Ch.6: Circular Motion and Newton's Laws

Physics 103: Classical Mechanics

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Outline

1. Newton’s Second Law Applied to Uniform Circular Motion

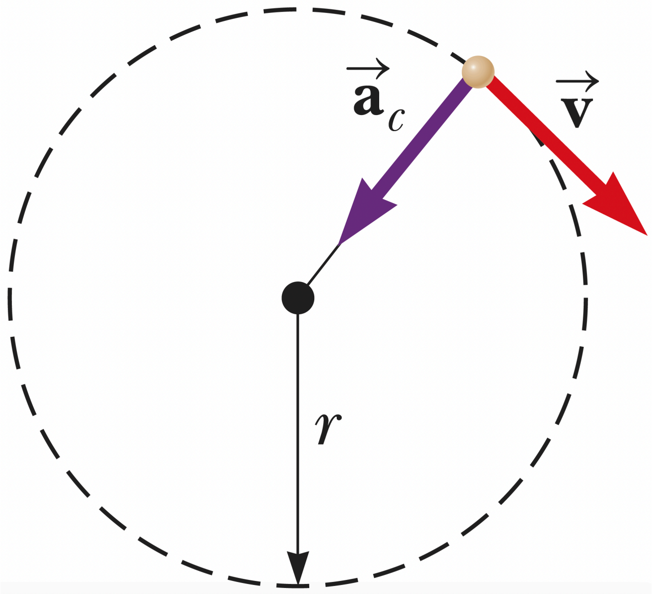
2. Examples

1.1 Remember Uniform Circular Motion

- The acceleration of an object in uniform circular motion is called *centripetal acceleration* and is always directed toward the center of the circle.

- The magnitude of the centripetal acceleration is given by:

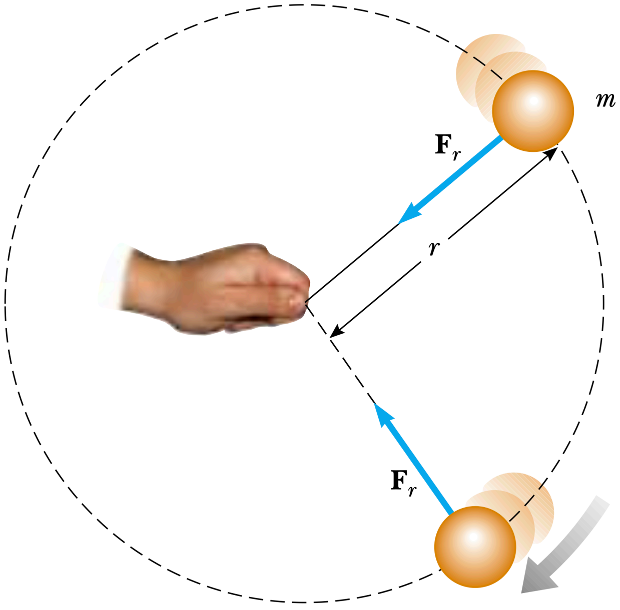
$$a_c = \frac{v^2}{r}$$

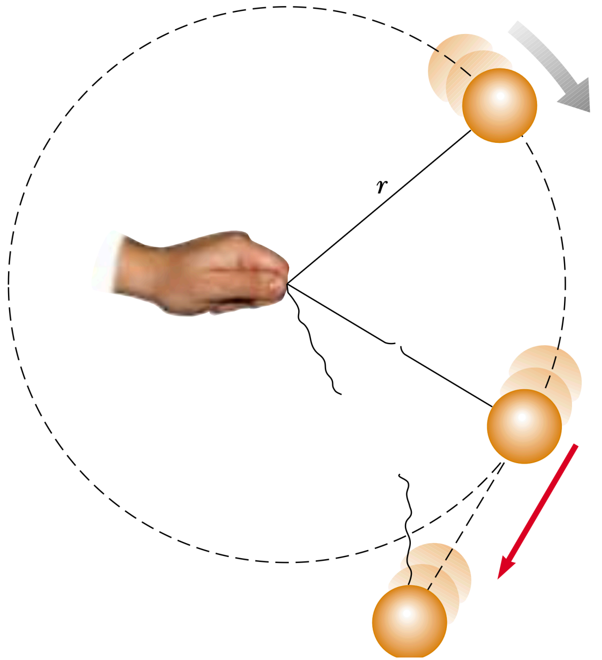


1.2 Newton's Second Law in Circular Motion

- Applying Newton's second law along the radial direction, we find the net force causing the centripetal acceleration to be:

$$\sum F = ma_c = m\frac{v^2}{r}$$





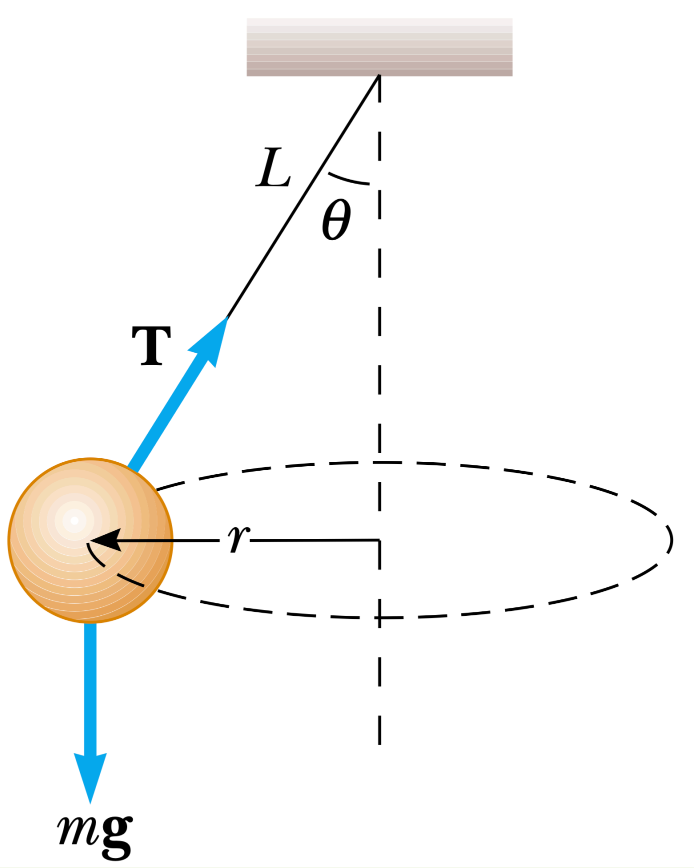
1. Newton's Second Law Applied to Uniform Circular Motion

2. Examples

2.1 Example: The Conical Pendulum

A small object of mass m is suspended from a string of length L. The object revolves with constant speed v in a horizontal circle of radius r, as shown in the Figure. (Because the string sweeps out the surface of a cone, the system is known as a conical pendulum.)

Find an expression for v.



2.1 Example: The Conical Pendulum

Applying Newton's second law:

$$\sum \vec{F}_y = ma$$

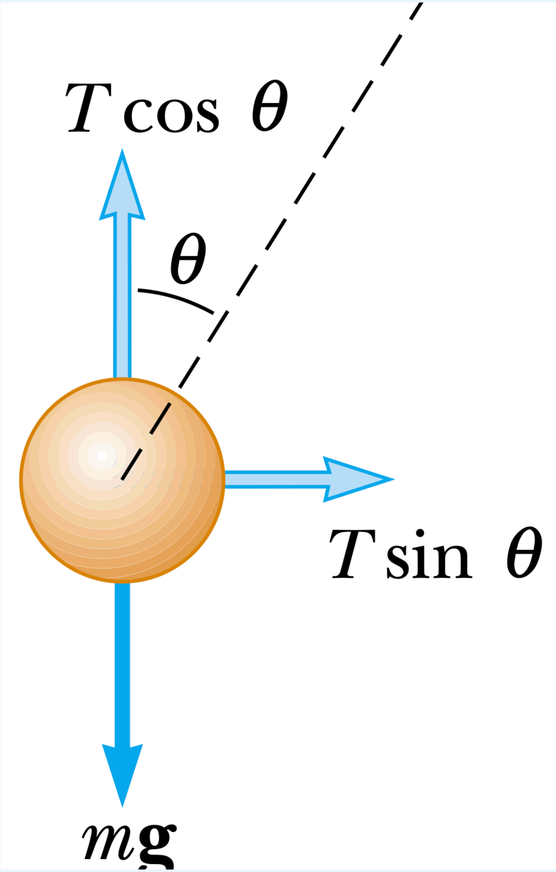
$$T\cos\theta - mg = 0$$

$$\implies T\cos\theta = mg \qquad (1)$$

$$\sum F_r = m\frac{v^2}{r}$$

$$T\sin\theta = m\frac{v^2}{r}$$

$$\implies T\sin\theta = m\frac{v^2}{r} \qquad (2)$$



2.1 Example: The Conical Pendulum

- Dividing Eq (2) by Eq (1), we get:

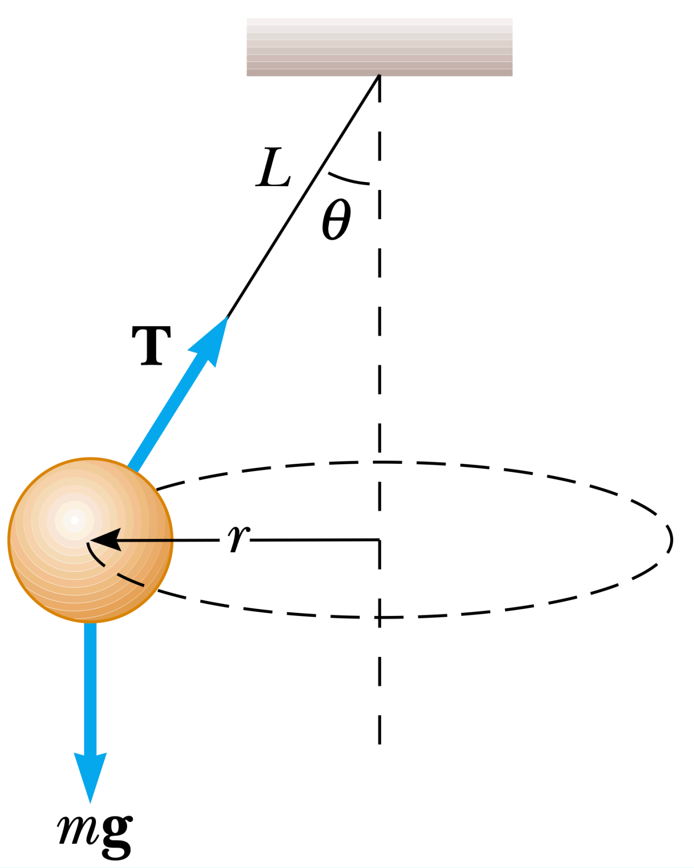
$$\tan\theta = \frac{v^2}{rg}$$

$$\implies v = \sqrt{rg\tan\theta}$$

- Using $r = L\sin\theta$, we get:

$$v = \sqrt{\tan\theta\sin\theta\ g\ L}$$

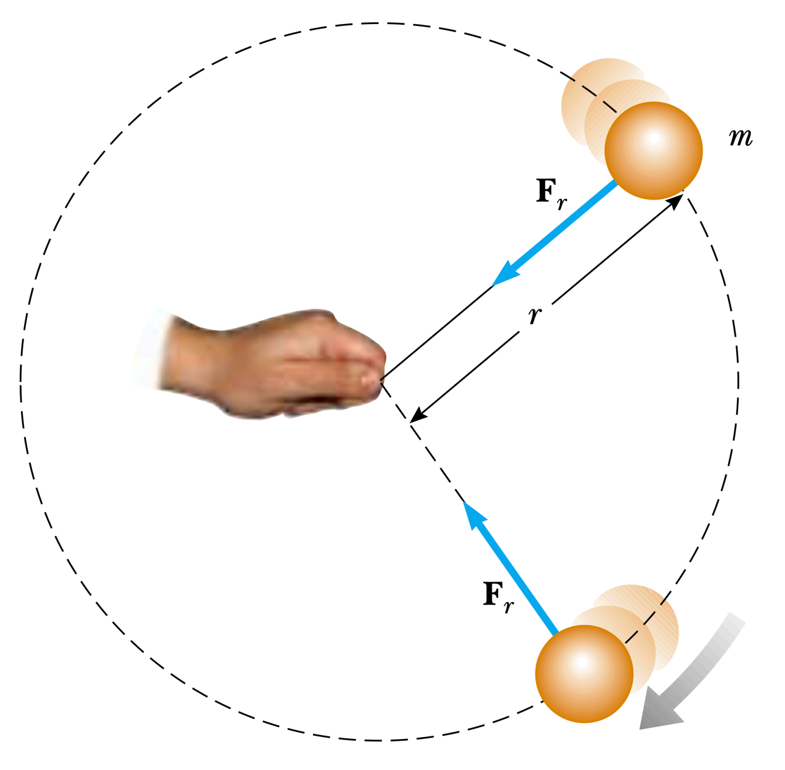
- Note that the velocity of the object does not depend on its mass.



2.2 Example: How Fast Can It Spin?

A ball of mass 0.5 kg is attached to the end of a cord 1.5m long. The ball is whirled (أُديرت) in a horizontal circle as shown in the Figure. If the cord can withstand (يتحمل) a maximum tension of 50 N,

what is the maximum speed at which the ball can be whirled before the cord breaks? Assume that the string remains horizontal during the motion.



2.2 Example: How Fast Can It Spin?

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Solution 2.2

$$\sum F_r = m\frac{v^2}{r}$$

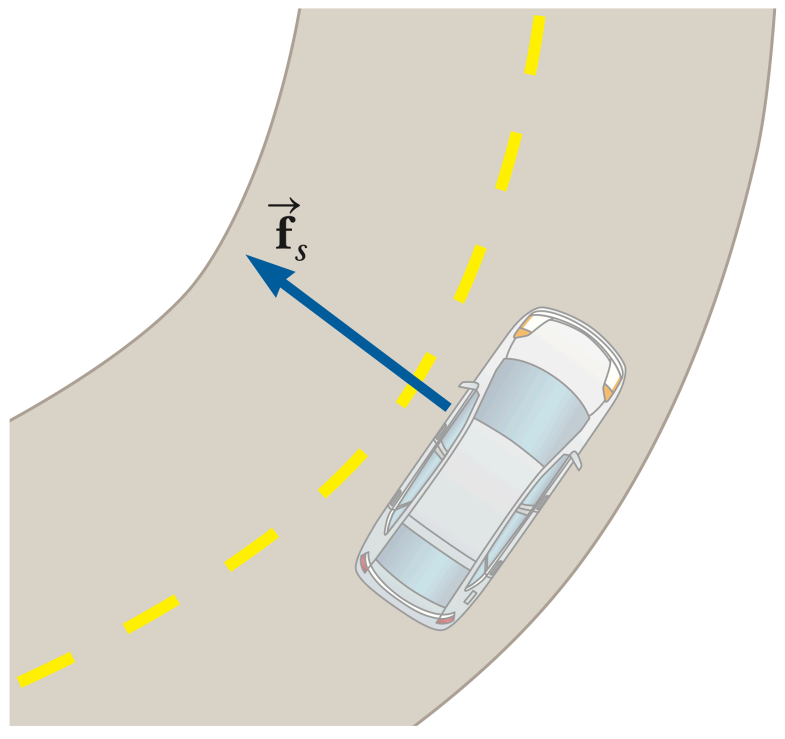
$$T = m\frac{v^2}{r}$$

$$\implies v_{\max} = \sqrt{\frac{T_{\max} r}{m}} = \sqrt{\frac{50N\ 1.5m}{0.5\text{kg}}} = 12.2\ \text{m/s}$$

2.3 Example: What Is the Maximum Speed of the Car?

A 1500kg car moving on a flat, horizontal road following a curve, as shown in the Figure. If the radius of the curve is 35m and the coefficient of static friction between the tires and dry pavement (سطح الطريق) is 0.5,

find the maximum speed the car can have and still make the turn successfully.



2.3 Example: What Is the Maximum Speed of the Car?

$$\sum F_r = m\frac{v^2}{r}$$

$$f_s = m\frac{v^2}{r}$$

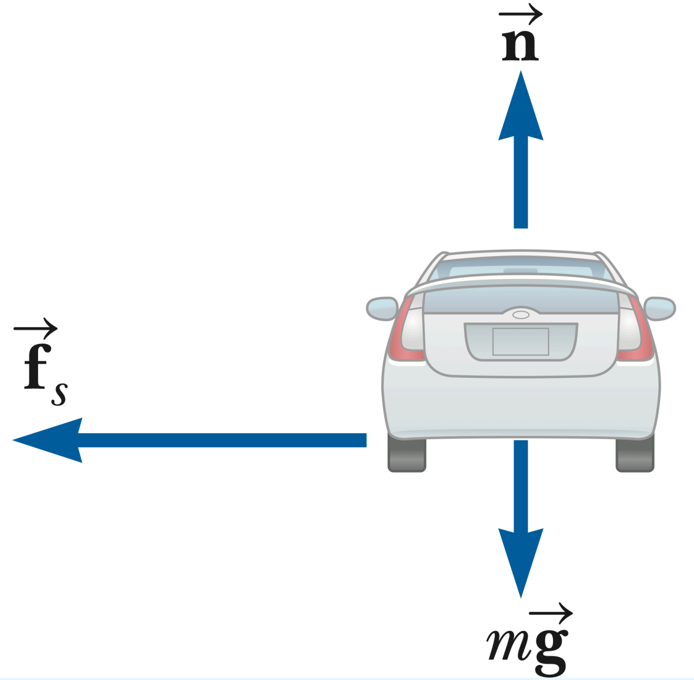
$$\implies v^2 = \frac{f_s r}{m}$$

$$\implies v^2 = \frac{(\mu_s n) r}{m}$$

$$\sum F_y = ma$$

$$n - mg = 0$$

$$\implies n = mg$$



2.3 Example: What Is the Maximum Speed of the Car?

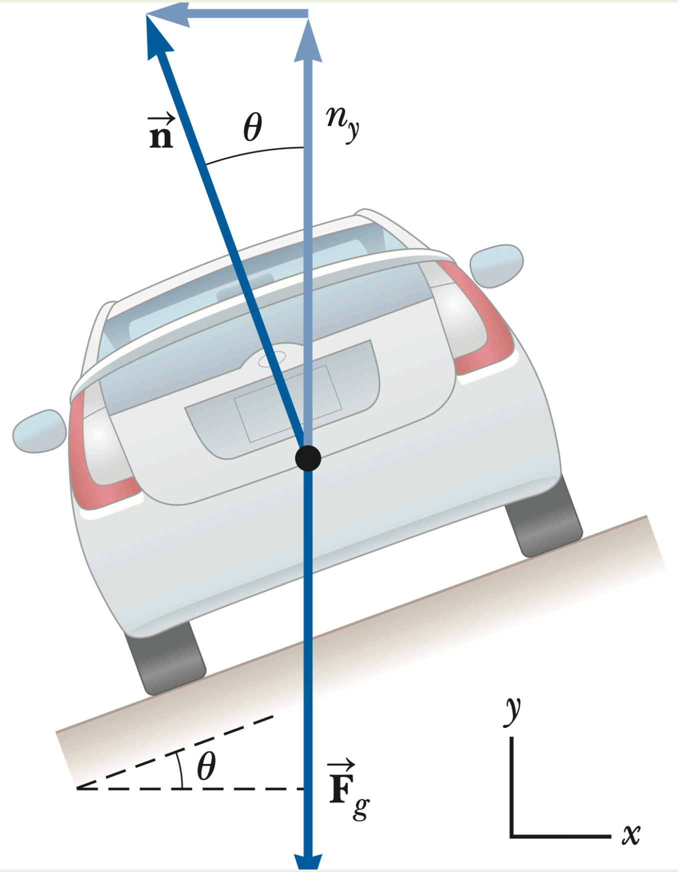
$$\Longrightarrow v^2 = \frac{\mu_s mgr}{m} = \mu_s gr$$

$$\Longrightarrow v_{\text{max}} = \sqrt{\mu_s gr} = \sqrt{(0.5)(9.8)(35)} = 13.1 \text{ m/s}$$

2.4 Example: The Banked Roadway

A civil engineer wishes to design a curved exit ramp for a highway in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can follow the curve even when the road is covered with ice. Such a ramp is usually banked; this means the roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be 13.4 m/s and the radius of the curve is 50 m.

At what angle should the curve be banked?



2.4 Example: The Banked Roadway

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$$\sum F_y = ma$$

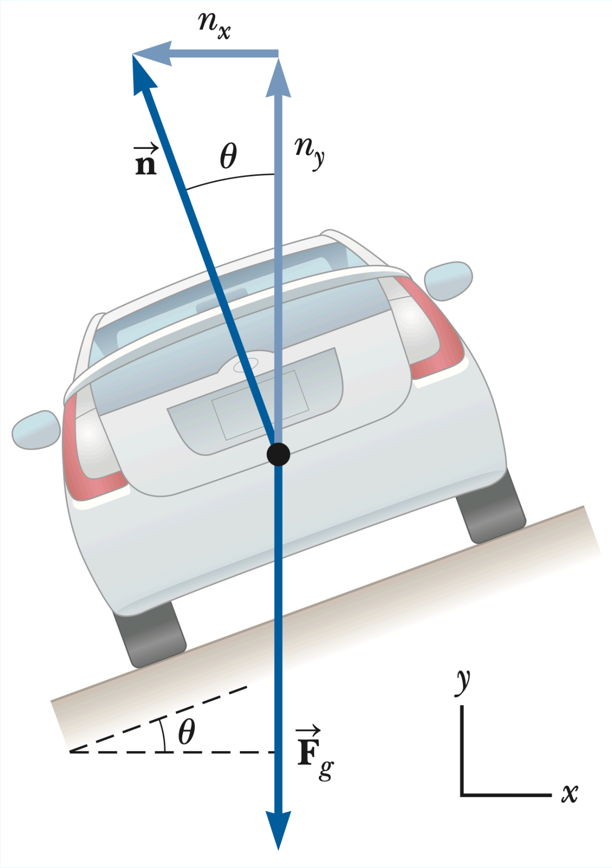
$$n\cos\theta - mg = 0$$

$$\implies n\cos\theta = mg \qquad (1)$$

$$\sum F_r = m\frac{v^2}{r}$$

$$n\sin\theta = m\frac{v^2}{r}$$

$$\implies n\sin\theta = m\frac{v^2}{r} \qquad (2)$$



2.4 Example: The Banked Roadway

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By Dividing Eq (2) by Eq (1), we get:

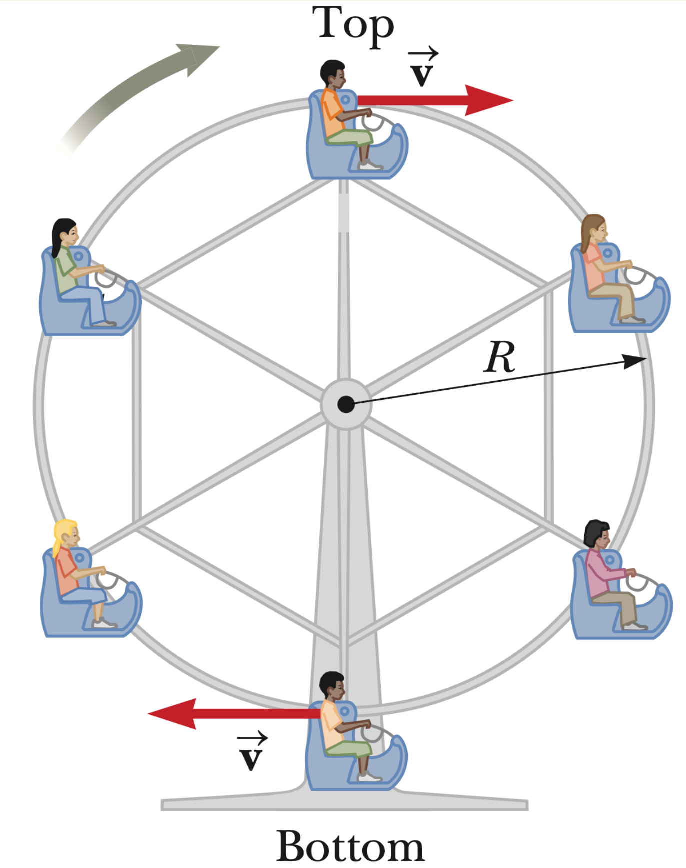
$$\tan\theta = \frac{v^2}{rg}$$

$$\Longrightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{(13.4\ \mathrm{m/s})^2}{(50m)(9.8\ \mathrm{m/s^2})}\right) = 20.1^\circ$$

2.5 Example: Riding the Ferris Wheel

A child of mass m rides on a Ferris wheel as shown in the Figure. The child moves in a vertical circle of radius 10 m at a constant speed of 3 m/s.

(A) Determine the force exerted by the seat on the child at the bottom of the ride. Express your answer in terms of the weight of the child, mg.



2.5 Example: Riding the Ferris Wheel

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$$\sum F = m\frac{v^2}{r}$$

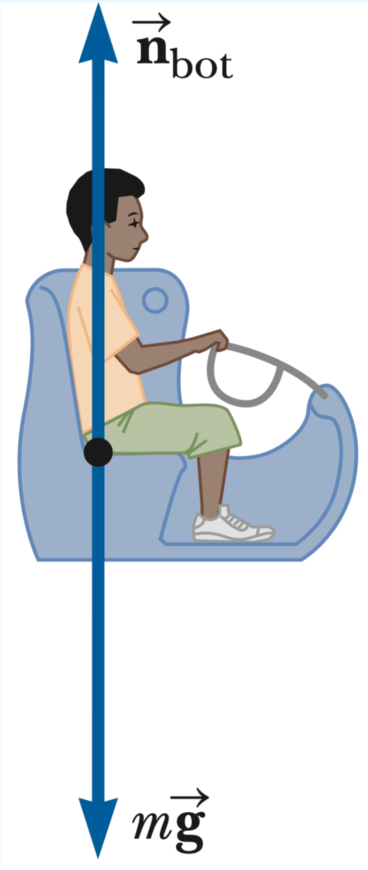
$$n_{\text{bot}} - mg = m\frac{v^2}{r}$$

Therefore,

$$\implies n_{\text{bot}} = mg + m\frac{v^2}{r}$$

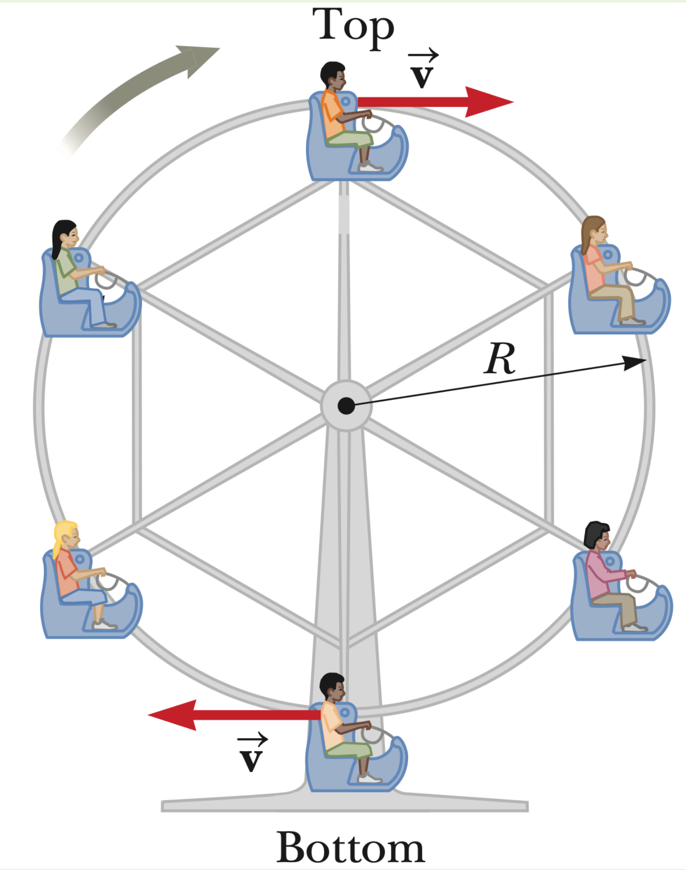
$$= mg\left(1 + \frac{v^2}{rg}\right)$$

$$= mg\left(1 + \frac{(3\ \ \text{m/s})^2}{(10m)(9.8\ \ \text{m/s}^2)}\right) = 1.09mg$$



2.5 Example: Riding the Ferris Wheel

(B) Determine the force exerted by the seat on the child at the top of the ride.



2.5 Example: Riding the Ferris Wheel

$$\sum F = m\frac{v^2}{r}$$

$$mg - n_{\text{top}} = m\frac{v^2}{r}$$

Therefore,

$$\implies n_{\text{top}} = mg - m\frac{v^2}{r}$$

$$= mg\left(1 - \frac{v^2}{rg}\right)$$

$$= mg\left(1 - \frac{(3\ \text{m/s})^2}{(10m)(9.8\ \text{m/s}^2)}\right) = 0.908mg$$