Ch.8: Potential Energy

Physics 103: Classical Mechanics

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Version 2

Outline

1. Potential Energy of a System

2. The Isolated System Conservation of Mechanical Energy

3. Conservative and Nonconservative Forces

4. Changes in Mechanical Energy for Nonconservative Forces

5. Relationship Between Conservative Forces and Potential Energy

6. Additional Problems

1.1 Gravitational Potential Energy

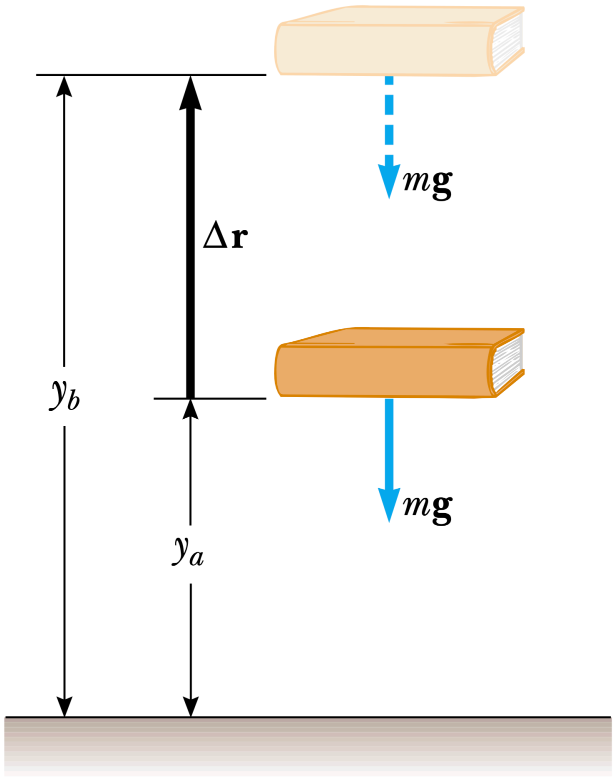
- If a particle of mass m is at a distance y above the Earth's surface, the gravitational potential energy of the particle-Earth system is

$$\boxed{U = mgy} \tag{1}$$

- When a work W_{ext} is done by an external force to move a particle from an initial position y_a to an *upper* final position y_b, the change in gravitational potential energy of the particle-Earth system is:

$$\begin{aligned} W_{\text{ext}} &= mg(y_b - y_a) \\ &= U_b - U_a \end{aligned}$$

$$\boxed{W_{\text{ext}} = \Delta U} \tag{2}$$



1.1 Gravitational Potential Energy

Example 1.1

A 2 kg object is lifted from the ground to the top of a building 50 m high. (A) What is the change in the gravitational potential energy of the object-Earth system, and (B) what is the work done by the external force?

Solution 1.1

Given: $m = 2$ kg, $y_a = 0m$, $y_b = 50m$,

$$\Delta U = mg(y_b - y_a) = (2\text{ kg})(9.8\text{ m/s}^2)(50\text{ m} - 0\text{ m}) = +980\text{ J}$$

.

$$W_{\text{ext}} = \Delta U = +980\text{ J}$$

1. Potential Energy of a System

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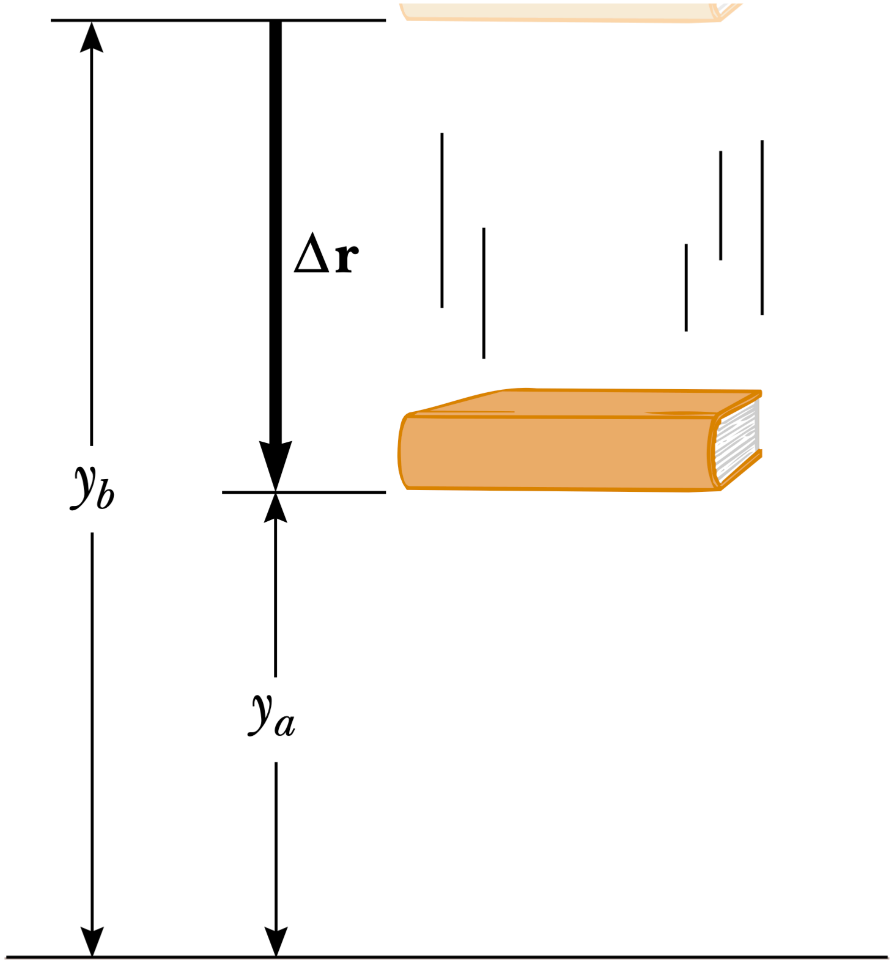
6. Additional Problems

2.1 Work Kinetic and Potential Energy

- The work done on the book alone by the gravitational force as the book falls back to its original height:

$$W_g = mg(y_b - y_a)$$

- The work is positive because the force and the displacement are in the same direction.



2.1 Work Kinetic and Potential Energy

- The work done by the gravitational force is equal to the negative change in potential energy Because the book looses potential energy as it falls.

$$\boxed{W_g = -\Delta U} \tag{3}$$

where

$$\Delta U = U_f - U_i = mg(y_a - y_b).$$

- The loss of potential energy means the book gains kinetic energy, we can write:

$$\boxed{W_g = \Delta K} \tag{4}$$

and

$$\boxed{\Delta K = -\Delta U} \tag{5}$$

- notice that, $\Delta k + \Delta U = 0$

2.1 Work Kinetic and Potential Energy

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- From Equation 5, we can write:

$$\boxed{K_f + U_f = K_i + U_i} \tag{6}$$

- This equation expresses the conservation of energy for an isolated system.
- An isolated system is one for which there are no energy transfers across the boundary. The energy is conserved as the sum of the kinetic and potential energies remains constant.

- Expanding Eqation 6, we get:

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

- The sum of the kinetic energy and the potential energy is called the mechanical energy E of the system:

$$\boxed{E = K + U} \tag{7}$$

Which is constant for an isolated system.

$$\Delta E = 0$$

2.2 Elastic Potential Energy of a Spring

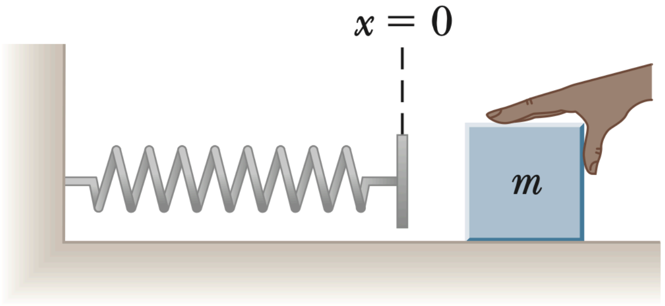
- As discussed in the previous chapter, when a spring is stretched or compressed from its equilibrium position, it exerts a restoring force F_s that is proportional to the displacement x from equilibrium:

$$F_s = -kx$$

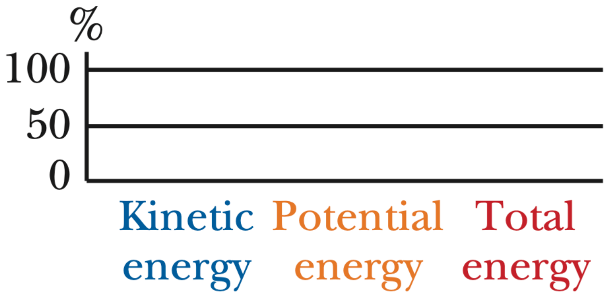
- The elastic potential energy at a displacement x is:

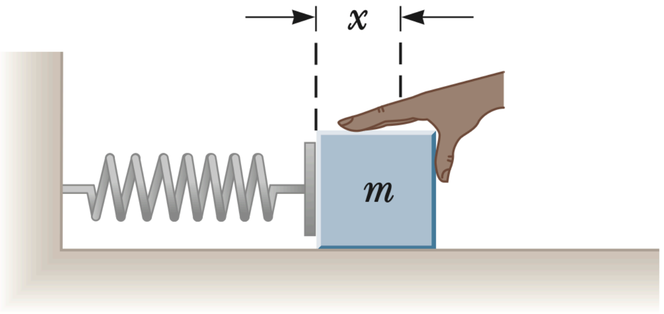
$$\boxed{U = \frac{1}{2}kx^2} \tag{8}$$

2.2 Elastic Potential Energy of a Spring

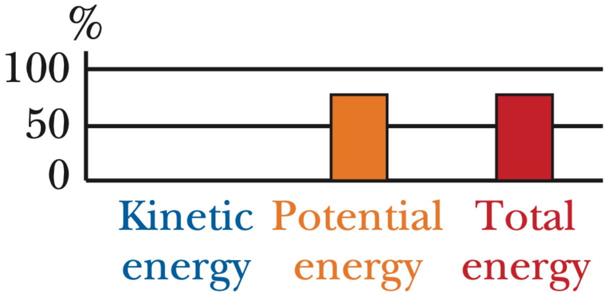


Before the spring is compressed, there is no energy in the spring–block system.

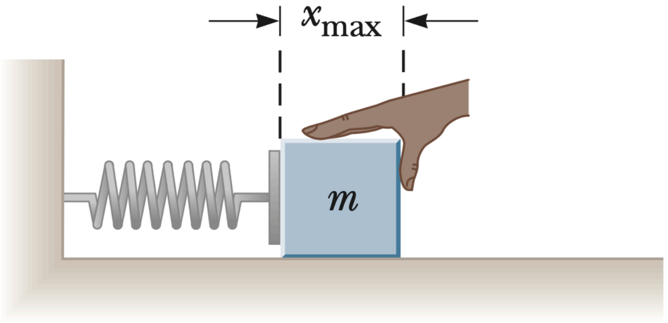




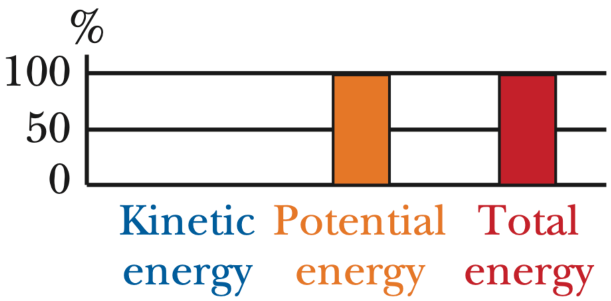
When the spring is partially compressed, the total energy of the system is elastic potential energy.

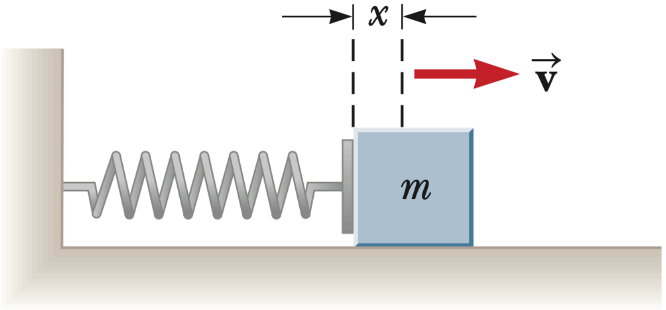


2.2 Elastic Potential Energy of a Spring

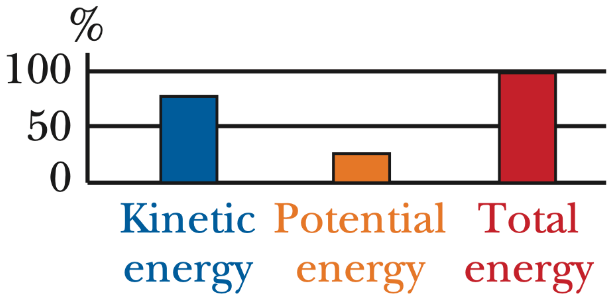


The spring is compressed by a maximum amount, and the block is held steady; there is elastic potential energy in the system and no kinetic energy.





After the block is released, the elastic potential energy in the system decreases and the kinetic energy increases.



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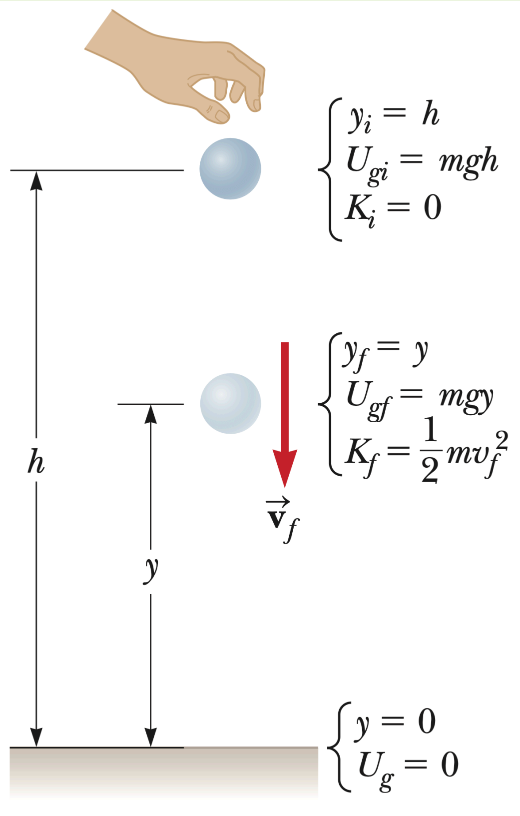
Example 2.2

A ball of mass m is dropped from a height h above the ground, as shown in the Figure.

(A) Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground.

(B) Determine the speed of the ball at y if at the instant of release it already has an initial upward speed v_i at the initial altitude h.



2.2 Elastic Potential Energy of a Spring

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Solution 2.2

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}mv_f^2 + mgy = 0 + mgh$$

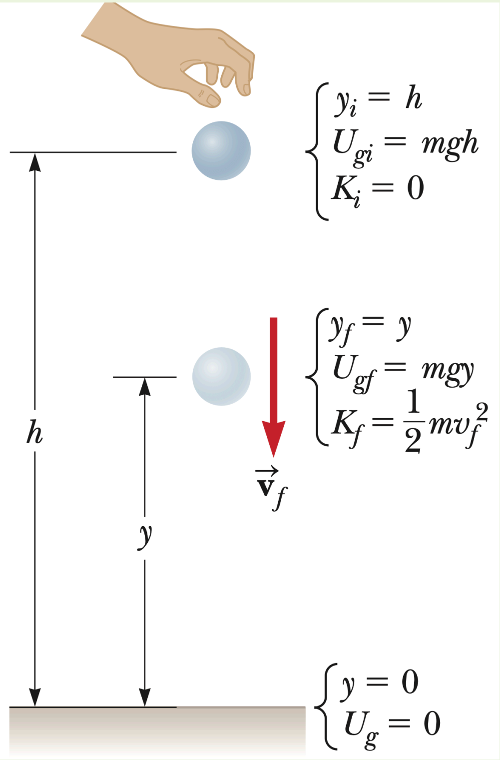
Solving for v_f, we get:

$$v_f = \sqrt{2g(h-y)}$$

2.2 Elastic Potential Energy of a Spring

Example 2.2

(B) Determine the speed of the ball at y if at the instant of release it already has an initial upward speed v_i at the initial altitude h.



2.2 Elastic Potential Energy of a Spring

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Solution 2.2

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}mv_f^2 + mgy = \frac{1}{2}mv_i^2 + mgh$$

Solving for v_f, we get:

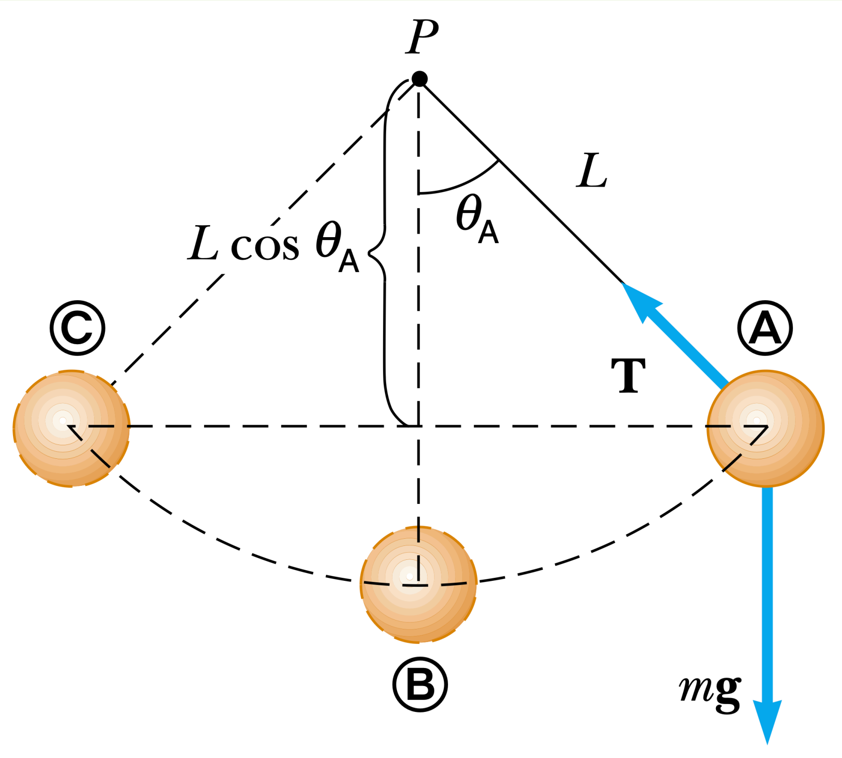
$$v_f = \sqrt{v_i^2 + 2g(h - y)}$$

2.2 Elastic Potential Energy of a Spring

Example 2.3

A pendulum consists of a sphere of mass m attached to a light cord of length L, as shown in the Figure. The sphere is released from rest at point when the cord makes an angle θ with the vertical, and the pivot at P is frictionless.

(A) Find the speed of the sphere when it is at the lowest point.



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Solution 2.3

- At point B: The position of the sphere is the lowest at

$$y_B = -L,$$

Therefore,

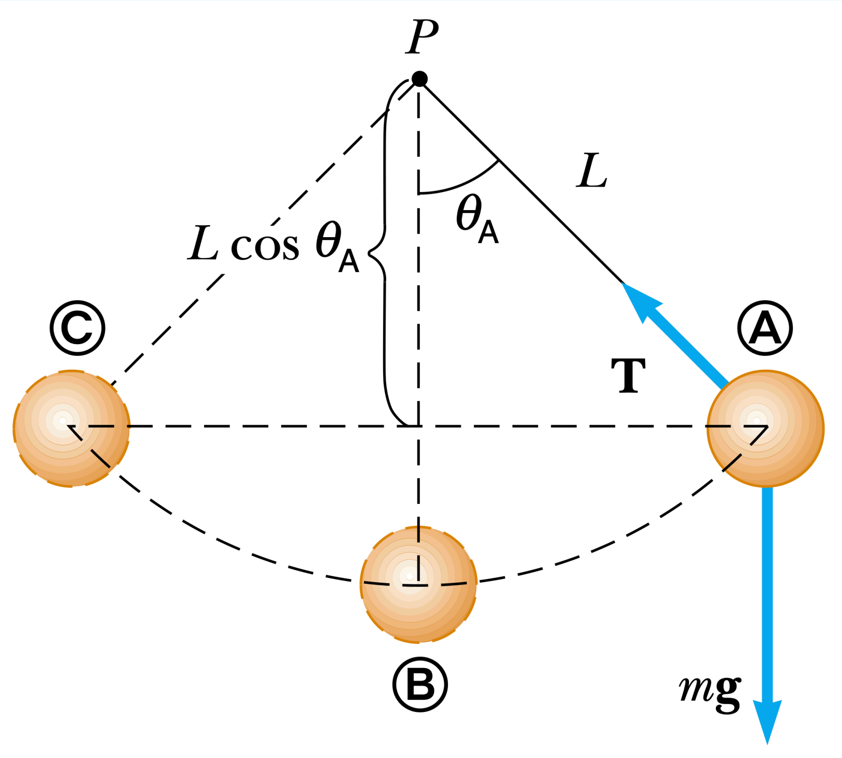
$$U_B = -mgL.$$

- At point A: the position is

$$y_A = -L\cos\theta_A,$$

Therefore,

$$U_A = -mgL\cos\theta_A.$$



2.2 Elastic Potential Energy of a Spring

- Applying the conservation of mechanical energy between points A and B:

$$K_B + U_B = K_A + U_A$$

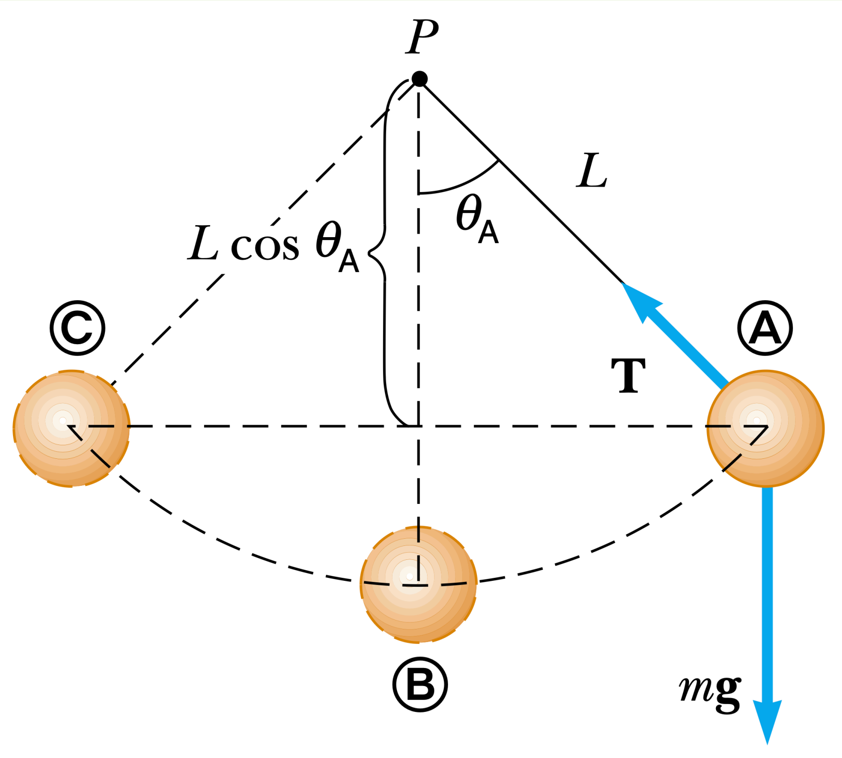
$$\frac{1}{2}mv_B^2 - mgL = 0 - mgL\cos\theta_A$$

$$\frac{1}{2}mv_B^2 = mgL(1 - \cos\theta_A)$$

$$\Longrightarrow v_B = \sqrt{2gL(1 - \cos\theta_A)}$$

Example 2.3

(B) What is the tension T_B in the cord at point B?



2.2 Elastic Potential Energy of a Spring

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- At point B, the forces acting on the sphere are the tension T_B in the cord (directed upward along the cord) and the weight mg (directed vertically downward). Applying Newton's second law in the radial direction at point B, we have:

$$\sum F_r = ma_r$$
$$T_B - mg = ma_r$$
$$T_B - mg = m\frac{v_B^2}{L}$$

- Substituting the expression for v_B obtained in part (A) into the above equation, we get:

$$T_B = mg + \frac{m}{L}[2gL(1 - \cos\theta_A)]$$
$$T_B = mg + 2mg(1 - \cos\theta_A)$$
$$T_B = mg + 2mg - 2mg\cos\theta_A$$

$$\boxed{T_B = mg(3 - 2\cos\theta_A)} \qquad (9)$$

At: $\theta_A = 0° \Longrightarrow T_B = mg,$

$\theta_A = 90° \Longrightarrow T_B = 3mg$

2.2 Elastic Potential Energy of a Spring

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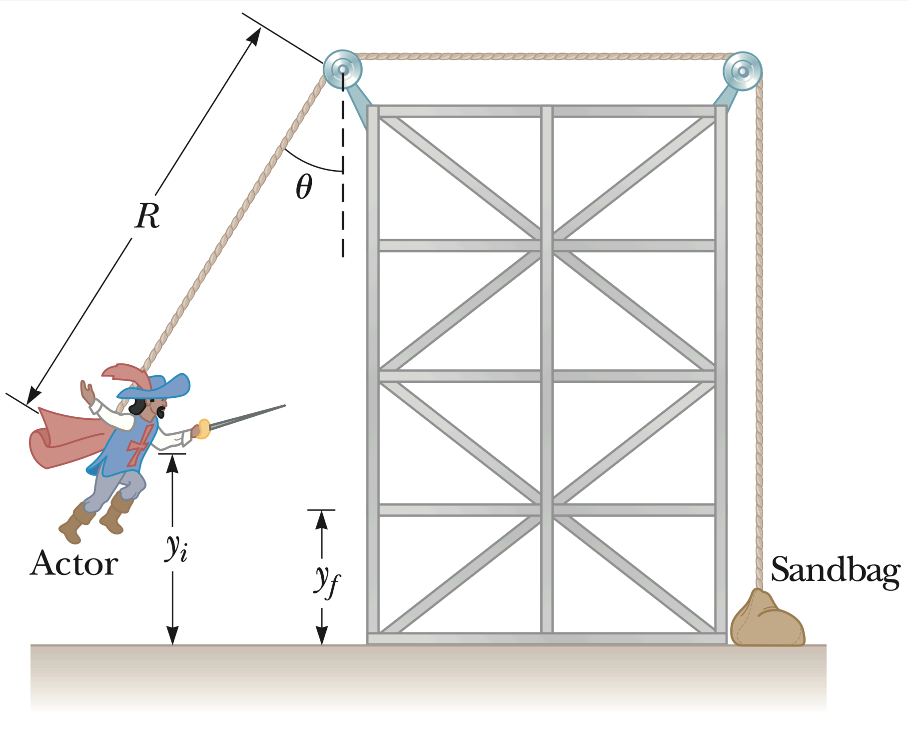
Example 2.4

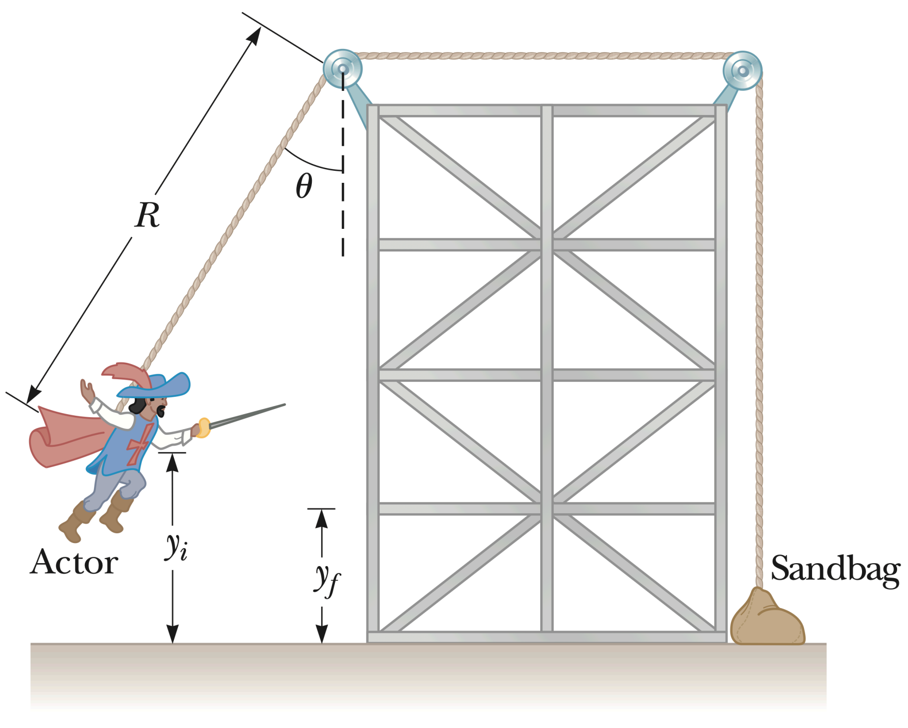
You are designing an apparatus to support an actor of mass 65 kg who is to “fly” down to the stage during the performance of a play. You attach the actor’s harness to a 130 kg sandbag by means of a lightweight steel cable running Interactive smoothly over two frictionless pulleys, as in the Figure.



2.2 Elastic Potential Energy of a Spring

You need 3 m of cable between the harness and the nearest pulley so that the pulley can be hidden behind a curtain. For the apparatus to work successfully, the sandbag must never lift above the floor as the actor swings from above the stage to the floor. Let us call the initial angle that the actor's cable makes with the vertical θ.

What is the maximum value θ can have before the sandbag lifts off the floor?



2.2 Elastic Potential Energy of a Spring

Solution 2.5

- From Equation 9, we have:

$$T = m_{\text{actor}}\, g(3 - 2\cos\theta)$$

- The tension T in the cable must be equal to the weight of the sandbag when it is about to lift off the floor:

$$T = m_{\text{bag}}\, g$$

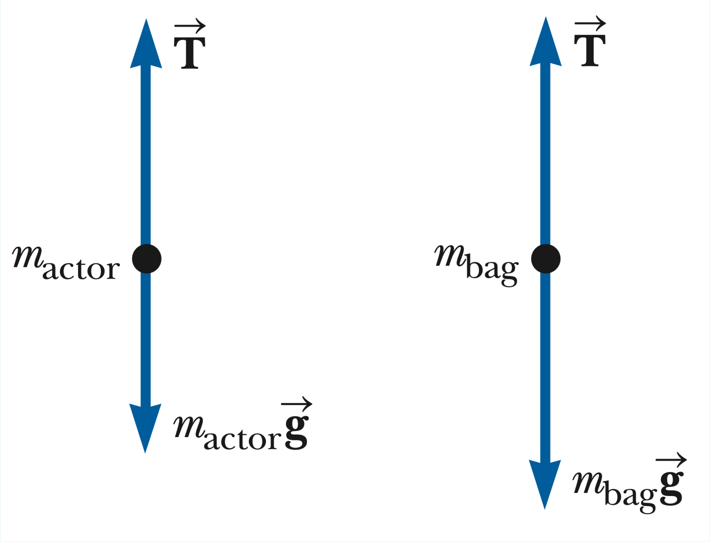
- Therefore,

$$m_{\text{bag}}\, g = m_{\text{actor}}\, g(3 - 2\cos\theta)$$

$$m_{\text{bag}} = 3m_{\text{actor}} - 2m_{\text{actor}}\cos\theta$$

$$\cos\theta = \frac{3m_{\text{actor}} - m_{\text{bag}}}{2m_{\text{actor}}}$$

$$\implies \theta = 60^\circ$$



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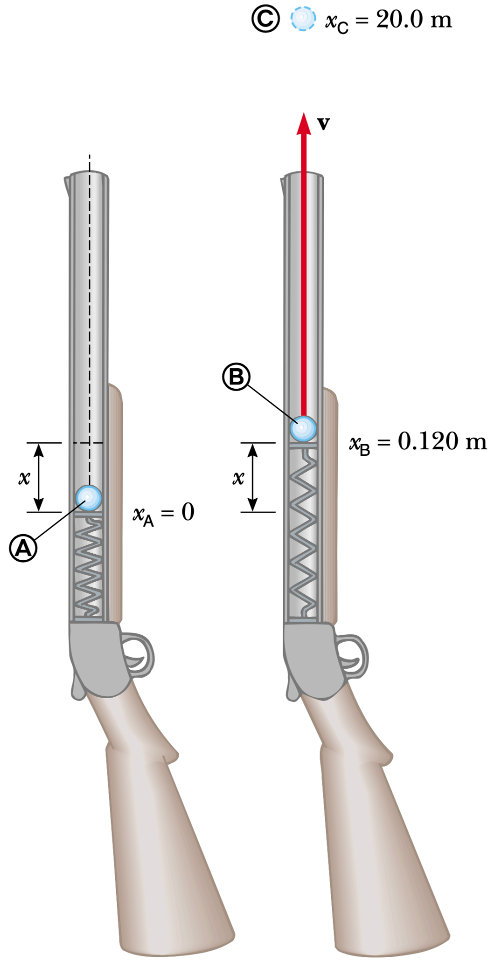
Example 2.6

The launching mechanism of a toy gun consists of a spring of unknown spring constant. When the spring is compressed 0.12 m, the gun, when fired vertically, is able to launch a 35 g projectile to a maximum height of 20 m above the position of the projectile before firing.

(A) Neglecting all resistive forces, determine the spring constant.



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Solution 2.6

$$E_C = E_A$$

$$K_C + U_{\mathrm{gC}} + U_{\mathrm{sC}} = K_A + U_{\mathrm{gA}} + U_{\mathrm{sA}}$$

$$0 + mgh + 0 = 0 + 0 + \frac{1}{2}kx^2$$

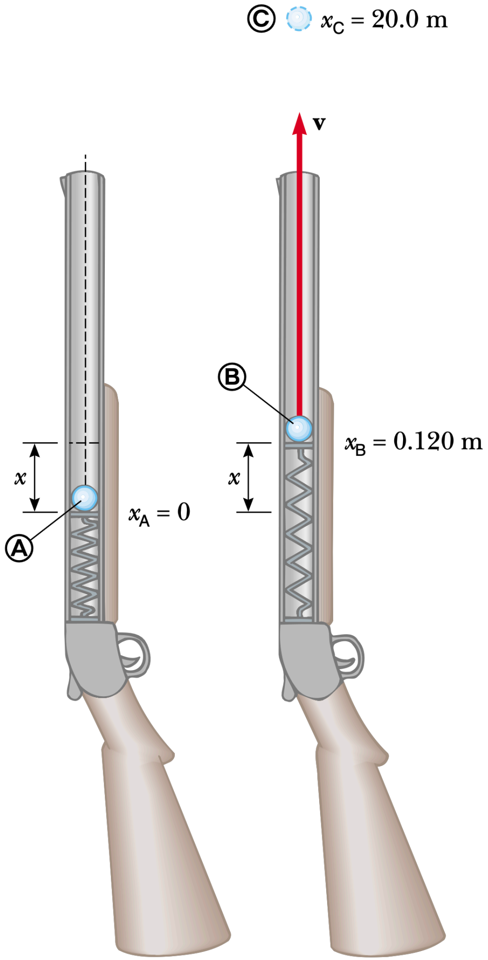
$$\Rightarrow k = \frac{2mgh}{x^2} = \frac{2(0.035\ \mathrm{kg})(9.81\ \mathrm{m/s^2})(20\ \mathrm{m})}{(0.12\ \mathrm{m})^2}$$

$$k = 953\ N/m$$

2.2 Elastic Potential Energy of a Spring

Example 2.6

(B) Find the speed of the projectile as it moves through the equilibrium position of the spring (where $x_B = 0.12\ \text{m}$) as shown in the Figure.



2.2 Elastic Potential Energy of a Spring

Solution 2.6

$$E_B = E_A$$

$$K_B + U_{\mathrm{gB}} + U_{\mathrm{sB}} = K_A + U_{\mathrm{gA}} + U_{\mathrm{sA}}$$

$$\frac{1}{2}mv_B^2 + mgx_B + 0 = 0 + 0 + \frac{1}{2}kx^2$$

Solving for v_B, we get:

$$v_B = \sqrt{\frac{kx^2}{m} - 2gx_B} = 19.7\ m/s$$

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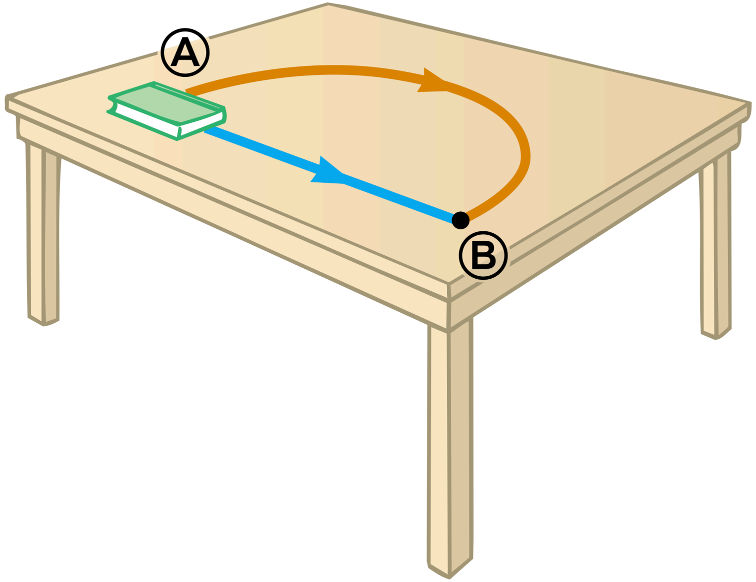
3.1 Conservative Forces

Conservative forces have two properties:

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.
2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical.)

$$\Delta E_{\text{mechanical}} = 0$$

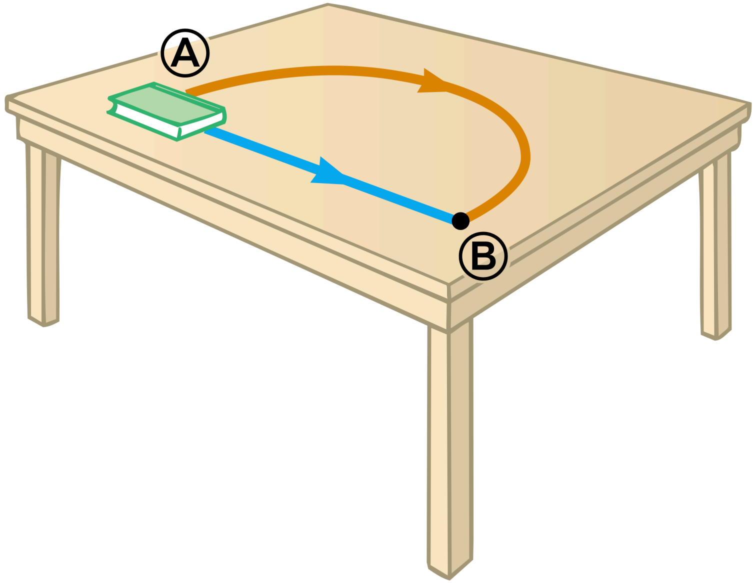


Examples: gravitational force, spring force,

3.2 Nonconservative Forces

1. The work done by a nonconservative force on a particle moving between two points depends on the path taken by the particle.
2. The work done by a nonconservative force on a particle moving through any closed path is not zero.

$$\Delta E_{\text{mechanical}} \neq 0$$



Examples: frictional force, air resistance,

1. Potential Energy of a System

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4. **Changes in Mechanical Energy for Nonconservative Forces**

5. Relationship Between Conservative Forces and Potential Energy

6. Additional Problems

4.1 Work Done by Nonconservative Forces

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When nonconservative forces are present, the change in mechanical energy of a system decreases, and for the case of friction, we can write:

$$\boxed{\Delta E = \Delta K + \Delta U = -f_k d} \tag{9}$$

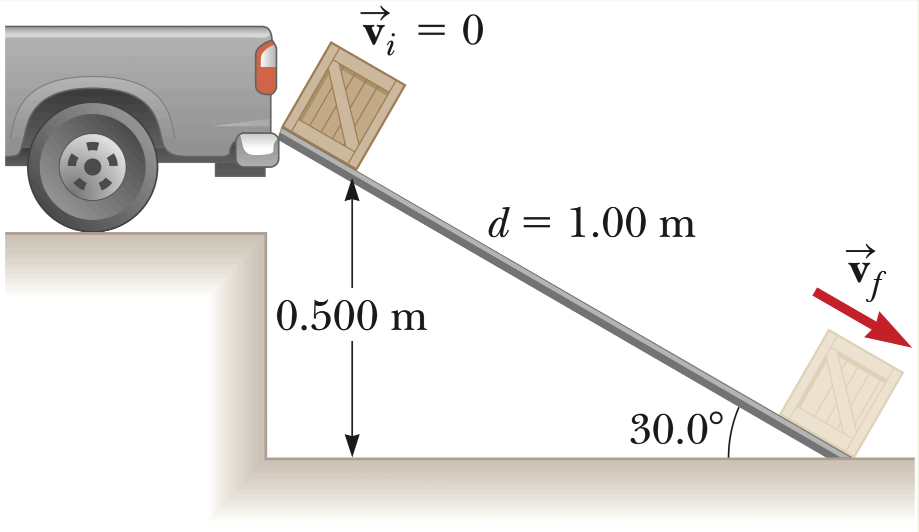
Notice that when the $f_k = 0$, we get back the conservation of mechanical energy.

4.1 Work Done by Nonconservative Forces

Example 4.7

A 3 kg crate slides down a ramp. The ramp is 1 m in length and inclined at an angle of 30°. The crate starts from rest at the top, experiences a constant friction force of magnitude 5 N, and continues to move a short distance on the horizontal floor after it leaves the ramp.

Use energy methods to determine the speed of the crate at the bottom of the ramp.



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Solution 4.7

Since there is a friction force acting on the crate,

$$\Delta E = E_f - E_i = -f_k d$$

$$E_i = K_i + U_i = 0 + mgh$$

$$E_f = K_f + U_f = \frac{1}{2}mv_f^2 + 0$$

Therefore,

$$E_f - E_i = \frac{1}{2}mv_f^2 - mgh = -f_k d$$

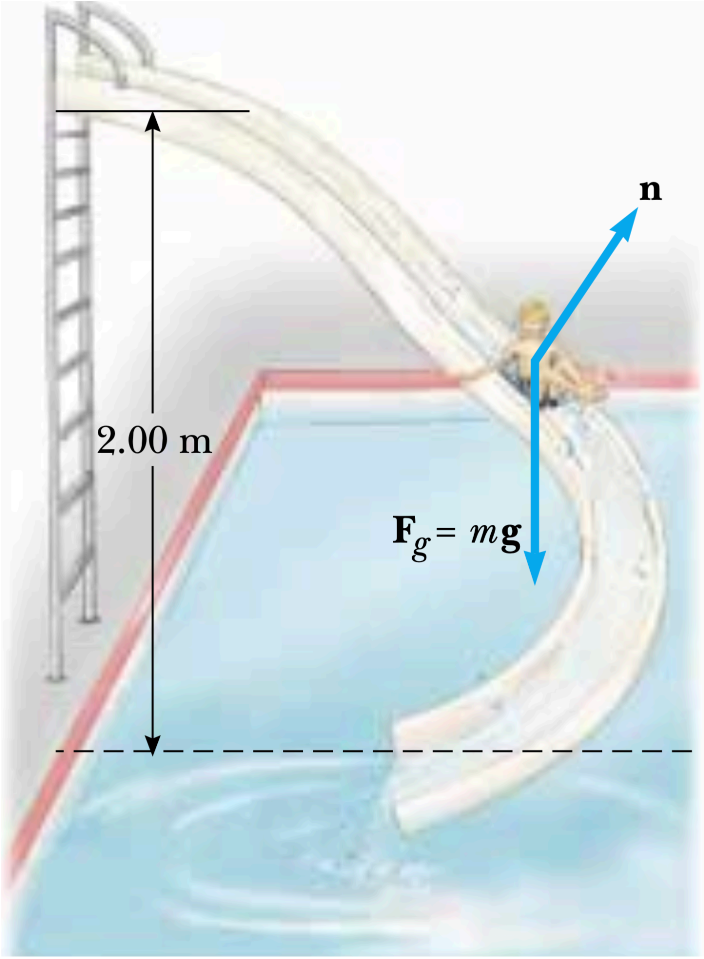
$$v_f = \sqrt{2\,\frac{mgh - f_k d}{m}} = 2.54\ \mathrm{m/s}$$

4.1 Work Done by Nonconservative Forces

Example 4.8

A child of mass m rides on an irregularly curved slide of height $h = 2$ m, as shown in the Figure. The child starts from rest at the top.

(A) Determine his speed at the bottom, assuming no friction is present.



4.1 Work Done by Nonconservative Forces

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Solution 4.8

- Since there is no friction force acting on the child, we can apply the conservation of mechanical energy between the top and the bottom of the slide:

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}mv_f^2 + 0 = 0 + mgh$$

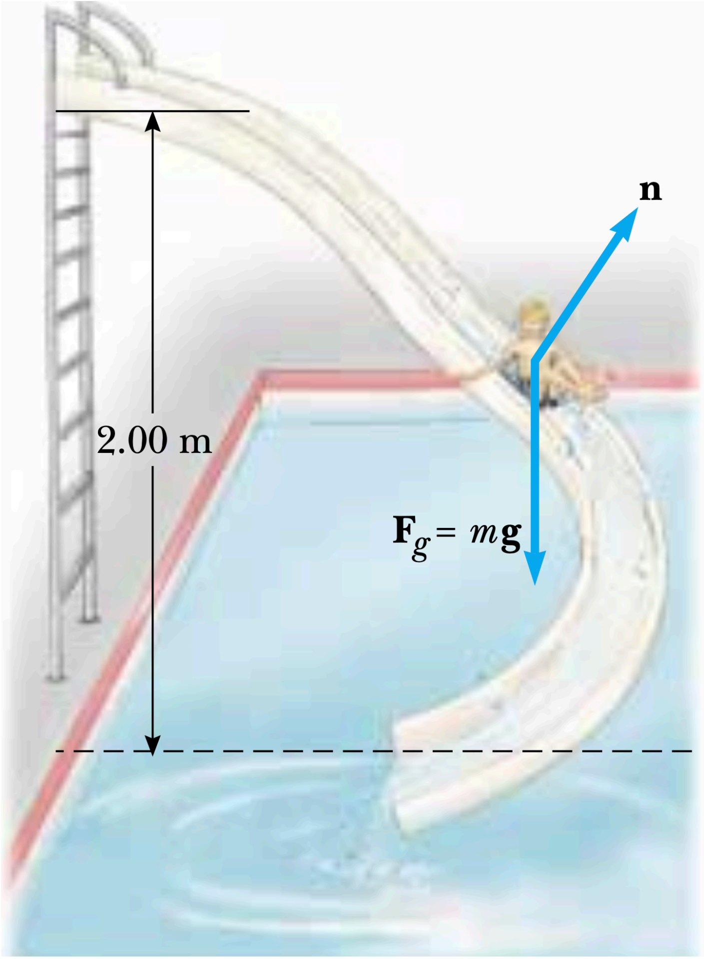
$$\frac{1}{2}mv_f^2 = mgh$$

$$\Longrightarrow v_f = \sqrt{2gh} = 6.26 \text{ m/s}$$

4.1 Work Done by Nonconservative Forces

Example 4.9

(B) If a force of kinetic friction acts on the child, **how** much mechanical energy does the system lose? Assume that $v_f = 3$ m/s and $m = 20$ kg.



4.1 Work Done by Nonconservative Forces

Solution 4.9

- Since there is a friction force acting on the child,

$$\Delta E = E_f - E_i = -f_k d$$

$$E_i = K_i + U_i = 0 + mgh$$

$$E_f = K_f + U_f = \frac{1}{2}mv_f^2 + 0$$

Therefore,

$$\Delta E = \frac{1}{2}mv_f^2 - mgh = \frac{1}{2}(20\ \text{kg})(3\ \text{m/s})^2 - (20\ \text{kg})(9.8\ \text{m/s}^2)(2\ \text{m})$$

$$\Delta E = -302\quad \text{J}$$

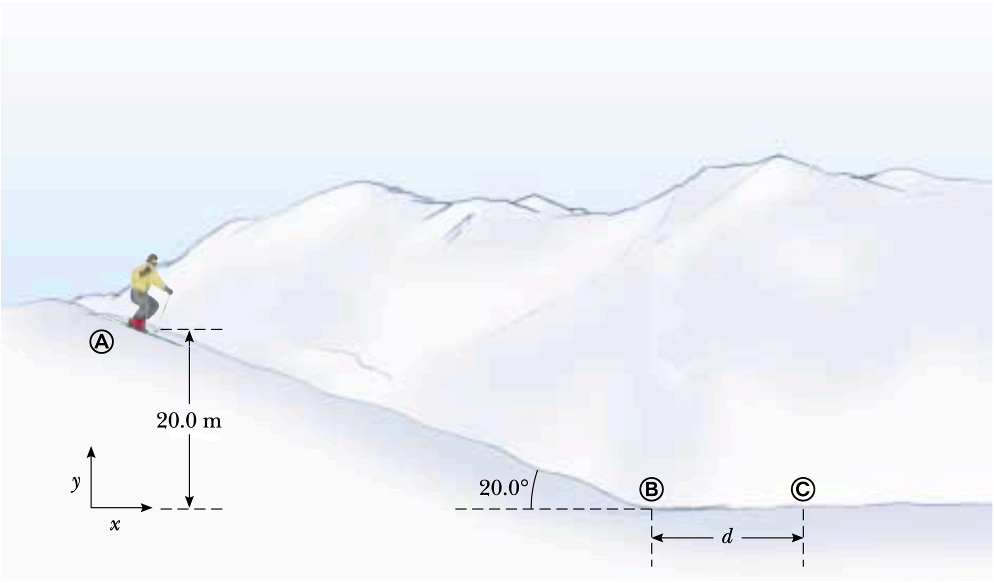
The mechanical energy **lost** due to friction is **negative** 302 J.

4.1 Work Done by Nonconservative Forces

Example 4.10

A skier starts from rest at the top of a frictionless incline of height 20 m. At the bottom of the incline, she encounters a horizontal surface where the coefficient of kinetic friction between the skis and the snow is 0.21.

How far does she travel on the horizontal surface before coming to rest, if she simply coasts to a stop?



Solution 4.10

Using conservation of mechanical energy from the top of the incline to the bottom, as we had in the previous example,

$$v_B = \sqrt{2gh} = 19.8 \text{ m/s}$$

Now, applying the work-energy principle on the horizontal surface:

$$\Delta E = E_C - E_B = -f_k d$$

$$= (0+0) - \left(\frac{1}{2}mv_B^2 + 0\right) = -(\mu_k mg)d$$

$$\Longrightarrow d = \frac{v_B^2}{2\mu_k g} = 95.2 \text{ m}$$

4.1 Work Done by Nonconservative Forces

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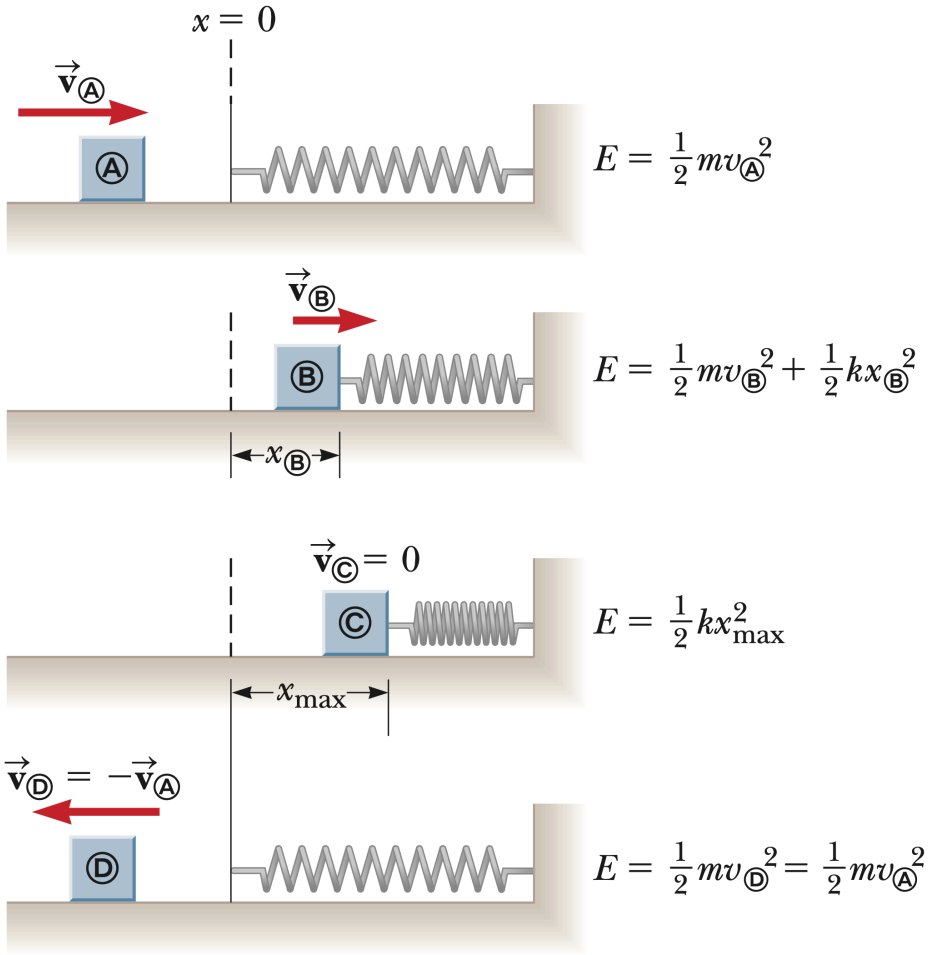
Example 4.11

A block having a mass of 0.8 kg is given an initial velocity $v_A = 1.2$ m/s to the right and collides with a spring of negligible mass and force constant $k = 50$ N/m, as shown in the Figure.

(A) Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.



4.1 Work Done by Nonconservative Forces

Solution 4.11

$$E_C = E_A$$

$$K_C + U_{\mathrm{sC}} = K_A + U_{\mathrm{sA}}$$

$$0 + \frac{1}{2}kx_{\mathrm{max}}^2 = \frac{1}{2}mv_A^2 + 0$$

$$\Longrightarrow x_{\mathrm{max}} = v_A\sqrt{\frac{m}{k}} = 0.15\ \mathrm{m}$$

4.1 Work Done by Nonconservative Forces

(B) Suppose a constant force of kinetic friction acts between the block and the surface, with $\mu_k = 0.5$. If the speed of the block at the moment it collides with the spring is $v_A = 1.2\ \mathrm{m/s}$, what is the maximum compression x_C in the spring?

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Solution 4.12

First, we find the friction force acting on the block:

$$f_k = \mu_k mg = 3.92 \text{ N}$$

Now, applying the work-energy principle between points A and C:

$$\Delta E = E_C - E_A = \left(0 + \frac{1}{2}kx_c^2\right) - \left(\frac{1}{2}mv_A^2 + 0\right) = -f_k x_C$$

$$= 25x_C^2 - 0.576 = -3.92x_C$$

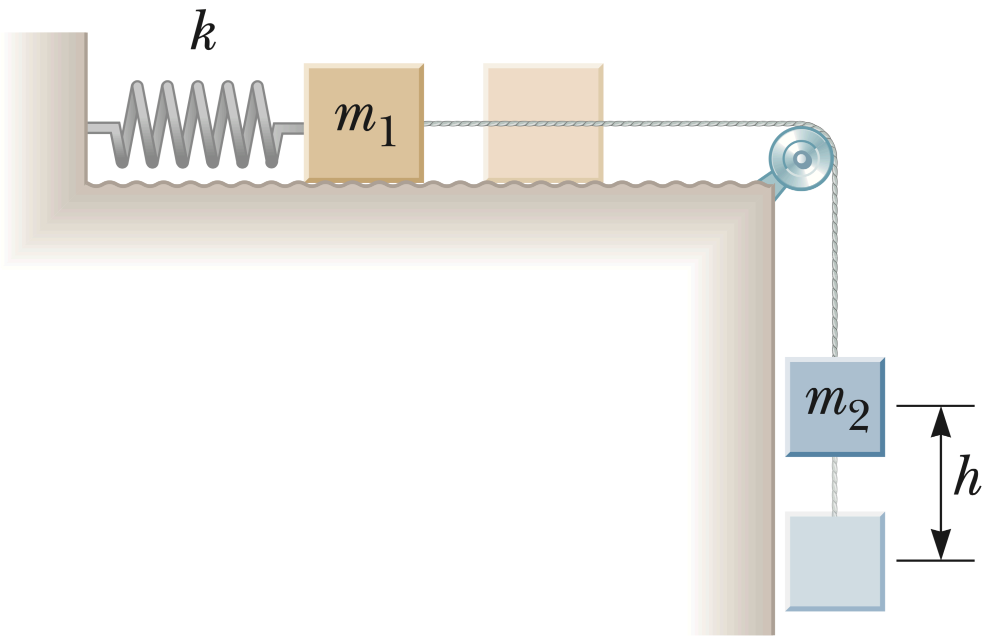
$$= 25x_C^2 + 3.92x_C - 0.576 = 0$$

$$\implies x_C = +0.092 \text{ m}$$

4.1 Work Done by Nonconservative Forces

Example 4.13

Two blocks are connected by a light string that passes over a frictionless pulley. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.



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Solution 4.13

- Since the blocks start from rest and come to rest again, then $K_i = K_f = 0$.
- Applying the work-energy principle between points A and B:

$$\begin{aligned} \Delta E = \Delta U_g + \Delta U_s &= -f_k h \\ &= \left(U_{gf} - U_{gi}\right) + \left(U_{sf} - U_{si}\right) = -(\mu_k m_1 g)h \\ &= (0 - m_2 g h) + \left(\frac{1}{2}kh^2 - 0\right) = -\mu_k m_1 g h \end{aligned}$$

- Rearranging the above equation to solve for μ_k, we get:

$$\mu_k = \frac{m_2 g - \frac{1}{2}kh}{m_1 g}$$

1. Potential Energy of a System

2. The Isolated System Conservation of Mechanical Energy

3. Conservative and Nonconservative Forces

4. Changes in Mechanical Energy for Nonconservative Forces

5. Relationship Between Conservative Forces and Potential Energy

6. Additional Problems

5.1 Deriving Force from Potential Energy

- A conservative force can be derived from the potential energy function $U(x)$ by taking the negative derivative of the potential energy with respect to position:

$$F(x) = -\frac{\mathrm{d}U}{\mathrm{d}x}$$

- This relationship indicates that the force exerted by a conservative force field is directed towards decreasing potential energy.
- The derivation comes from the work-energy theorem,

$$W_c = -\Delta U = \int_a^b F(x)\,\mathrm{d}x$$

5.1 Deriving Force from Potential Energy

- In three dimensions, the force components can be derived from the potential energy function $U(x, y, z)$ as follows:

$$\boxed{F_x = -\frac{\partial U}{\partial x}, \quad F_y = -\frac{\partial U}{\partial y}, \quad F_z = -\frac{\partial U}{\partial z}} \tag{10}$$

- The force vector can be expressed as:

$$\vec{\boldsymbol{F}}(x, y, z) = F_x \hat{\boldsymbol{\imath}} + F_y \hat{\boldsymbol{\jmath}} + F_z \hat{\boldsymbol{k}}$$

5.1 Deriving Force from Potential Energy

Example 5.14

Find the conservative force of the following potential energy functions:

$$1. \qquad U = \frac{1}{2}kx^2$$

$$2. \qquad U = mgy$$

Solution 5.14

$$1. \qquad F = -\frac{\partial U}{\partial x} = -kx$$

$$2. \qquad F = -\frac{\partial U}{\partial y} = -mg$$

5.1 Deriving Force from Potential Energy

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Problem 5.1

A potential-energy function for a two-dimensional force is of the form

$$U = 3x^3y - 7x$$

Find the force that acts at the point (x, y)

Answer 5.1

$$F_x = -\frac{\partial U}{\partial x} = -(9x^2y - 7) = -9x^2y + 7$$

$$F_y = -\frac{\partial U}{\partial y} = -(3x^3) = -3x^3$$

$$\Longrightarrow \vec{\boldsymbol{F}}(x, y) = (-9x^2y + 7)\hat{\boldsymbol{\imath}} - 3x^3\hat{\boldsymbol{\jmath}}$$

6. Additional Problems

2, 5, 6, 11, 13, 17, 31, 33, 36, 38, 42, 55, 57, 59, 60

6.1 Selected Problems

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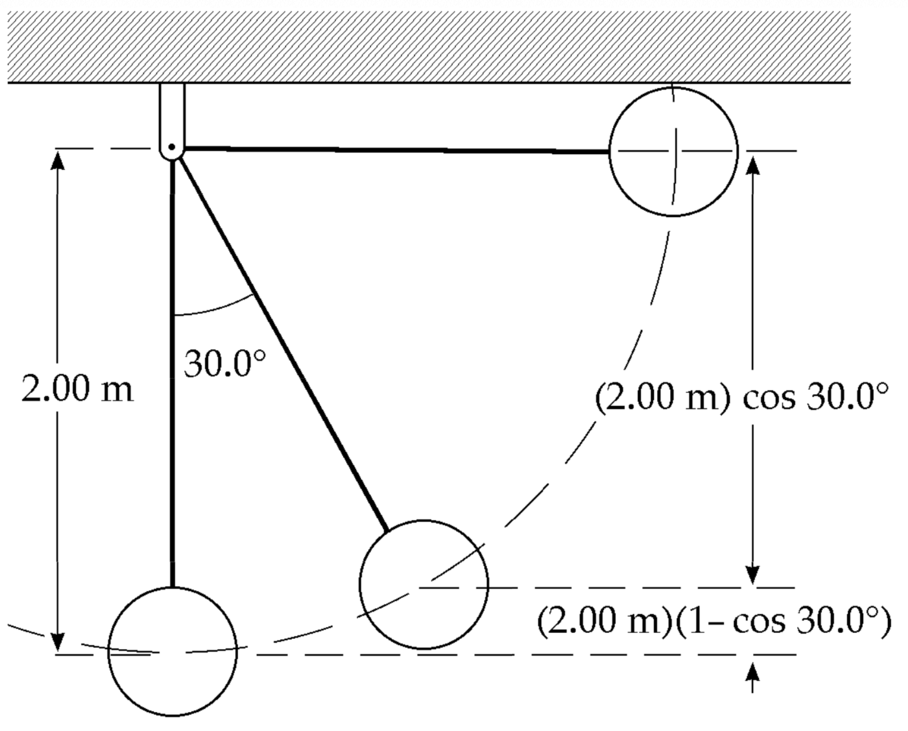
Problem 6.2

A 400N child is in a swing that is attached to ropes 2 m long. Find the gravitational potential energy of the child-Earth system relative to the child's lowest position when:

(a) the ropes are horizontal,
(b) the ropes make a 30° angle with the vertical, and
(c) the child is at the bottom of the circular arc.



6.1 Selected Problems

Answer 6.2

(a)

$$U_g = (mg)h = (400 \text{ N})(2 \text{ m}) = 800 \text{ J}$$

(B)

$$U_g = (mg)h = (400 \text{ N})(2 \text{ m})(1 - \cos 30°) = 107 \text{ J}$$

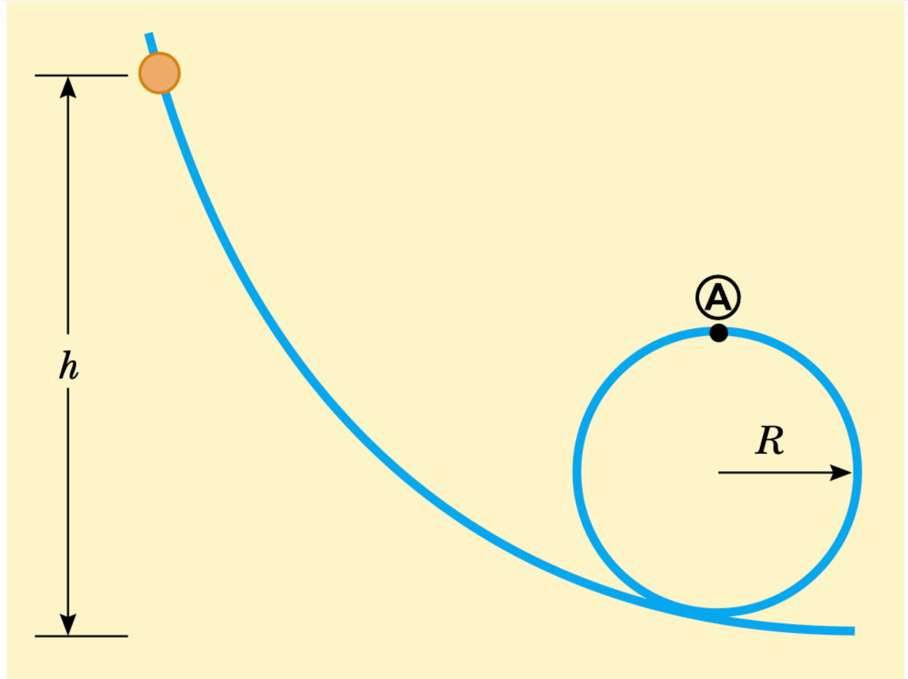
(C)

$$U_g = (mg)h = (400 \text{ N})(0 \text{ m}) = 0 \text{ J}$$

Problem 6.3

A bead (خرزة) slides without friction around a loop-the-loop. The bead is released from a height $h = 3.5R$.

(a) What is its speed at point A?
(b) How large is the normal force on it if its mass is 5 g?



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Answer 6.3

(a)

$$E_i = E_f$$

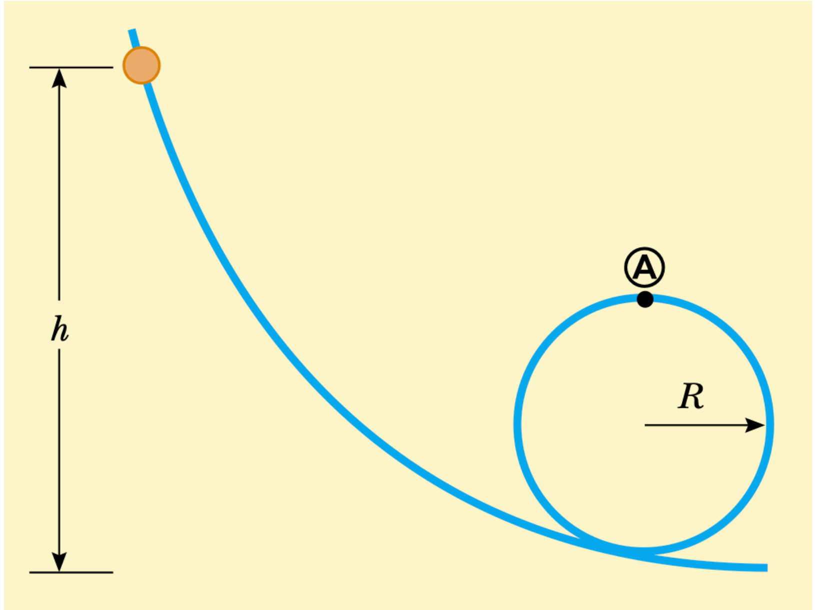
$$U_i + K_i = U_f + K_f$$

$$mgh + 0 = mg(2R) + \frac{1}{2}mv_A^2$$

$$\implies \frac{1}{2}v_A^2 = gh - 2gR = g(h - 2R) = g(3.5R - 2R) = 1.5gR$$

$$v_A = \sqrt{3gR}$$

(b)



$$\sum F_r = m\frac{v_A^2}{R}$$

Since both the normal force n_A and the weight mg act toward the center of the loop at point A **parallel to the central acceleration**, we have:

$$n_A + mg = m\frac{v_A^2}{R}$$

$$n_A = m\left(\frac{v_A^2}{R} - g\right)$$

$$n_A = m\left(\frac{3gR}{R} - g\right)$$

$$= 2mg = 0.098\ \mathrm{N}$$

6.1 Selected Problems

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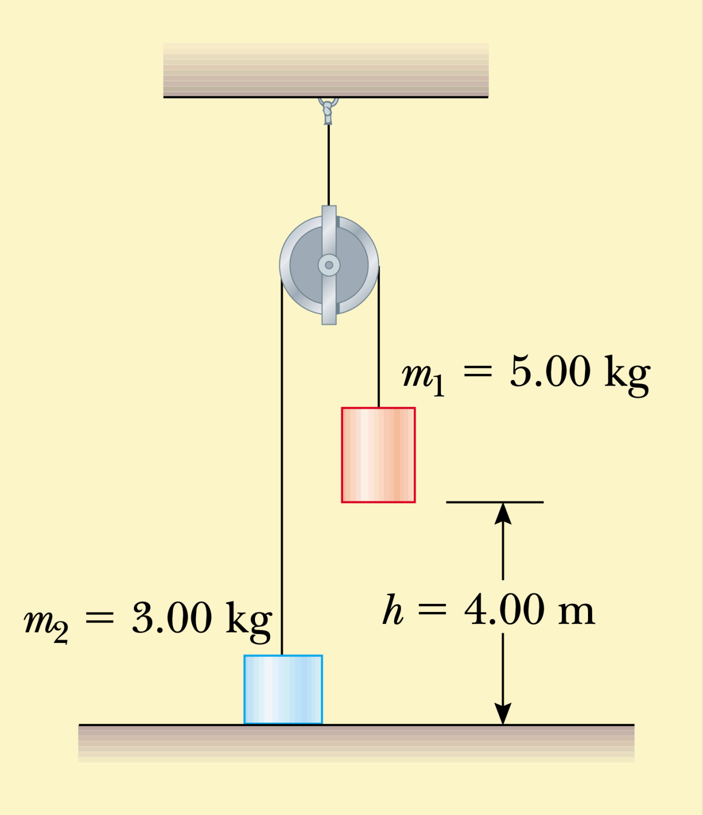
Problem 6.4

Two objects are connected by a light string passing over a light frictionless pulley as shown in the Figure. The object of mass 5 kg is released from rest. Using the principle of conservation of energy,

(a) determine the speed of the 3 kg object just as the 5 kg object hits the ground.

(b) Find the maximum height to which the 3 kg object rises.



6.1 Selected Problems

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Answer 6.4

(a)

$$E_1 = E_2$$
$$U_i + K_i = U_f + K_f$$
$$m_1 gh + m_2 g(0) + 0 + 0 = m_2 gh + m_1 g(0) + \frac{1}{2}(m_1 + m_2)v^2$$
$$m_1 gh = m_2 gh + \frac{1}{2}(m_1 + m_2)v^2$$
$$\implies v = 4.43 \text{ m/s}$$

6.1 Selected Problems

(b) Since the 3 kg object rises to a maximum height h_f, at which point its speed is zero, we can apply the conservation of mechanical energy between the position $h = 4$ m and the maximum height:

$$E_i = E_f \qquad \text{(Only for the 3 Kg object)}$$

$$U_i + K_i = U_f + K_f$$

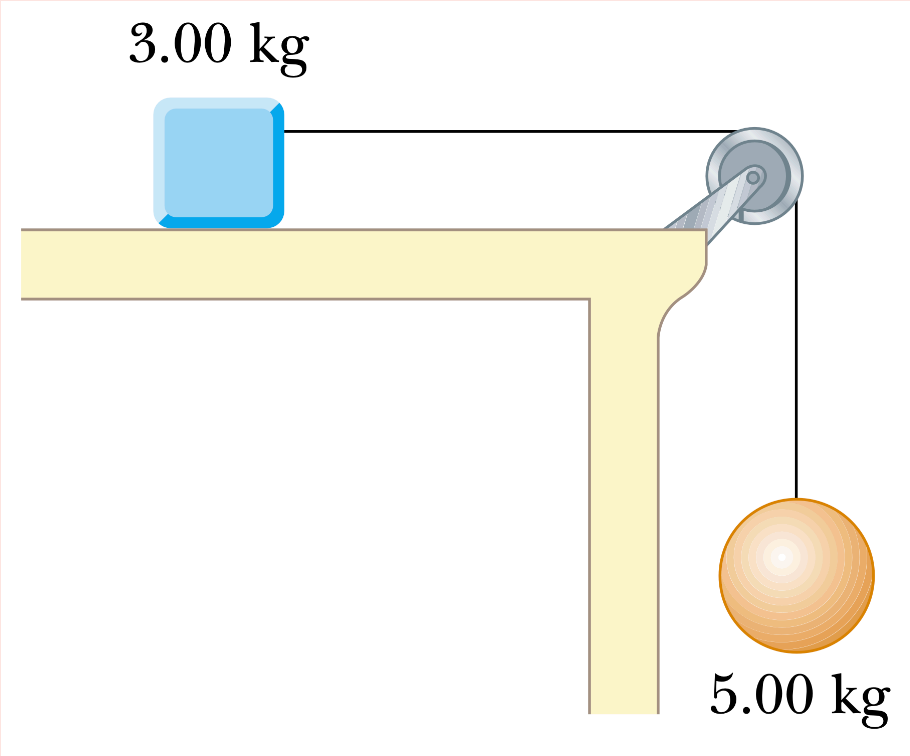
$$m_2 g h + \frac{1}{2} m_2 v^2 = m_2 g h_f + 0$$

$$\Longrightarrow h_f = h + \frac{v^2}{2g} = 4 \text{ m} + \frac{(4.43 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 5 \text{ m}$$

6.1 Selected Problems

Problem 6.5

The coefficient of friction between the $m_1 = 3$ kg block and the surface in the Figure is 0.4. The system starts from rest. What is the speed of the $m_2 = 5$ kg ball when it has fallen 1.5 m?



6.1 Selected Problems

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Answer 6.5

Since there is a friction, the conservation of energy of the system becomes:

$$\Delta E = -f_k d$$

$$E_f - E_i = -(\mu_k m_1 g)h$$

$$U_f + K_f - U_i - K_i = -(\mu_k m_1 g)h$$

$$\left[(0+0) + \left(\frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2\right)\right] - [(m_2 gh + 0) - (0+0)] = -(\mu_k m_1 g)h$$

$$\frac{1}{2}(m_1 + m_2)v^2 = m_2 gh - (\mu_k m_1 g)h$$

$$\Longrightarrow v = 3.74 \text{ m/s}$$

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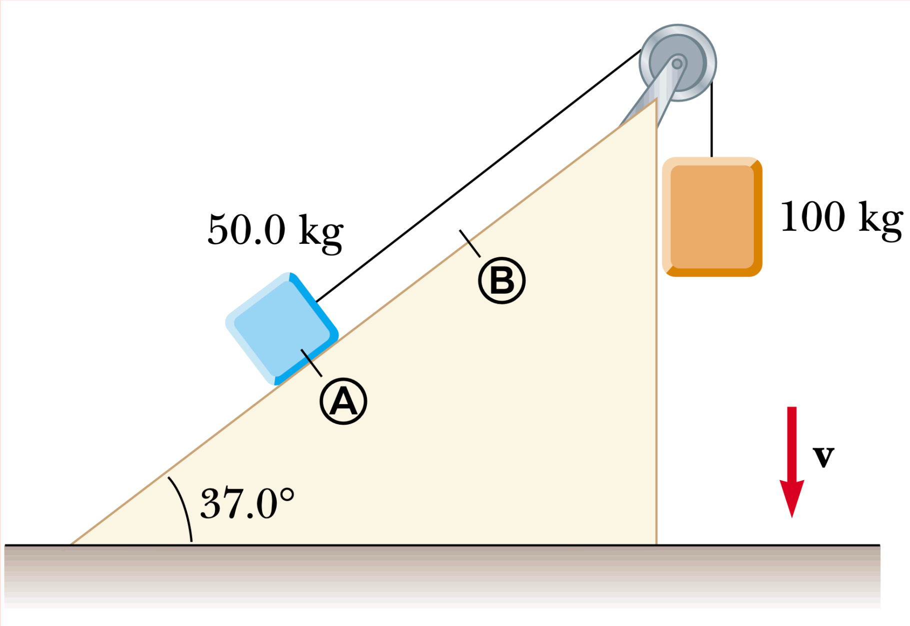
Problem 6.6

An $m_1 = 50$ kg block and an $m_2 = 100$ kg block are connected by a string as in the Figure. The pulley is frictionless and of negligible mass. The coefficient of kinetic friction between the 50 kg block and incline is 0.25.

Determine the change in the kinetic energy of the 50 kg block as it moves from (A) to (B) a distance of 20 m.



6.1 Selected Problems

Answer 6.6

Since there is a friction, the conservation of energy of the system becomes:

$$\Delta E = -f_k d$$

$$E_f - E_i = -(\mu_k m_1 g \cos\theta) d$$

$$U_f + K_f - U_i - K_i = -(\mu_k m_1 g \cos\theta) d$$

$$\left[(m_1 g h_1 + 0) + \left(\frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2\right)\right] - [(m_2 g h_2 + 0) - (0)] = -(\mu_k m_1 g \cos\theta) d$$

$$\left[m_1 g h_1 + \frac{1}{2}(m_1 + m_2) v^2\right] - [m_2 g h_2] = -(\mu_k m_1 g \cos\theta) d$$

$$\frac{1}{2} m_1 v^2 = -\frac{1}{2} m_2 v^2 - m_1 g h_1 + m_2 g h_2 - (\mu_k m_1 g \cos\theta) d \qquad \text{(Eq1)}$$

6.1 Selected Problems

Since $m_2 = 2m_1$, $K_1 = \frac{1}{2}m_1 v^2$, and $h_1 = h_2 \sin\theta$, we can rewrite Eq1 as:

$$K_1 = -2K_1 - m_1 g(h_2 \sin\theta) + (2m_1 g h_2) - (\mu_k m_1 g \cos\theta)d$$
$$3K_1 = m_1 g h_2 (2 - \sin\theta) - (\mu_k m_1 g \cos\theta)d$$

$$\implies K_1 = \frac{m_1 g h_2 (2 - \sin\theta) - (\mu_k m_1 g \cos\theta)d}{3}$$

$$K_1 = 3915 \quad \text{J}$$

Note that: $d = h_2 = 20$ m