4.2 EXERCISES

In Problems 1–12, find a general solution to the given differential equation.

3. $y'' + 5y' + 6y = 0$

In Problems 13–20, solve the given initial value problem.

17. $y'' - 6y' + 9y = 0; \quad y(0) = 2, \quad y'(0) = 25/3$

For each of the following, determine whether the given three functions are linearly dependent or linearly independent on $(-\infty, \infty)$:

(a) $y_1(t) = 1, \quad y_2(t) = t, \quad y_3(t) = t^2.$

(b) $y_1(t) = -3, \quad y_2(t) = 5\sin^2 t, \quad y_3(t) = \cos^2 t.$

(c) $y_1(t) = e^t, \quad y_2(t) = te^t, \quad y_3(t) = t^2e^t.$

(d) $y_1(t) = e^t, \quad y_2(t) = e^{-t}, \quad y_3(t) = \cosh t.$

4.3 EXERCISES

In Problems 9–20, find a general solution.

13. $y'' - 2y' + 26y = 0$

19. $y''' + y'' + 3y' - 5y = 0$

In Problems 21–27, solve the given initial value problem.

27. $y''' - 4y'' + 7y' - 6y = 0; \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 0$

29. Find a general solution to the following higher-order equations.

(a) $y''' - y'' + y' + 3y = 0$

(b) $y''' + 2y'' + 5y' - 26y = 0$

(c) $y^{\text{iv}} + 13y'' + 36y = 0$

4.4 EXERCISES

In Problems 9–26, find a particular solution to the differential equation.

13. $y'' - y' + 9y = 3\sin 3t$

15. $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = xe^x$

19. $4y'' + 11y' - 3y = -2te^{-3t}$

25. $y'' + 2y' + 4y = 111e^{2t}\cos 3t$

In Problems 33–36, use the method of undetermined coefficients to find a particular solution to the given higher-order equation.

33. $y''' - y'' + y = \sin t$

35. $y''' + y'' - 2y = te^t$

4.5 EXERCISES

In Problems 31–36, determine the form of a particular solution for the differential equation. Do not solve.

31. $y'' + y = \sin t + t\cos t + 10^t$

33. $x'' - x' - 2x = e^t\cos t - t^2 + \cos^3 t$

35. $y'' - 4y' + 5y = e^{5t} + t\sin 3t - \cos 3t$

In Problems 37–40, find a particular solution to the given higher-order equation.

39. $y''' + y'' - 2y = te^t + 1$

4.6 EXERCISES

In Problems 1–8, find a general solution to the differential equation using the method of variation of parameters.

4. $y'' + 2y' + y = e^{-t}$

4.7 EXERCISES

In Problems 9 through 14, find a general solution to the given Cauchy–Euler equation for $t > 0$.

9. $t^2y''(t) + 7ty'(t) - 7y(t) = 0$

11. $t^2\dfrac{d^2z}{dt^2} + 5t\dfrac{dz}{dt} + 4z = 0$

In Problems 15 through 18, find a general solution for $t < 0$.

17. $t^2y''(t) + 9ty'(t) + 17y(t) = 0$

In Problems 37 through 39, find general solutions to the nonhomogeneous Cauchy–Euler equations using variation of parameters.

37. $t^2z'' + tz' + 9z = -\tan(3\ln t)$

39. $t^2z'' - tz' + z = t\left(1 + \dfrac{3}{\ln t}\right)$

In Problems 41 through 44, a differential equation and a nontrivial solution f are given. Find a second linearly independent solution using reduction of order.

43. $tx'' - (t+1)x' + x = 0, \quad t > 0; \quad f(t) = e^t$