Chapter 9:

part I: One-sample estimation problems:

Population: Normal $X \sim (\mu, \sigma)$

μ, σ unknown parameters

* Statistical inferences:

1- Estimation of parameters:

$\longrightarrow$ point estimate

$\longrightarrow$ interval estimation (Confidence interval) C.I.

2- Tests of Hypotheses.

point estimation: single value of a statistic

$\hat{\theta}$ of $\hat{\Theta}$

interval estimation: $\theta \in (\theta_L, \theta_U)$ $(1-\alpha)100\%$ C.I.

$$\boxed{P(\theta_L < \theta < \theta_U) = 1-\alpha} \quad , 0<\alpha<1$$

* Point estimate of the mean μ: $\bar{X}$ sample mean

* Interval estimation of the mean μ:

$$\boxed{\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})}$$

i- σ^2 is known

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$

ii- σ^2 is unknown

$$T = \frac{\bar{X} - \mu}{S / \sqrt{n}} \sim t(n-1)$$

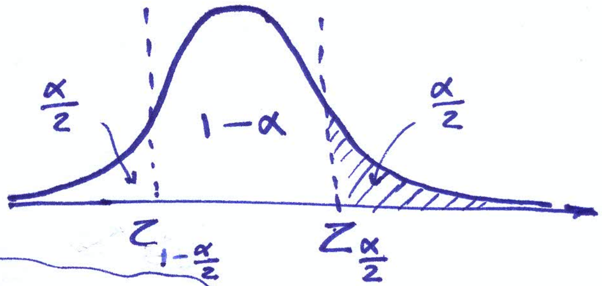
i- $(1-\alpha)100\%$ C.I. for μ (σ^2 known)

$$\left(\bar{X} - Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} \,,\, \bar{X} + Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right)$$

$Z_{\frac{\alpha}{2}}$ ... Z-value leaving area $\frac{\alpha}{2}$ to the right.

$P(Z > Z_{\frac{\alpha}{2}}) = \frac{\alpha}{2}$

$P(Z < Z_{\frac{\alpha}{2}}) = 1 - \frac{\alpha}{2}$



$Z_{1-\frac{\alpha}{2}} = -Z_{\frac{\alpha}{2}}$

Max. error of estimation

$$\boxed{e = Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}}$$ with $(1-\alpha)100\%$ Confidence level

then, no. of observations (sample size) to be $(1-\alpha)100\%$ Confident that error will not exceed e

$$\boxed{n = \left(Z_{\frac{\alpha}{2}}\frac{\sigma}{e}\right)^2}$$ rounded up

ii- $(1-\alpha)100\%$ C.I. for μ (σ^2 unknown)

$$\left(\bar{X} - t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}} \,,\, \bar{X} + t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}}\right)$$

$t_{\frac{\alpha}{2}}$ ... t-value with $\nu = n-1$, $P(T > t_{\frac{\alpha}{2}}) = \frac{\alpha}{2}$

from table

note: $s.e.(\bar{X}) = \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

standard error

Important Ex 11-1 CQ2 مكرر في الامتحانات

Ch. 9 Part II

* Estimating the difference betn. two means:

$$\mu_1 - \mu_2$$

Recall $\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2 , \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

* Point estimate for $\mu_1 - \mu_2$ is: $\boxed{\bar{X}_1 - \bar{X}_2}$

* Confidence interval for $\mu_1 - \mu_2$

i- σ_1^2, σ_2^2 are known:

$(1-\alpha)100\%$ C.I. for $\mu_1 - \mu_2$:

$$\left((\bar{X}_1 - \bar{X}_2) - Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} , (\bar{X}_1 - \bar{X}_2) + Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$$

ii- σ_1^2, σ_2^2 are unknown but $\sigma_1^2 = \sigma_2^2 = \sigma^2$

$$S_p^2 = \frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}$$ (pooled estimate for σ^2)

$(1-\alpha)100\%$ C.I. for $\mu_1 - \mu_2$:

$$\left((\bar{X}_1 - \bar{X}_2) - t_{\frac{\alpha}{2}} S_p \sqrt{1/n_1 + 1/n_2} , (\bar{X}_1 - \bar{X}_2) + t_{\frac{\alpha}{2}} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

t- distribution with $\nu = n_1 + n_2 - 2$ degrees of freedom

* Estimating of a proportion p

Recall: sample proportion $\hat{p} = \frac{x}{n}$

$$\hat{p} \sim N\left(p, \sqrt{\frac{pq}{n}}\right) \quad \text{for large } n$$

$$Z = \frac{\hat{p} - p}{\sqrt{pq/n}} \sim N(0,1)$$

* Point estimate for p : $\boxed{\hat{p}}$

* $(1-\alpha)100\%$ C.I. for p :

$$\left(\hat{p} - Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} \, , \, \hat{p} + Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$$

* Estimating the difference betn. two proportions

$\boxed{P_1 - P_2}$ P_1 ... 1st population proportion
P_2 ... 2nd ~ ~

Recall $X_1 \sim \text{binomial}(n_1, P_1)$ & $X_2 \sim \text{binomial}(n_2, P_2)$

$\hat{P}_1 = \frac{X_1}{n_1}$ & $\hat{P}_2 = \frac{X_2}{n_2}$

$$\hat{P}_1 - \hat{P}_2 \sim N\left(P_1 - P_2 \, , \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}\right) \quad \text{for large } n_1, n_2$$

$$Z = \frac{(\hat{P}_1 - \hat{P}_2) - (P_1 - P_2)}{\sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}} \sim N(0,1)$$

* Point estimate for $P_1 - P_2$: $\boxed{\hat{P}_1 - \hat{P}_2 = \frac{X_1}{n_1} - \frac{X_2}{n_2}}$

* $(1-\alpha)100\%$ C.I. for $P_1 - P_2$:

$$\left((\hat{P}_1 - \hat{P}_2) - Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{P}_1\hat{q}_1}{n_1} + \frac{\hat{P}_2\hat{q}_2}{n_2}} \, , \, (\hat{P}_1 - \hat{P}_2) + Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{P}_1\hat{q}_1}{n_1} + \frac{\hat{P}_2\hat{q}_2}{n_2}}\right)$$

Ch. 10 One- and Two-Sample Test of Hypotheses:

Consider a population with unknown parameter θ. We are interested in (confirming or denying) some conjectures about θ.

A statistical hypothesis is a conjecture about the population.

Null hypothesis: $H_0: \theta = \theta_0$

Alternative hypothesis: $H_1: \begin{cases} \theta \neq \theta_0 \\ \theta > \theta_0 \\ \theta < \theta_0 \end{cases}$

	H_0 true	H_0 false
Accept H_0	correct	Type II error
Reject H_0	Type I error	correct

significance level

$P(\text{Type I error}) = P(\text{Rejecting } H_0 \mid H_0 \text{ true}) = \alpha$

$P(\text{Type II error}) = P(\text{Accepting } H_0 \mid H_0 \text{ false}) = \beta$

One-sided Alternative test

$\rightarrow$ $H_0: \theta = \theta_0$, $H_1: \theta > \theta_0$ or $\rightarrow$ $H_0: \theta = \theta_0$, $H_1: \theta < \theta_0$

Two-sided alternative test

$H_0: \theta = \theta_0$

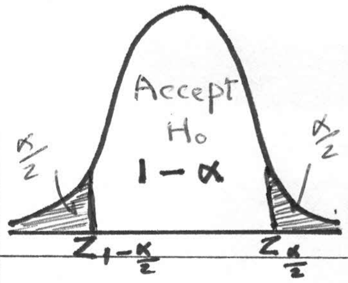
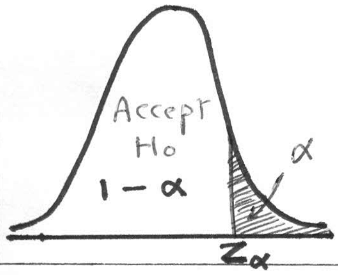
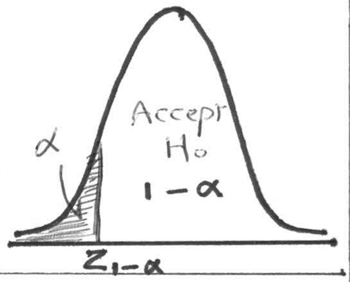
$H_1: \theta \neq \theta_0$

Steps:

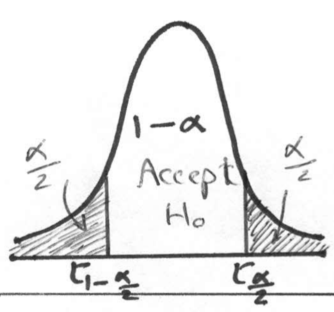
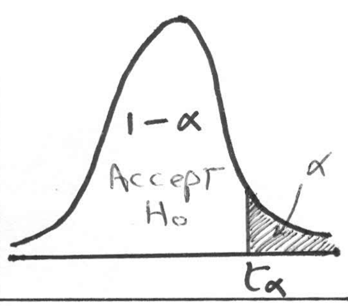
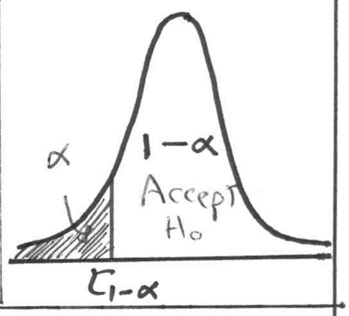
1. Test statistic
2. level of significance α
3. Determine R.R. rejection region & A.R. acceptance region of H_0
4. Accept or Reject H_0

* Single sample: Tests concerning single mean μ

i- known σ^2

Hypotheses:	$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$	$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_1: \mu < \mu_0$
Test statistic	$Z = \dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} \sim N(0,1)$		
A.R & R.R. of H_0			
Decision:	Reject H_0 with significance level α if:		
	$Z > Z_{\frac{\alpha}{2}}$ or $Z < -Z_{\frac{\alpha}{2}}$ two-sided test	$Z > Z_\alpha$ one-sided test	$Z < -Z_\alpha$ one-sided test

ii- unknown σ^2

Hypotheses:	$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$	$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_1: \mu < \mu_0$
Test Statistic	$T = \dfrac{\bar{X} - \mu_0}{S / \sqrt{n}} \sim t(n-1)$		
A.R & R.R. of H_0			
Decision:	Reject H_0 with significance level α if:		
	$T > t_{\frac{\alpha}{2}}$ or $T < -t_{\frac{\alpha}{2}}$ two-sided test	$T > t_\alpha$ one-sided test	$T < -t_\alpha$ one-sided test

Reject H_0 : (accept H_1) claim is not correct

Accept H_0 : (reject H_1) claim is correct

Two-samples: Tests on Two means:

i- σ_1^2 and σ_2^2 are known:

Hypotheses: $H_0: \mu_1 - \mu_2 = d$ $\quad H_0: \mu_1 - \mu_2 = d$ $\quad H_0: \mu_1 - \mu_2 = d$

$H_1: \mu_1 - \mu_2 \neq d$ $\quad H_1: \mu_1 - \mu_2 > d$ $\quad H_1: \mu_1 - \mu_2 < d$

Test statistic: $$Z = \frac{(\bar{X}_1 - \bar{X}_2) - d}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

Decision: Reject H_0 at significance level α if:

$Z \in$ R.R. of H_0

ii- σ_1^2 and σ_2^2 are unknown, but $\sigma_1^2 = \sigma_2^2 = \sigma^2$

Hypotheses: $H_0: \mu_1 - \mu_2 = d$ $\quad H_0: \mu_1 - \mu_2 = d$ $\quad H_0: \mu_1 - \mu_2 = d$

$H_1: \mu_1 - \mu_2 \neq d$ $\quad H_1: \mu_1 - \mu_2 > d$ $\quad H_1: \mu_1 - \mu_2 < d$

Test statistic: $$T = \frac{(\bar{X}_1 - \bar{X}_2) - d}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1 + n_2 - 2)$$

Decision: Reject H_0 at significance level α if:

$T \in$ R.R. of H_0

note: $H_0: \mu_1 = \mu_2 \iff H_0: \mu_1 - \mu_2 = 0$

One sample: Test on a single proportion:

Hypotheses: $H_0: p = p_0$ $\quad$ $H_0: p = p_0$ $\quad$ $H_0: p = p_0$

$H_1: p \neq p_0$ $\quad$ $H_1: p > p_0$ $\quad$ $H_1: p < p_0$

Test statistic:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{X - np_0}{\sqrt{np_0 q_0}} \sim N(0,1)$$

Decision: Reject H_0 at significance level α, if

$Z \in$ R.R. of H_0

Two samples: Test on two proportion:

Hypotheses: $H_0: p_1 - p_2 = 0$ $\quad$ $H_0: p_1 - p_2 = 0$ $\quad$ $H_0: p_1 - p_2 = 0$

$H_1: p_1 - p_2 \neq 0$ $\quad$ $H_1: p_1 - p_2 > 0$ $\quad$ $H_0: p_1 - p_2 < 0$

Test statistic:

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0,1)$$

Decision: Reject H_0 at significance level α, if:

$Z \in$ R.R. of H_0

note: under $H_0: p_1 = p_2 = p$, then

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2}, \quad \hat{q} = 1 - \hat{p}$$

or

$$\hat{p} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$$