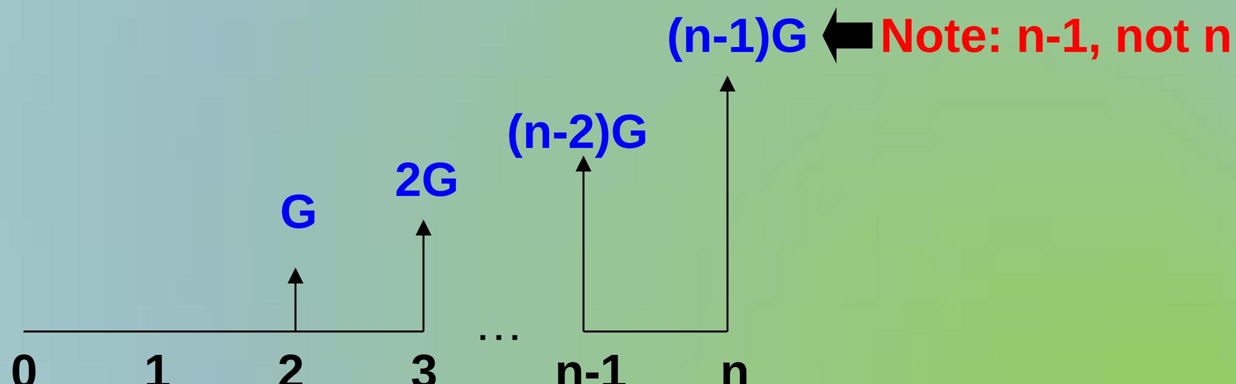
# ***Gradient Series***

# Gradient Series

$$A_t = \begin{cases} 0 & t = 1 \\ A_{t-1} + G & t = 2, \ldots, n \end{cases}$$

or

$$A_t = (t-1)G \qquad t = 1, \ldots, n$$



# Converting Gradient Series

## *Converting gradient series to present worth*

$$\mathbf{P = G}\left[\frac{1-(1+ni)(1+i)^{-n}}{i^2}\right] \quad \textbf{(2.35)}$$

$$\mathbf{P = G}\left[\frac{(P \mid A\ i\%,\ n) - n(P \mid F\ i\%,\ n)}{i}\right] \quad \textbf{(2.36)}$$

**P = G(P|G i%, n)** **(2.37)**

# ***Converting gradient series to annual worth***

$$\mathbf{A = G}\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

$$\mathbf{A = G}\left[\frac{1 - n(A \mid F\ i\%,\ n)}{i}\right]$$

**A = G(A|G i%, n)** **(2.38)**

## Converting gradient series to future worth

$$\mathbf{F = G} \left[ \frac{(1+i)^n - (1+ni)}{i^2} \right] \qquad \textbf{(2.39)}$$

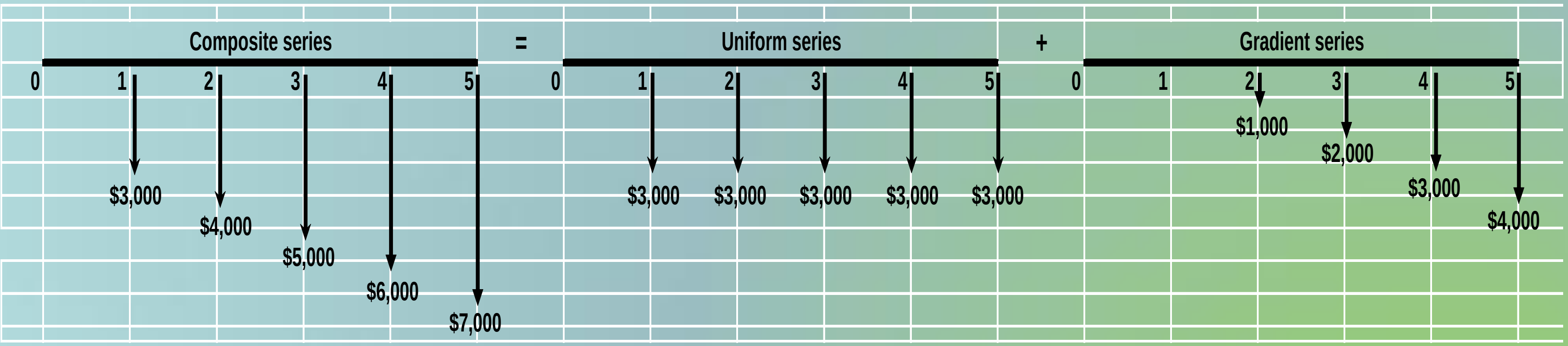
$$\mathbf{F = G} \left[ \frac{(F \mid A\ i\%,\ n) - n}{i} \right]$$

**F = G(F|G i%, n)**

***(not provided in the tables)***

# Example 2.28

**Maintenance costs for a particular production machine increase by $1,000/year over the 5-yr life of the machine. The initial maintenance cost is $3,000. Using an interest rate of 8% compounded annually, determine the present worth equivalent for the maintenance costs.**



**P = $3,000(P|A 8%,5) + $1,000(P|G 8%,5)**

**P = $3,000(3.99271) + $1,000(7.372.43) = $19,350.56**

**or**

**P = ($3,000 + $1,000(A|G 8%,5))(P|A 8%,5)**

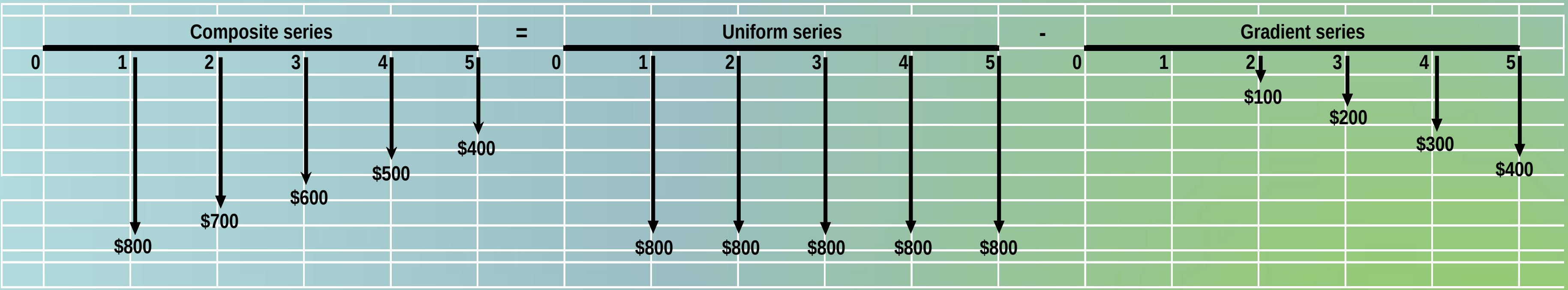
**P = ($3,000 + $1,000(1.846.47))(3.99271) = $19,350.55**

**or**

**P =1000*NPV(8%,3,4,5,6,7) = $19,350.56**

# Example 2.29

**Amanda Dearman made 5 annual deposits into a fund that paid 8% compound annual interest. Her first deposit was $800; each successive deposit was $100 less than the previous deposit. How much was in the fund immediately after the 5th deposit?**



**A = \$800 - \$100(A|G 8%,5)**

**= \$800 - \$100(1.84647) = \$615.35**

**F = \$615.35(F|A 8%,5)**

**= \$615.35(5.86660) = \$3,610.01**

**F =(FV(8%,5,,-NPV(8%,800,700,600,500,400))**

**F = \$3,610.03**

$$P = G\left[\frac{1 - (1 + ni)(1 + i)^{-n}}{i^2}\right]$$

gradient series, present worth factor

$= G(P|G\ i\%,n)$

$$A = G\left[\frac{(1 + i)^n - (1 + ni)}{i[(1 + i)^n - 1]}\right]$$

gradient-to-uniform series conversion factor

$= G(A|G\ i\%,n)$

$$F = G\left[\frac{(1 + i)^n - (1 + ni)}{i^2}\right]$$

gradient series, future worth factor

$= A(F|G\ i\%,n)$

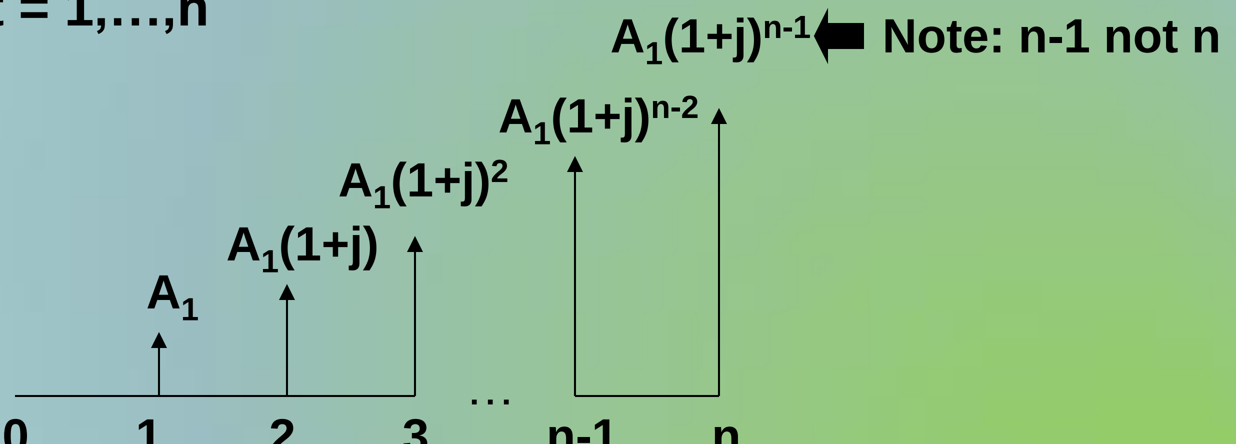
# ***Geometric Series***

# Geometric Series

$A_t = A_{t-1}(1+j) \quad t = 2,\ldots,n$

or

$A_t = A_1(1+j)^{t-1} \quad t = 1,\ldots,n$



# Converting Geometric Series – I

$$P = A_1\left[\frac{1-(1+j)^n(1+i)^{-n}}{i-j}\right] \qquad i \neq j$$ **(2.42)**

$$P = A_1\left[\frac{1-(F \mid P\ j\%, n)(P \mid F\ i\%, n)}{i-j}\right] \qquad i \neq j \quad j > 0$$ **(2.44)**

$$P = nA_1/(1+i) \qquad i = j$$ **(2.42)**

$$P = A_1(P \mid A_1\ i\%, j\%, n)$$ **(2.43)**

# Converting Geometric Series – II

$$F = A_1\left[\frac{(1+i)^n - (1+j)^n}{i-j}\right] \qquad i \neq j$$ **(2.45)**

$$F = A_1\left[\frac{(F \mid P\ i\%,\ n) - (F \mid P\ j\%,\ n)}{i-j}\right] \qquad i \neq j \quad j > 0$$

$F = nA_1(1+i)^{n\text{-}1}$ $\qquad$ $i = j$

$F = A_1(F|A_1\ i\%,j\%,n)$

**Note:** $(F|A_1\ i\%,j\%,n) = (F|A_1\ j\%,i\%,n)$

**Notice the symmetry**

# Example 2.30

A firm is considering purchasing a new machine. It will have maintenance costs that increase 8% per year. An initial maintenance cost of $1,000 is expected. Using a 10% interest rate, what present worth cost is equivalent to the cash flows for maintenance of the machine over its 15-year expected life?

$A_1$ = $1,000, i = 10%, j = 8%, n = 15, P = ?

P = $1,000(P|$A_1$10%,8%,15)

= $1,000(12.03040) = $12,030.40

P =1000*NPV(10%,1,1.08,1.08^2,1.08^3,1.08^4,1.08^5,
1.08^6,1.08^7,1.08^8,1.08^9,1.08^10,1.08^11,1.08^12,
1.08^13,1.08^14)

P = $12,030.40

# Example 2.31

**Mattie Bookhout deposits her annual bonus in a savings account that pays 8% compound annual interest. Her annual bonus is expected to increase by 10% each year. If her initial deposit is $500, how much will be in her account immediately after her 10th deposit?**

$A_1$ **= $500, i = 8%, j = 10%, n = 10, F = ?**

**F = $500(F|**$A_1$ **8%,10%,10) = $500(21.74087)**

**F = $10,870.44**

# Example 2.32

**Julian Stewart invested $100,000 in a limited partnership in a natural gas drilling project. His net revenue the 1st year was $25,000. Each year, thereafter, his revenue decreased 10%/yr. Based on a 12% TVOM, what is the present worth of his investment over a 20-year period?**

**$A_1$ = $25,000, i = 12%, j = -10%, n = 20, P = ?**

**P = -$100,000 + $25,000(P|$A_1$ 12%,-10%,20)**

**P = -$100,000 + $25,000[1 – $(0.90)^{20}$$(1.12^{)-20}$]/(0.12 + 0.10)**

**P = $13,636.36**

| End of Year (n) | Cash Flow (CF) | |
|---|---|---|
| 0 | -$100,000 | |
| 1 | $25,000 | |
| 2 | $22,500 | |
| 3 | $20,250 | |
| 4 | $18,225 | |
| 5 | $16,403 | |
| 6 | $14,762 | |
| 7 | $13,286 | |
| 8 | $11,957 | |
| 9 | $10,762 | |
| 10 | $9,686 | |
| 11 | $8,717 | |
| 12 | $7,845 | =C14*0.9 |
| 13 | $7,061 | |
| 14 | $6,355 | |
| 15 | $5,719 | |
| 16 | $5,147 | |
| 17 | $4,633 | |
| 18 | $4,169 | |
| 19 | $3,752 | |
| 20 | $3,377 | |
| P = | $12,204.15 | =NPV(12%,C4:C23)+C3 |

$$P = A_1\left[\frac{1 - (1 + j)^n(1 + i)^{-n}}{i - j}\right]$$

geometric series, present worth factor

$i \neq j$

$$P = nA_1/(1 + i)$$

$i = j$

$$P = A_1(P|A_1\ i\%,j\%,n)$$

$$F = A_1\left[\frac{(1 + i)^n - (1 + j)^n}{i - j}\right]$$

geometric series, future worth factor

$i \neq j$

$$F = nA_1(1 + i)^{n-1}$$

$i = j$

$$F = A_1(F|A_1\ i\%,j\%,n)$$