Ch.7: Energy and Energy Transfer

Physics 103: Classical Mechanics

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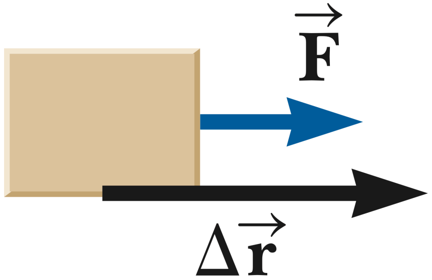
Version 1

Outline

1. **Work Done by a Constant Force**
2. The Scalar Product of Two Vectors
3. Work Done by a Varying Force
4. Kinetic Energy and the Work-Kinetic Energy Theorem
5. The Nonisolated System—Conservation of Energy
6. Situations Involving Kinetic Friction
7. Power
8. Additional Problems

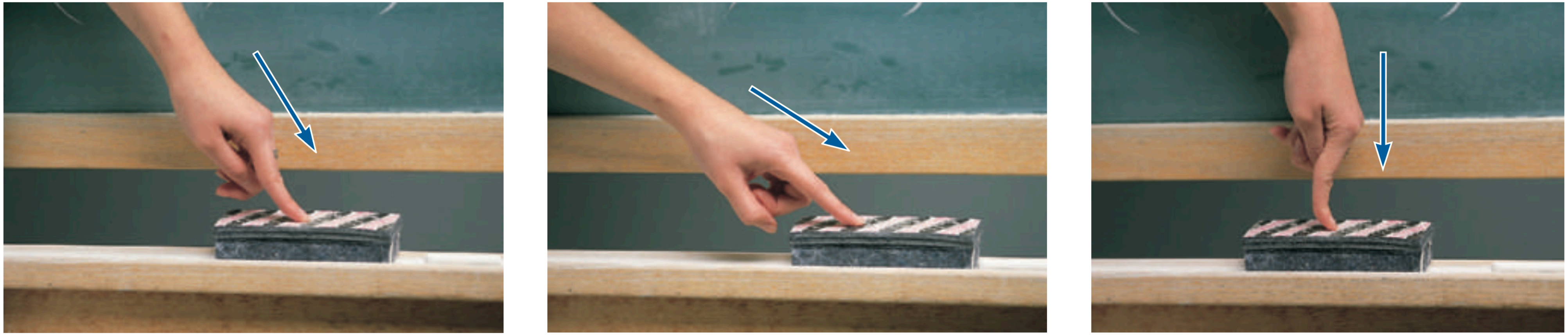
1.1 What is Work?

- Work (W) is the energy transferred to an object by a Force ($\vec{F}$) that causes the object to move a distance ($\Delta\vec{r}$).



- The more distance the object moves, the more energy is transferred to the object and the greater the work done on the object.
- The greater the force applied to the object, the greater the work and energy transferred to the object.

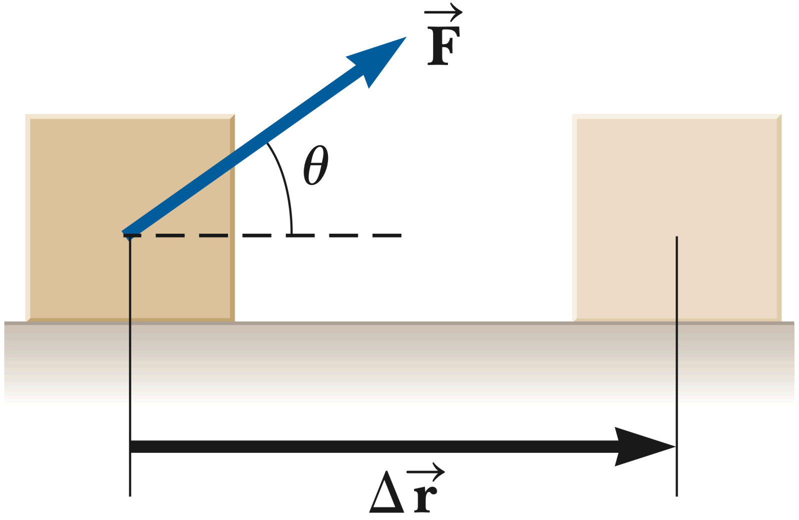
1.2 What if the Force is Not in the Direction of Motion?



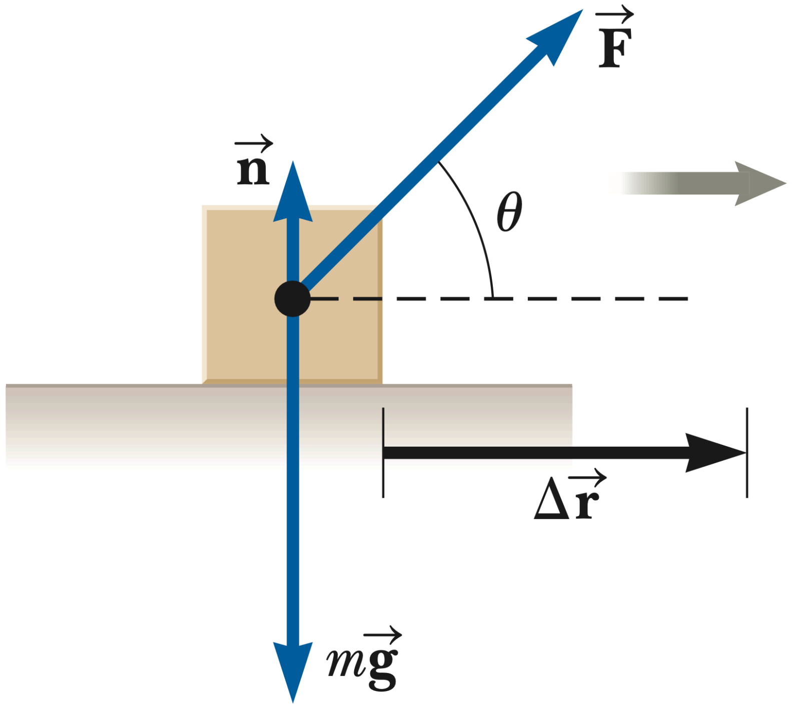
If the force is equal for all the three figures, which force is more effective in moving the box?

1.3 The Mathematical Definition of Work

$$\boxed{W = F\Delta r\cos\theta}$$

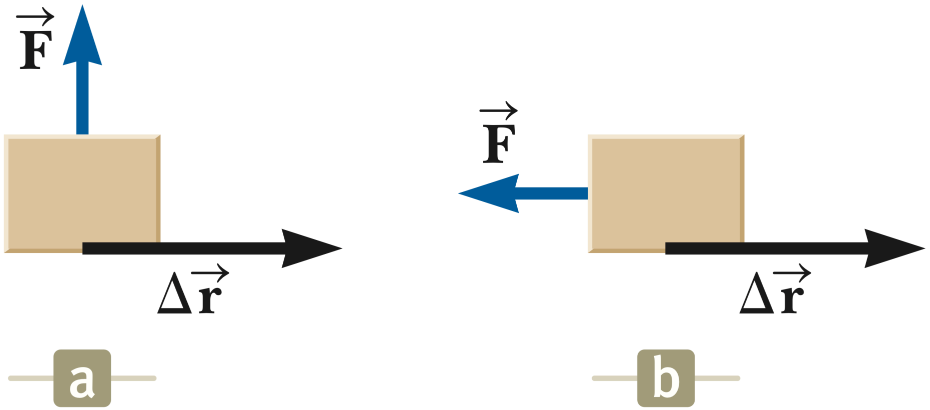


1.4 Cases



Which force results in non-zero work?

Only $\vec{F}$ along the direction of motion.



What is the work done by the force in each case?

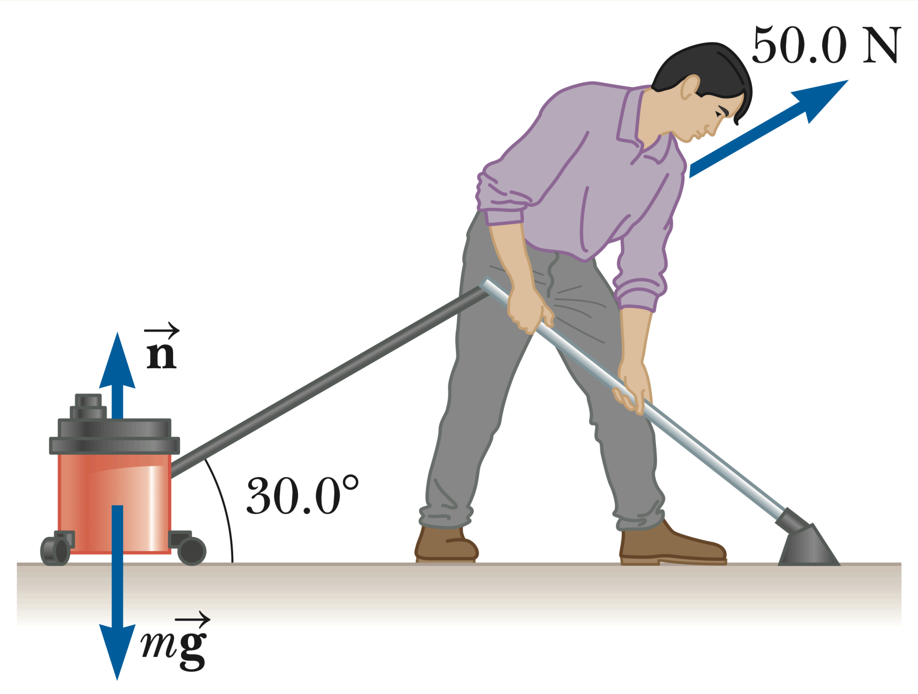
(a) Zero work

(b) Negative work

1.5 Examples

Example 1.1

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F = 50$ N at an angle of 30° with the horizontal. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3 m to the right.



1.5 Examples

Solution 1.1

$$W = F\Delta r\cos\theta = (50\ \mathrm{N})(3\ \mathrm{m})\cos 30° = 130\ \mathrm{J}$$

1. Work Done by a Constant Force

2. The Scalar Product of Two Vectors

3. Work Done by a Varying Force

4. Kinetic Energy and the Work-Kinetic Energy Theorem

5. The Nonisolated System—Conservation of Energy

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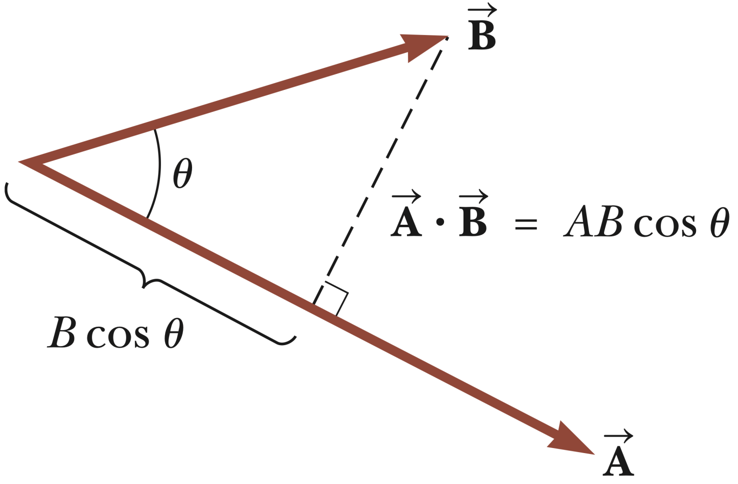
7. Power

8. Additional Problems

2.1 What is the Scalar Product?

- It is helpful to use a convenient notation for multiplying two vectors to get a scalar quantity taking into account the angle between them.
- When two vectors $\vec{A}$ and $\vec{B}$ are multiplied to get a scalar quantity, the operation is called the scalar product (or dot product) and is denoted by a dot $(\cdot)$ between the two vectors.



2.2 Rewriting the Work Formula

- The work done by a constant force can be expressed in terms of the scalar product of two vectors as:

$$\boxed{W = F\Delta r\cos\theta = \vec{\boldsymbol{F}} \cdot \Delta\vec{\boldsymbol{r}}}$$

- Notice that $F\cos\theta$ is the component of the force in the direction of the displacement. Or the projection of the force vector onto the displacement vector.

2.3 Properties of the Scalar Product

- The scalar product of two vectors is commutative:

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}}$$

- The scalar product of a vector with itself is equal to the square of the magnitude of the vector:

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{A}} = A^2$$

- The scalar product of two perpendicular ($\perp$) vectors is zero:

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = 0 \; , \qquad \text{If} \;\; \vec{\boldsymbol{A}} \perp \vec{\boldsymbol{B}}$$

2.3 Properties of the Scalar Product

- The scalar product of two parallel ($\rightrightarrows$) vectors is equal to the product of their magnitudes:

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = AB \,, \qquad \text{If} \;\; \vec{\boldsymbol{A}} \rightrightarrows \vec{\boldsymbol{B}}$$

- The scalar product of two anti-parallel vectors is equal to the negative product of their magnitudes:

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = -AB \,, \qquad \text{If} \;\; \vec{\boldsymbol{A}} \rightleftarrows \vec{\boldsymbol{B}}$$

- The scalar product is distributive over vector addition:

$$\vec{\boldsymbol{A}} \cdot \left(\vec{\boldsymbol{B}} + \vec{\boldsymbol{C}} \right) = \vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} + \vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{C}}$$

2.3 Properties of the Scalar Product

- The scalar product of a vector and a scalar multiple of another vector is:

$$\vec{A} \cdot \left(k\vec{B}\right) = k\left(\vec{A} \cdot \vec{B}\right)$$

where k is a scalar.

- The scalar product of two vectors can be expressed in terms of their components:

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

where $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ are the unit vectors in the x, y, and z directions, respectively.

2.3 Properties of the Scalar Product

$$\hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\imath}} = \hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{\jmath}} = \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}} = 1$$

$$\hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\jmath}} = \hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{k}} = \hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{k}} = 0$$

- The scalar product of two vectors can be used to find the angle between them:

$$\cos\theta = \frac{\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}}}{AB}$$

where θ is the angle between the vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$.

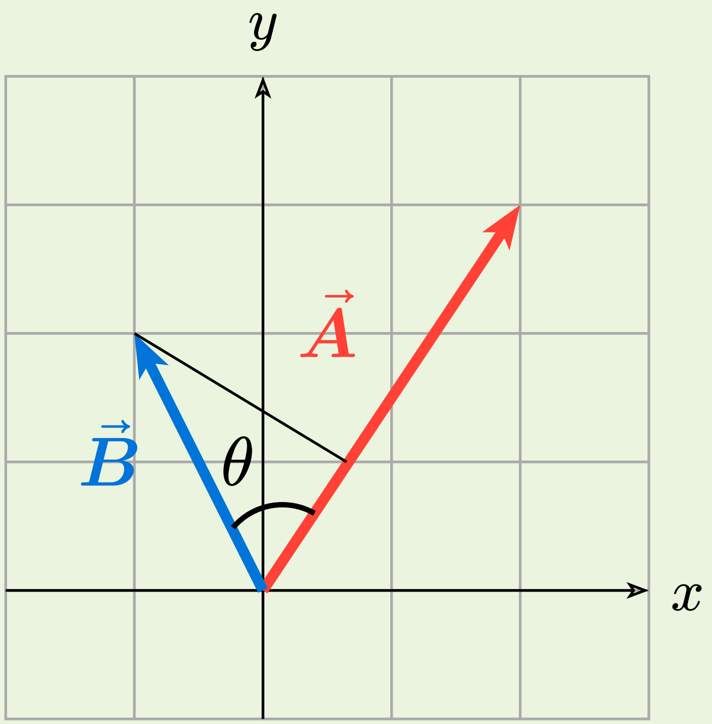
Example 2.2

The vectors $\vec{A}$ and $\vec{B}$ are given by

$$\vec{A} = 2\hat{\imath} + 3\hat{\jmath}$$
$$\vec{B} = -\hat{\imath} + 2\hat{\jmath}$$

(A) Determine the scalar product $\vec{A} \cdot \vec{B}$

(B) Find the angle θ between $\vec{A}$ and $\vec{B}$.



Solution 2.2 **(A)**

$$\vec{A}\cdot\vec{B} = (2\hat{\imath} + 3\hat{\jmath})\cdot(-\hat{\imath} + 2\hat{\jmath})$$

$$\vec{A}\cdot\vec{B} = [(2)(-1)(\hat{\imath}\cdot\hat{\imath})] + [(2)(2)(\hat{\imath}\cdot\hat{\jmath})] + [(3)(-1)(\hat{\jmath}\cdot\hat{\imath})] + [(3)(2)(\hat{\jmath}\cdot\hat{\jmath})]$$

$$\vec{A}\cdot\vec{B} = \quad (2)(-1)(1) \quad + \quad (2)(2)(0) \quad + \quad (3)(-1)(0) \quad + \quad (3)(2)(1)$$

$$\vec{A}\cdot\vec{B} = (2)(-1) + (3)(2) = -2 + 6 = 4$$

- The same answer can be obtained using the component form of the scalar product:

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y = (2)(-1) + (3)(2) = -2 + 6 = 4$$

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Solution 2.2 (B)

$$\cos\theta = \frac{\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}}}{AB}$$

$$\cos\theta = \frac{4}{\sqrt{2^2 + 3^2} \times \sqrt{(-1)^2 + 2^2}}$$

$$\theta = \cos^{-1}\left(\frac{4}{\sqrt{65}}\right) = 60.3^\circ$$

2.4 Examples

Example 2.3

A particle moving in the xy plane undergoes a displacement given by

$$\Delta\vec{\boldsymbol{r}} = (2\hat{\boldsymbol{\imath}} + 3\hat{\boldsymbol{\jmath}}) \quad \text{m}$$

as a constant force

$$\vec{\boldsymbol{F}} = (5\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}}) \quad \text{N}$$

acts on the particle.

Calculate the work done by $\vec{\boldsymbol{F}}$ on the particle.

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Solution 2.3

$$W = \vec{\boldsymbol{F}} \cdot \Delta\vec{\boldsymbol{r}} = (5\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}}) \cdot (2\hat{\boldsymbol{\imath}} + 3\hat{\boldsymbol{\jmath}})$$

$$W = (5)(2) + (2)(3) = 10 + 6 = 16 \text{ J}$$

1. Work Done by a Constant Force

2. The Scalar Product of Two Vectors

3. **Work Done by a Varying Force**

4. Kinetic Energy and the Work-Kinetic Energy Theorem

5. The Nonisolated System—Conservation of Energy

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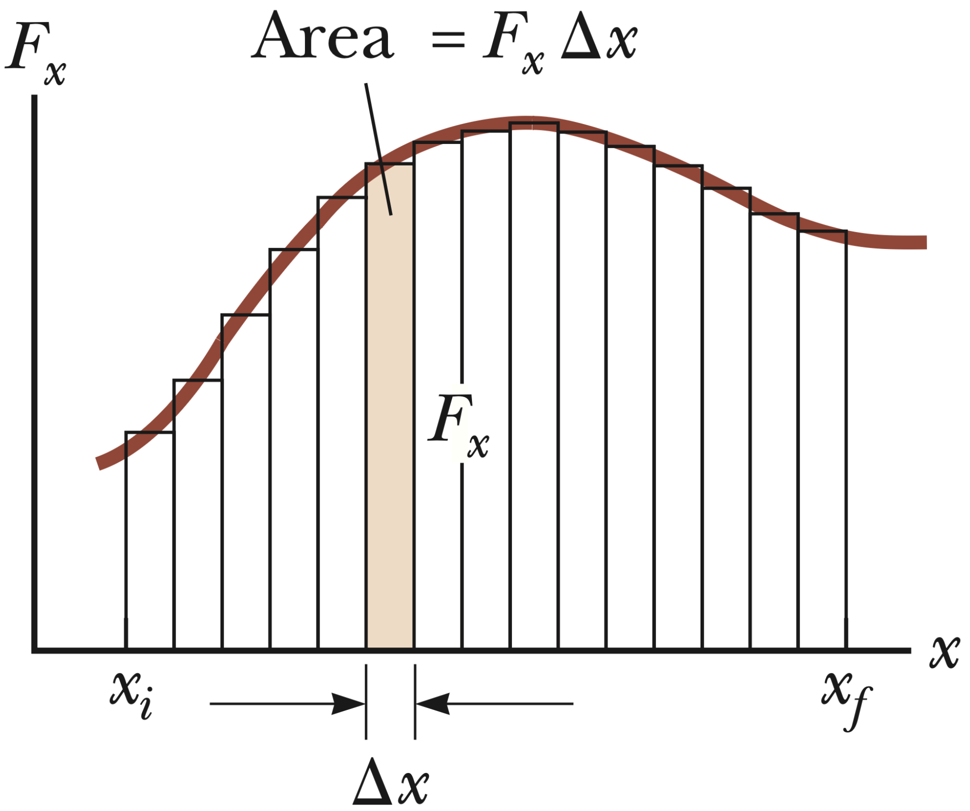
8. Additional Problems

3.1 Definition

- When a force varies in magnitude and/or direction as an object moves, the work done by the force can be found by dividing the total displacement into small segments over which the force can be considered constant.
- The total work done by the force is the sum of the work done over each segment.

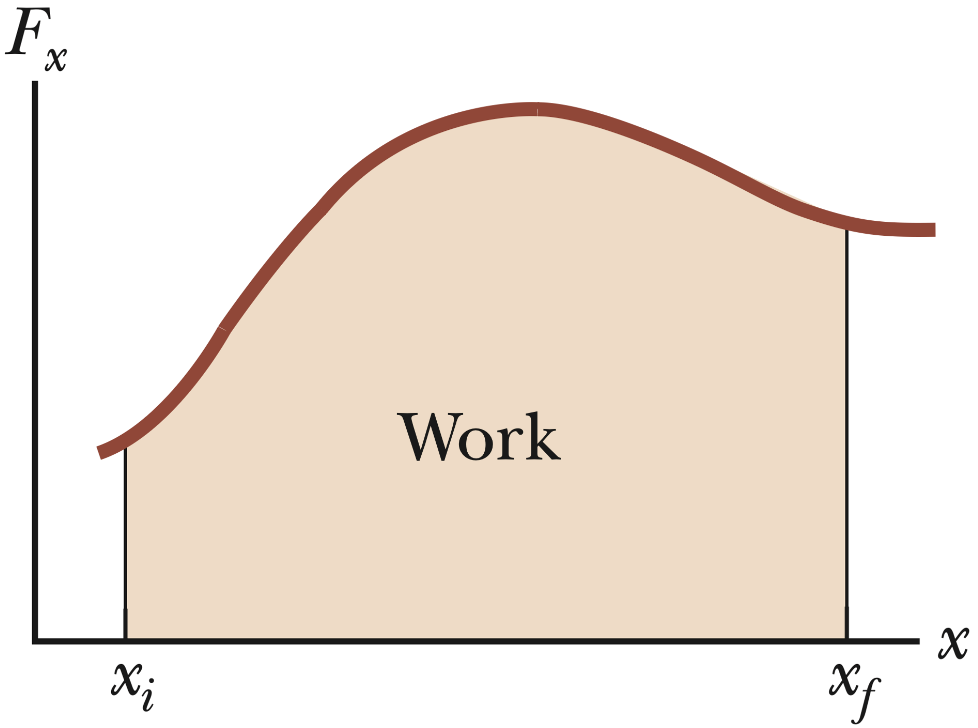
$$W \approx \sum_{x_i}^{x_f} F_x \Delta x_i$$



3.1 Definition

- As the number of segments increases and their length decreases, the approximation becomes more accurate.
- In the limit as the segment length approaches zero, the sum becomes an integral.

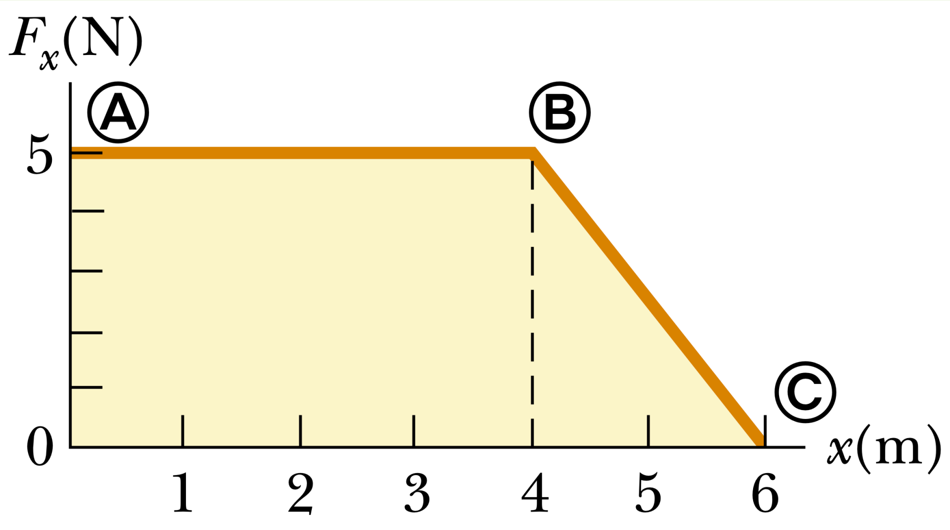
$$\boxed{W = \int_{x_i}^{x_f} F_x \, \mathrm{d}x}$$



3.2 Examples

Example 3.4

A force acting on a particle varies with x, as shown in the figure. Calculate the work done by the force as the particle moves from $x = 0$ to $x = 6$ m.



3.2 Examples

Solution 3.4

- The work done by the force is equal to the area under the curve of the force-position graph as shown in the figure.
- The area under the curve can be calculated by dividing it into simple geometric shapes (rectangles and triangles) and summing their areas. Therefore,

$$W = \text{Area}_{0\to4} + \text{Area}_{4\to6}$$

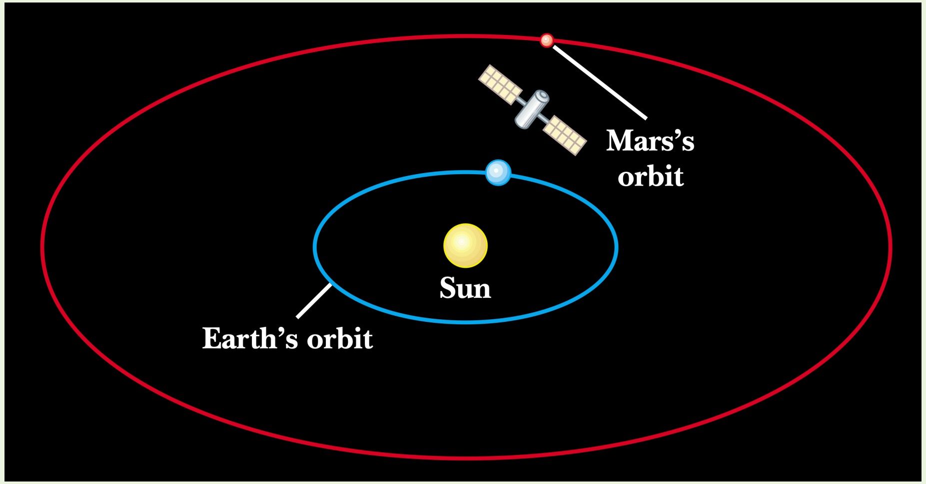
$$W = (5 \times 4) + \left(\frac{1}{2} \times 2 \times 5\right) = 25 \text{ J}$$

Example 3.5

The interplanetary probe shown in the Figure is attracted to the Sun by a force given by

$$F = -\frac{1.3 \times 10^{22}}{x^2} \quad \mathrm{N}$$

where x is the Sun-probe separation distance. Graphically and analytically determine how much work is done by the Sun on the probe as the probe–Sun separation changes from 1.5×10^{11} m to 2.3×10^{11} m.



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Solution 3.5

Analytical Solution

$$W = \int_{1.5\times10^{11}}^{2.3\times10^{11}} -\frac{1.3\times10^{22}}{x^2}\,\mathrm{d}x = -1.3\times10^{22}\int_{1.5\times10^{11}}^{2.3\times10^{11}} x^{-2}\,\mathrm{d}x$$

$$= -1.3\times10^{22}\left[-\frac{1}{x}\right]_{1.5\times10^{11}}^{2.3\times10^{11}}$$

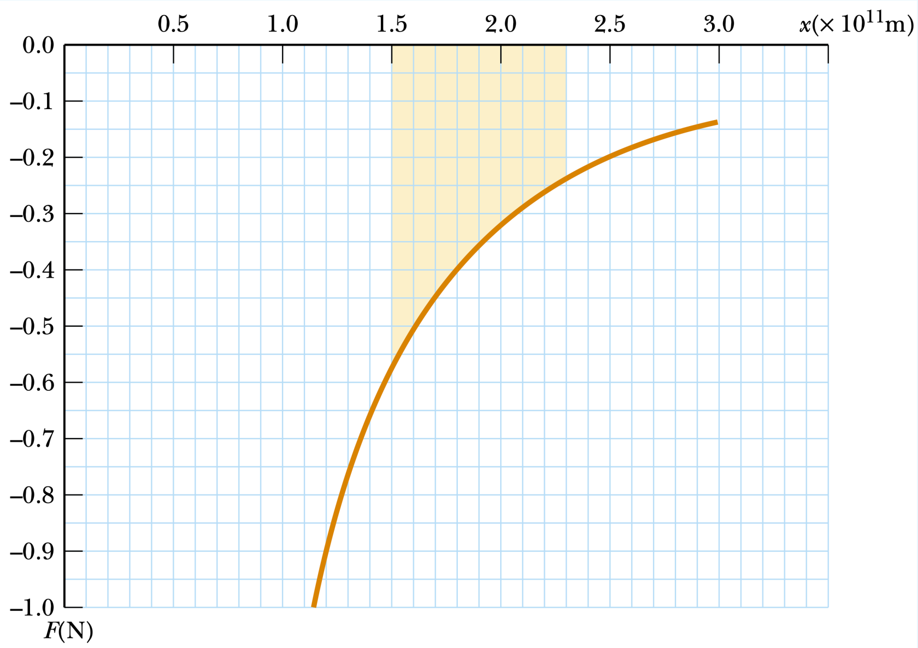
$$= -1.3\times10^{22}\left[-\frac{1}{2.3\times10^{11}} + \frac{1}{1.5\times10^{11}}\right]$$

$$= -3\times10^{10}\ \mathrm{J}$$

Solution 3.5

Graphical Solution

- The work done by the force is equal to the area under the curve of the force-position graph as shown in the figure.
- The area under the curve is equal to -3×10^{10} J, which is the same as the analytical solution.



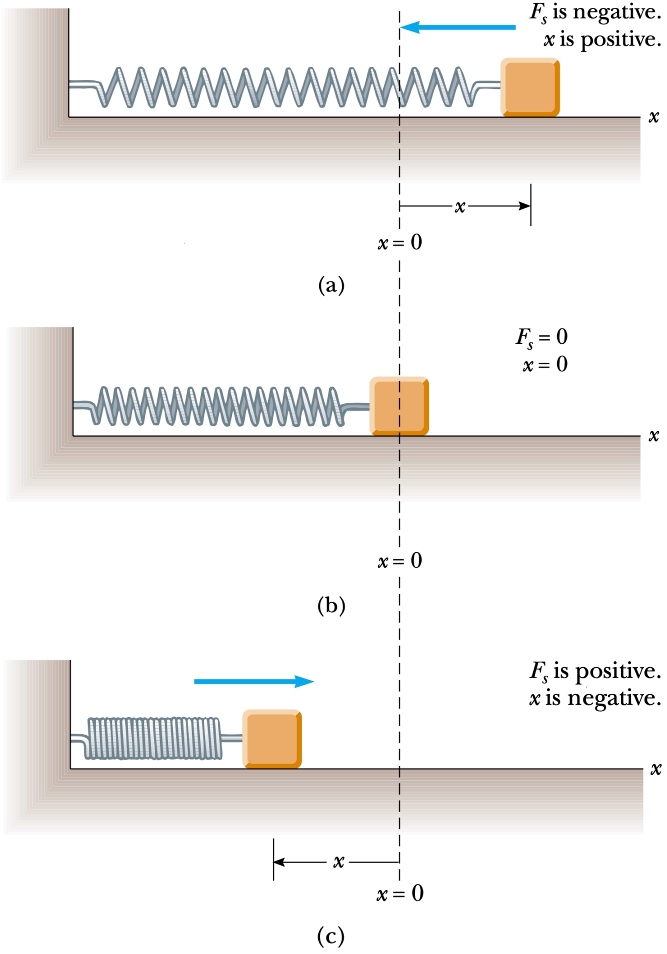
3.3 Work Done by a Spring

- A spring exerts a restoring force that is proportional to the displacement from its equilibrium position (Hooke's Law):

$$F_s = -kx$$

where k is the spring constant and x is the displacement from the equilibrium position.

- The negative sign indicates that the force exerted by the spring is always directed opposite to the displacement. When x is negative (compression), the force is positive (to the right), and vice versa.



3.3 Work Done by a Spring

- The work done by the spring force as the spring is stretched or compressed from an initial position x_i to a final position x_f is given by:

$$\boxed{W_s = \int_{x_i}^{x_f} (-kx)\,\mathrm{d}x = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2}$$

3.4 What is the Total Work Done by the Spring?

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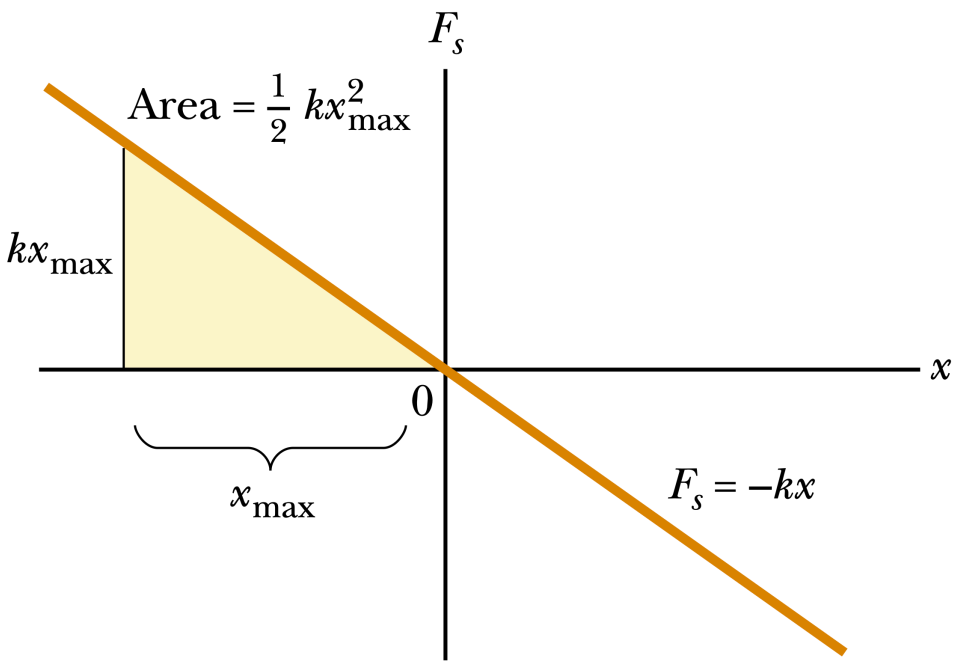
$$-x_{\max} \to 0: \quad W_s = +\frac{1}{2}kx_{\max}^2$$

$$0 \to +x_{\max}: \quad W_s = -\frac{1}{2}kx_{\max}^2$$

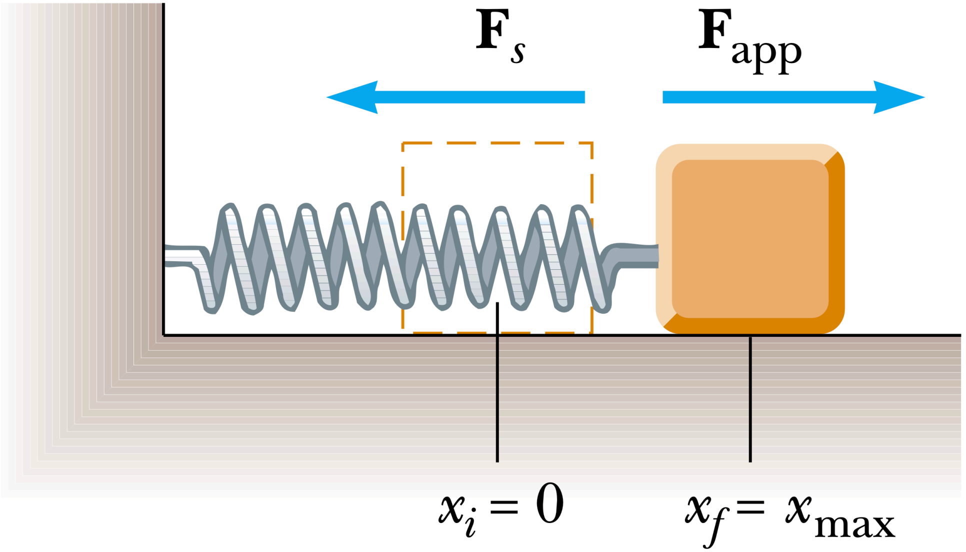
$$-x_{\max} \to +x_{\max}: \quad W_s = 0$$

$$+x_{\max} \to -x_{\max}: \quad W_s = 0$$



3.5 External Force Applied to the Spring-Block System

- When an external force $\vec{F}_{\text{app}}$ is applied to the block attached to the spring and the block is moved from x_i to x_f, the work done by the external force (assuming to be almost equal to the spring force) is:



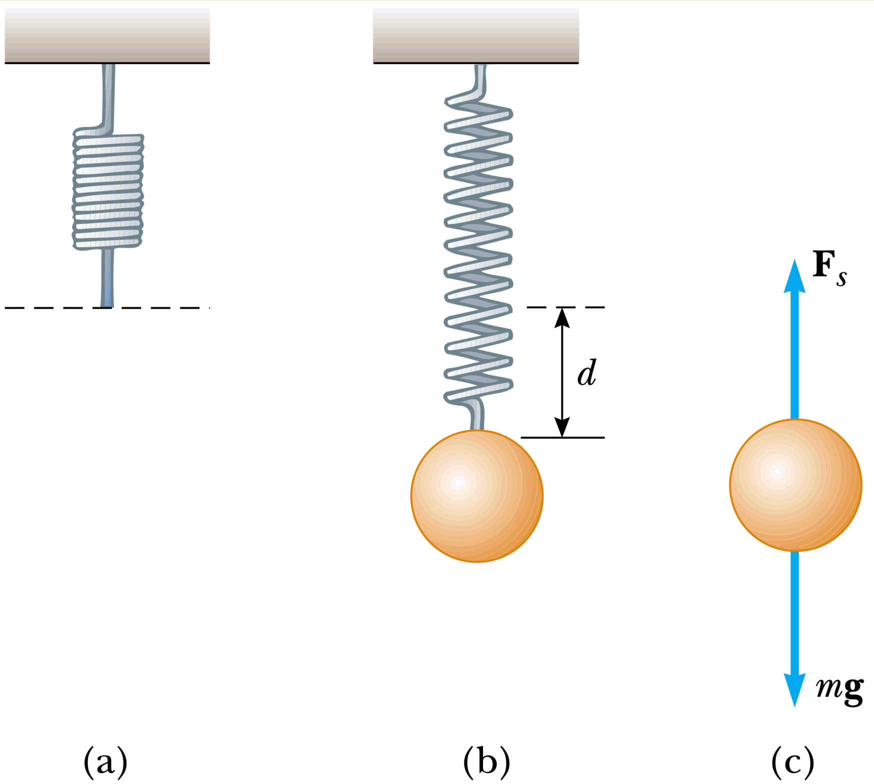
$$\boxed{W_{\text{ext}} = \int_{x_i}^{x_f} (kx)\,\mathrm{d}x = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2}$$

3.6 Examples

Example 3.6

A common technique used to measure the force constant of a spring is demonstrated by the setup in the Figure. The spring is hung vertically, and an object of mass m is attached to its lower end. Under the action of the "load" mg, the spring stretches a distance d from its equilibrium position.

(A) If a spring is stretched 2 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?



3.6 Examples

Solution 3.6

$$kd = mg$$

$$\Rightarrow k = \frac{mg}{d} = \frac{(0.55\ \mathrm{kg})(9.8\ \mathrm{m/s^2})}{2 \times 10^{-2}\ \mathrm{m}} = 270\ \mathrm{N/m}$$

3.6 Examples

Example 3.6

(B) How much work is done by the spring as it stretches through this distance?

Solution 3.6

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

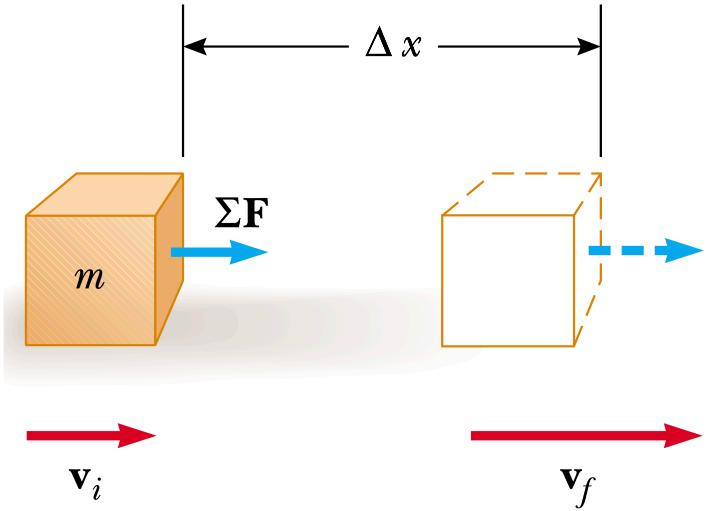
$$W_s = 0 - \frac{1}{2}(270\ \mathrm{N/m})(2 \times 10^{-2}\ \mathrm{m})^2 = -5.4 \times 10^{-2}\ \mathrm{J}$$

4.1 Kinetic Energy

- One outcome of doing work on a system is that it changes its speed.
- To find a quantity that measures the energy of a moving object, we start with Newton's second law:

$$\sum W = \int_{x_i}^{x_f} \sum F \, \mathrm{d}x = \int_{x_i}^{x_f} ma \, \mathrm{d}x$$

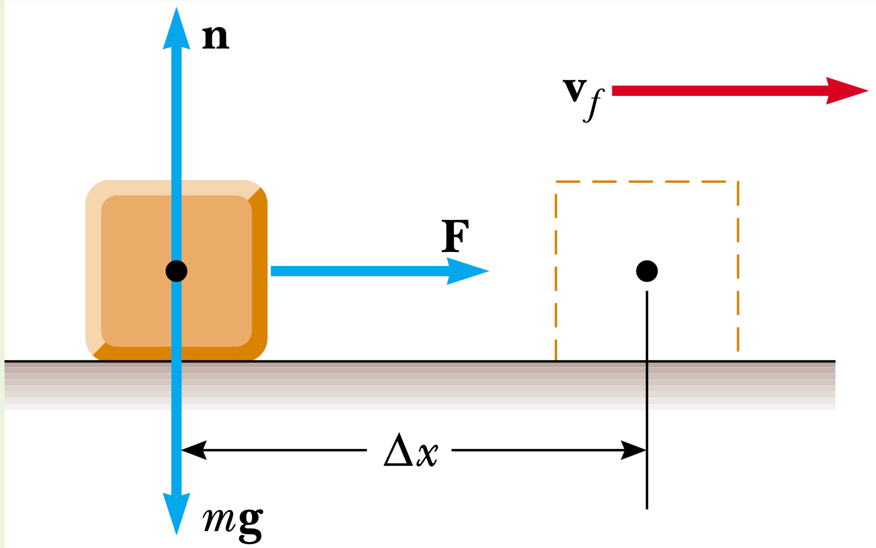
$$= \int_{x_i}^{x_f} m \frac{\mathrm{d}v}{\mathrm{d}t} \mathrm{d}x = \int_{v_i}^{v_f} mv \, \mathrm{d}v$$



$$\boxed{\begin{aligned} \sum W &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ &= K_f - K_i = \Delta K \end{aligned}}$$

Example 4.7

A 6 kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force of 12 N. Find the speed of the block after it has moved 3.0 m.



4.1 Kinetic Energy

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Solution 4.7

- First, we find the work done by the force on the block:

$$W = F\Delta x = (12\ \mathrm{N})(3\ \mathrm{m}) = 36\ \mathrm{J}$$

- Then, we use the work-kinetic energy theorem to find the final speed:

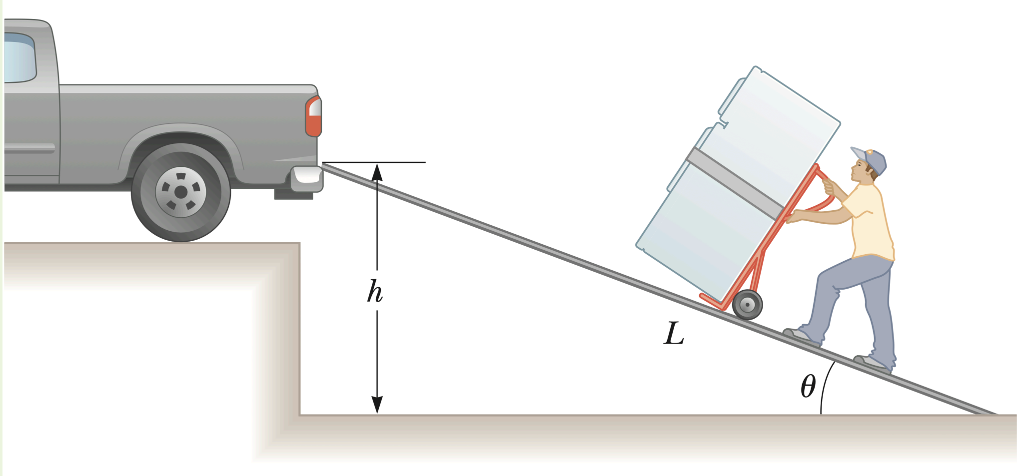
$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$36\ \mathrm{J} = \frac{1}{2}(6\ \mathrm{kg})v_f^2 - 0$$

$$v_f = \sqrt{\frac{2\times 36\ \mathrm{J}}{6.0\ \mathrm{kg}}} = 3.5\quad \mathrm{m/s}$$

Example 4.8

A man wishes to load a refrigerator onto a truck using a ramp at angle θ as shown in the Figure. He claims that less work would be required to load the truck if the length L of the ramp were increased so that the angle θ would be smaller. Is his claim valid?



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Solution 4.8

No, the claim is wrong.

- The man pushes the refrigerator up the ramp at a constant speed, so the net work and the change in kinetic energy are both zero:

$$W_{\text{net}} = W_{\text{man}} - W_{\text{gravity}} = \Delta K = 0.$$

- Therefore, the work done by the man is equal to the work done by gravity:

$$W_{\text{man}} = W_{\text{gravity}} = mgh$$

- Thus, the work done by the man is independent of the length of the ramp L and the angle θ.

5.1 Reading Assignment

Read Section 7.6 (page 196) in Serway's book (6th edition), and make sure to understand the following concepts:

- Isolated system
- Nonisolated system
- Types of energy and energy transfer

1. Work Done by a Constant Force

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8. Additional Problems

6.1 Frictional Force and Work

- Friction always acts to oppose the motion of an object, therefore, the work done by friction is always negative, and take out energy from the system:

$$\boxed{\Delta K = +W - f_k d}$$

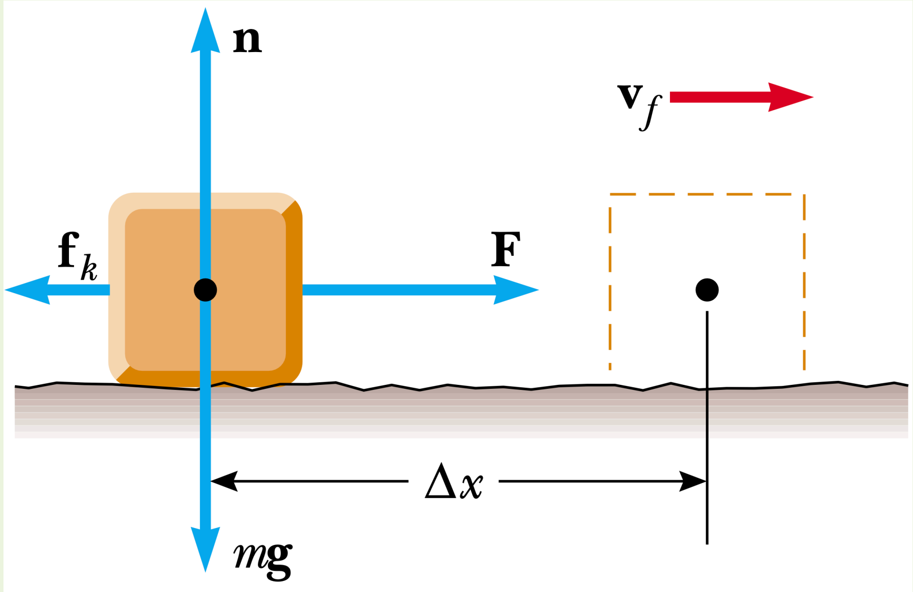
- Also we can write:

$$\boxed{\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + F_{\text{net}}d - f_k d}$$

Example 6.9

A 6 kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 12 N.

(A) Find the speed of the block after it has moved 3 m if the surfaces in contact have a coefficient of kinetic friction of 0.15.



6.1 Frictional Force and Work

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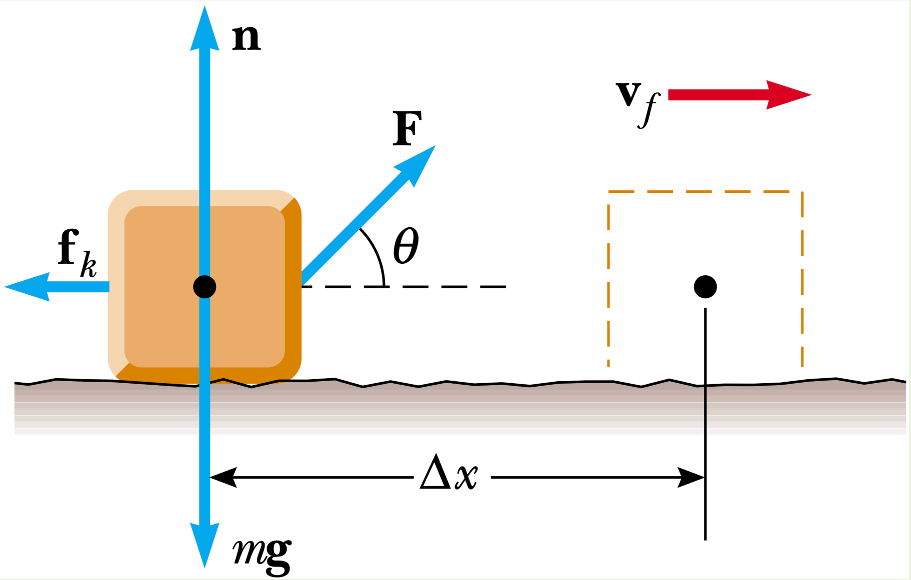
Solution 6.9

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + F_{\text{net}}d - f_kd$$
$$= 0 + Fd - (\mu_k mg)d$$
$$\frac{1}{2}(6\text{ kg})v_f^2 = 0 + (12\text{ N})(3\text{ m}) - (0.15)(6\text{ kg})(9.8\text{ m/s}^2)(3\text{ m})$$
$$\Longrightarrow v_f = 1.8\text{ m/s}$$

Example 6.9

(B) Suppose the force F is applied at an angle as shown in the Figure. At what angle should the force be applied to achieve the largest possible speed after the block has moved 3.0 m to the right?



6.1 Frictional Force and Work

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Solution 6.9

- To find the angle that maximizes the final speed, we first write the equation for the final speed in terms of the angle θ:

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + (F\cos\theta)d - f_k d$$

- To find the frictional force, we apply Newton's second law in the vertical direction:

$$n + F\sin\theta - mg = 0$$

$$\Rightarrow n = mg - F\sin\theta$$

$$\Rightarrow f_k = \mu_k n = \mu_k(mg - F\sin\theta)$$

6.1 Frictional Force and Work

Using this expression for f_k and with $v_i = 0$, we get:

$$\frac{1}{2}mv_f^2 = (F\cos\theta)\ d - \mu_k(mg - F\sin\theta)\ d$$

- To maximize $\frac{1}{2}mv_f^2$, we take the derivative with respect to θ and set it to zero:

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{1}{2}mv_f^2\right) &= \frac{d}{d\theta}\Big[(F\cos\theta)\ d - \mu_k(mg - F\sin\theta)\ d\Big] = 0 \\
&= -(F\sin\theta)d - \mu_k(0 - F\cos\theta)d = 0 \\
&\Rightarrow -\cancel{Fd}\sin\theta + \mu_k\cancel{Fd}\cos\theta = 0 \\
&\Rightarrow \sin\theta = \mu_k\cos\theta \\
&\Rightarrow \tan\theta = \mu_k \\
&\Rightarrow \theta = \tan^{-1}(\mu_k) = \tan^{-1}(0.15) = 8.5^\circ
\end{aligned}$$

6.1 Frictional Force and Work

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Example 6.10

A car traveling at an initial speed v slides a distance d to a halt after its brakes lock. Assuming that the car's initial speed is instead $2v$ at the moment the brakes lock, estimate the distance it slides.

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Solution 6.10

- Using the work-kinetic energy theorem:

$$\begin{aligned}\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 &= -f_k d_1 \\ 0 - \frac{1}{2}mv^2 &= -f_k d_1\end{aligned} \tag{1}$$

- If the initial speed is $2v$, then:

$$0 - \frac{1}{2}m(2v)^2 = -f_k d_2 \tag{2}$$

- Dividing the second equation by the first:

$$\frac{\frac{1}{2}m(2v)^2}{\frac{1}{2}mv^2} = \frac{-f_k d_2}{-f_k d_1}$$

$$\Rightarrow 4 = \frac{d_2}{d_1}$$

- Therefore, the distance the car slides is:

$$d_2 = 4d_1$$

6.1 Frictional Force and Work

Example 6.11

A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1×10^3 N/m. The spring is compressed 2 cm and is then released from rest.

(A) Calculate the speed of the block as it passes through the equilibrium position ($x = 0$) if the surface is frictionless.

6.1 Frictional Force and Work

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Solution 6.11

- We saw earlier that the work done by the spring as it moves from x_i to 0 is:

$$W = \frac{1}{2}kx_i^2 = \frac{1}{2}(1 \times 10^3 \text{ N/m})(-2 \times 10^{-2} \text{ m})^2 = 0.2 \text{ J}$$

- Using the work-kinetic energy theorem:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W$$

$$\Rightarrow \frac{1}{2}(1.6 \text{ kg})v_f^2 - 0 = 0.2 \text{ J}$$

$$\Rightarrow v_f = 0.5 \text{ m/s}$$

6.1 Frictional Force and Work

Example 6.11

(B) Calculate the speed of the block as it passes through the equilibrium position if a constant friction force of 4 N opposes its motion from the moment it is released.

6.1 Frictional Force and Work

Solution 6.11

- The work done by friction as the block moves from x_i to 0 is:

$$-f_k d = -(4\ \mathrm{N})(2 \times 10^{-2}\ \mathrm{m}) = -0.08\ \mathrm{J}$$

- Using the work-kinetic energy theorem:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W - f_k d$$

$$\Rightarrow \frac{1}{2}(1.6\ \mathrm{kg})v_f^2 - 0 = 0.2\ \mathrm{J} - 0.08\ \mathrm{J}$$

$$\Rightarrow v_f = 0.39\ \mathrm{m/s}$$

1. Work Done by a Constant Force

2. The Scalar Product of Two Vectors

3. Work Done by a Varying Force

4. Kinetic Energy and the Work-Kinetic Energy Theorem

5. The Nonisolated System—Conservation of Energy

6. Situations Involving Kinetic Friction

7. Power

8. Additional Problems

7.1 Definition

- **Power** is the time rate of energy transfer.
- The **average power** $\bar{P}$ is defined as the work done W divided by the time interval Δt during which the work is done:

$$\boxed{\bar{P} = \frac{W}{\Delta t}}$$

- The **instantaneous power** P is the limit of the average power as the time interval approaches zero, or the derivative of work with respect to time:

$$\boxed{P = \frac{\mathrm{d}W}{\mathrm{d}t}}$$

- Using $\mathrm{d}W = \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$, we can write:

$$\boxed{P = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{v}}}$$

7.1 Definition

- Since there are many types of energy, power can be defined as the time rate of change of any type of energy E,

$$P = \frac{\mathrm{d}E}{\mathrm{d}t}.$$

- The SI unit of power is **watt** (W):

$$1W = 1\ \mathrm{J/s}.$$

- Another common unit of power is the horsepower (hp):

$$1\ \mathrm{hp} = 746\ \mathrm{W}.$$

- One **kilowatt-hour** (kWh) is a common unit of *energy* used by electric companies:

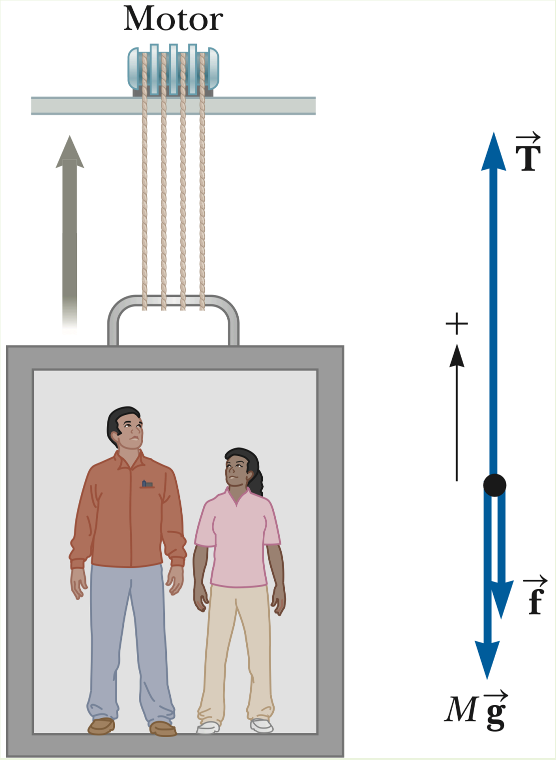
$$\begin{aligned} 1\ \mathrm{kWh} &= 10^3\ \mathrm{W} \times 3600\ \mathrm{s} \\ &= 3.6 \times 10^6\ \mathrm{J}. \end{aligned}$$

7.1 Definition

Example 7.12

An elevator car has a mass of 1600 kg and is carrying passengers having a combined mass of 200 kg. A constant friction force of 4000 N retards its motion upward, as shown in the Figure.

(A) What power delivered by the motor is required to lift the elevator car at a constant speed of 3 m/s?



Solution 7.12

- The motor must supply the force of magnitude T that pulls the elevator car upward.
- Using Newton's second law in the vertical direction, we find:

$$\sum F = T - f - (m_{\text{car}} + m_{\text{pass}})g = 0$$

$$T = f + (m_{\text{car}} + m_{\text{pass}})g$$

$$T = (4000\ \text{N}) + (1600\text{kg} + 200\text{kg})(9.8\ \text{m/s}^2) = 2.16 \times 10^4\ \text{N}$$

- The power delivered by the motor is:

$$P = \vec{\boldsymbol{T}} \cdot \vec{\boldsymbol{v}} = (2.16 \times 10^4\ \text{N})(3\ \text{m/s}) = 6.48 \times 10^4\ \text{W}$$

Example 7.12

(B) What power must the motor deliver at the instant the speed of the elevator is v if the motor is designed to provide the elevator car with an upward acceleration of $1\ \mathrm{m/s}$?

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Solution 7.12

- Similar to part (A), we apply Newton's second law in the vertical direction to find the tension in the cable:

$$\sum F_y = T - f - M_{\text{tot}} g = M_{\text{tot}} a$$
$$\Rightarrow T = M_{\text{tot}}(g + a) + f$$
$$\Rightarrow T = (1800\text{kg})(9.8\ \text{m/s}^2 + 1\ \text{m/s}^2) + 4000\ \text{N} = 2.34 \times 10^4\ \text{N}$$

- The power delivered by the motor at v is:

$$P = \vec{T} \cdot \vec{v} = 2.34 \times 10^4\ v$$

7.1 Definition

Example 7.13

Your bill may state that you used 900 kWh of energy during a month and that you are being charged at the rate of 11¢ per kilowatt-hour.

(A) What is the energy, in joules, that you used during the month?

(B) How much is your bill for the month?

7.1 Definition

Solution 7.13

- (A) The energy used during the month is:
$$E = (900\ \mathrm{kWh})(3.6 \times 10^6\ \mathrm{J/kWh}) = 3.24 \times 10^9\ \mathrm{J}$$

- (B) The cost of the energy used during the month is:
$$\mathrm{cost} = (900\ \mathrm{kWh})(0.11\ \$/\mathrm{kWh}) = 99\ \$$$

7.1 Definition

Example 7.14

Suppose an electric bulb is rated at 100 W. In 1 h of operation, how much energy does it convert to heat and light in joules and kilowatt-hours?

- The energy converted by the bulb in 1 h is:

$$E = P \times t = (100\ \mathrm{W})(3600\ \mathrm{s}) = 3.6 \times 10^5\ \mathrm{J}$$

- In kilowatt-hours:

$$E = (0.1\ \mathrm{kW})(1h) = 0.1\ \mathrm{kWh}$$

8. Additional Problems

1, 4, 7, 13, 14, 15, 16, 19, 21, 24, 25, 26, 28, 31, 32, 33, 35, 37, 40