Ch.32: Alternating Current Circuits

Physics 104: Electricity and Magnetism

Dr. Abdulaziz Alqasem

Physics and Astronomy Department
King Saud University

2026

Ref.: Serway, R. A., & Jewett, J. W. (2018). Physics for Scientists and Engineers (10th ed.)

Version 1

Outline

Remember From Previous Chapters

Classical Mechanics

- Equations of motion:

$$\vec{\boldsymbol{v}}_f = \vec{\boldsymbol{v}}_i + \vec{\boldsymbol{a}}t$$

$$\vec{\boldsymbol{r}}_f = \vec{\boldsymbol{r}}_i + \vec{\boldsymbol{v}}_i t + \frac{1}{2}\vec{\boldsymbol{a}}t^2$$

$$\vec{\boldsymbol{v}}_f^2 = \vec{\boldsymbol{v}}_i^2 + 2\vec{\boldsymbol{a}} \cdot \left(\vec{\boldsymbol{r}}_f - \vec{\boldsymbol{r}}_i\right)$$

- Newton's Second Law:

$$\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$$

- Work-Energy Theorem:

$$W = \Delta K$$

$$W = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{d}}$$

$$K = \frac{1}{2}m\vec{\boldsymbol{v}}^2$$

Electric Field

- Coulomb's Law:

$$\vec{\boldsymbol{F}}_e = k_e \frac{q_1 q_2}{r^2}\hat{\boldsymbol{r}}$$

- Electric Field:

$$\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}_e}{q_0} = k_e \frac{q}{r^2}\hat{\boldsymbol{r}}$$

$$\vec{\boldsymbol{a}} = \left(\frac{q}{m}\right)\vec{\boldsymbol{E}}$$

Flux

- Gauss's Law:

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot \mathrm{d}\vec{\boldsymbol{A}} = \frac{q_{\text{in}}}{\varepsilon_0}$$

Remember From Previous Chapters

Electric Potential and Energy

- Electric Potential:

$$V = k_e \frac{q}{r}$$

- Potential Energy:

$$U_E = k_e \frac{q_1 q_2}{r}$$

- Relation to Electric Field:

$$\Delta V = -\vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{d}}$$

- Potential and Energy:

$$\Delta U_E = q \Delta V$$

Capacitance and Dielectrics

- Capacitance:

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$

- Capacitors in Parallel:

$$C_{\text{eq}} = \sum C_i$$

- Capacitors in Series:

$$\frac{1}{C_{\text{eq}}} = \sum \left(\frac{1}{C_i} \right)$$

- Energy Stored in Capacitor:

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$

Remember From Previous Chapters

- Energy Density of Electric Field:

$$u_E = \frac{U_E}{V} = \frac{1}{2}\varepsilon_0 E^2$$

- Dielectric Constant:

$$\Delta V = \Delta V_0/\kappa$$

$$C = \kappa C_0$$

$$Q = \kappa Q_0$$

$$U_E = U_0/\kappa$$

Current and Resistance

- Current:

$$I = \frac{\Delta Q}{\Delta t}$$

$$I_{\mathsf{avg}} = nAv_d q$$

- Ohm's Relation:

$$\Delta V = IR$$

- Resistance:

$$R = \rho\frac{L}{A}$$

- conductivity:

$$\sigma = \frac{1}{\rho}$$

- Temperature Effect

$$R = R_0[1 + \alpha(T - T_0)]$$

- Electrical Power:

$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

$$\text{Energy} = P\Delta t$$

Remember From Previous Chapters

Direct-Current Circuits

- Electromotive Force:

$$\Delta V = \varepsilon - Ir = IR,$$

- Resistors in Series:

$$R_{\text{eq}} = R_1 + R_2 + \ldots + R_n$$

- Resistors in Parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

- Kirchhoff's Rules:

1. Junction Rule:

$$\sum_{\text{node}} I = 0$$

2. Loop Rule:

$$\sum_{\text{loop}} \Delta V = 0$$

Magnetic Fields

- Magnetic Force on a Moving Charge:

$$\vec{\boldsymbol{F}}_B = q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$$

$$\left|\vec{\boldsymbol{F}}_B\right| = qvB\sin\theta$$

- Charges in a circular path under a magnetic field:

$$r = \frac{mv}{qB}$$

Remember From Previous Chapters

- Angular Velocity:

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

- Period of Revolution:

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

- Magnetic Force on a Current-Carrying Wire:

$$\vec{\boldsymbol{F}}_B = I\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}}$$

$$\left|\vec{\boldsymbol{F}}_B\right| = ILB\sin\theta$$

Sources of the Magnetic Field

- Biot-Savart Law:

$$\mathrm{d}\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{I}{r^2}(\mathrm{d}\vec{\boldsymbol{s}} \times \hat{\boldsymbol{r}})$$

- Magnetic Force Between Two Parallel Currents:

$$\frac{F}{\ell} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{a}$$

- Ampere's Law:

$$\oint \vec{\boldsymbol{B}} \cdot \mathrm{d}\vec{\boldsymbol{s}} = \mu_0 I$$

- Magnetic Field of a solenoid

$$B = \mu_0 n I$$

- Magnetic Flux and Gauss's Law:

$$\Phi_B = \oint \vec{\boldsymbol{B}} \cdot \mathrm{d}\vec{\boldsymbol{A}} = 0$$

Remember From Previous Chapters

Faraday's Law

- Induced emf:

$$\varepsilon = -N\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

- Motional emf:

$$\varepsilon = -B\ell v$$

Inductance

- Self-Induced:

$$\varepsilon_L = -L\,\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$L = N\frac{\Phi_B}{i}$$

- Inductance of a Solenoid:

$$L = \frac{\mu_0 N^2 A}{\ell} = \mu_0 n^2 A\ell = \mu_0 n^2 V$$

- Energy Stored in an Inductor:

$$U_B = \frac{1}{2}Li^2$$

- Energy Density Stored in a Solenoid:

$$u_B = \frac{U_B}{V} = \frac{B^2}{2\mu_0}$$

1. AC Sources

2. Resistors in an AC Circuit

3. Inductors in an AC Circuit

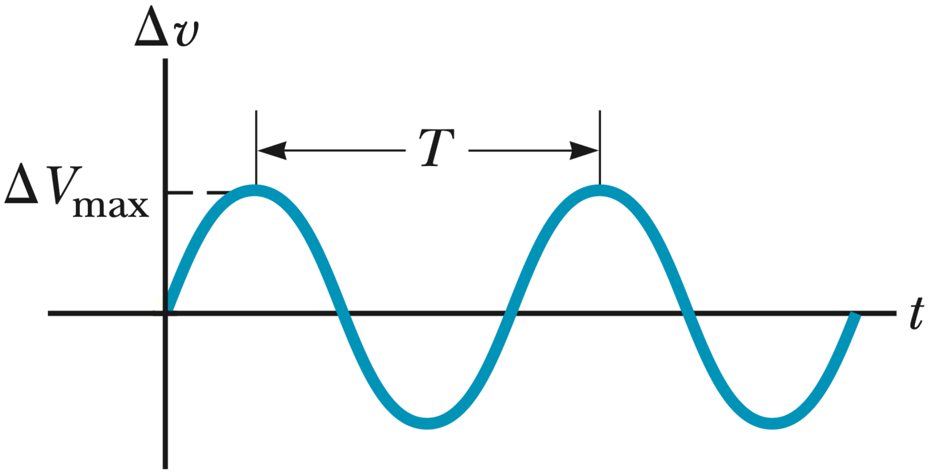
4. Capacitors in an AC Circuit

5. The RLC Series Circuit

6. Power in an AC Circuit

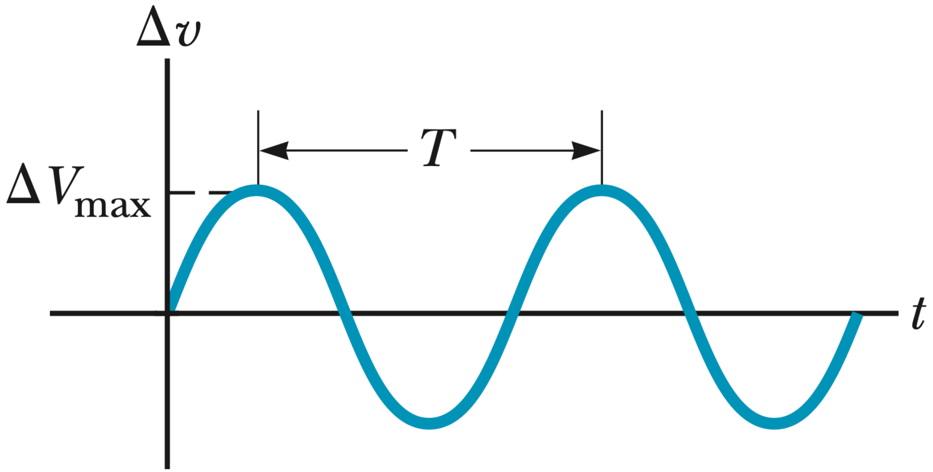
7. Resonance in a Series RLC Circuit

1.1 What is Alternating Current (AC)?



- An alternating current (AC) is a current that periodically reverses direction, in contrast to direct current (DC) which flows only in one direction.
- AC is the form of electric power that is delivered to homes and businesses, and it is generated by power plants using various methods such as turbines, solar panels, and combustion engines.

1.2 Voltage in an AC Circuit

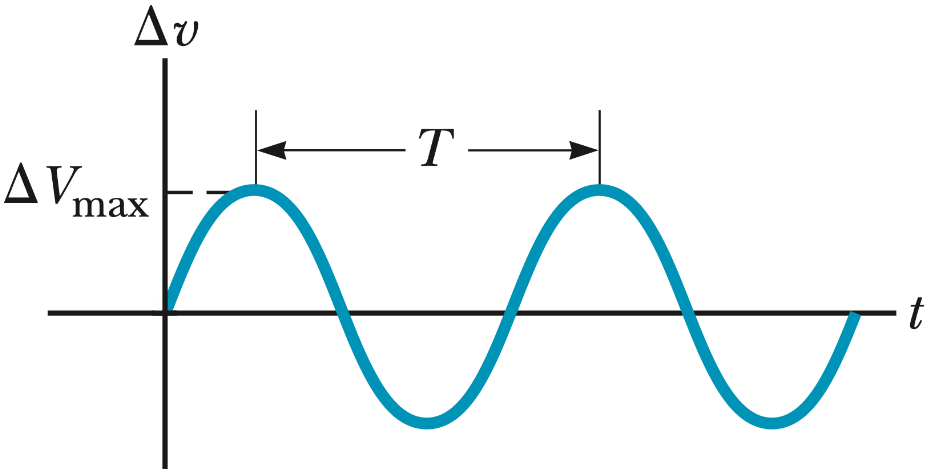


The voltage in an AC circuit varies with time, typically in a sinusoidal manner, and can be expressed as:

$$\Delta v = \Delta V_{max} \sin(\omega t)$$

where ΔV_{max} is the maximum voltage from the source (also called the voltage amplitude), ω is the angular frequency, and t is time.

1.3 Frequency and Period of AC



- The frequency of the AC is denoted by f and is related to the angular frequency ω by the equation:

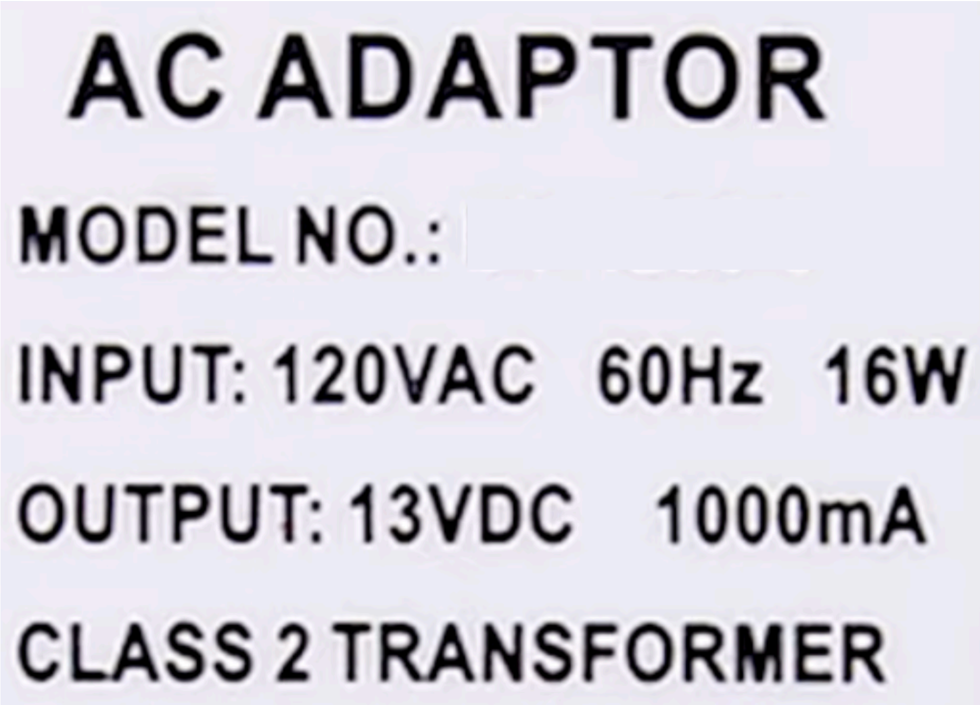
$$\omega = 2\pi f = \frac{2\pi}{T}$$

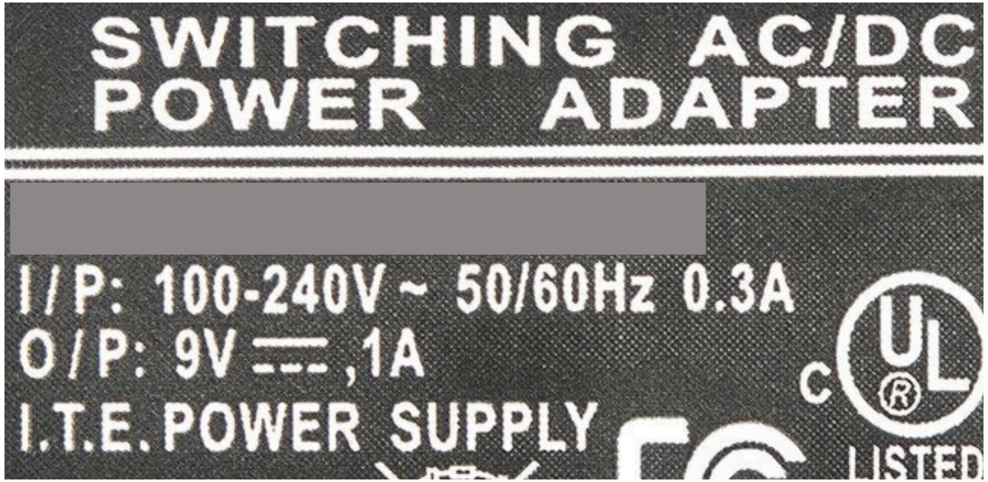
where T is the period of the AC, which is the time it takes for one complete cycle of the waveform.

1.4 What is the Frequency of AC in Saudi Arabia?

- Commercial AC power typically has a frequency of 50 Hz or 60 Hz, depending on the region; it is 60 Hz (377 rad/s) in Saudi Arabia.
- Therefore, all appliances and devices that are designed to operate on AC power in Saudi Arabia are built to work with this frequency, and they will NOT function properly if connected to a power source with a different frequency.

1.5 Reading Electronics Labels





1. AC Sources

2. Resistors in an AC Circuit

3. Inductors in an AC Circuit

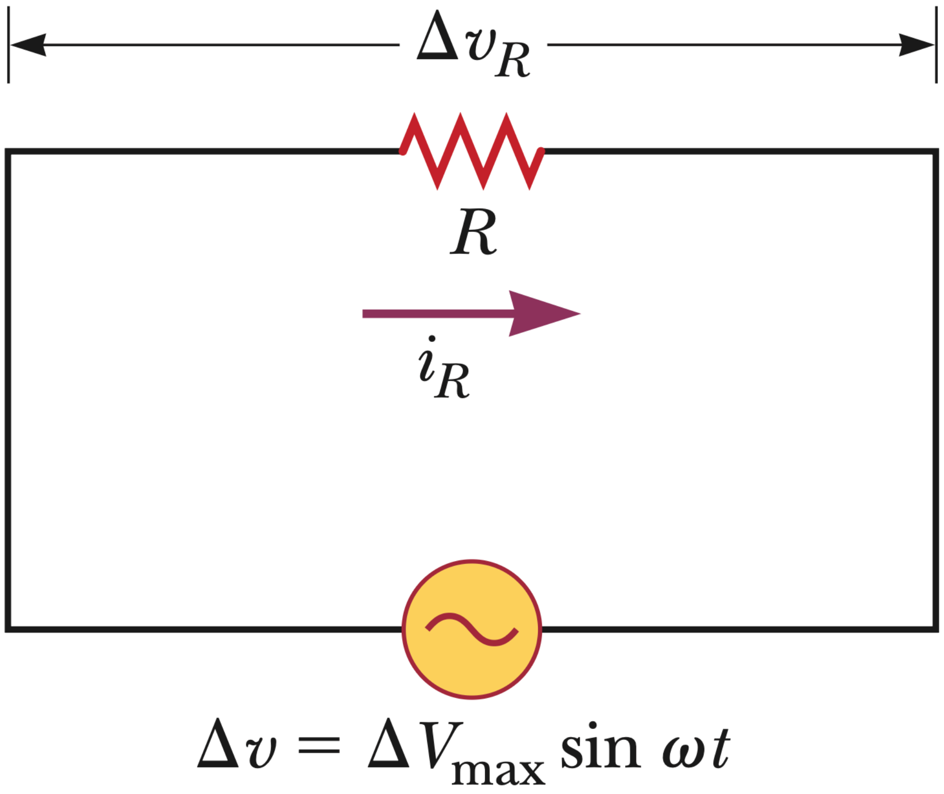
4. Capacitors in an AC Circuit

5. The RLC Series Circuit

6. Power in an AC Circuit

7. Resonance in a Series RLC Circuit

2.1 Resistor in an AC Circuit



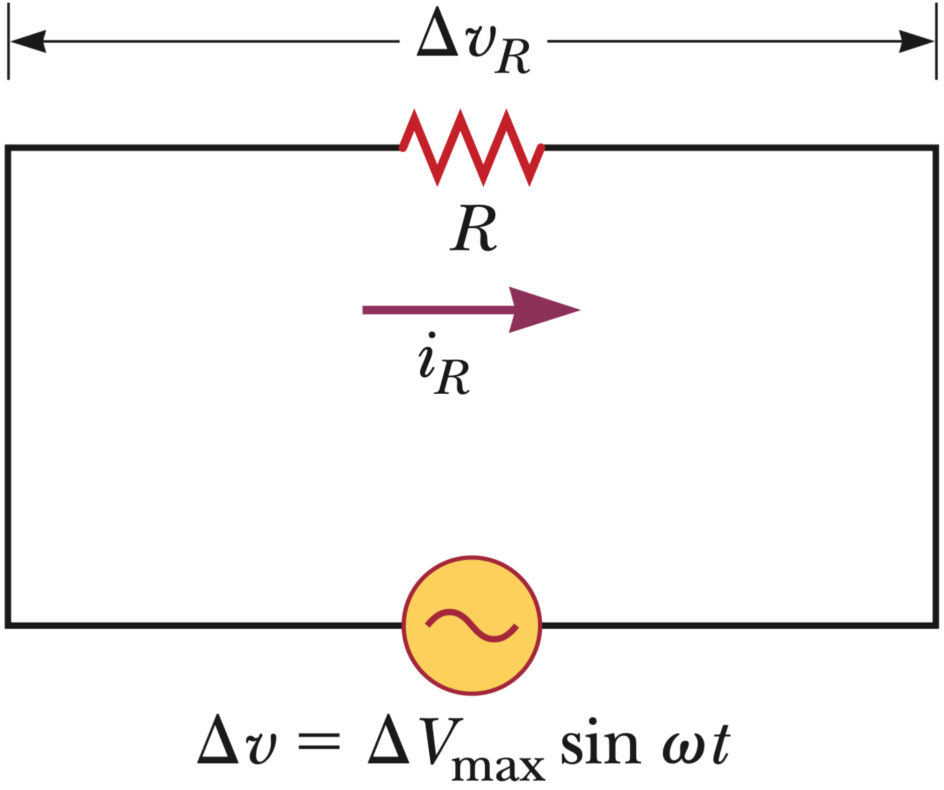
When a resistor is connected to an AC source, the current through the resistor also varies sinusoidally with time, and it can be expressed as:

$$i_R = \frac{\Delta v}{R} = \frac{\Delta V_{max}}{R} \sin(\omega t)$$

$$i_R = I_{max} \sin(\omega t)$$

Notice that i_R has the same frequency as the voltage from the AC source,

2.1 Resistor in an AC Circuit



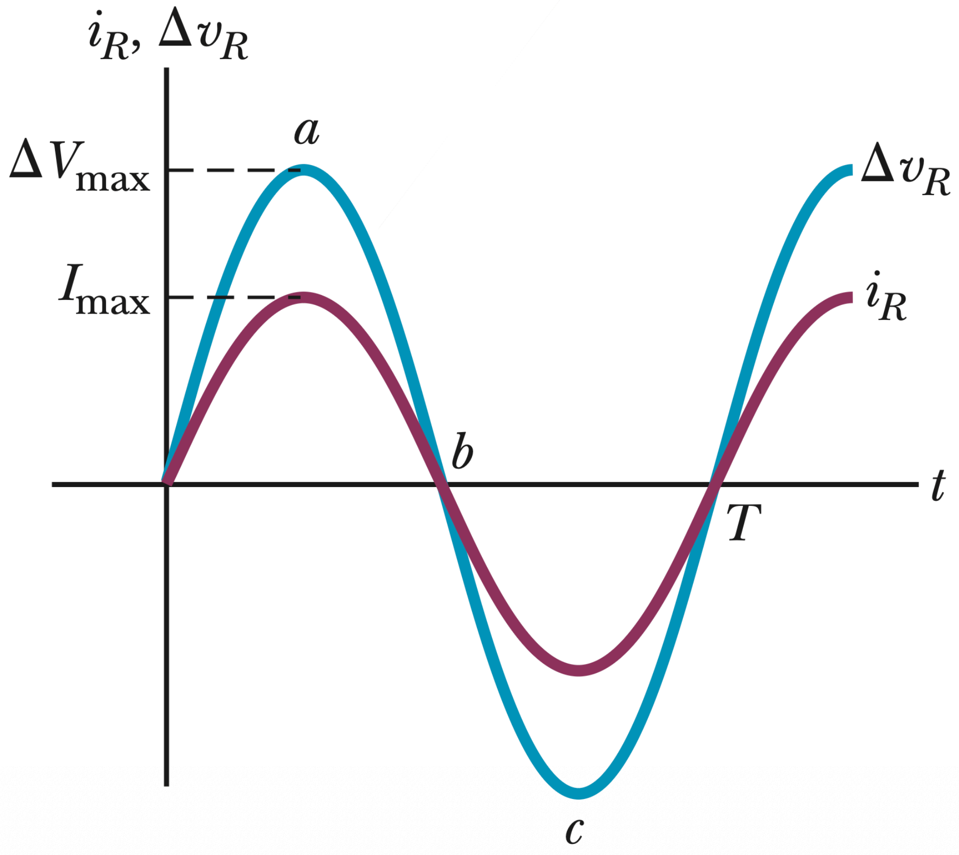
I_{max} is the maximum current through the resistor, which is given by:

$$I_{max} = \frac{\Delta V_{max}}{R}$$

The instantaneous voltage across the resistor is,

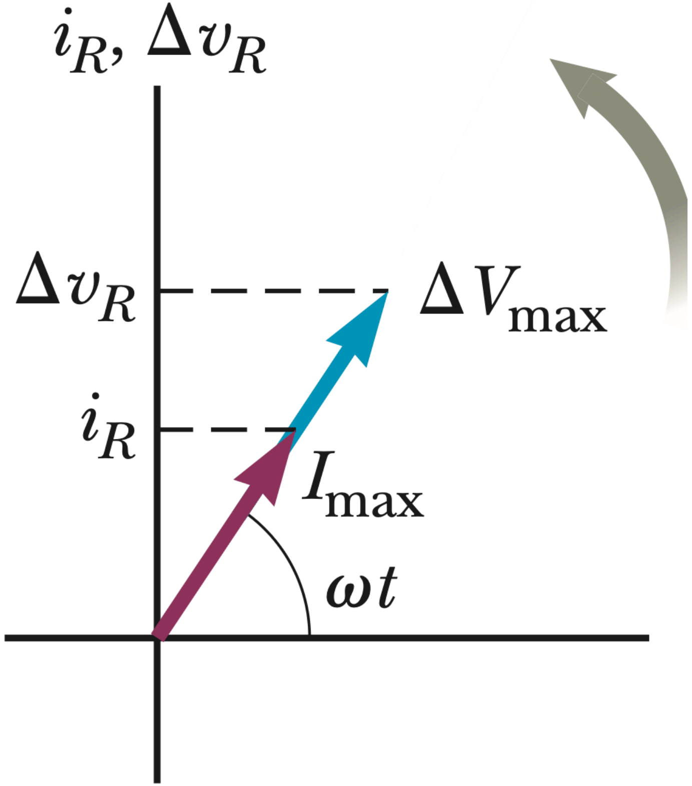
$$\Delta v = i_R R = I_{max} R \sin(\omega t)$$

2.1 Resistor in an AC Circuit



A plot of the voltage and current in a resistor connected to an AC source would show that they are **in phase**, meaning that they reach their maximum and minimum values at the same time.

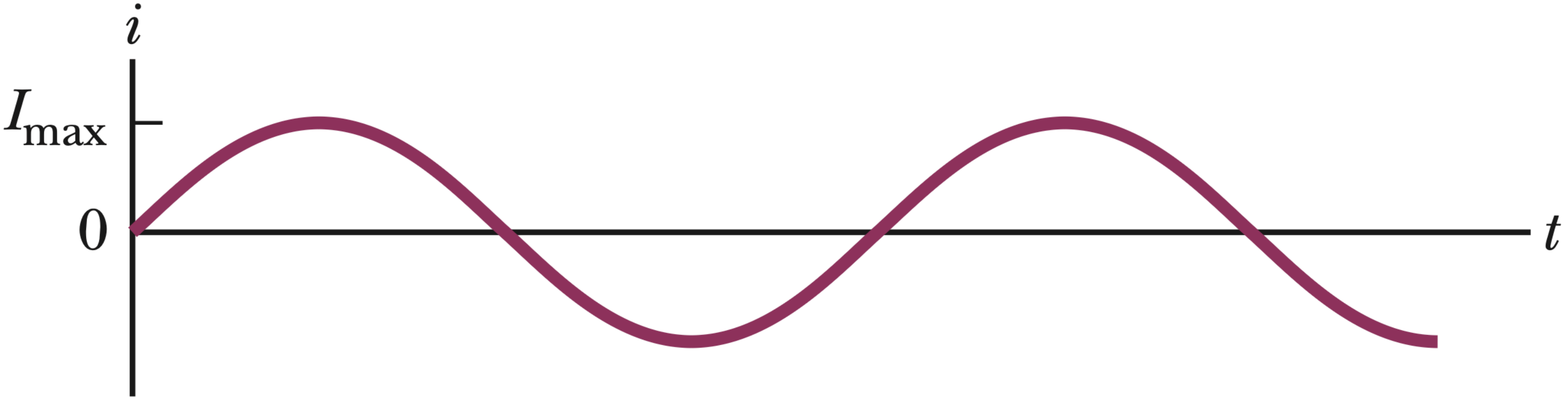
2.2 A Phasor Diagram



A phasor diagram is a graphical representation of the voltage and current in an AC circuit, where the voltage and current are represented as vectors (phasors) that rotate in the two-dimensional plane. In a resistor connected to an AC source, the voltage and current phasors are aligned along the same axis, indicating that they are **in phase**. The length of the phasor represents the amplitude of the voltage or current, while the angle of the phasor represents the **phase** ωt. In this case, since they are in phase, the angle between the voltage and current phasors is **zero**.

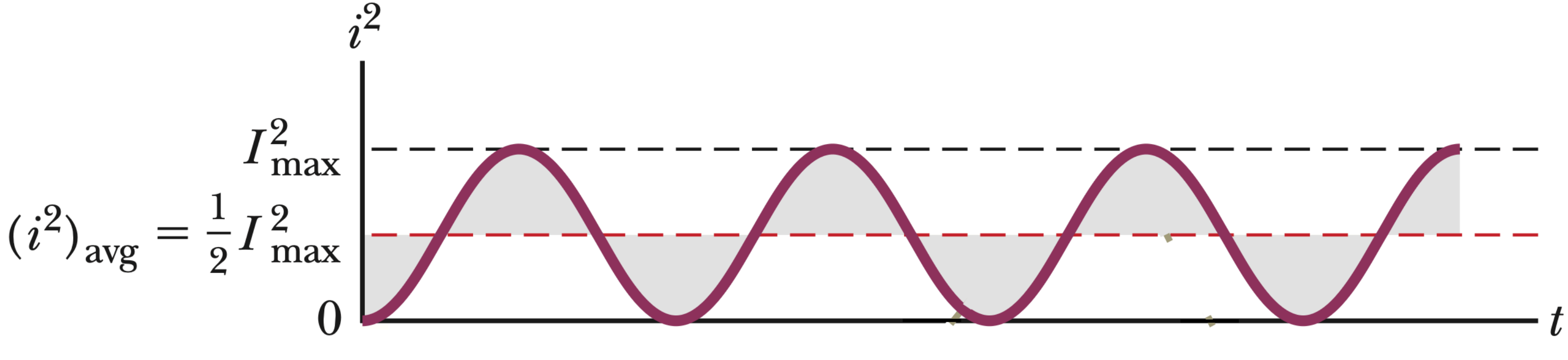
2.3 Average Value of AC

What is the average value of the following AC?



Zero!

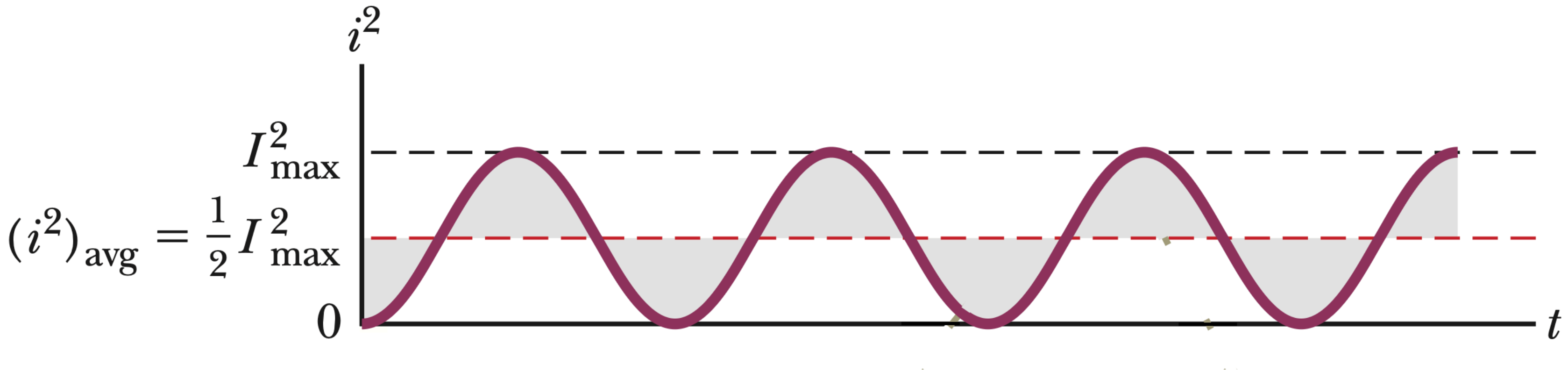
2.4 Root-Mean-Square (RMS) Values



The root-mean-square (RMS) value of an AC quantity is a measure of the effective value of the quantity, which is equivalent to the value of a DC quantity that would produce the same amount of power in a resistor. The RMS value of a sinusoidal AC voltage or current can be calculated using the following formulas:

$$(i^2)_{\text{avg}} = (I^2_{\text{max}} \sin^2 \omega t)_{\text{avg}} = I^2_{\text{max}} \times (1/2) = \frac{I^2_{\text{max}}}{2}$$

2.4 Root-Mean-Square (RMS) Values



The average value of $\sin^2 \omega t$ over one complete cycle is 1/2. Taking the square root, we get:

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707\ I_{max}$$

2.5 Power in a Resistor Connected to an AC Source

- The alternating current with maximum value $I_{\text{max}} = 2\ \text{A}$ delivers to a resistor the same power as a direct current that has a value of (0.707)(2 A) = 1.4 A.

- The average power delivered to a resistor that carries an alternating current is:

$$P_{\text{avg}} = I_{\text{rms}}^2 R$$

2.6 Voltage in an AC Circuit

- The alternating voltage also has an RMS value, which is given by:

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}} = 0.707\ \Delta V_{\text{max}}$$

- The voltage delivered to homes in Saudi Arabia is typically 220 V RMS, which corresponds to a maximum voltage of about 311 V.
- Voltmeters and ammeters are designed to measure the RMS values of voltage and current in AC circuits, so they will display the RMS values directly.

Example 2.1

The voltage output of an AC source is given by the expression $\Delta v = 200\sin(\omega t)$, where Δv is in volts.
Find the rms current in the circuit when this source is connected to a 47 Ω resistor.

$$I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}R}$$

$$I_{\text{rms}} = \frac{200}{\sqrt{2} * 47} = 3 \text{ A}$$

1. AC Sources

2. Resistors in an AC Circuit

3. Inductors in an AC Circuit

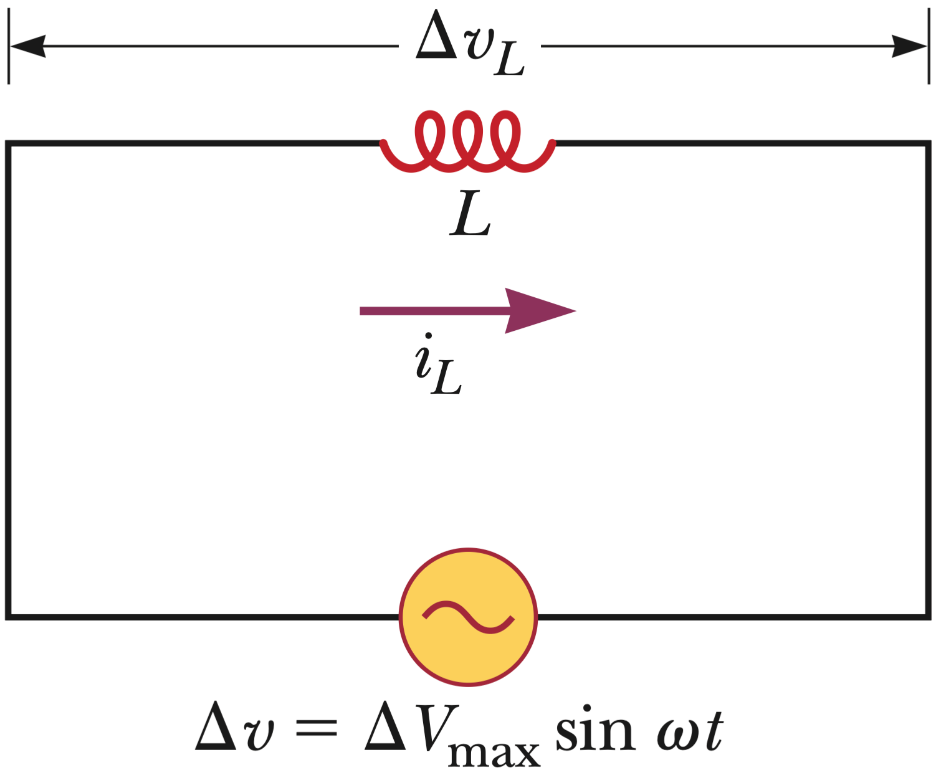
4. Capacitors in an AC Circuit

5. The RLC Series Circuit

6. Power in an AC Circuit

7. Resonance in a Series RLC Circuit

3.1 Inductor in an AC Circuit: Current

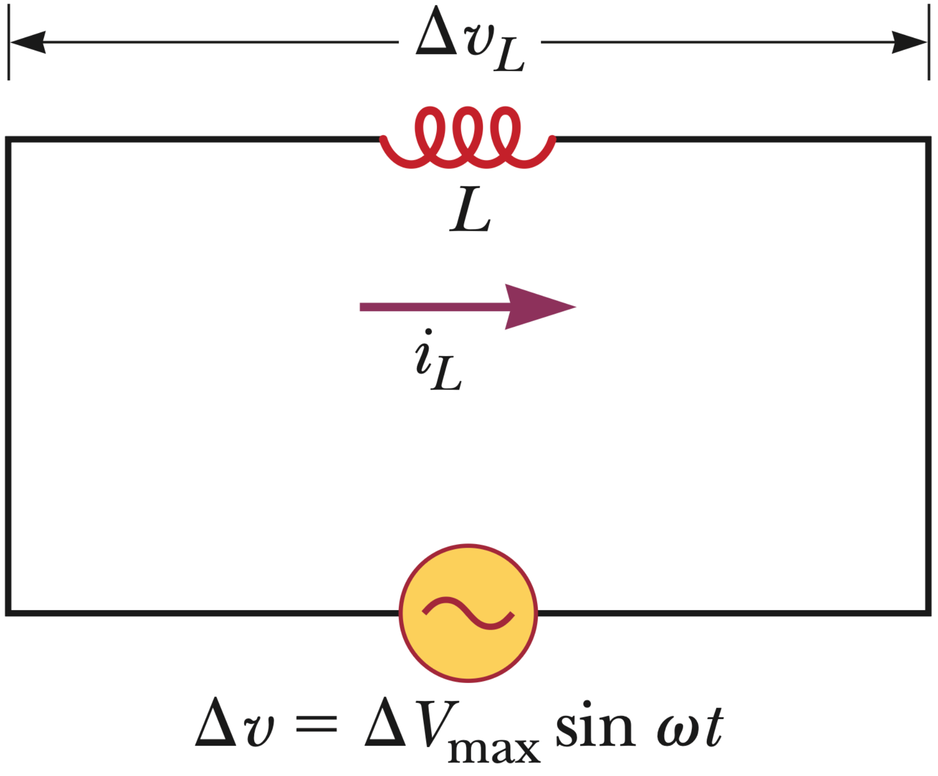


When an inductor is connected to an AC source, the current through the inductor also varies sinusoidally with time, but it **lags behind** the voltage by **90 degrees**. The current can be expressed as:

$$i_L = I_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$$

Therefore, the current reaches its maximum value a quarter of a cycle **after** the voltage reaches its maximum value.

3.2 Inductor in an AC Circuit: Voltage

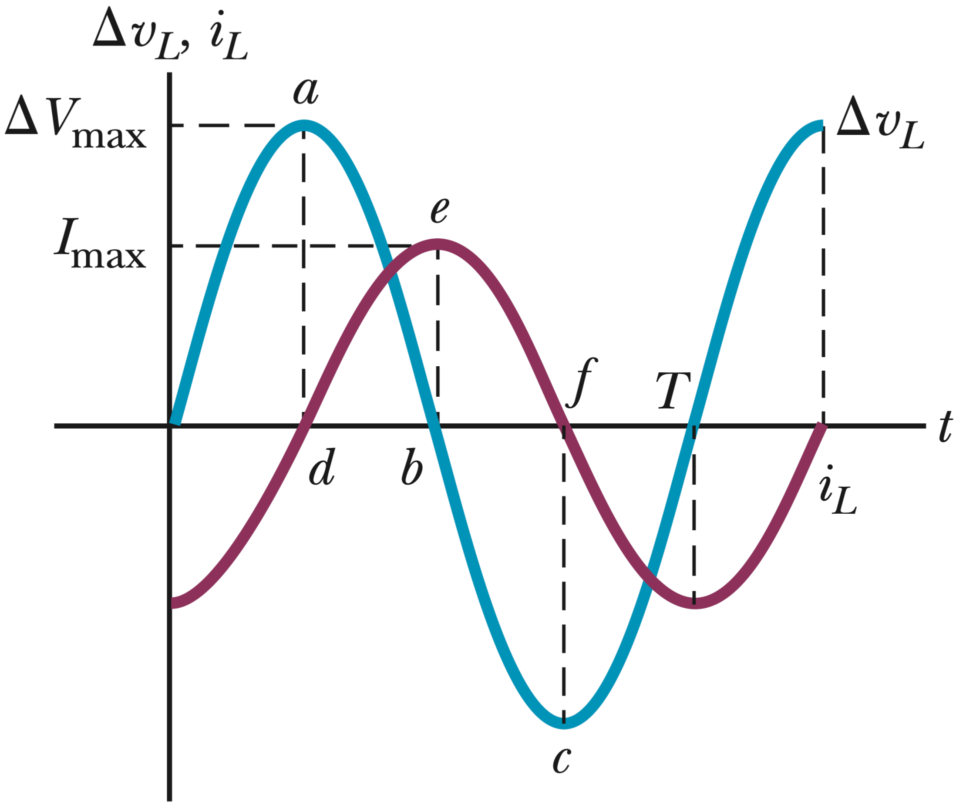


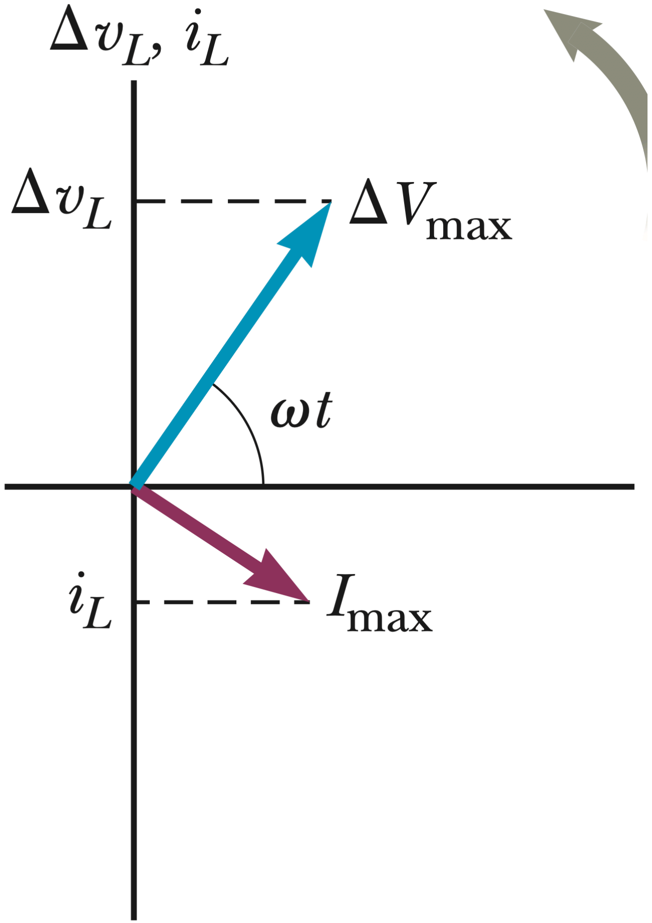
The voltage across the inductor which is given by $(-L\, di_L / dt)$ is:

$$\Delta v_L = -\Delta V_{max} \sin(\omega t)$$

Notice that the induced voltage across the inductor is **out of phase** with the source voltage by 180 degrees. However, the amplitude of both voltages is the same, which is $|\Delta V_{max}|$.

3.3 Inductor in an AC Circuit: Phasor Diagram





3.4 Inductive Reactance

King Saud University

The maximum current through the inductor is:

$$I_{\max} = \frac{\Delta V_{\max}}{\omega L}$$

This expression is similar to Ohm's relation, where ωL plays the role of resistance. Therefore, we define the **inductive reactance** X_L as:

$$X_L \equiv \omega L$$

The inductive reactance X_L is a measure of the opposition that an inductor presents to the flow of alternating current, and it has the unit of **ohm** (Ω). Therefore,

$$I_{\max} = \frac{\Delta V_{\max}}{X_L}$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L}$$

3.5 Example: Inductor in an AC Circuit

جامعة الملك سعود
King Saud University

Example 3.2

In a purely inductive AC circuit, L = 25 mH and the rms voltage is 150 V. Calculate the inductive reactance and rms current in the circuit if the frequency is 60 Hz.

$$X_L = \omega L = 2\pi f L = 2\pi(60)(25 \times 10^{-3}) = 9.42\ \Omega$$

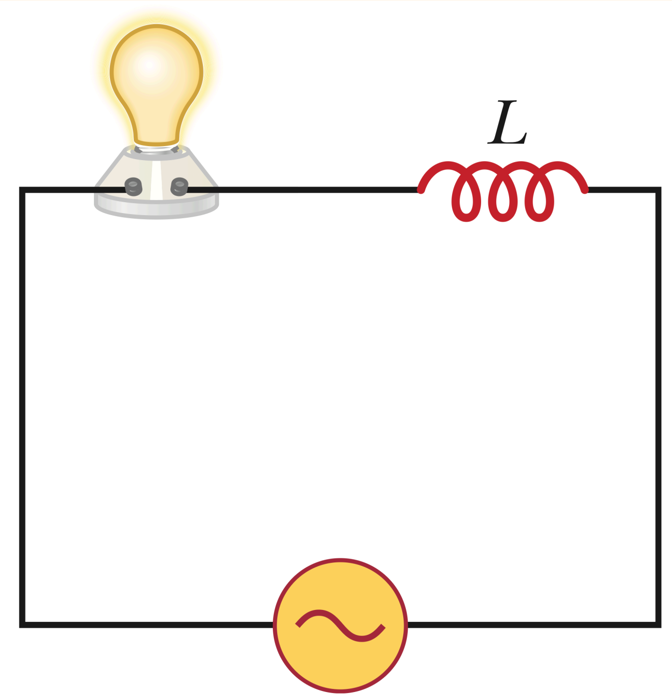
$$I_{\mathrm{rms}} = \frac{\Delta V_{\mathrm{rms}}}{X_L} = \frac{150}{9.42} = 15.9\ \mathrm{A}$$

3.6 Quiz: Inductor in an AC Circuit

King Saud University

Quiz

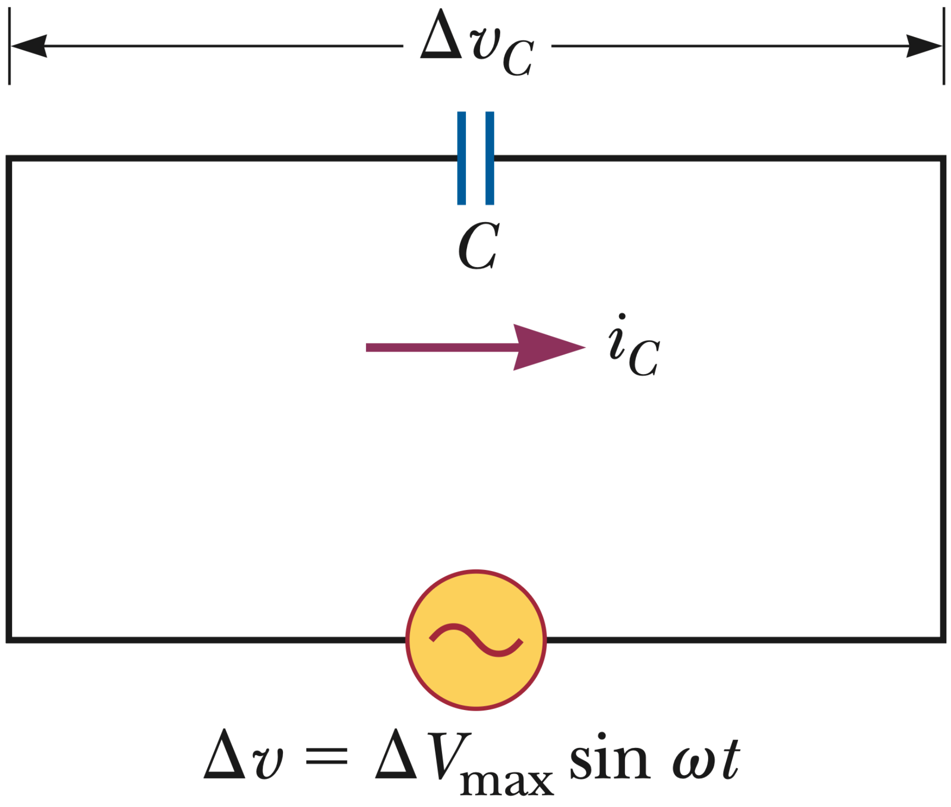
Consider the AC circuit in the Figure. The frequency of the AC source is adjusted while its voltage amplitude is held constant. When does the lightbulb glow the brightest? (a) It glows brightest at high frequencies. (b) It glows brightest at low frequencies. (c) The brightness is the same at all frequencies.



Answer is (b): At low frequencies, X_L is low, and I is high.

جامعة الملك سعود
King Saud University

4.1 Capacitor in an AC Circuit: Current

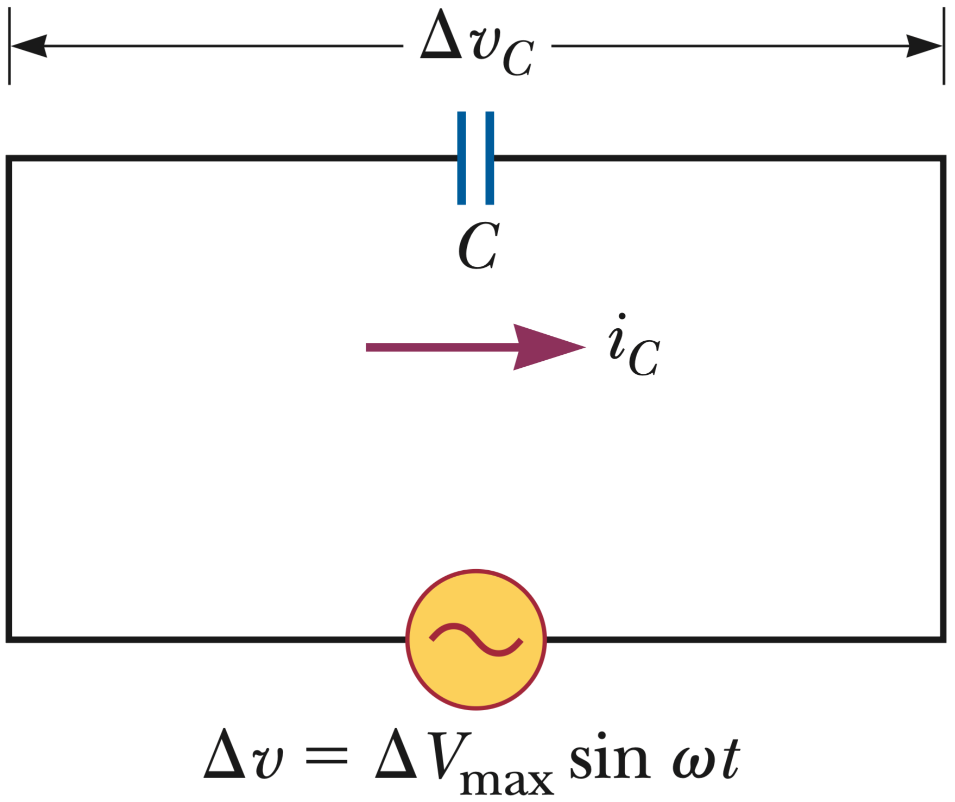


When a capacitor is connected to an AC source, the current through the capacitor also varies sinusoidally with time, but it **leads** (**ahead**) of the voltage by **90 degrees**. The current can be expressed as:

$$i_C = I_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

Therefore, the current reaches its maximum value a quarter of a cycle **before** the voltage reaches its maximum value.

4.2 Capacitor in an AC Circuit: Voltage and Charge



The voltage across the capacitor is:

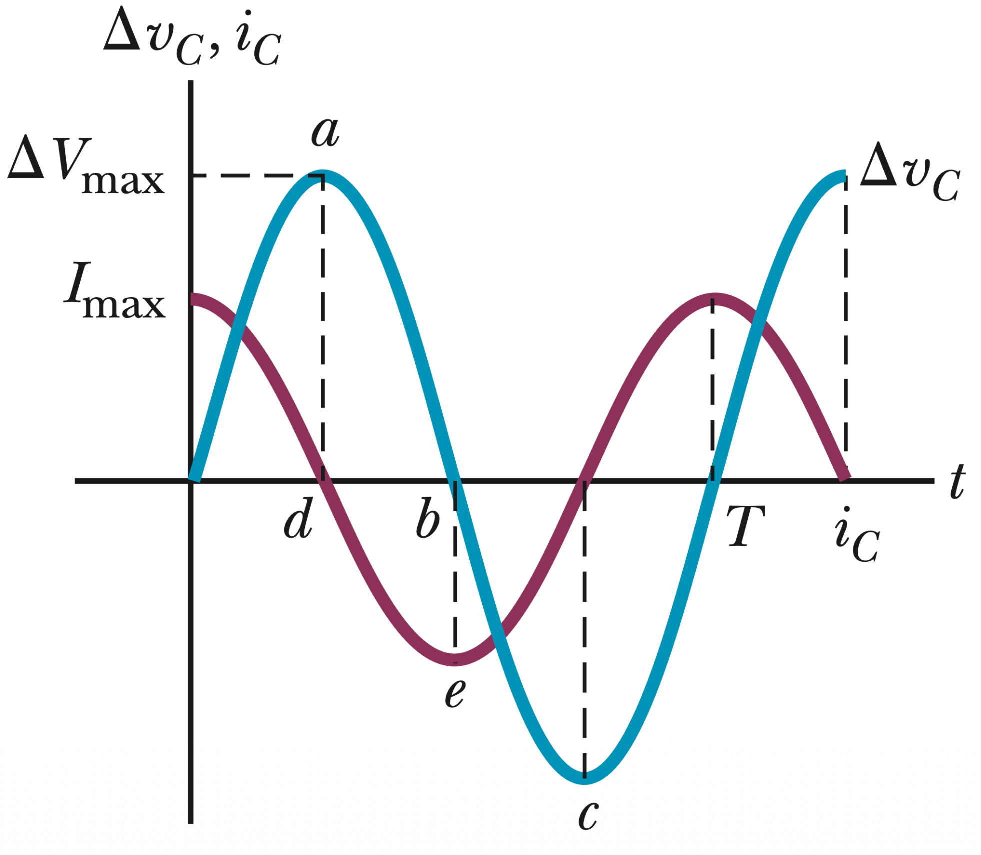
$$\Delta v_C = \Delta V_{\max} \sin(\omega t)$$

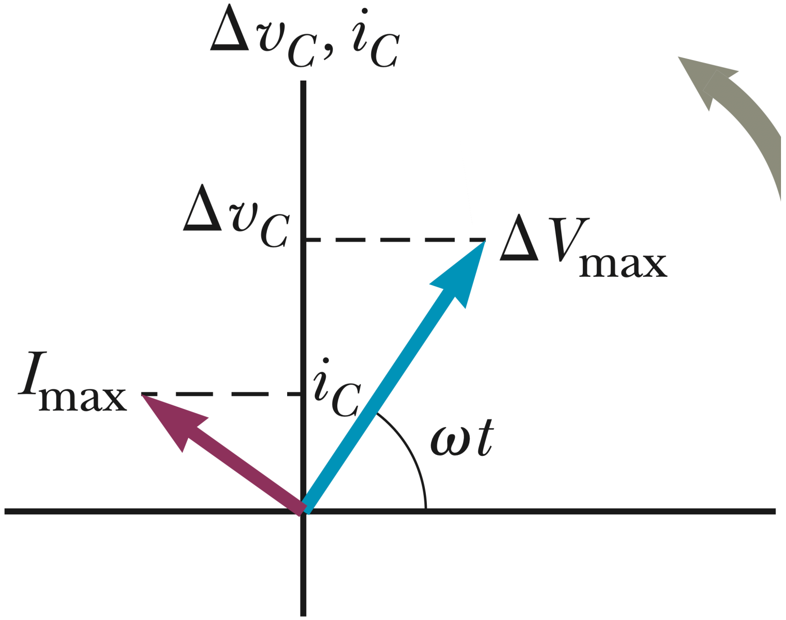
The charge on the capacitor is:

$$q = C\Delta v_C$$

Both the voltage and charge are **In phase** with the source voltage.

4.3 Capacitor in an AC Circuit: Phasor Diagram





4.4 Capacitive Reactance

The maximum current through the capacitor is:

$$I_{\max} = \omega C \Delta V_{\max} = \frac{\Delta V_{\max}}{1/(\omega C)}$$

This expression is similar to Ohm's relation, where $1/(\omega C)$ plays the role of resistance. Therefore,

$$X_C \equiv \frac{1}{\omega C}$$

The **capacitive reactance** X_C has the unit of **ohm** (Ω).

The maximum and rms current then are:

$$I_{\max} = \frac{\Delta V_{\max}}{X_C}$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_C}$$

King Saud University

Example 4.3

An 8 μF capacitor is connected to the terminals of a 60 Hz AC source whose rms voltage is 150 V. Find the capacitive reactance and the rms current in the circuit.

Solution 4.3

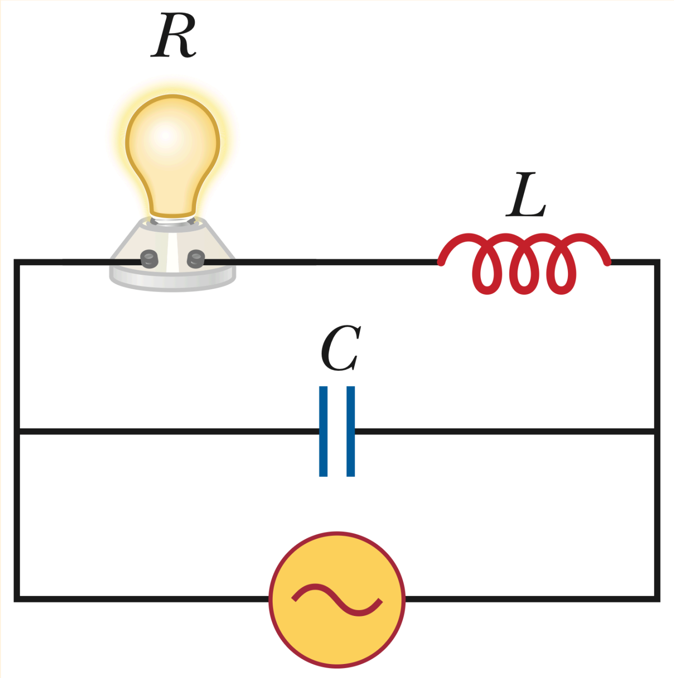
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi(60)(8 \times 10^{-6})} = 332\ \Omega$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_C} = \frac{150}{332} = 0.45\ \text{A}$$

4.6 Quiz: Capacitor in an AC Circuit

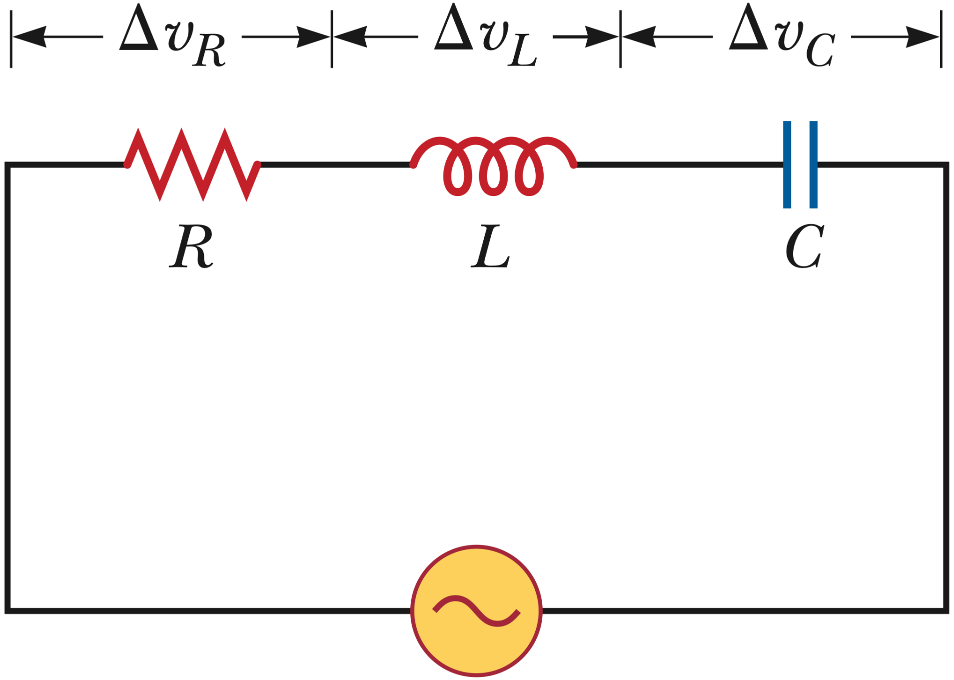
Quiz

Consider the AC circuit in the Figure. The frequency of the AC source is adjusted while its voltage amplitude is held constant. When does the lightbulb glow the brightest? (a) It glows brightest at high frequencies. (b) It glows brightest at low frequencies. (c) The brightness is the same at all frequencies.



Answer is (b): At low frequencies, X_L is low and X_C is high, therefore I at the bulb is high.

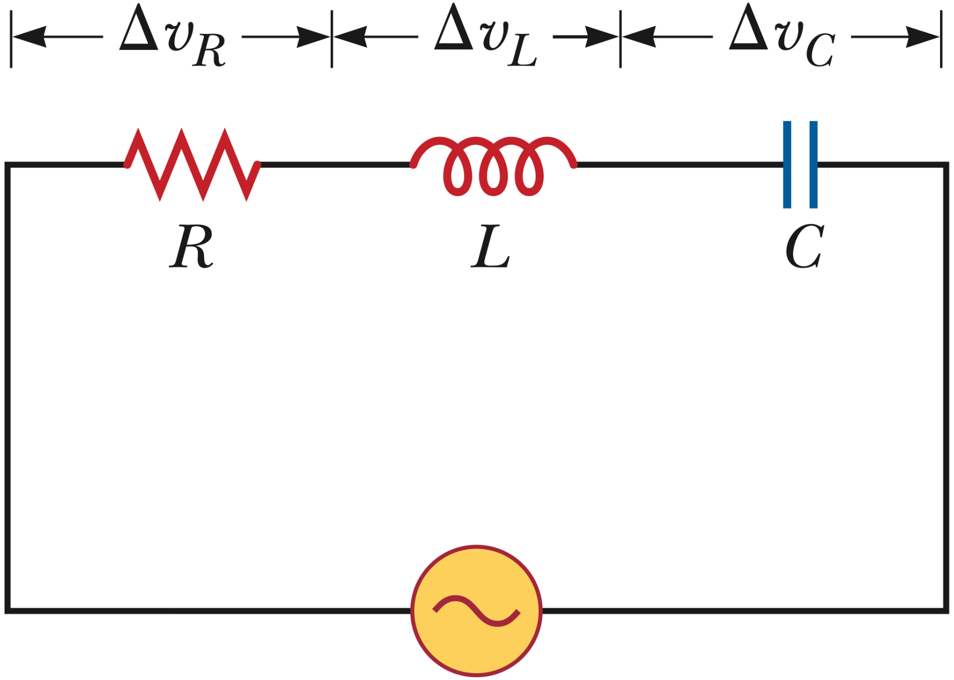
5.1 RLC Series Circuit



- An RLC series circuit consists of a resistor (R), an inductor (L), and a capacitor (C) connected in series to an AC source.
- The current through the circuit is the **same** for all elements.
- The voltage different across each element is generally **different** and depends on the reactance of the inductor and capacitor, as well as the resistance of the resistor.

5.2 RLC Series Circuit: Current is the Same Everywhere

King Saud University

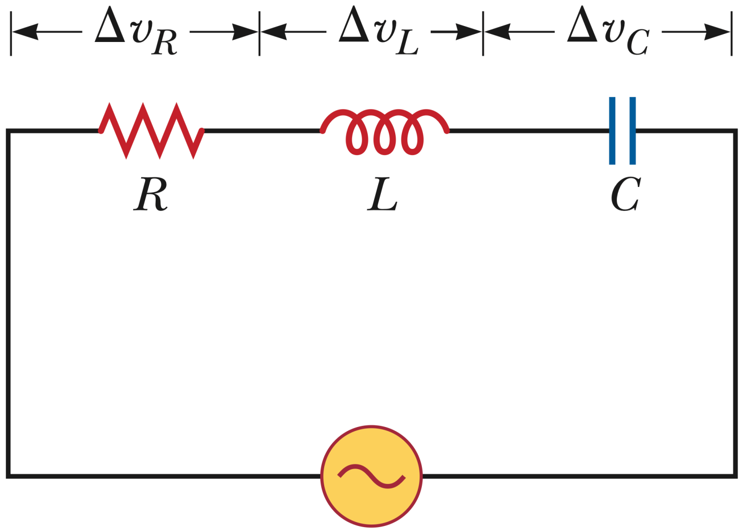


Since the current through the circuit is the same at every point, we can write:

$$i = i_R = i_L = i_C = I_{\max} \sin(\omega t - \varphi)$$

where φ is the phase angle between the current and the voltage from the AC source.

5.3 RLC Series Circuit: Voltages



Recall that the voltage across each element is given by:

$$\Delta v_R = I_{\max} R \sin(\omega t) = \Delta V_R \sin(\omega t)$$

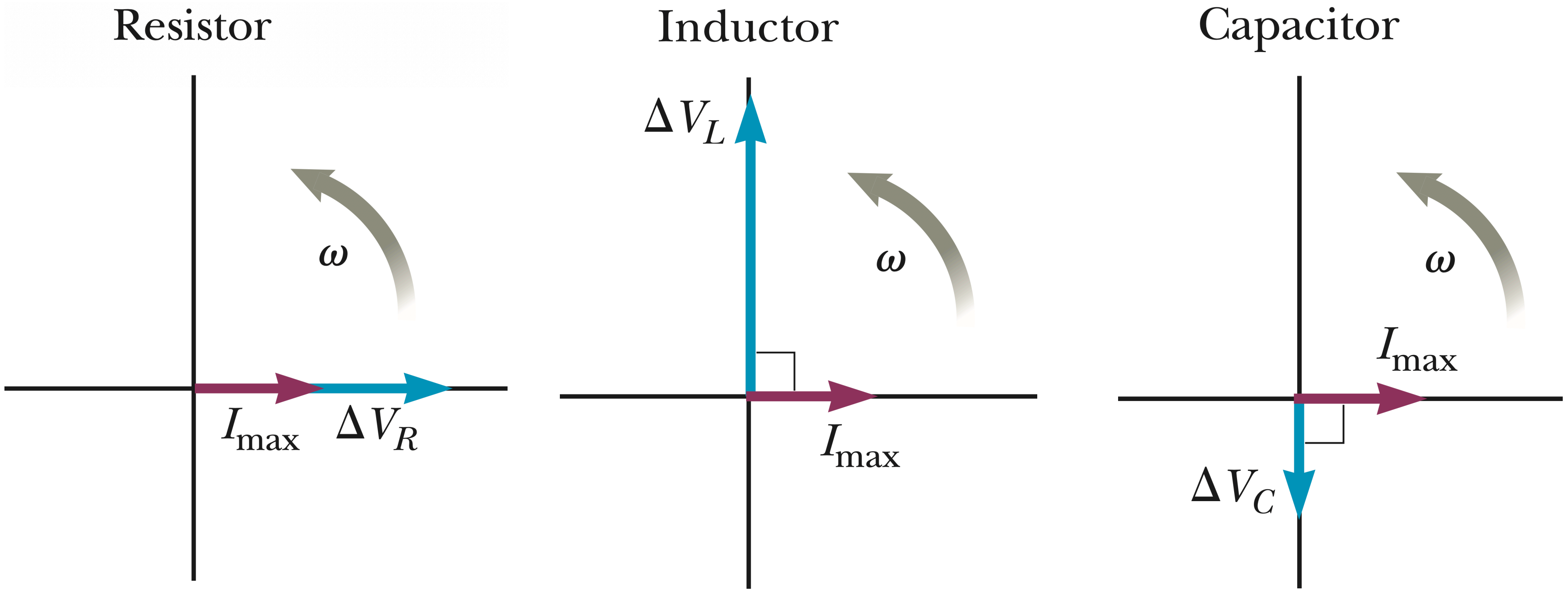
$$\Delta v_L = I_{\max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos(\omega t)$$

$$\Delta v_C = I_{\max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos(\omega t)$$

The sum of all voltages must equal the voltage from the AC source, which is:

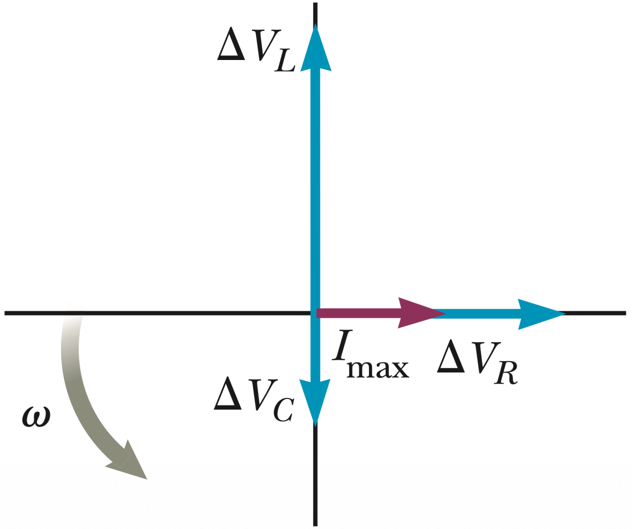
$$\Delta v = \Delta V_{\max} \sin(\omega t)$$

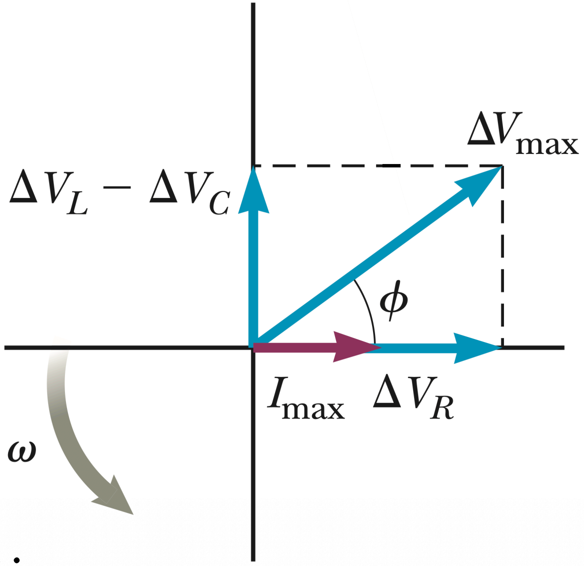
5.4 RLC Series Circuit: Phasor Diagram



The three voltages have different phase relation with the current; therefore, they cannot be added directly. Instead ...

5.5 RLC Series Circuit: Phasor Diagram





... Instead, they can be combined using vector addition.

$$\Delta V_{max} = \sqrt{(\Delta V_R)^2 + (\Delta V_L - \Delta V_C)^2} = \sqrt{(I_{max}R)^2 + (I_{max}X_L - I_{max}X_C)^2}$$

$$\Delta V_{max} = I_{max}\sqrt{R^2 + (X_L - X_C)^2} = I_{max}\ Z$$

5.6 RLC Series Circuit: Impedance and Current

The quantity Z is called the **impedance** of the circuit, and it is a measure of the total opposition that the circuit presents to the flow of alternating current. It has the unit of **ohm** (Ω) and is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

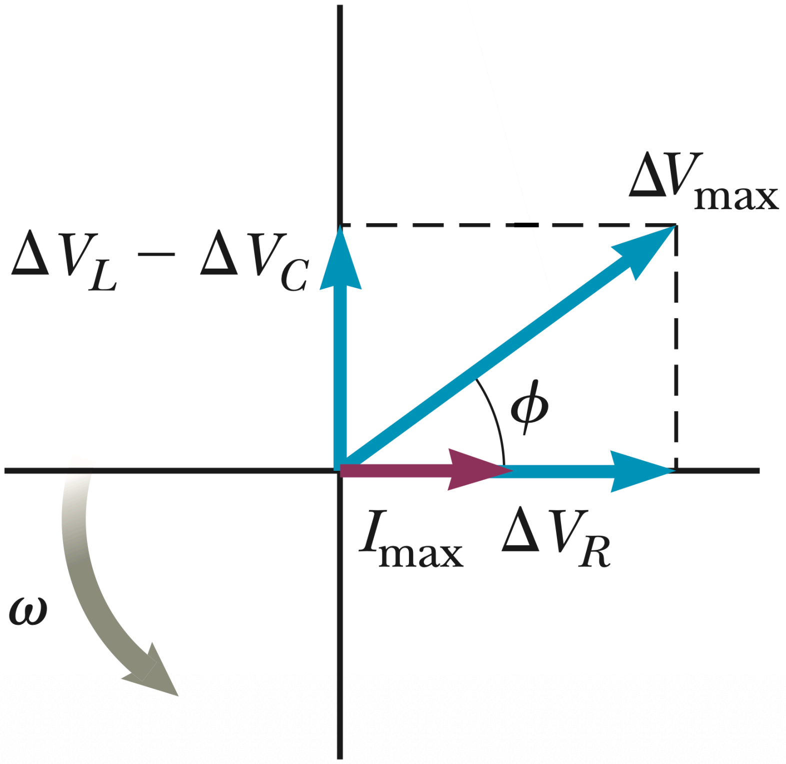
The maximum current in the circuit is:

$$I_{\max} = \frac{\Delta V_{\max}}{Z}$$

The rms current in the circuit is:

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z}$$

5.7 RLC Series Circuit: Phase Angle



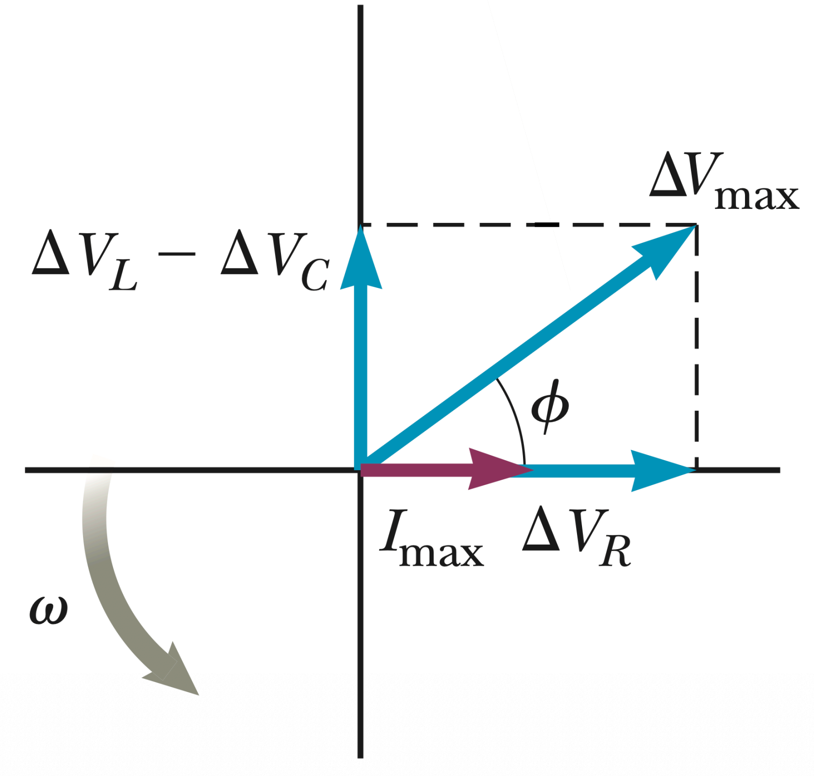
The phase angle φ between the current and the voltage from the AC source is given by:

$$\varphi = \tan^{-1}\left(\frac{\Delta V_L - \Delta V_C}{\Delta V_R}\right)$$

$$= \tan^{-1}\left(\frac{I_{max} X_L - I_{max} X_C}{I_{max} R}\right)$$

$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

5.7 RLC Series Circuit: Phase Angle

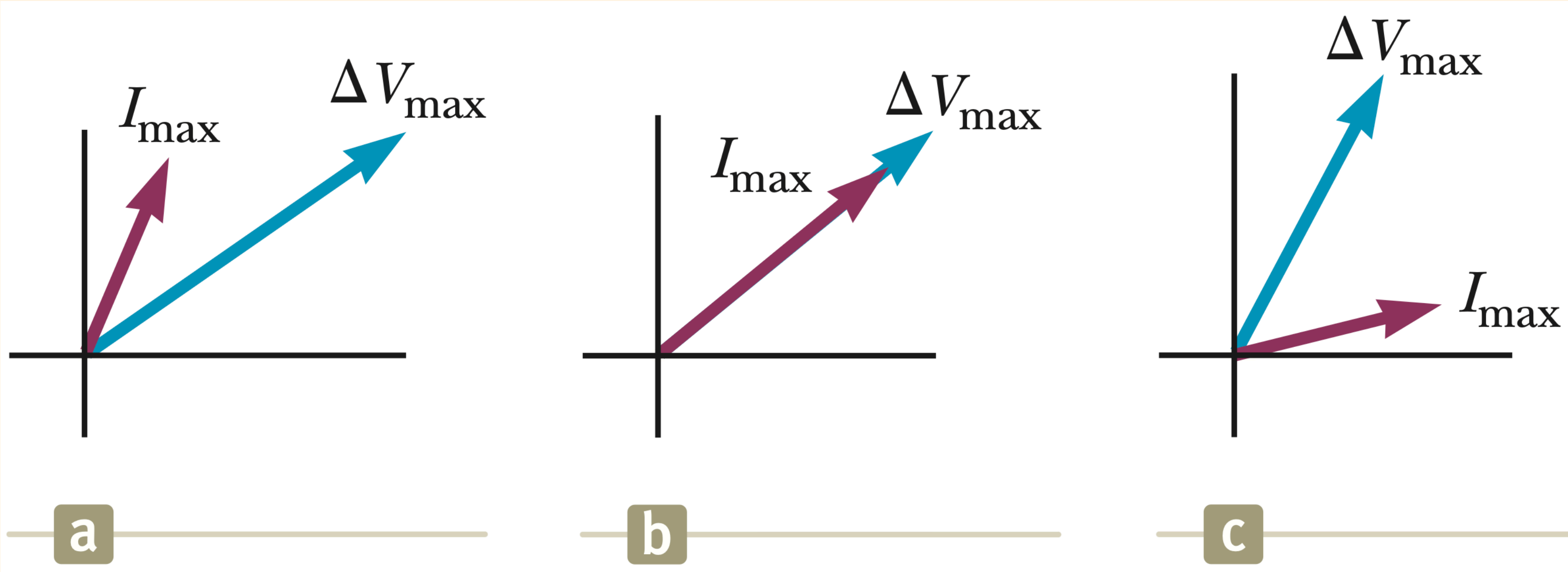


- If $X_L > X_C$, then φ is positive, and the current **lags** behind the voltage, therefore the circuit is more **inductive** than capacitive.
- If $X_L < X_C$, then φ is negative, and the current **leads** the voltage, therefore the circuit is more **capacitive** than inductive.
- If $X_L = X_C$, then φ is zero, and the current and voltage are **in phase**, therefore the circuit is purely **resistive**.

5.8 RLC Series Circuit: Quiz

Quiz

Label each part of the phasor diagrams as $X_L > X_C$, $X_L < X_C$ or $X_L = X_C$



Answer: (a) $X_C > X_L$; (b) $X_L = X_C$; (c) $X_L > X_C$.

5.9 Example: RLC Series Circuit

جامعة الملك سعود
King Saud University

Example 5.4

A series RLC circuit has $R = 425\ \Omega$, $L = 1.25$ H, and $C = 3.5\mu$F. It is connected to an AC source with $f = 60$ Hz and $\Delta V_{\text{max}} = 150\ $ V.
(A) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit.

Solution 5.4

$$\omega = 2\pi f = 2\pi(60) = 377 s^{-1}$$

$$X_L = \omega L = 377(1.25) = 471\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(377)(3.5 \times 10^{-6})} = 758\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(425)^2 + (471 - 758)^2} = 513\ \Omega$$

5.9 Example: RLC Series Circuit

King Saud University

(B) Find the maximum current in the circuit.

$$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{150}{513} = 0.29\ \text{A}$$

(C) Find the phase angle between the current and voltage.

$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{471 - 758}{425}\right) = -34^\circ$$

5.9 Example: RLC Series Circuit

(D) Find the maximum voltage across each element.

$$\Delta V_R = I_{\max} R = (0.29)(425) = 124\ \text{V}$$
$$\Delta V_L = I_{\max} X_L = (0.29)(471) = 138\ \text{V}$$
$$\Delta V_C = I_{\max} X_C = (0.29)(758) = 222\ \text{V}$$

5.9 Example: RLC Series Circuit

(E) What replacement value of L should an engineer analyzing the circuit choose such that the current leads the applied voltage by 30° rather than 34°? All other values in the circuit stay the same.

$$X_L = X_C + R\tan\varphi$$

$$\omega L = \frac{1}{\omega C} + R\tan\varphi$$

$$L = \frac{1}{\omega}\left(\frac{1}{\omega C} + R\tan\varphi\right)$$

$$= \frac{1}{377}\left(\frac{1}{(377)(3.5\times10^{-6})} + (425)\tan(-30°)\right) = 1.36\ \mathrm{H}$$

1. AC Sources

2. Resistors in an AC Circuit

3. Inductors in an AC Circuit

4. Capacitors in an AC Circuit

5. The RLC Series Circuit

6. Power in an AC Circuit

7. Resonance in a Series RLC Circuit

6.1 Power in an AC Circuit

The instantaneous power delivered to a circuit element is given by the product of the instantaneous voltage across the element and the instantaneous current through the element:

$$p = i\Delta v$$
$$= [I_{\text{max}} \sin(\omega t - \varphi)][\Delta V_{\text{max}} \sin(\omega t)]$$

This is a complicated expression that varies with time, and it is not very useful for calculating the power delivered to the circuit. Instead, we can calculate the **average** power delivered to the circuit over one complete cycle of the AC waveform, which is given by:

$$P_{\text{avg}} = I_{\text{rms}} \Delta V_{\text{rms}} \cos\varphi$$

where I_{rms} and ΔV_{rms} are the root-mean-square values of the current and voltage, respectively, and φ is the phase angle between the current and voltage.

6.1 Power in an AC Circuit

- The quantity $\cos\varphi$ is called the **power factor** of the circuit, and it represents the fraction of the total power that is actually delivered to the circuit as useful work.
- The power factor can range from 0 to 1, where a power factor of 1 indicates that all the power is delivered to the resistor.
- The power factor can be expressed as:

$$\cos\varphi = \frac{R}{Z}$$

Substituting the expressions into the formula for average power, we find that the average power delivered to a resistor in an AC circuit is:

$$P_{\text{avg}} = I_{\text{rms}}^2 R$$

6.1 Power in an AC Circuit

King Saud University

Example 6.5

Calculate the average power delivered to the series RLC circuit described in Example 4 ($\Delta V_{\max} = 150$ V, $I_{\max} = 0.29$ A, and $\varphi = -34°$).

Solution 6.5

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\max}}{\sqrt{2}} = \frac{150}{\sqrt{2}} = 106 \text{ V}$$

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}} = \frac{0.29}{\sqrt{2}} = 0.21 \text{ A}$$

$$P_{\text{avg}} = I_{\text{rms}} \Delta V_{\text{rms}} \cos\varphi = (0.21)(106)\cos(-34°) = 18.2 \text{ W}$$

1. AC Sources

2. Resistors in an AC Circuit

3. Inductors in an AC Circuit

4. Capacitors in an AC Circuit

5. The RLC Series Circuit

6. Power in an AC Circuit

7. Resonance in a Series RLC Circuit

7.1 Resonance in a Series RLC Circuit

King Saud University

- Resonance occurs in a series RLC circuit when an external AC source drives the circuit at a frequency that matches the natural frequency of the circuit (ω_0), which is determined by the values of the inductance and capacitance.
- The induced current in the circuit

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

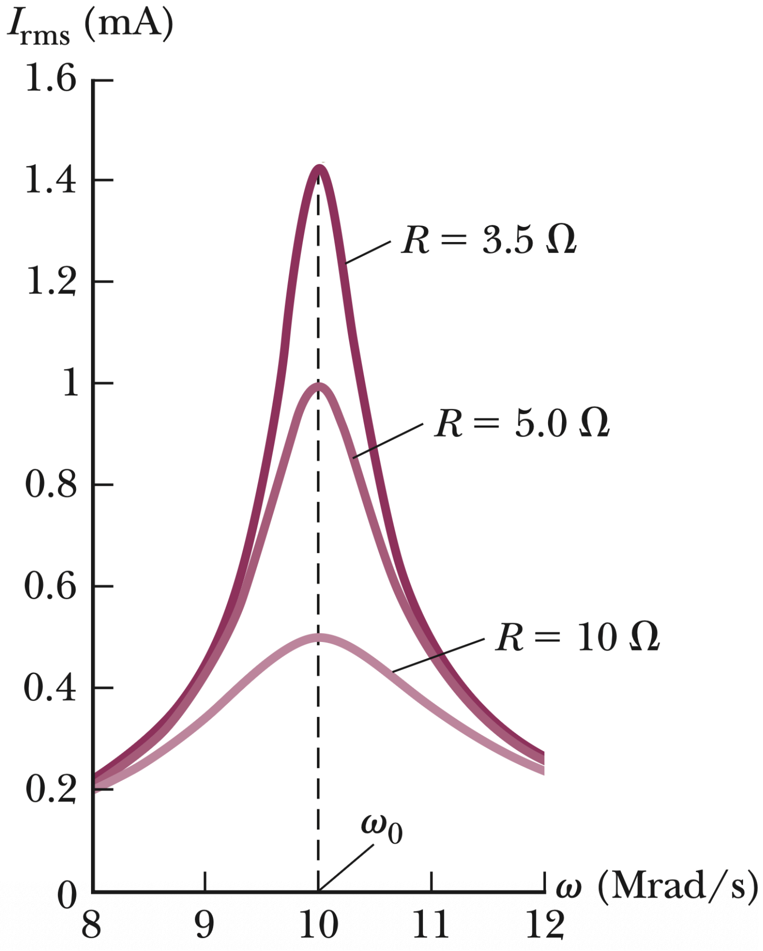
reaches its maximum value (maximum response) only at the resonance frequency ω_0. Therefore

$$X_L - X_C = 0 \quad \Longrightarrow \quad \omega_0 L = \frac{1}{\omega_0 C}.$$

Solving for ω_0, we find that the resonance frequency is:

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

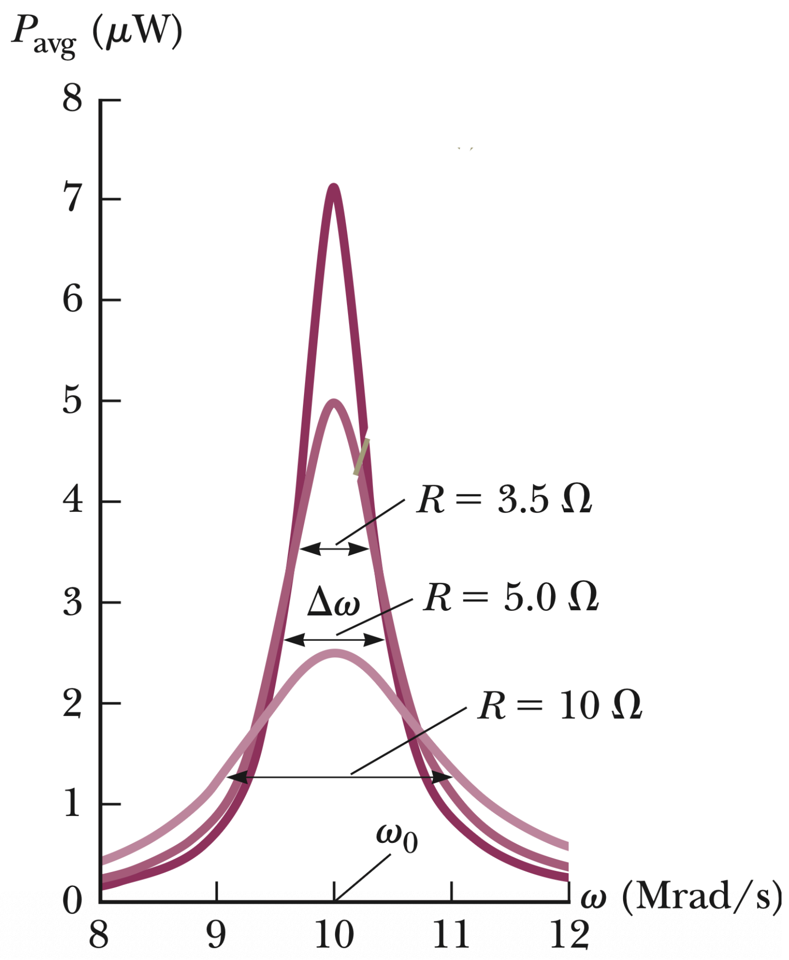
7.2 Power at Resonance



- At **resonance**, the impedance of the circuit is equal to the resistance R, and therefore, the average power delivered to the circuit is at its maximum value.

- At **off-resonance** frequencies, the impedance of the circuit is greater than R, and the average power is less than the maximum value.

7.3 Quality Factor

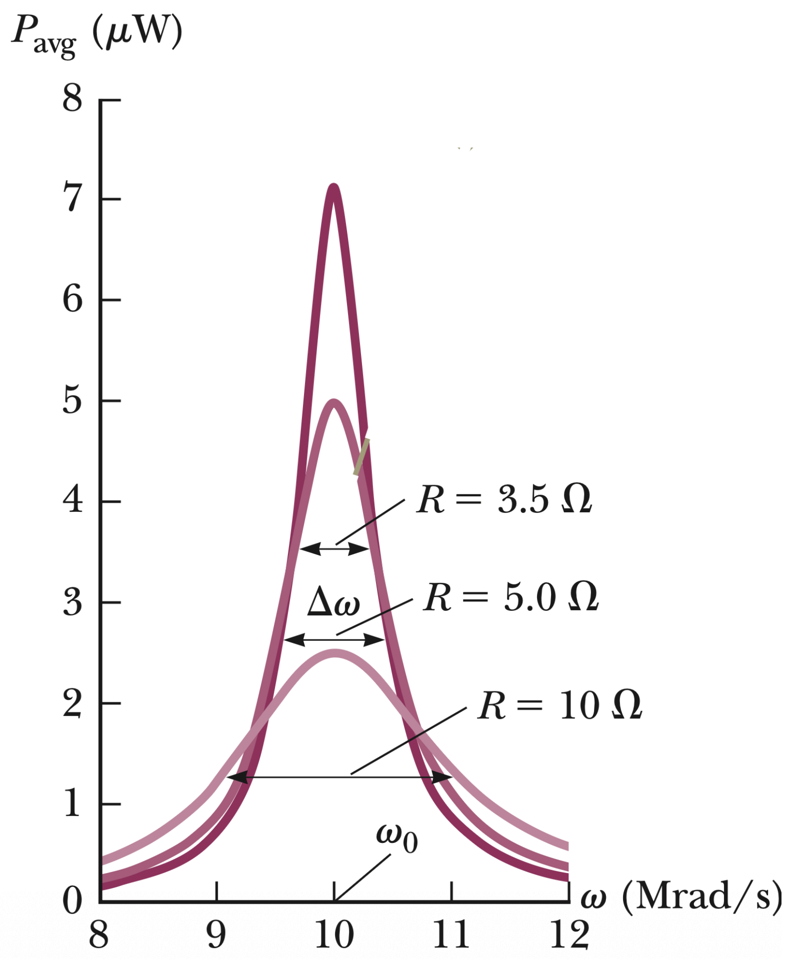


The dimensionless parameter **quality factor** Q of RLC circuit is a measure of how **sharp** the resonance is, and it is defined as the ratio of the resonance frequency ω_0 to the bandwidth of the resonance $\Delta\omega$.

$$Q = \frac{\omega_0}{\Delta\omega}$$

where $\Delta\omega$ is the width of the curve measured between the two values of ω for which $P_{avg} = P_{avg}^{max}/2$, known as the *half-power points*.

7.3 Quality Factor



$\Delta\omega$ can be calculated using the following formula:

$$\Delta\omega = \frac{R}{L}$$

Therefore, the quality factor can be expressed as:

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0 L}{R} = \frac{X_L}{R}$$

7.3 Quality Factor

Example 7.6

Consider a series RLC circuit for which $R = 150\ \Omega$, $L = 20$ mH, $\Delta V_{\text{rms}} = 20$ V, and $\omega_0 = 5\ 000 s^{-1}$. Determine the value of the capacitance for which the current is a maximum.

Solution 7.6

$$\omega_0 = \frac{1}{\sqrt{LC}} \Longrightarrow C = \frac{1}{\omega_0^2 L}$$

$$C = \frac{1}{(5000)^2(20 \times 10^{-3})} = 2\mu\text{F}$$

Suggested Problems

1, 4, 6, 9, 12, 13, 16, 19, 20, 21, 23, 24, 25

Book: Serway, R. A., & Jewett, J. W. (2018). Physics for Scientists and Engineers (10th ed.)

Chapter: 32 - Alternating Current Circuits