Ch.9: Linear Momentum and Collisions

Physics 103: Classical Mechanics

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Version 1

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1. Linear Momentum and Its Conservation

2. Impulse and Momentum

3. Collisions in One Dimension

4. Two-Dimensional Collisions

5. Suggested Problems

1.1 Derivation of Momentum

- From Newton's third law, we have:

$$\vec{\boldsymbol{F}}_{12} = -\vec{\boldsymbol{F}}_{21}$$
$$\Rightarrow \vec{\boldsymbol{F}}_{12} + \vec{\boldsymbol{F}}_{21} = 0$$
$$m\vec{\boldsymbol{a}}_{12} + m\vec{\boldsymbol{a}}_{21} = 0$$
$$m\frac{d\vec{\boldsymbol{v}}_1}{dt} + m\frac{d\vec{\boldsymbol{v}}_2}{dt} = 0$$
$$\Longrightarrow \frac{d}{dt}(m\vec{\boldsymbol{v}}_1 + m\vec{\boldsymbol{v}}_2) = 0$$

- Therefore, the quantity

$$m\vec{\boldsymbol{v}}_1 + m\vec{\boldsymbol{v}}_2 = \text{Constant.} \tag{1}$$

- This quantity is called the **total linear momentum** of the system of two particles.
- The fact that the total linear momentum is constant is known as the **law of conservation of linear momentum**.
- Therefore, the total linear momentum before an interaction **is equal** to the total linear momentum after the interaction.

1.2 Linear Momentum of a Particle

- The linear momentum (or simply momentum) of a particle of mass m moving with velocity $\vec{\boldsymbol{v}}$ is defined as the product of the mass and velocity:

$$\boxed{\vec{\boldsymbol{p}} \equiv m\vec{\boldsymbol{v}}} \tag{2}$$

- where $\vec{\boldsymbol{p}}$ is the linear momentum **vector** of a particle and has the same direction as the velocity vector $\vec{\boldsymbol{v}}$. Therefore, in three dimensions:

$$p_x = mv_x, \quad p_y = mv_y, \quad p_z = mv_z$$

- The SI unit of momentum is **(kg m/s)**.

1.3 The Momentum and Newton's Second Law

- From the definition of momentum, we can write Newton's second law as:

$$\sum \vec{\boldsymbol{F}} = m\frac{d\vec{\boldsymbol{v}}}{dt} = \frac{d\vec{\boldsymbol{p}}}{dt} \tag{3}$$

- This shows that the time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.

1.4 The Conservation of Linear Momentum

- We saw earlier in equation 1 that for a system of two particles, the total linear momentum is conserved. This result can be generalized to a system of n particles,

$$\boxed{\begin{gathered}\vec{\boldsymbol{P}}_i = \vec{\boldsymbol{P}}_f \\ \vec{\boldsymbol{p}}_{1i} + \vec{\boldsymbol{p}}_{2i} + \ldots = \vec{\boldsymbol{p}}_{1f} + \vec{\boldsymbol{p}}_{2f} + \ldots\end{gathered}} \tag{4}$$

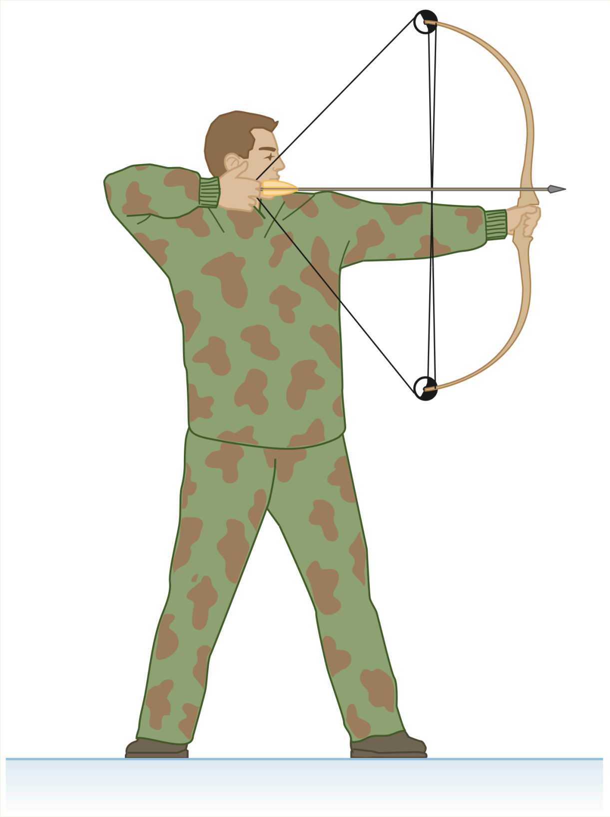
- This implies that momentum is conserved for every dimension separately:

$$\boxed{p_{xi} = p_{xf}, \quad p_{yi} = p_{yf}, \quad p_{zi} = p_{zf}} \tag{5}$$

Example 1.1

A 60 kg archer stands at rest on frictionless ice and fires a 0.5 kg arrow horizontally at 50 m/s.

With what velocity does the archer move across the ice after firing the arrow?



1.4 The Conservation of Linear Momentum

Solution 1.1

- The horizontal momentum for the two objects is conserved as:

$$\vec{\boldsymbol{p}}_i = \vec{\boldsymbol{p}}_f$$

$$m_1\vec{\boldsymbol{v}}_{1i} + m_2\vec{\boldsymbol{v}}_{2i} = m_1\vec{\boldsymbol{v}}_{1f} + m_2\vec{\boldsymbol{v}}_{2f}$$

$$0 + 0 = m_1\vec{\boldsymbol{v}}_{1f} + m_2\vec{\boldsymbol{v}}_{2f}$$

$$\implies \vec{\boldsymbol{v}}_{1f} = -\left(\frac{m_2}{m_1}\right)\vec{\boldsymbol{v}}_{2f}$$

$$= -\left(\frac{0.5}{60}\right)(50\ \hat{\boldsymbol{\imath}}) = -0.42\ \text{m/s}\ \hat{\boldsymbol{\imath}}$$

1.4 The Conservation of Linear Momentum

What if the arrow were shot in a direction that makes an angle with the horizontal? How will this change the recoil velocity of the archer?

- Since the relevant momentum is only in the horizontal direction, we take the horizontal component of the arrow's velocity:

$$m_1\vec{\boldsymbol{v}}_{1f} + m_2\vec{\boldsymbol{v}}_{2f}\cos\theta = 0$$

$$\implies v_{x1f} = -\left(\frac{m_2}{m_1}\right)v_{x2f}\cos\theta$$

1.4 The Conservation of Linear Momentum

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Problem 1.1

A 3 kg particle has a velocity of $(3\hat{\imath} - 4\hat{\jmath})$ m/s.

(a) Find its x and y components of momentum.
(b) Find the magnitude and direction of its momentum.

Answer 1.1

- (a) The x and y components of momentum are given by:

$$p_x = mv_x = (3\text{ kg})(3\text{ m/s}) = 9\text{ kg m/s}$$
$$p_y = mv_y = (3\text{ kg})(-4\text{ m/s}) = -12\text{ kg m/s}$$

- (b) The magnitude of the momentum is:

$$|\vec{\boldsymbol{p}}| = \sqrt{p_x^2 + p_y^2} = \sqrt{(9\text{ kg m/s})^2 + (-12\text{ kg m/s})^2} = 15\text{ kg m/s}$$

- The direction of the momentum is given by the angle θ:

$$\theta = \tan^{-1}\left(\frac{p_y}{p_x}\right) = 307^\circ$$

1. Linear Momentum and Its Conservation

2. Impulse and Momentum

3. Collisions in One Dimension

4. Two-Dimensional Collisions

5. Suggested Problems

2.1 Impulse

- From Newton's second law in equation 3, we have:

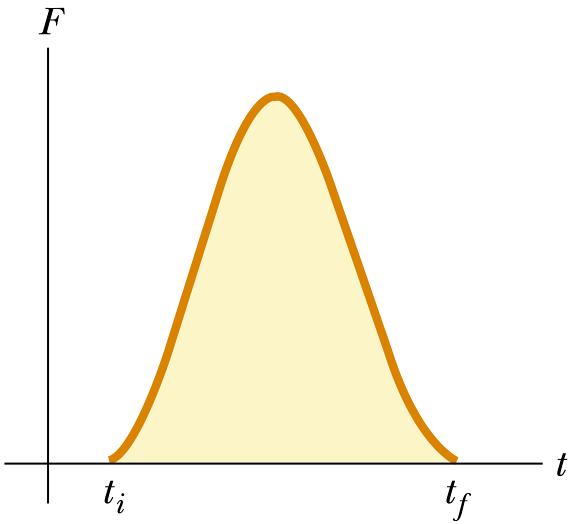
$$\vec{\boldsymbol{F}} = \frac{d\vec{\boldsymbol{p}}}{dt}$$

$$d\vec{\boldsymbol{p}} = \vec{\boldsymbol{F}}dt$$

- Integrating both sides from an initial time t_i to a final time t_f, we obtain:

$$\Delta\vec{\boldsymbol{p}} = \vec{\boldsymbol{p}}_f - \vec{\boldsymbol{p}}_i = \int_{t_i}^{t_f} \vec{\boldsymbol{F}}dt$$

- The quantity on the right side of the equation is called the **impulse** $\vec{\boldsymbol{I}}$ delivered to the particle by the force $\vec{\boldsymbol{F}}$ during the time interval $\Delta t = t_f - t_i$,



2.1 Impulse

- Therefore, impulse is defined as:

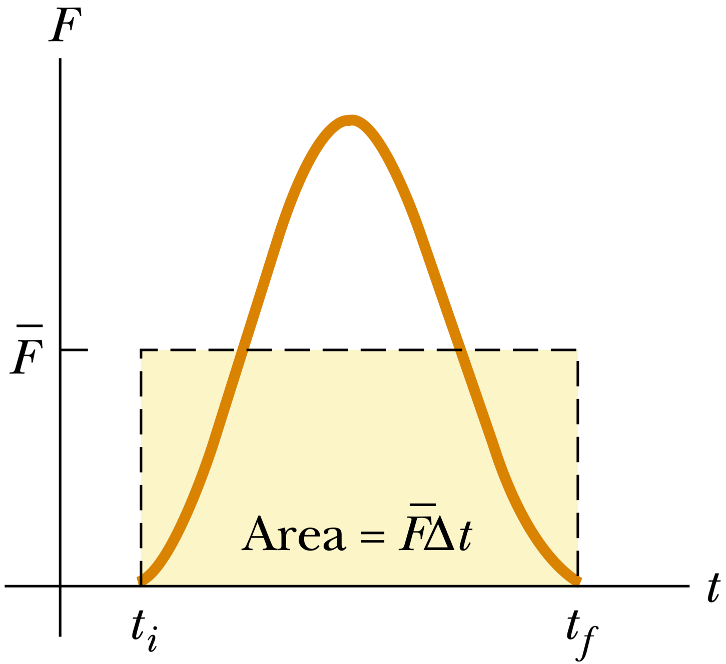
$$\boxed{\vec{I} \equiv \int_{t_i}^{t_f} \vec{F} dt = \Delta\vec{p}} \qquad (6)$$

- where $\Delta\vec{p}$ is the change in momentum of the particle during the time interval Δt.
- Impulse has the same unit as momentum, which is **(kg m/s)**.
- Impulse is a vector quantity and has the same direction as the force $\vec{F}$ and the change in momentum $\Delta\vec{p}$.
- Examples of impulse is the force experienced by a car during a collision, or the force exerted by a bat on a baseball during a hit.

2.2 Time Average Force

- The average force $\vec{\bar{F}}$ exerted on a particle during the time interval Δt is defined as:

$$\boxed{\vec{\bar{F}} \equiv \frac{1}{\Delta t}\int_{t_i}^{t_f} \vec{F} dt} \quad (7)$$



- Therefore, from the definition of impulse in equation 6, we have:

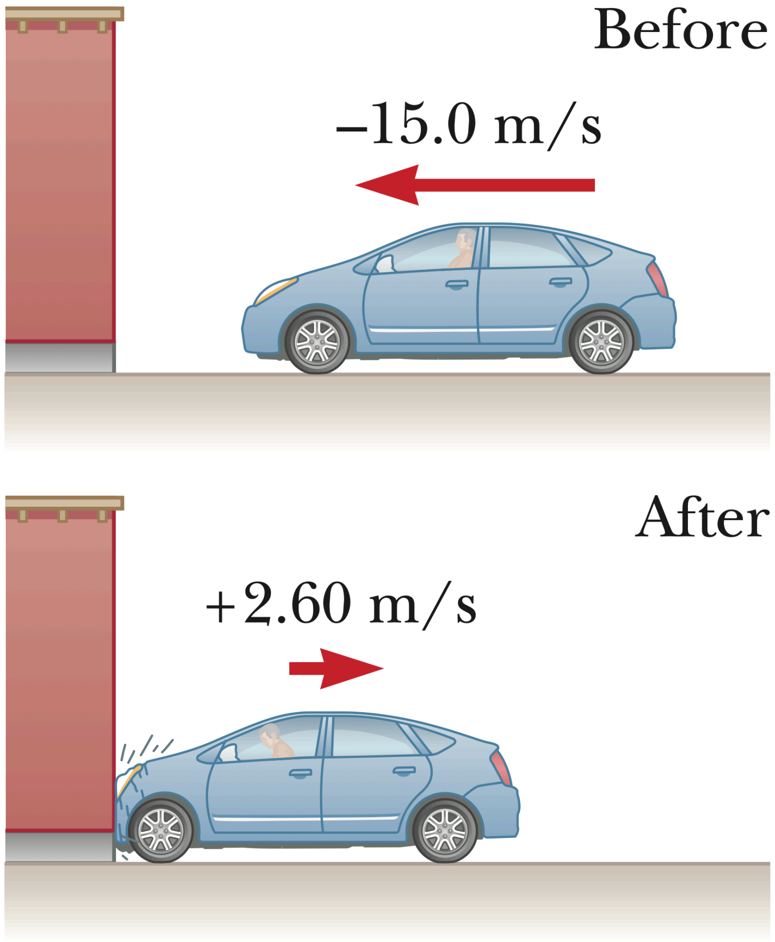
$$\boxed{\vec{I} \equiv \vec{\bar{F}}\Delta t} \quad (8)$$

2.3 Example

Example 2.2

In a particular crash test, a car of mass 1500 kg collides with a wall. The initial and final velocities of the car are $v_i = -15\hat{\imath}$ m/s and $v_f = 2.6\hat{\imath}$ m/s, respectively. If the collision lasts for 0.15 s,

find the impulse caused by the collision and the average force exerted on the car.



2.3 Example

Solution 2.2

- The impulse experienced by the car is given by:

$$\vec{\boldsymbol{I}} = \Delta\vec{\boldsymbol{p}} = \vec{\boldsymbol{p}}_f - \vec{\boldsymbol{p}}_i$$

$$= m\vec{\boldsymbol{v}}_f - m\vec{\boldsymbol{v}}_i$$

$$= (1500 \text{ kg})(2.6\hat{\boldsymbol{\imath}} \text{ m/s}) - (1500 \text{ kg})(-15\hat{\boldsymbol{\imath}} \text{ m/s})$$

$$= 2.64 \times 10^4 \text{ kg m/s } \hat{\boldsymbol{\imath}}$$

- The average force exerted on the car during the collision is given by:

$$\bar{\vec{\boldsymbol{F}}} = \frac{\vec{\boldsymbol{I}}}{\Delta t} = \frac{2.64 \times 10^4 \text{ kg m/s } \hat{\boldsymbol{\imath}}}{0.15 \text{ s}} = 1.76 \times 10^5 \text{ N } \hat{\boldsymbol{\imath}}$$

1. Linear Momentum and Its Conservation

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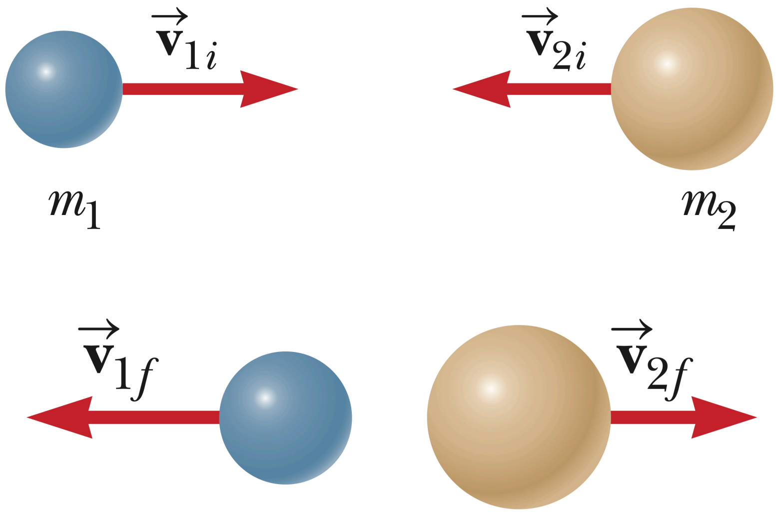
3.1 Definition and Types of Collisions

- A collision is an event in which two or more bodies exert forces on each other for a relatively short time.



- Collisions are classified into two main types:
  - **Elastic collisions**, and
  - **Inelastic collisions**.
- In both types of collisions, the total linear momentum is conserved.
- In an **inelastic collision**, the kinetic energy is **not** conserved because some of the energy is transformed into other forms of energy, such as heat or sound.

3.2 Perfectly Inelastic Collisions

- In a perfectly inelastic collision, the colliding objects stick together after the collision.
- Applying the conservation of linear momentum to a perfectly inelastic collision between two objects of masses m_1 and m_2 with initial velocities $\vec{v}_{1i}$ and $\vec{v}_{2i}$, we have:

$$\boxed{m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f} \tag{9}$$

- where $\vec{v}_f$ is the common velocity of the two objects after the collision:

$$\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2}$$

3.3 Elastic Collisions

- In an elastic collision, both momentum and kinetic energy are conserved.
- Applying the conservation of linear momentum to an elastic collision, we get:

$$\boxed{m_1\vec{\boldsymbol{v}}_{1i} + m_2\vec{\boldsymbol{v}}_{2i} = m_1\vec{\boldsymbol{v}}_{1f} + m_2\vec{\boldsymbol{v}}_{2f}} \tag{10}$$

- Applying the conservation of kinetic energy to the same collision, we have:

$$\boxed{\frac{1}{2}m_1\vec{\boldsymbol{v}}_{1i}^2 + \frac{1}{2}m_2\vec{\boldsymbol{v}}_{2i}^2 = \frac{1}{2}m_1\vec{\boldsymbol{v}}_{1f}^2 + \frac{1}{2}m_2\vec{\boldsymbol{v}}_{2f}^2} \tag{11}$$

3.3 Elastic Collisions

- Solving equations 10 and 11 simultaneously, we obtain the final velocities after an elastic collision:

$$\boxed{\begin{aligned} \vec{v}_{1f} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{2i} \\ \vec{v}_{2f} &= \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{2i} \end{aligned}} \tag{12}$$

3.4 Special Cases for Elastic Collisions

- If $m_1 = m_2$, then from equation 12:

$$\vec{\boldsymbol{v}}_{1f} = \vec{\boldsymbol{v}}_{2i}, \quad \vec{\boldsymbol{v}}_{2f} = \vec{\boldsymbol{v}}_{1i}$$

- This means that the two objects simply exchange their velocities.
- If $m_1 \gg m_2$, then from equation 12:

$$\vec{\boldsymbol{v}}_{1f} \approx \vec{\boldsymbol{v}}_{1i}, \quad \vec{\boldsymbol{v}}_{2f} \approx 2\vec{\boldsymbol{v}}_{1i}$$

- This means that the first object continues with nearly its original velocity, while the second object rebounds with approximately twice the initial velocity of the first object.

3.4 Special Cases for Elastic Collisions

- If $m_1 \ll m_2$, then from equation 12:

$$\vec{\boldsymbol{v}}_{1f} \approx -\vec{\boldsymbol{v}}_{1i}, \quad \vec{\boldsymbol{v}}_{2f} \approx \vec{\boldsymbol{v}}_{2i}$$

- This means that the first object rebounds with nearly its original speed in the opposite direction, while the second object continues with nearly its original velocity.
- If particle 2 is initially at rest ($\vec{\boldsymbol{v}}_{2i} = 0$), then from equation 12:

$$\vec{\boldsymbol{v}}_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{\boldsymbol{v}}_{1i}$$

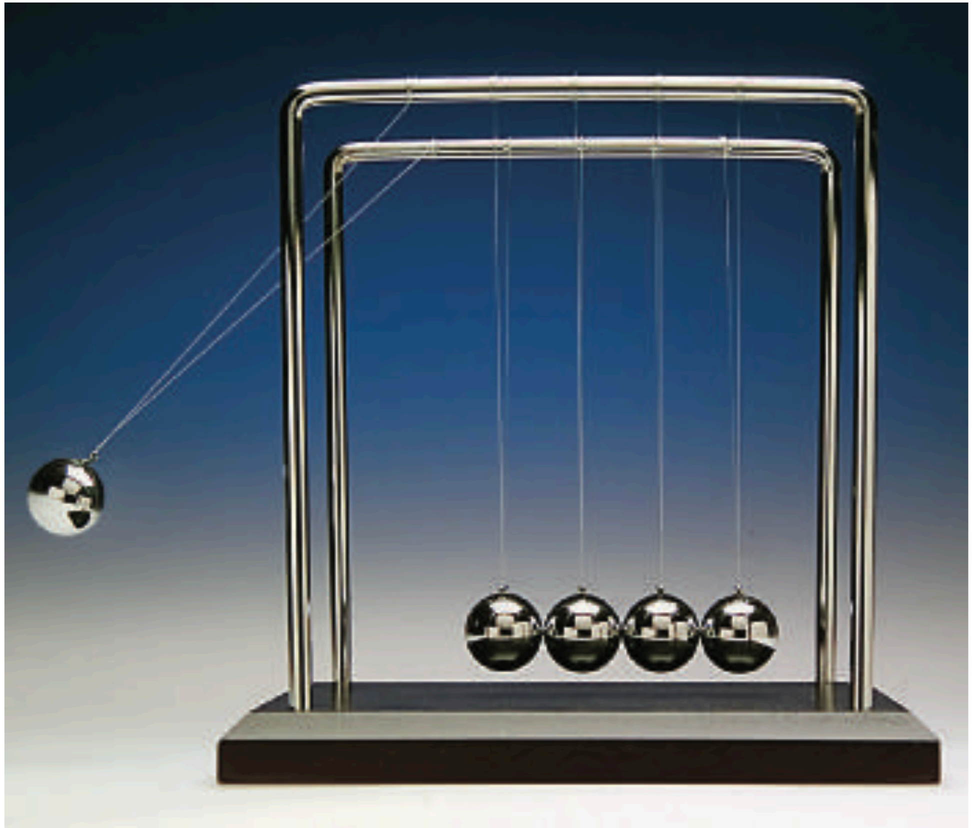
$$\vec{\boldsymbol{v}}_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{\boldsymbol{v}}_{1i}$$

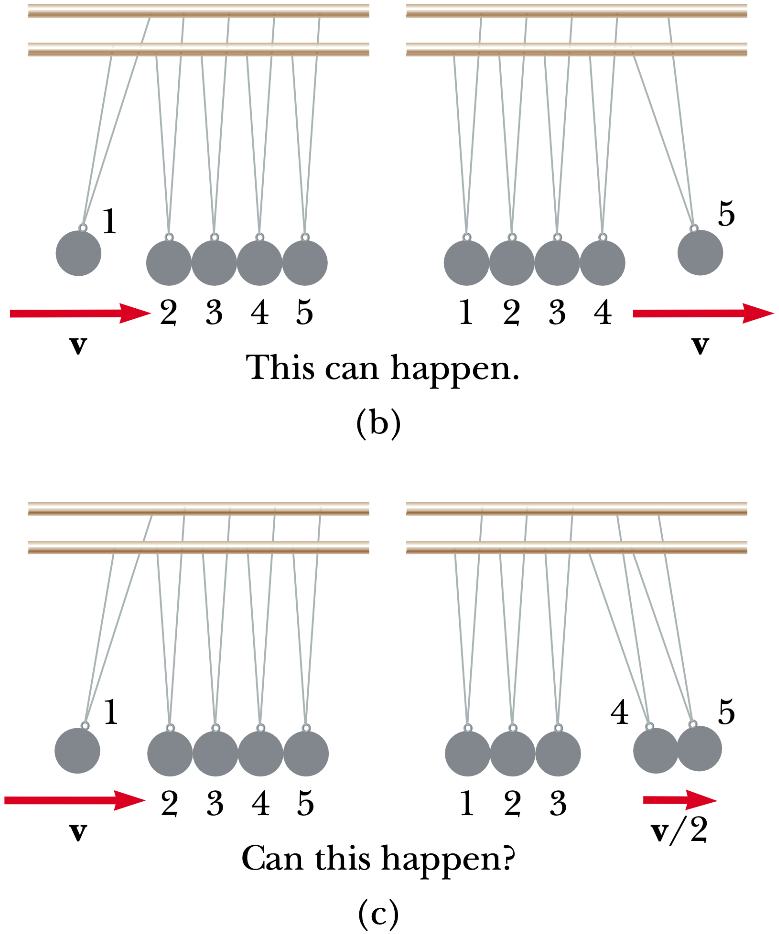
3.4 Special Cases for Elastic Collisions

Example 3.3

An ingenious device that illustrates conservation of momentum and kinetic energy is shown in the Figure. It consists of five identical hard balls supported by strings of equal lengths. When ball 1 is pulled out and released, after the almost-elastic collision between it and ball 2, ball 5 moves out, as shown in the Figure. If balls 1 and 2 are pulled out and released, balls 4 and 5 swing out, and so forth. Is it ever possible that when ball 1 is released, balls 4 and 5 will swing out on the opposite side and travel with half the speed of ball 1,as in the Figure?

3.4 Special Cases for Elastic Collisions





3.4 Special Cases for Elastic Collisions

Solution 3.3

- No, such movement can never occur if we assume the collisions are elastic.
- The conservation of momentum and energy for one ball is expressed as:

$$\vec{\boldsymbol{p}}_f = \vec{\boldsymbol{p}}_i$$

$$m\vec{\boldsymbol{v}}_{5f} = m\vec{\boldsymbol{v}}_{1i}$$

$$\implies \vec{\boldsymbol{v}}_{5f} = \vec{\boldsymbol{v}}_{1i}$$

$$\frac{1}{2}m\vec{\boldsymbol{v}}_{5f}^2 = \frac{1}{2}m\vec{\boldsymbol{v}}_{1i}^2$$

$$\implies \vec{\boldsymbol{v}}_{5f} = \vec{\boldsymbol{v}}_{1i}$$

- But if two balls are involved,

$$\vec{\boldsymbol{p}}_f = \vec{\boldsymbol{p}}_i$$

$$m\left(\frac{v}{2}\right) + m\left(\frac{v}{2}\right) = mv$$

But the conservation of kinetic energy gives:

3.4 Special Cases for Elastic Collisions

$$\frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 \neq \frac{1}{2}mv^2$$

Therefore, such a movement is impossible.

3.4 Special Cases for Elastic Collisions

Example 3.4

An 1800 kg car stopped at a traffic light is struck from the rear by a 900 kg car, and the two become entangled, moving along the same path as that of the originally moving car. If the smaller car were moving at 20 m/s before the collision, what is the velocity of the entangled cars after the collision?

3.4 Special Cases for Elastic Collisions

Solution 3.4

- The phrase "become entangled" tells us that this is a perfectly inelastic collision.
- Applying the conservation of linear momentum, we have:

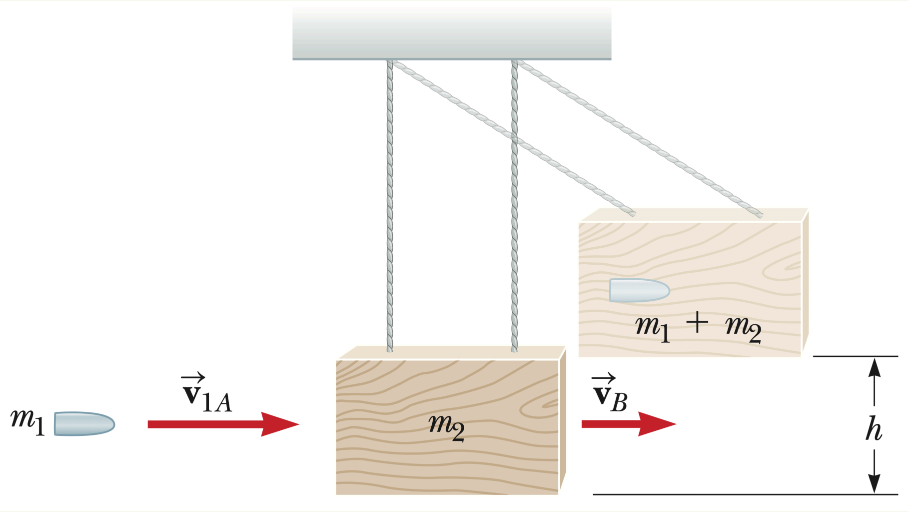
$$\vec{\boldsymbol{v}}_f = \frac{m_1\vec{\boldsymbol{v}}_{1i} + m_2\vec{\boldsymbol{v}}_{2i}}{m_1 + m_2} = \frac{(900\ \text{kg})(20\ \text{m/s}) + (1800\ \text{kg})(0\ \text{m/s})}{900\ \text{kg} + 1800\ \text{kg}}$$

$$= 6.67\ \text{m/s}\ \hat{\boldsymbol{\imath}}$$

3.4 Special Cases for Elastic Collisions

Example 3.5

The ballistic pendulum is an apparatus used to measure the speed of a fast-moving projectile, such as a bullet. A bullet of mass m_1 is fired into a large block of wood of mass m_2 suspended from some light wires. The bullet embeds in the block, and the entire system swings through a height h. How can we determine the speed of the bullet from a measurement of h?



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Solution 3.5

- The collision is perfectly inelastic because the bullet becomes embedded in the block. Therefore, the kinetic energy is not conserved during the collision.
- **However**, the mechanical energy is conserved **after the collision** since the system is isolated and under a conservative force (gravity).

$$E_f = E_i$$

$$K_f + U_f = K_i + U_i$$

$$0 + (m_1 + m_2)gh = \frac{1}{2}m_1 v_b^2 + 0$$

$$v_b^2 = \frac{2gh(m_1 + m_2)}{m_1} \tag{E1}$$

3.4 Special Cases for Elastic Collisions

- To find the bullet speed v_b after the collision, we apply the conservation of linear momentum during the collision:

$$v_b = \frac{m_1 v_{1A} + m_2 v_{2i}}{m_1 + m_2} = \frac{m_1 v_{1A} + 0}{m_1 + m_2} \tag{E2}$$

- By substituting equations E2 into E1, we obtain:

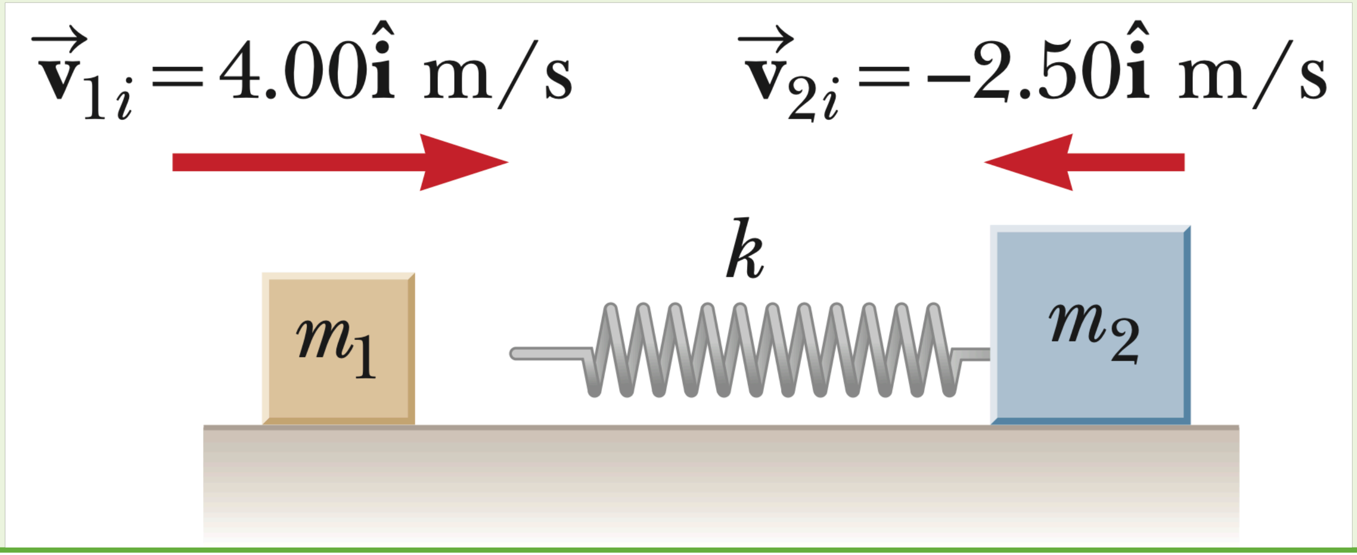
$$\frac{(m_1 v_{1A})^2}{(m_1 + m_2)^2} = \frac{2gh(m_1 + m_2)}{m_1}$$

- Solving for v_{1A}, we get:

$$v_{1A} = \frac{m_1 + m_2}{m_1} \sqrt{2gh}$$

Example 3.6

A block of mass $m_1 = 1.6$ kg initially moving to the right with a speed of 4 m/s on a frictionless horizontal track collides with a spring attached to a second block of mass $m_2 = 2.1$ kg initially moving to the left with a speed of 2.5 m/s. The spring constant is 600 N/m. **(A)** Find the velocities of the two blocks after the collision.



3.4 Special Cases for Elastic Collisions

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Solution 3.6

- First the type of collision is **elastic** because the spring force is conservative and there is no energy loss to heat, sound, or deformation.
- Using Equation 12, we find the final velocities of the two blocks after the collision:

$$\vec{v}_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{2i}$$

$$= \left(\frac{1.6\text{ kg} - 2.1\text{ kg}}{1.6\text{ kg} + 2.1\text{ kg}}\right)(4\text{ m/s}) + \left(\frac{2(2.1\text{ kg})}{1.6\text{ kg} + 2.1\text{ kg}}\right)(-2.5\text{ m/s})$$

$$= -3.38\text{ m/s }\hat{\imath}$$

3.4 Special Cases for Elastic Collisions

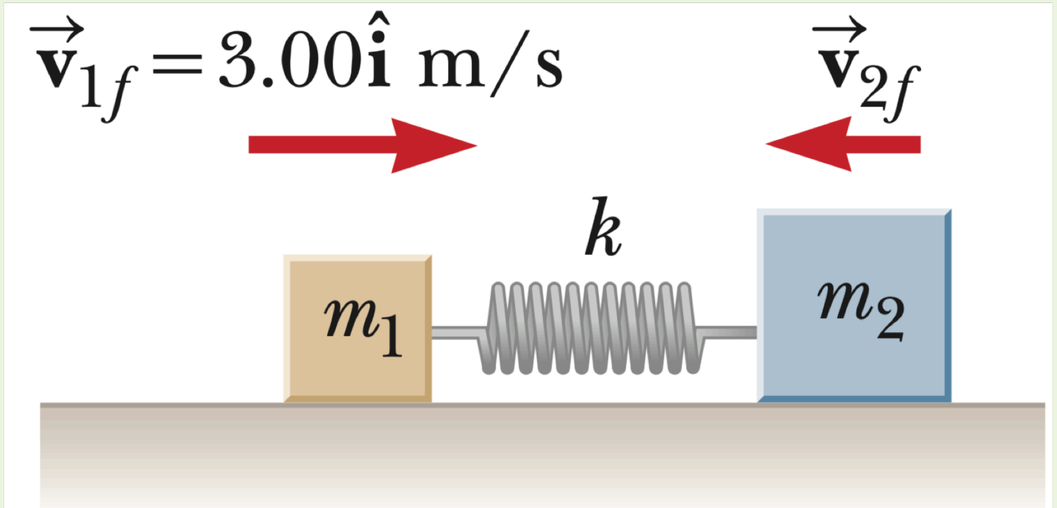
$$\vec{\boldsymbol{v}}_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{\boldsymbol{v}}_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{\boldsymbol{v}}_{2i}$$

$$= \left(\frac{2(1.6\ \mathrm{kg})}{1.6\ \mathrm{kg} + 2.1\ \mathrm{kg}}\right)(4\ \mathrm{m/s}) + \left(\frac{2.1\ \mathrm{kg} - 1.6\ \mathrm{kg}}{1.6\ \mathrm{kg} + 2.1\ \mathrm{kg}}\right)(-2.5\ \mathrm{m/s})$$

$$= 3.12\ \mathrm{m/s}\ \hat{\boldsymbol{\imath}}$$

3.4 Special Cases for Elastic Collisions

(B) During the collision, at the instant block 1 is moving to the right with a velocity of 3 m/s, as in the Figure, determine the velocity of block 2.



3.4 Special Cases for Elastic Collisions

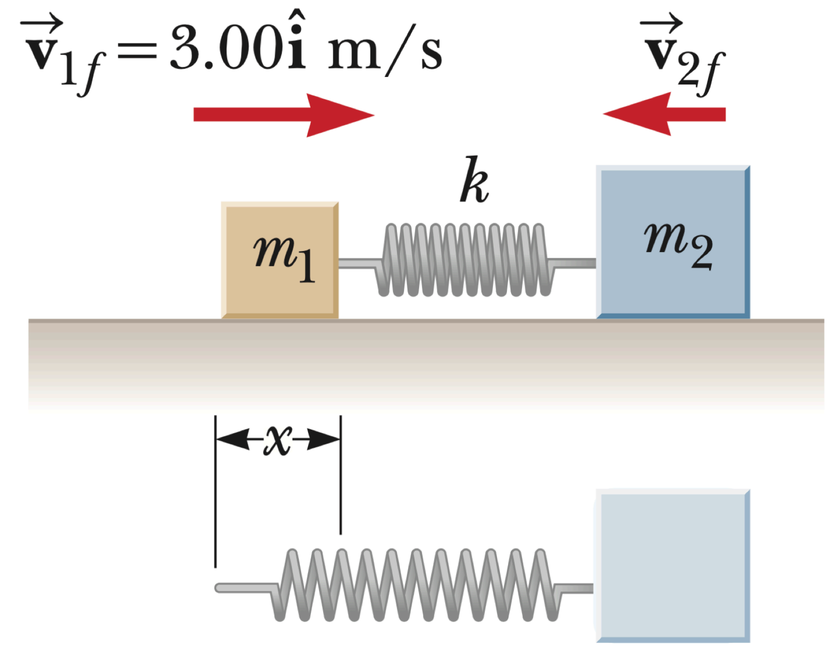
- Using the conservation of linear momentum, we have:

$$m_1\vec{\boldsymbol{v}}_{1i} + m_2\vec{\boldsymbol{v}}_{2i} = m_1\vec{\boldsymbol{v}}_{1f} + m_2\vec{\boldsymbol{v}}_{2f}$$

$$\begin{aligned}
\Rightarrow \vec{\boldsymbol{v}}_{2f} &= \frac{m_1\vec{\boldsymbol{v}}_{1i} + m_2\vec{\boldsymbol{v}}_{2i} - m_1\vec{\boldsymbol{v}}_{1f}}{m_2} \\
&= \frac{(1.6\ \mathrm{kg})(4\ \mathrm{m/s}) + (2.1\ \mathrm{kg})(-2.5\ \mathrm{m/s}) - (1.6\ \mathrm{kg})(3\ \mathrm{m/s})}{2.1\ \mathrm{kg}} \\
&= -1.74\ \mathrm{m/s}\ \hat{\boldsymbol{\imath}}
\end{aligned}$$

3.4 Special Cases for Elastic Collisions

(C) What is the compression of the spring at this instant?



3.4 Special Cases for Elastic Collisions

$$\frac{1}{2}m_1\vec{v}_{1f}^2 + \frac{1}{2}m_2\vec{v}_{2f}^2 + \frac{1}{2}kx^2 = \frac{1}{2}m_1\vec{v}_{1i}^2 + \frac{1}{2}m_2\vec{v}_{2i}^2 + 0$$

Solving for x, we get:

$$x = \sqrt{\frac{m_1\vec{v}_{1i}^2 + m_2\vec{v}_{2i}^2 - m_1\vec{v}_{1f}^2 - m_2\vec{v}_{2f}^2}{k}}$$

$$x = \sqrt{\frac{(1.6)(4)^2 + (2.1)(-2.5)^2 - (1.6)(3)^2 - (2.1)(-1.74)^2}{600}}$$

$$= 0.173 \text{ m}$$

3.4 Special Cases for Elastic Collisions

(D) What is the maximum compression of the spring during the collision?

3.4 Special Cases for Elastic Collisions

- The maximum compression would occur when the two blocks are moving with the same velocity.

$$\vec{\boldsymbol{v}}_f = \frac{m_1\vec{\boldsymbol{v}}_{1i} + m_2\vec{\boldsymbol{v}}_{2i}}{m_1 + m_2}$$

$$= 0.311 \text{ m/s}$$

- Using the conservation of mechanical energy, we have:

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}(m_1 + m_2)v_f^2 + \frac{1}{2}kx_{\text{max}}^2$$

$$\Longrightarrow x_{\text{max}} = 0.253 \text{ m}$$

1. Linear Momentum and Its Conservation

2. Impulse and Momentum

3. Collisions in One Dimension

4. **Two-Dimensional Collisions**

5. Suggested Problems

4.1 Conservation of Momentum in Two Dimensions

- In two-dimensional collisions, both the x and y components of momentum are conserved separately.

$$\boxed{m_1\vec{v}_{1i,x} + m_2\vec{v}_{2i,x} = m_1\vec{v}_{1f,x} + m_2\vec{v}_{2f,x}} \tag{13}$$

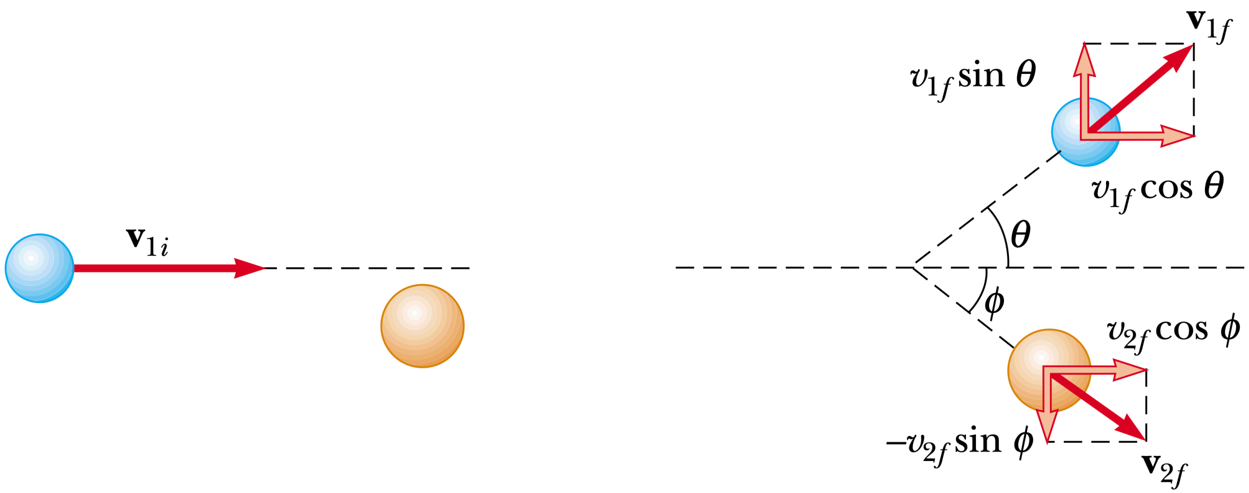
$$\boxed{m_1\vec{v}_{1i,y} + m_2\vec{v}_{2i,y} = m_1\vec{v}_{1f,y} + m_2\vec{v}_{2f,y}} \tag{14}$$

4.2 Conservation of Kinetic Energy in Two Dimensions

- If the collision is elastic, then the kinetic energy is also conserved:

$$\boxed{\frac{1}{2}m_1\left|\vec{\boldsymbol{v}}_{1i}\right|^2+\frac{1}{2}m_2\left|\vec{\boldsymbol{v}}_{2i}\right|^2=\frac{1}{2}m_1\left|\vec{\boldsymbol{v}}_{1f}\right|^2+\frac{1}{2}m_2\left|\vec{\boldsymbol{v}}_{2f}\right|^2} \tag{15}$$

4.3 Example of Two-Dimensional Collision



(a) Before the collision (b) After the collision

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\varphi$$

$$0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\varphi$$

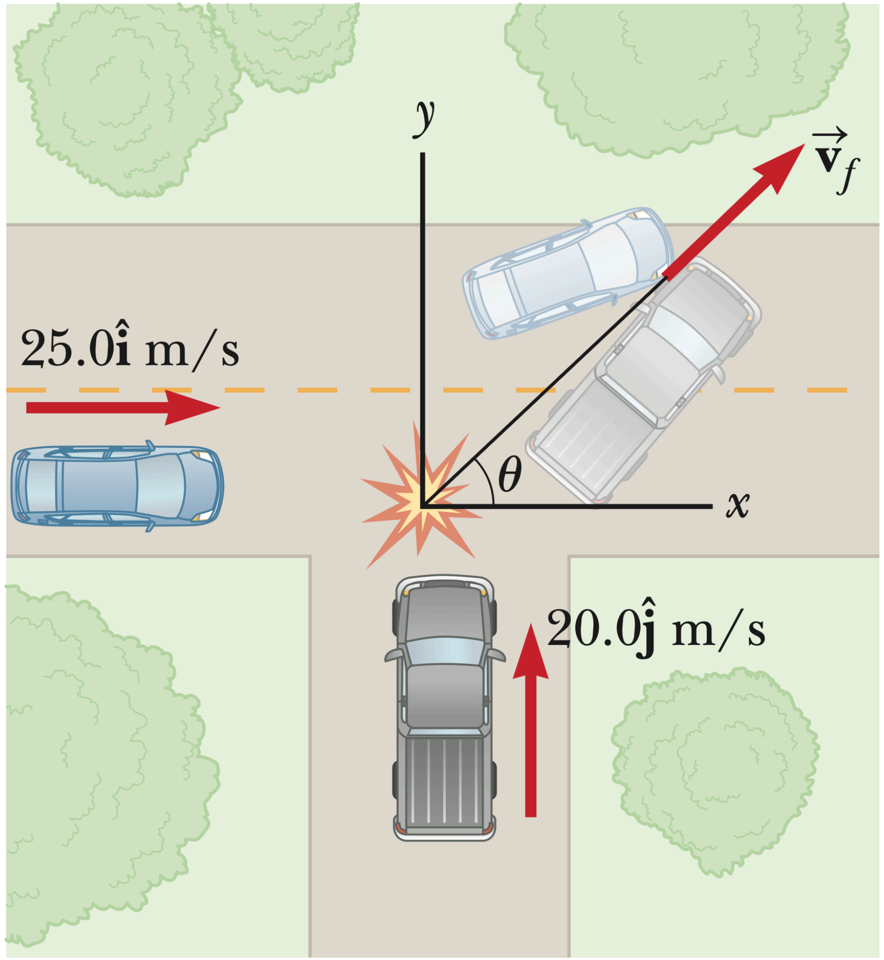
$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

4.3 Example of Two-Dimensional Collision

Example 4.7

A 1500 kg car traveling east with a speed of 25 m/s collides at an intersection with a 2500 kg van traveling north at a speed of 20 m/s, as shown in the Figure.

Find the direction and magnitude of the velocity of the wreckage (حطام) after the collision, assuming that the vehicles undergo a perfectly inelastic collision (that is, they stick together).



4.3 Example of Two-Dimensional Collision

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Solution 4.7

- Applying the conservation of linear momentum in the x and y directions, we have:

$$\begin{aligned} \text{x:} \qquad & m_1 v_{1i} + 0 = (m_1 + m_2) v_f \cos\theta \\ \text{y:} \qquad & 0 + m_2 v_{2i} = (m_1 + m_2) v_f \sin\theta \end{aligned}$$

- Dividing the y equation by the x equation, we get:

$$\tan\theta = \frac{m_2 v_{2i}}{m_1 v_{1i}} = 1.33 \quad \Longrightarrow \quad \theta = 53.1^\circ$$

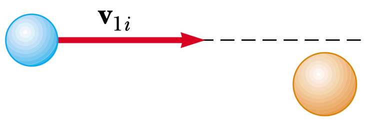
- Substituting θ back into the x equation (or y equation), we obtain:

$$v_f = \frac{m_1 v_{1i}}{(m_1 + m_2)\cos\theta} = 15.6 \text{ m/s}$$

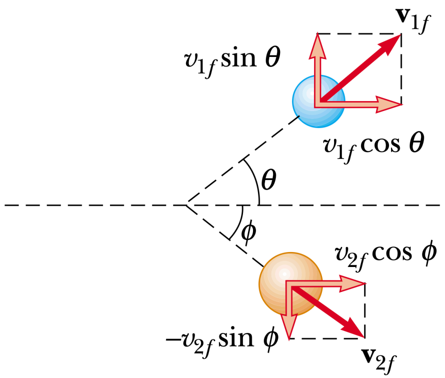
4.3 Example of Two-Dimensional Collision

Example 4.8

A proton collides elastically with another proton that is initially at rest. The incoming proton has an initial speed of 3.5×10^5 m/s and makes a glancing collision with the second proton, as in the Figure. After the collision, one proton moves off at an angle of $\theta = 37°$ to the original direction of motion, and the second deflects at an angle of φ to the same axis.
Find the final speeds of the two protons and the angle φ.



(a) Before the collision



(b) After the collision

4.3 Example of Two-Dimensional Collision

Solution 4.8

- We have four givens:

$$v_{1i} = 3.5 \times 10^5 \text{ m/s} \quad v_{2i} = 0 \quad m_1 = m_2 = m_p \quad \theta = 37^\circ$$

- We have three equations from the conservation of momentum and energy:

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\varphi$$

$$0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\varphi$$

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

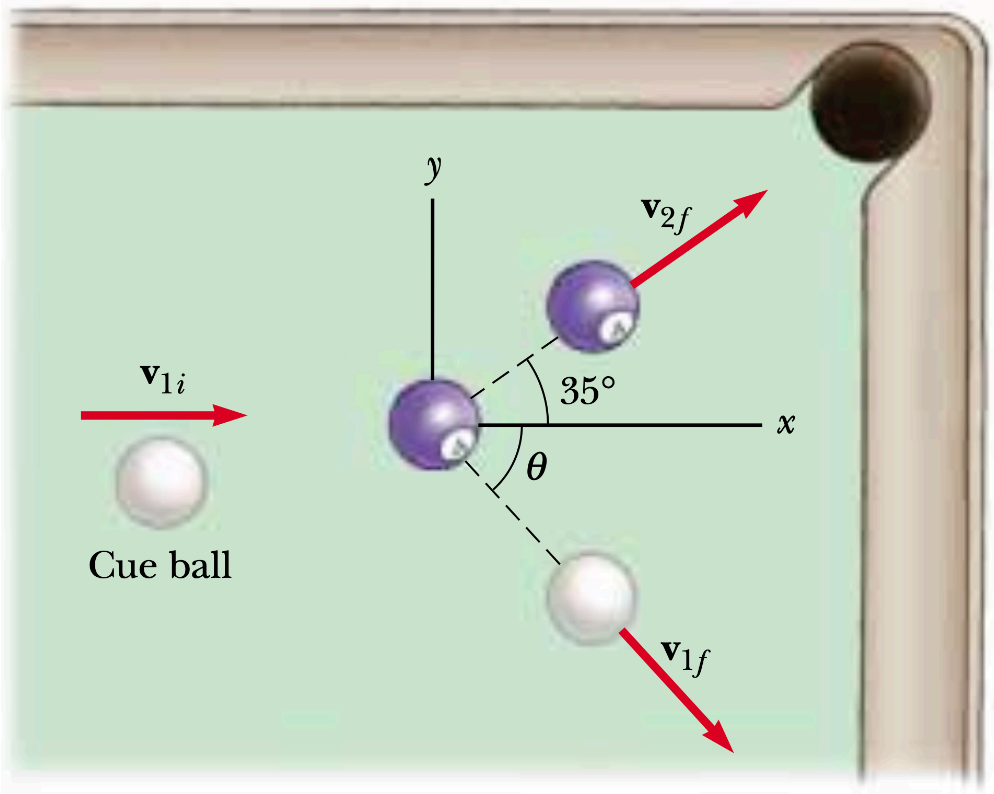
- Solving three equations simultaneously to find three unknowns v_{1f}, v_{2f}, and φ, we obtain:

$$v_{1f} = 2.8 \times 10^5 \text{ m/s}, \quad v_{2f} = 2.11 \times 10^5 \text{ m/s}, \quad \varphi = 53^\circ$$

4.3 Example of Two-Dimensional Collision

Example 4.9

In a game of billiards, a player wishes to sink a target ball in the corner pocket, as shown in the figure.
If the angle to the corner pocket is 35°, at what angle is the cue ball deflected?
Assume that friction and rotational motion are unimportant and that the collision is elastic. Also assume that all billiard balls have the same mass m.



4.3 Example of Two-Dimensional Collision

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Solution 4.9

$$\begin{aligned} \text{x:} \qquad m_1 v_{1i} &= m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\varphi \\ \text{y:} \qquad 0 &= m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\varphi \\ \text{KE:} \qquad \frac{1}{2}m_1 v_{1i}^2 &= \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 \end{aligned}$$

- Solving the three equations simultaneously, we find the angle φ:

$$\varphi = 55.1^\circ$$

5. Suggested Problems

1, 2, 4, 5, 7, 8, 9, 10, 13, 15, 16, 17, 18, 21, 25, 27, 32, 33, 35

Problem 5.2

A ball of mass 0.15 kg is dropped from rest from a height of 1.25 m. It rebounds from the floor to reach a height of 0.96 m.

What impulse was given to the ball by the floor?

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Answer 5.2

The impulse delivered to the ball by the floor is given by:

$$I = \Delta \vec{\boldsymbol{p}} = \vec{\boldsymbol{p}}_f - \vec{\boldsymbol{p}}_i = m\vec{\boldsymbol{v}}_f - m\vec{\boldsymbol{v}}_i = m\big(v_f - (-v_i)\big)$$

To find the initial and final velocities, we use the kinematic equation:

$$\frac{1}{2}mv_i^2 = mgh_i \implies v_i = \sqrt{2gh_i}$$

$$\frac{1}{2}mv_f^2 = mgh_f \implies v_f = \sqrt{2gh_f}$$

5.1 Selected Problems

Substituting the values, we get:

$$
\begin{aligned}
I &= m\left(\sqrt{2gh_f} + \sqrt{2gh_i}\right) \\
&= (0.15\ \mathrm{kg})\left(\sqrt{2(9.8\ \mathrm{m/s^2})(0.96\ \mathrm{m})} + \sqrt{2(9.8\ \mathrm{m/s^2})(1.25\ \mathrm{m})}\right) \\
\vec{\boldsymbol{I}} &= 1.39\ \mathrm{kg\ m/s}\ \hat{\boldsymbol{\jmath}} \quad \text{(upward direction)}
\end{aligned}
$$

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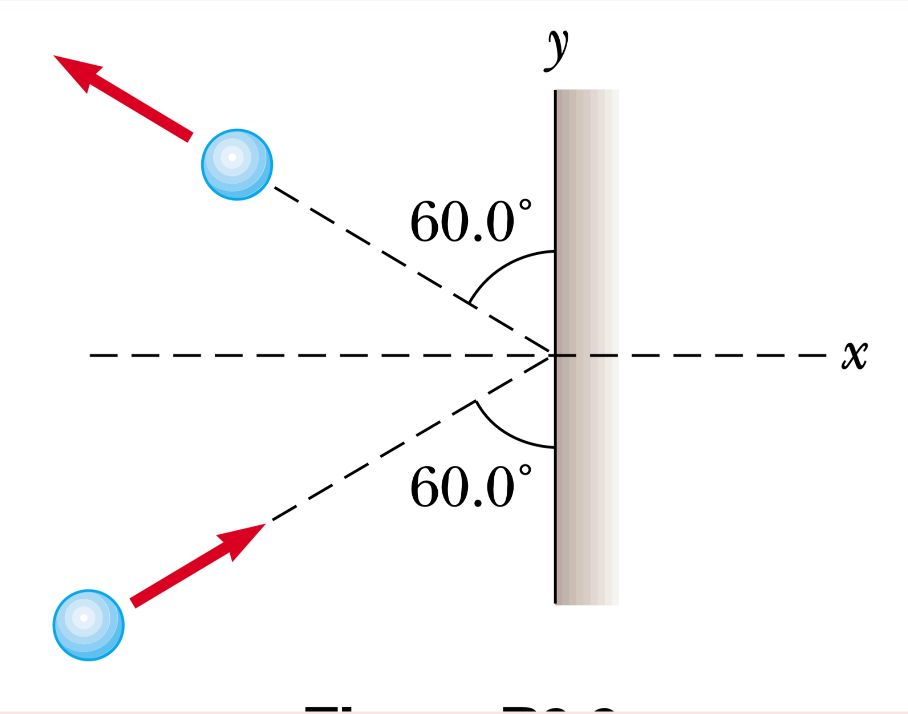
Problem 5.3

A 3 kg steel ball strikes a wall with a speed of 10 m/s at an angle of 60° with the surface. It bounces off with the same speed and angle. If the ball is in contact with the wall for 0.2 s,

what is the average force exerted by the wall on the ball ?



Answer 5.3

The average force exerted by the wall on the ball is given by:

$$\vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t}$$

Since the ball bounces off with the same speed and angle, the change in momentum in the y-direction is zero. Therefore, we only need to consider the x-direction:

$$\begin{aligned}\Delta\vec{p}_x &= \vec{p}_{xf} - \vec{p}_{xi} = m\vec{v}_{xf} - m\vec{v}_{xi} \\ &= m(-v\sin\theta) - m(v\sin\theta) = -2mv\sin\theta\end{aligned}$$

Thus, the average force exerted by the wall on the ball is:

$$\vec{F}_{\text{avg}} = \frac{-2mv\sin\theta}{\Delta t} = \frac{-2(3\ \text{kg})(10\ \text{m/s})(\sin 60^\circ)}{0.2\ \text{s}} = -260\ \text{N}\ \hat{\imath}$$

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Problem 5.4

High-speed stroboscopic photographs show that the head of a golf club (مضرب جولف) of mass 200 g is traveling at 55 m/s just before it strikes a 46 g golf ball at rest on a tee (نقطة الإنطلاق). After the collision, the club head travels (in the same direction) at 40 m/s.

Find the speed of the golf ball just after impact.

5.1 Selected Problems

Answer 5.4

From the conservation of linear momentum, we have:

$$m_1\vec{\boldsymbol{v}}_{1i} + m_2\vec{\boldsymbol{v}}_{2i} = m_1\vec{\boldsymbol{v}}_{1f} + m_2\vec{\boldsymbol{v}}_{2f}$$

$$(0.2\text{kg})(55\ \text{m/s}) + 0 = (0.2\text{kg})(40\ \text{m/s}) + (0.046\text{kg})v_{2f}$$

$$\Longrightarrow v_{2f} = 65.2\ \text{m/s}$$

Problem 5.5

A 10g bullet is fired into a stationary block of wood ($m = 5$ kg). The relative motion of the bullet stops inside the block. The speed of the bullet-plus-wood combination immediately after the collision is 0.6 m/s.

What was the original speed of the bullet?

5.1 Selected Problems

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Answer 5.5

Since this is a perfectly inelastic collision, we have:

$$m_1\vec{\boldsymbol{v}}_{1i} + m_2\vec{\boldsymbol{v}}_{2i} = (m_1 + m_2)\vec{\boldsymbol{v}}_f$$

$$\Longrightarrow \vec{\boldsymbol{v}}_{1i} = \frac{(m_1 + m_2)\vec{\boldsymbol{v}}_f}{m_1} = \frac{(0.01\ \mathrm{kg} + 5\ \mathrm{kg})(0.6\ \mathrm{m/s})}{0.01\ \mathrm{kg}} = 300.6\ \ \mathrm{m/s}$$

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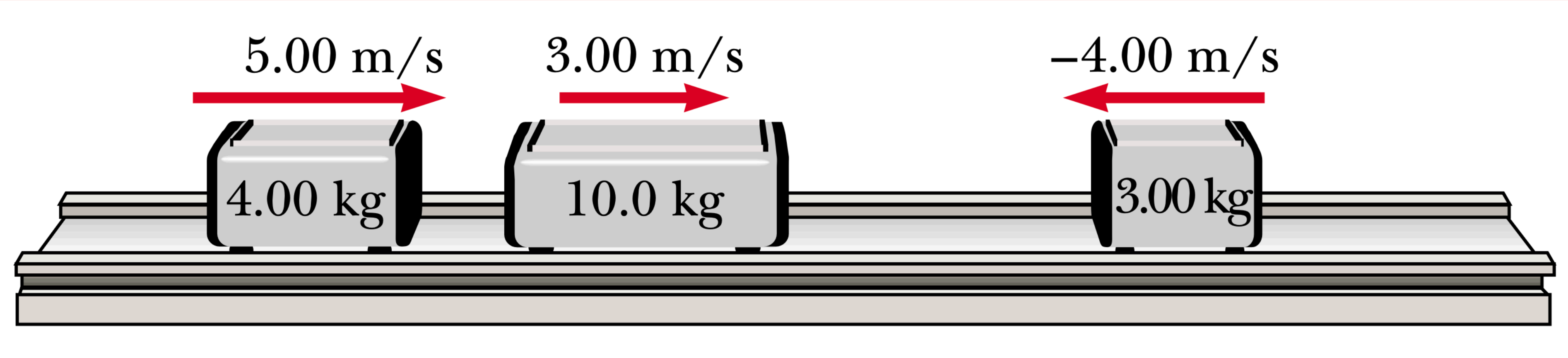
Problem 5.6

Three carts of masses 4 kg, 10 kg, and 3 kg move on a frictionless horizontal track with speeds of 5 m/s, 3 m/s, and 4 m/s, as shown in the Figure. Velcro couplers make the carts stick together after colliding.
Find the final velocity of the train of three carts.



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Answer 5.6

From the conservation of linear momentum, we have:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} + m_3\vec{v}_{3i} = (m_1 + m_2 + m_3)\vec{v}_f$$

$$\implies \vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i} + m_3\vec{v}_{3i}}{m_1 + m_2 + m_3}$$

$$= \frac{(4\text{ kg})(5\text{ m/s}) + (10\text{ kg})(3\text{ m/s}) + (3\text{ kg})(-4\text{ m/s})}{4\text{ kg} + 10\text{ kg} + 3\text{ kg}} = 2.24\text{ m/s}$$

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Problem 5.7

Two automobiles of equal mass approach an intersection. One vehicle is traveling with velocity 13 m/s toward the east, and the other is traveling north with speed v_{2i}. Neither driver sees the other. The vehicles collide in the intersection and stick together, leaving parallel skid marks at an angle of 55° north of east. The speed limit for both roads is 35 mi/h, and the driver of the northward-moving vehicle claims he was within the speed limit when the collision occurred.

Is he telling the truth?

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Answer 5.7

Since the vehicles stick together after the collision, this is a perfectly inelastic collision. From the conservation of linear momentum in the x and y directions, we have:

$$\text{x:} \qquad mv_{1i} + 0 = (m+m)v_f \cos\theta$$

$$\text{y:} \qquad 0 + mv_{2i} = (m+m)v_f \sin\theta$$

Dividing the y equation by the x equation, we get:

$$\tan\theta = \frac{v_{2i}}{v_{1i}} \implies v_{2i} = v_{1i}\tan\theta = (13)\tan(55°) = 18.6 \text{ m/s} \approx 42 \text{ mi/h}$$

Therefore, the driver is not telling the truth since 42 mi/h > 35 mi/h.

5.1 Selected Problems

Problem 5.8

A billiard ball moving at 5 m/s strikes a stationary ball of the same mass. After the collision, the first ball moves, at 4.33 m/s, at an angle of 30° with respect to the original line of motion. Assuming an elastic collision (and ignoring friction and rotational motion),

find the struck ball's velocity after the collision.

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Answer 5.8

From the conservation of linear momentum in the x and y directions, we have:

$$\text{x:} \qquad mv_{1i} + 0 = mv_{1f}\cos\theta + mv_{2f}\cos\varphi$$

$$v_{2f}\cos\varphi = v_{1i} - v_{1f}\cos\theta \qquad (1)$$

$$\text{y:} \qquad 0 + 0 = mv_{1f}\sin\theta - mv_{2f}\sin\varphi$$

$$v_{2f}\sin\varphi = v_{1f}\sin\theta \qquad (2)$$

- Squaring and adding equations (1) and (2), we get:

$$v_{2f}^2(\cos^2\varphi + \sin^2\varphi) = \left(v_{1i} - v_{1f}\cos\theta\right)^2 + \left(v_{1f}\sin\theta\right)^2$$

$$v_{2f} = \sqrt{v_{1f}^2 + v_{1i}^2 - 2v_{1i}v_{1f}\cos\theta} = 2.5 \text{ m/s}$$