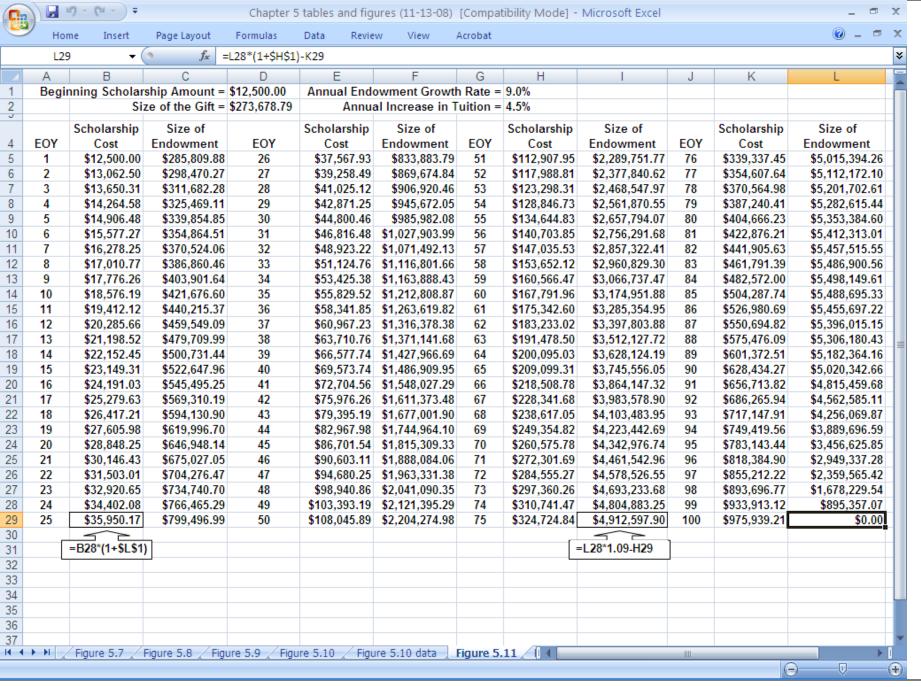
CW = \$12,500/0.045 = \$277,777.78

Suppose, instead of endowing a scholarship, you wish to establish a fund that will pay for the cost of a scholarship for 100 years. How much must you contribute to a fund that earns interest at an annual rate of 9%, if the size of the scholarship grows at an annual rate of 4.5%?





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Capitalized Worth Analysis

Multiple Alternatives

In a developing country, two alternatives are being considered for delivering water from a mountainous area to an arid area. A pipeline can be installed at a cost of \$125 million; major replacements every 15 years will cost \$10 million. Annual O&M costs are estimated to be \$5 million. Alternately, a canal can be constructed at a cost of \$200 million; annual O&M costs are estimated to be \$1 million; upgrades of the canal will be required every 10 years at a cost of \$5 million. Using a 5% MARR and a capitalized cost analysis, which alternative should be chosen?

Example 5.15 (Solution)

Pipeline

```
CC = $125,000,000 + [$10,000,000(A|F 5%,15)
+ $5,000,000]/0.05 = $234,268,000.00
=125000000+(PMT(5%,15,,-10000000)+5000000)/0.05
= $234,268,457.52
```

Canal

```
CC = $200,000,000 + [$5,000,000(A|F 5%,10)
+ $1,000,000]/0.05 = $227,950,000.00
= 20000000+(PMT(5%,10,,-5000000)+1000000)/.05
CC = $227,950,457.50
```

Pit Stop #5— Open Road Ahead!

- 1. True or False: Present worth analysis is the most popular *DCF* measure of economic worth.
- 2. True or False: Unless non-monetary considerations dictate otherwise, choose the mutually exclusive investment alternative that has the greatest present worth, regardless of the lives of the alternatives.
- 3. True or False: When using present worth analysis to evaluate the economic viability of mutually exclusive alternatives, use a common period of time in the comparison.
- 4. True or False: If PW > 0 and MARR = 20%, then DPBP < 5 years.
- 5. True or False: *DPBP* > *PBP*.
- 6. True or False: If CW > 0, then PW > 0.
- 7. True or False: If PW(A) > PW(B), then CW(A) > CW(B), DPBP(A) < DPBP(B), and PBP(A) < PBP(B).
- 8. True or False: PW, FW, AW, CW, and B/C are ranking methods; therefore, the alternative having the greatest PW, FW, AW, CW, or B/C should be recommended.
- 9. True or False: Either ranking or incremental analysis can be used with all four "worth" methods (*PW*, *FW*, *AW*, and *CW*).
- 10. True or False: The "do nothing" alternative always has negligible incremental costs and revenues.

Pit Stop #5— Open Road Ahead!

- 1. True or False: Present worth analysis is the most popular *DCF* measure of economic worth. TRUE
- 2. True or False: Unless non-monetary considerations dictate otherwise, choose the mutually exclusive investment alternative that has the greatest present worth, regardless of the lives of the alternatives. FALSE
- 3. True or False: When using present worth analysis to evaluate the economic viability of mutually exclusive alternatives, use a common period of time in the comparison. TRUE
- 4. True or False: If PW > 0 and MARR = 20%, then DPBP < 5 years. FALSE
- 5. True or False: *DPBP* > *PBP*. TRUE
- 6. True or False: If CW > 0, then PW > 0. TRUE
- 7. True or False: If PW(A) > PW(B), then CW(A) > CW(B), DPBP(A) < DPBP(B), and PBP(A) < PBP(B). FALSE
- 8. True or False: PW, FW, AW, CW, and B/C are ranking methods; therefore, the alternative having the greatest PW, FW, AW, CW, or B/C should be recommended. FALSE
- 9. True or False: Either ranking or incremental analysis can be used with all four "worth" methods (*PW*, *FW*, *AW*, and *CW*). TRUE
- 10. True or False: The "do nothing" alternative always has negligible incremental costs and revenues. FALSE