Chapter 3

Borrowing, Lending, and Investing

Section 3-7: Equivalence and Indifference .

Section 3-9: Variable Interest Rates

Equivalence and Indifference .
Section 3-7

Equivalence

Two cash flow streams are said to be equivalent at k% interest if and only if their present worth are equal at k% interest.

Example 3.21

What uniform series over periods [1,8] is equivalent at 15% to the following cash flow profile?

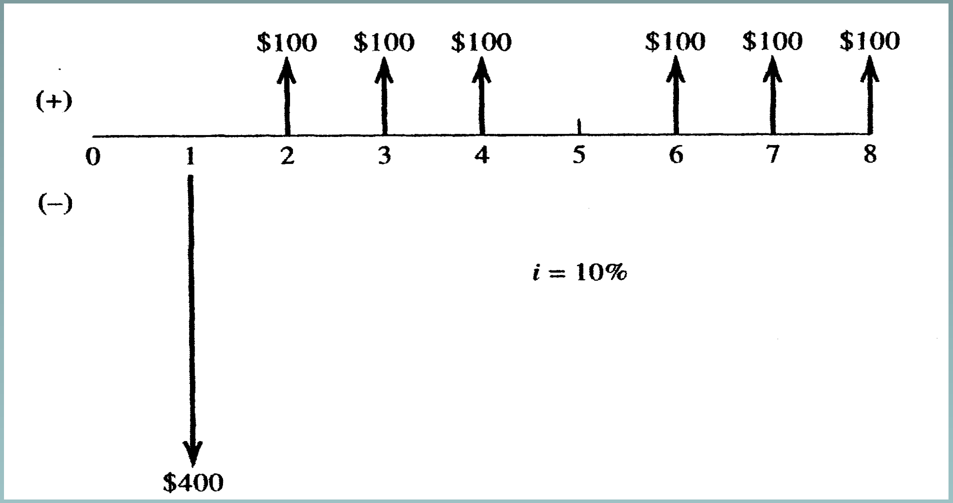
End of Period	Cash Flow
1	$100
3	$200
4	$100
5	$300

Solution:

[100(F|P15%,7)+200(F|P15%,5)+100(F|P15%,4) +300(F|P 15%,3)](A|F 15%,8) = $94.86

Example 3.22

What single sum at t=6 is equivalent at 10% to the following cash flow profile



Solution:

[-400+100(P|A 10%,3)](F|P 10%,5) +
100(P|A 10%,3)(F|P 10%,1) = $29.85

Alternative Solution:

F = [$100(F|A 10%,7)-$400(F|P 10%,7)-$100(F|P 10%,3)](P|F 10%,2)

F = [$100(9.48717)-$400(1.94872)-$100(1.33100)](0.82645)

F = $29.86

Example 3.23

What uniform series over [1,5] is equivalent to the following cash flow profile if i = 8%?

End of Period	Cash Flow
1	$0
2	$500
3	$400
4	$300
5	$200
6	$100
7	$0

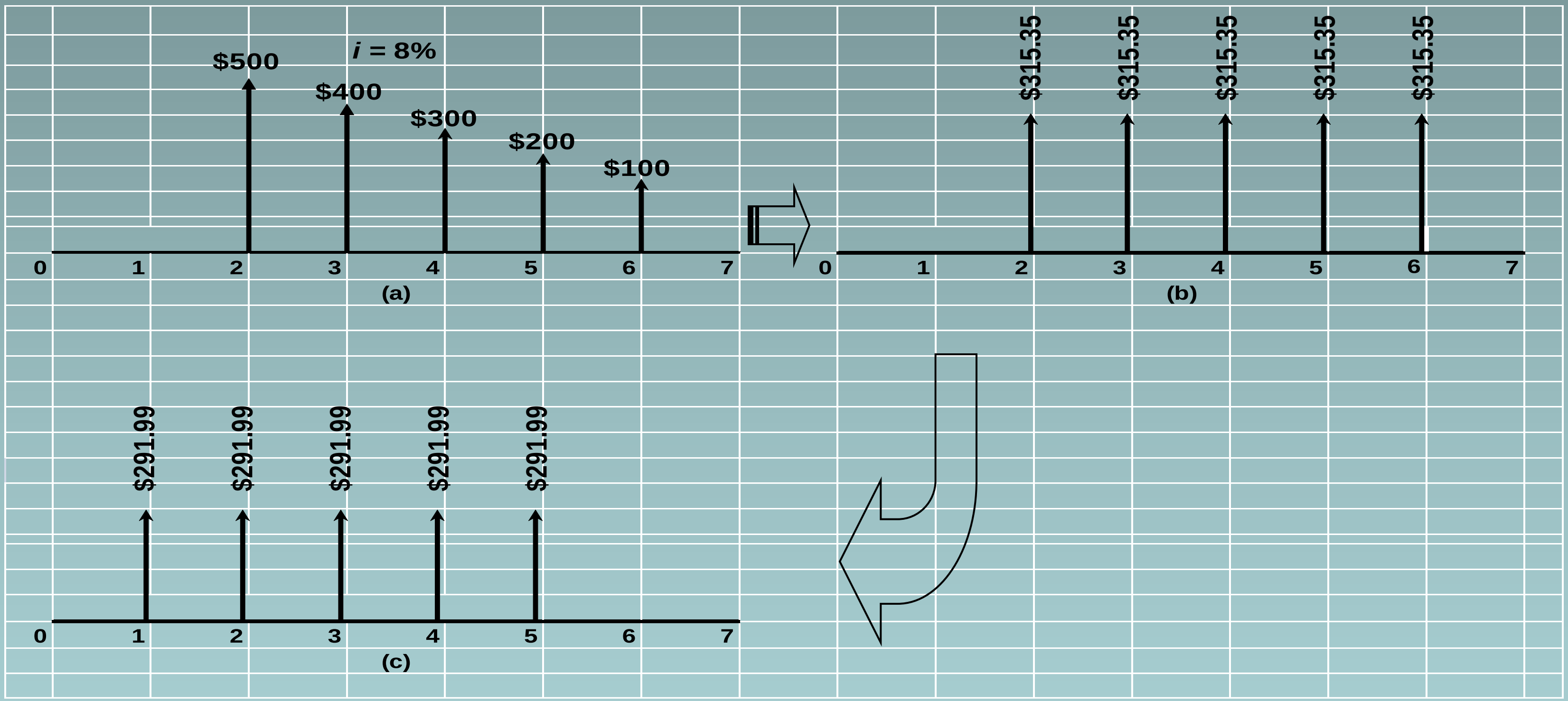
Solution:

The uniform series equivalent over [2,6] is

A = $500 - $100(A|G 8%,5) = $500 - $100(1.84647) = $315.35

The uniform series equivalent over [1,5] is

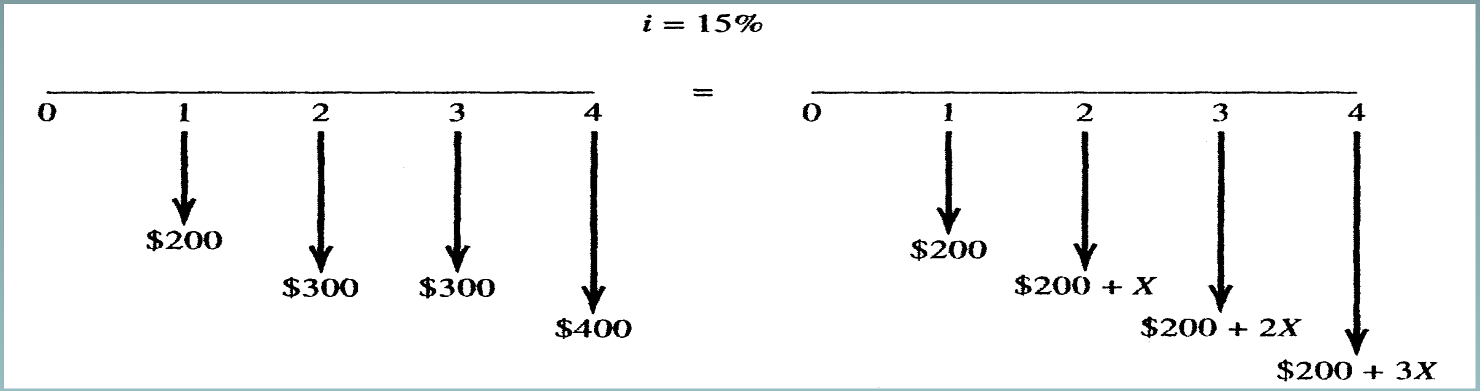
A = $315.35(P|F8%,1) = $315.35(0.92593) = $291.99



$500
$400
$300
$200
$100
i = 8%
0 1 2 3 4 5 6 7
(a)
$315.35
$315.35
$315.35
$315.35
$315.35
0 1 2 3 4 5 6 7
(b)
$291.99
$291.99
$291.99
$291.99
$291.99
0 1 2 3 4 5 6 7
(c)

Example 3.24

Determine the value of X that makes the two CFDs equivalent.



FW(LHS) = \$200(F|A 15%,4) + \$100(F|A 15%,3) + \$100

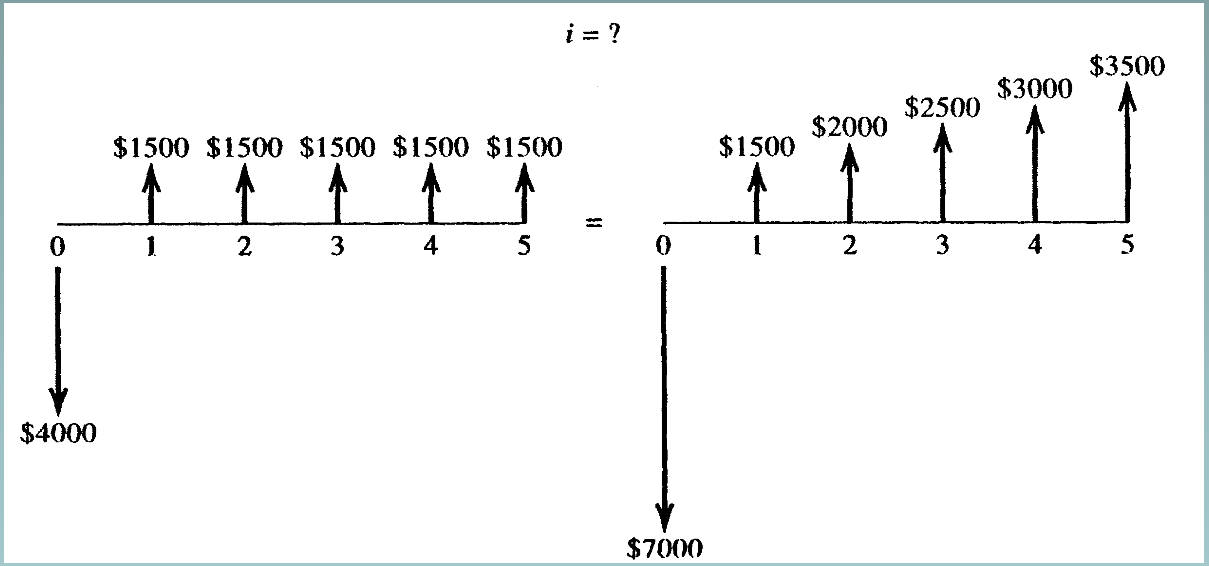
FW(RHS) = [\$200 + X(A|G 15%,4)](F|A 15%,4)

Equating the two and eliminating the common term of \$200(F|A 15%,4),
\$100(3.47250) + \$100 = X(1.32626)(4.99338)

Solving for X give a value of \$67.53.

Example 3.25

For what interest rate are the two cash flow diagrams equivalent?



Solution:

-$4000(A|P i%,5) + $1500 =

-$7000(A|P i%,5) + $1500 + $500(A|G i%,5)

i ≈ 13.8641% (by interpolation)

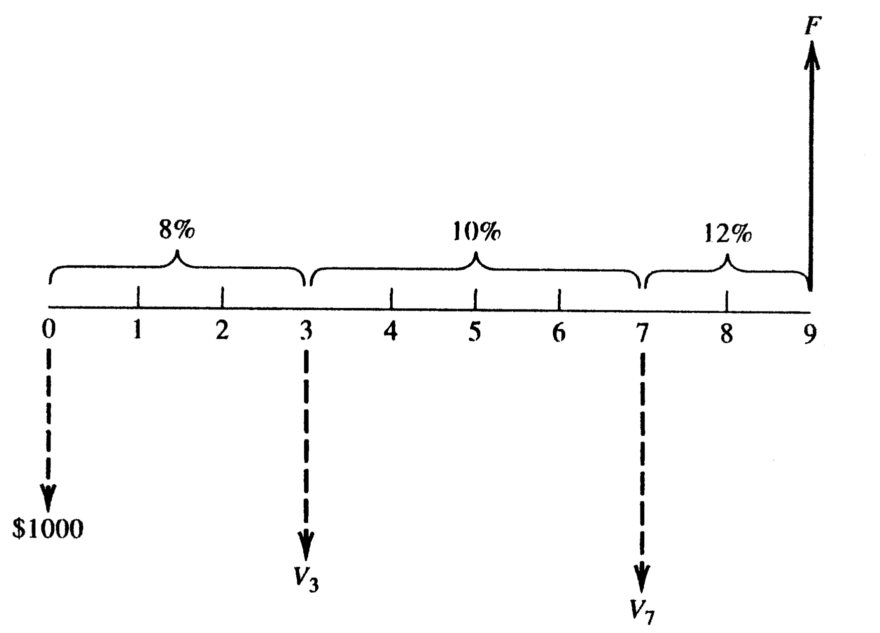
Section 3-9
Variable Interest Rates

Consider the case in which different interest rates apply for different time periods. Let A_t denote the magnitude of the cash flow at the end of time period t, t = 1, …, n. Let i_s denote the interest rate during time period s, s = 1, …, t. The present worth of $\{A_t\}$ is given by

$$P = \sum_{t=1}^{n} A_t \prod_{s=1}^{t} (1 + i_s)^{-1}$$

Example 3.30

You deposit $1000 in a fund paying 8% annual interest; after 3 years the fund increases its interest rate to 10%; after 4 years of paying 10% interest the fund begins paying 12%. How much will be in the fund 9 years after the initial deposit?



Solution:

let V_t = value of fund at time t

V_3 = \$1000.00(F|P 8%,3) = \$1259.71

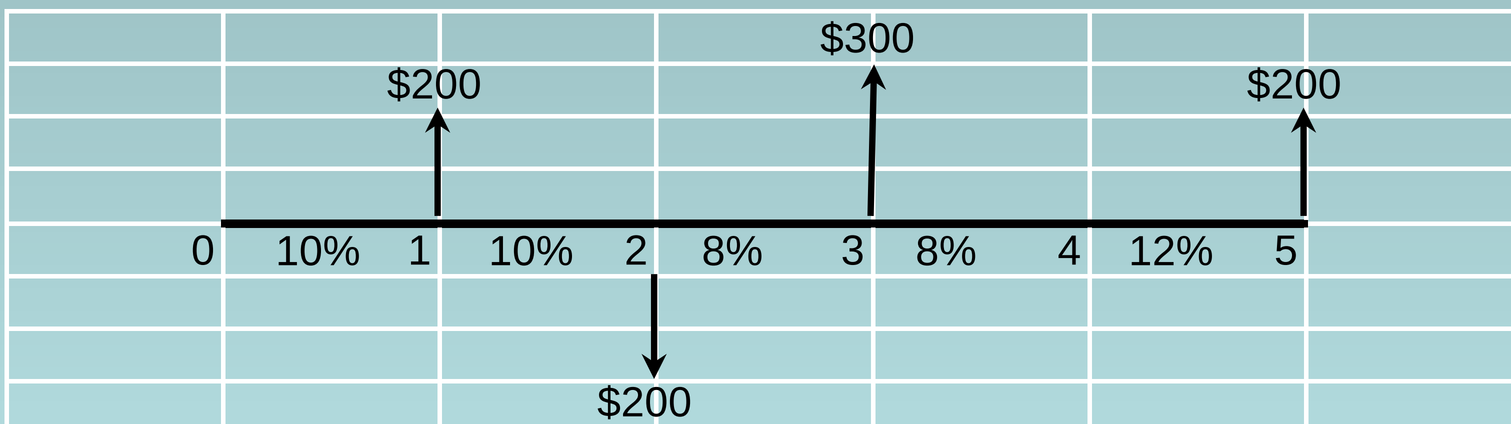
V_7 = \$1259.71(F|P 10%,4) = \$1844.34

V_9 = \$1844.34(F|P 12%,2) = \$2313.54

V_9=FVSCHEDULE(1000,{8%,8%,8%,10%,10%,10%,10%,12%,12%}) = \$2,313.55

Example 3.31

Consider a cash flow profile in which $200 is received at t=1, spent at t=2, and received at t=5, and $300 is received at t=3. Suppose the interest rate is 10% the first 2 periods, 8% the next two periods, and is 12% the 5th period. What are the equivalent *present worth*, *future worth*, and *uniform series* for the cash flow profile? [note: t denotes end of period t]



Solution:

P = \$200(P|F 10%,1) - \$200(P|F 10%,2) +

\$300(P|F 8%,1)(P|F 10%,2) +

\$200(P|F 12%,1)(P|F 8%,2)(P|F 10%,2)

P = \$372.63

F = \$200 + \$300(F|P 8%,1)(F|P 12%,1) –

\$200(F|P 8%,2)(F|P 12%,1) +

\$200(F|P 10%,1)(F|P 8%,2)(F|P 12%,1)

F = \$589.01

F =FVSCHEDULE(200,{0.1,0.08,0.08,0.12})

-FVSCHEDULE(200,{0.08,0.08,0.12})

+FVSCHEDULE(300,{0.08,0.12})+200

F = \$589.01

***To solve for the uniform series equivalent*, notice**

$$F = A[1+(F|P\ 12\%,1)+(F|P\ 8\%,1)(F|P\ 12\%,1)+ (F|P\ 8\%,2)(F|P\ 12\%,1)+ (F|P\ 10\%,1)(F|P\ 8\%,2)(F|P\ 12\%,1)]$$

$$= A[1+1.12+1.08(1.12)+1.1664(1.12) +1.1(1.08)(1.12)] = \$589.01$$

$$\$589.01 = 6.073A$$

$$A = \$589.01/6.073 = \$96.99$$