Ch.31: Inductance

Physics 104: Electricity and Magnetism

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Outline

Remember From Previous Chapters

Classical Mechanics

- Equations of motion:

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a} \cdot (\vec{r}_f - \vec{r}_i)$$

- Newton's Second Law:

$$\sum \vec{F} = m\vec{a}$$

- Work-Energy Theorem:

$$W = \Delta K$$

$$W = \vec{F} \cdot \vec{d}$$

$$K = \frac{1}{2}m\vec{v}^2$$

Electric Field

- Coulomb's Law:

$$\vec{F}_e = k_e \frac{q_1 q_2}{r^2}\hat{r}$$

- Electric Field:

$$\vec{E} = \frac{\vec{F}_e}{q_0} = k_e \frac{q}{r^2}\hat{r}$$

$$\vec{a} = \left(\frac{q}{m}\right)\vec{E}$$

Flux

- Gauss's Law:

$$\Phi_E = \oint \vec{E} \cdot \mathrm{d}\vec{A} = \frac{q_{\text{in}}}{\varepsilon_0}$$

Remember From Previous Chapters

Electric Potential and Energy

- Electric Potential:

$$V = k_e \frac{q}{r}$$

- Potential Energy:

$$U_E = k_e \frac{q_1 q_2}{r}$$

- Relation to Electric Field:

$$\Delta V = -\vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{d}}$$

- Potential and Energy:

$$\Delta U_E = q \Delta V$$

Capacitance and Dielectrics

- Capacitance:

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$

- Capacitors in Parallel:

$$C_{\text{eq}} = \sum C_i$$

- Capacitors in Series:

$$\frac{1}{C_{\text{eq}}} = \sum \left(\frac{1}{C_i} \right)$$

- Energy Stored in Capacitor:

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$

Remember From Previous Chapters

- Energy Density of Electric Field:

$$u_E = \frac{U_E}{V} = \frac{1}{2}\varepsilon_0 E^2$$

- Dielectric Constant:

$$\Delta V = \Delta V_0/\kappa$$

$$C = \kappa C_0$$

$$Q = \kappa Q_0$$

$$U_E = U_0/\kappa$$

Current and Resistance

- Current:

$$I = \frac{\Delta Q}{\Delta t}$$

$$I_{\text{avg}} = nAv_d q$$

- Ohm's Relation:

$$\Delta V = IR$$

- Resistance:

$$R = \rho\frac{L}{A}$$

- conductivity:

$$\sigma = \frac{1}{\rho}$$

- Temperature Effect

$$R = R_0[1 + \alpha(T - T_0)]$$

- Electrical Power:

$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

$$\text{Energy} = P\Delta t$$

Remember From Previous Chapters

Direct-Current Circuits

- Electromotive Force:

$$\Delta V = \varepsilon - Ir = IR,$$

- Resistors in Series:

$$R_{\text{eq}} = R_1 + R_2 + \ldots + R_n$$

- Resistors in Parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

- Kirchhoff's Rules:

1. Junction Rule:

$$\sum_{\text{node}} I = 0$$

2. Loop Rule:

$$\sum_{\text{loop}} \Delta V = 0$$

Magnetic Fields

- Magnetic Force on a Moving Charge:

$$\vec{\boldsymbol{F}}_B = q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$$

$$\left|\vec{\boldsymbol{F}}_B\right| = qvB\sin\theta$$

- Charges in a circular path under a magnetic field:

$$r = \frac{mv}{qB}$$

Remember From Previous Chapters

- Angular Velocity:

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

- Period of Revolution:

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

- Magnetic Force on a Current-Carrying Wire:

$$\vec{\boldsymbol{F}}_B = I\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}}$$

$$\left|\vec{\boldsymbol{F}}_B\right| = ILB\sin\theta$$

Sources of the Magnetic Field

- Biot-Savart Law:

$$\mathrm{d}\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{I}{r^2}(\mathrm{d}\vec{\boldsymbol{s}} \times \hat{\boldsymbol{r}})$$

- Magnetic Force Between Two Parallel Currents:

$$\frac{F}{\ell} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{a}$$

- Ampere's Law:

$$\oint \vec{\boldsymbol{B}} \cdot \mathrm{d}\vec{\boldsymbol{s}} = \mu_0 I$$

- Magnetic Field of a solenoid

$$B = \mu_0 n I$$

- Magnetic Flux and Gauss's Law:

$$\Phi_B = \oint \vec{\boldsymbol{B}} \cdot \mathrm{d}\vec{\boldsymbol{A}} = 0$$

Remember From Previous Chapters

Faraday's Law

- Induced emf:

$$\varepsilon = -N \frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

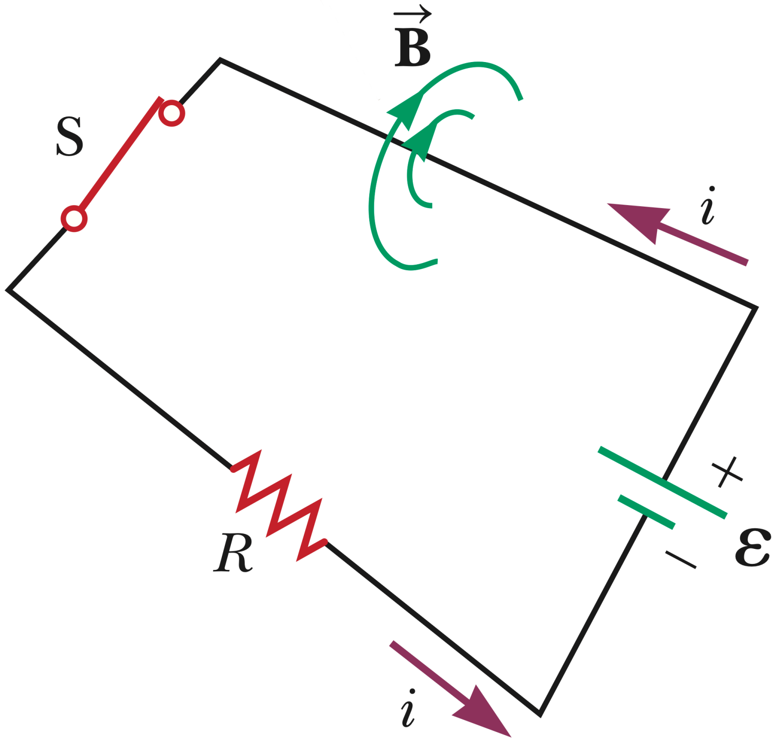
- Motional emf:

$$\varepsilon = -B\ell v$$

1. Self-Induction and Inductance

2. Energy in a Magnetic Field

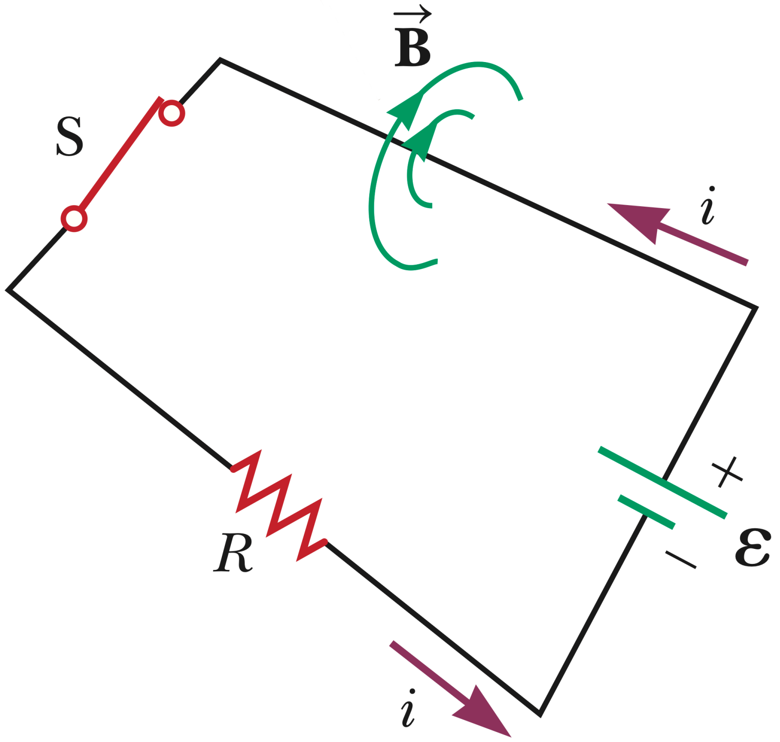
1.1 Self-Induction and Inductance



- Can the change in current in a circuit jumps *immediately* from $I = 0$ to $I = I_{\text{max}}$? The answer is **NO**, because of the **self-induction** phenomenon, which *slows* down the change in current.

- When the current in a circuit changes, the magnetic field it produces also changes, which induces an emf in the circuit itself. This phenomenon is called **self-induction**.

1.1 Self-Induction and Inductance

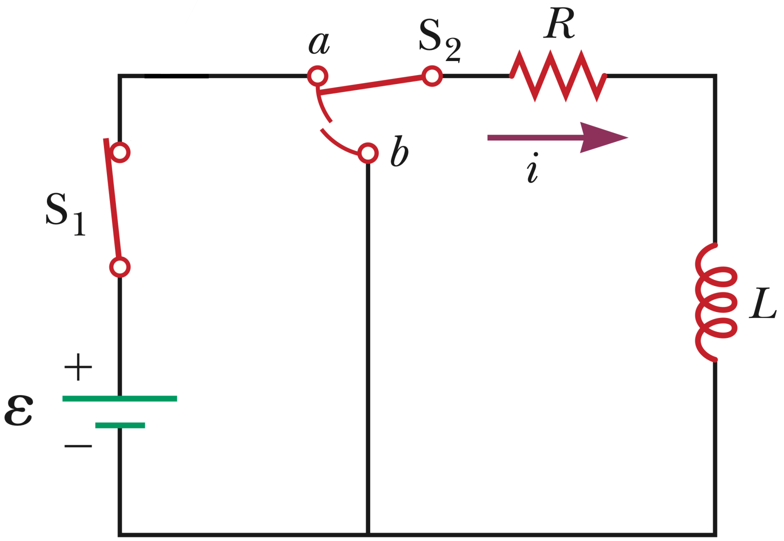


- The self-induced emf, denoted as ε_L, is proportional to the rate of change of current $\mathrm{d}i/\,\mathrm{d}t$:

$$\varepsilon_L = -L\,\frac{\mathrm{d}i}{\mathrm{d}t} \tag{1}$$

- The negative sign in Equation 1 means that the self-induced emf always opposes the change in current, therefore, slows the change in current.

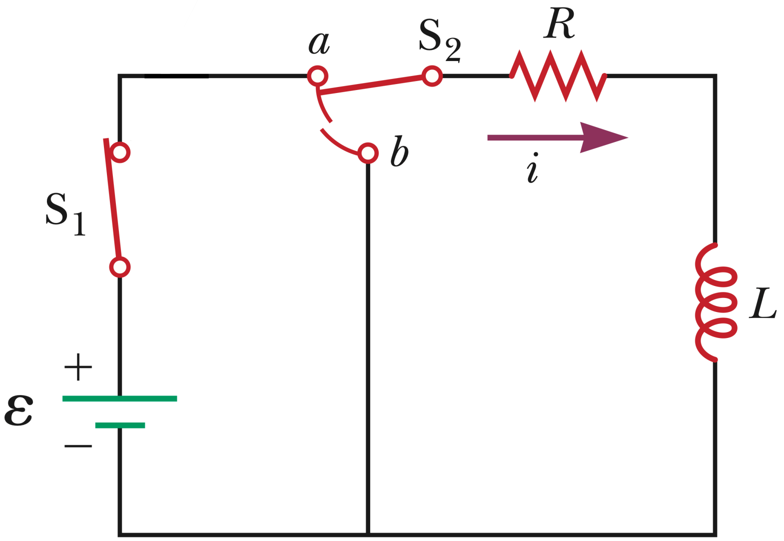
1.1 Self-Induction and Inductance



- The proportionality constant L is called the **inductance** of the circuit, and it depends on the geometry of the circuit and other physical factors.
- comparing Equation 1 with Faraday's law of induction ($\varepsilon = -N\frac{d\Phi_B}{dt}$), we can express the inductance as:

$$L = N\frac{\Phi_B}{i} \quad (2)$$

1.1 Self-Induction and Inductance



- Also, Equation 1 can be rewritten as:

$$L = -\frac{\varepsilon_L}{di/dt} \tag{3}$$

- The SI unit of inductance is henry (H), where

$$1 \text{ H} = 1 \text{ V} \cdot \text{s} / \text{A}$$

- The inductance is a measure of the opposition to a change in current.

1.2 Example

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Example 1.1

Consider a uniformly wound solenoid having N turns and length ℓ, Assume ℓ is much longer than the radius of the windings and the core of the solenoid is air. (A) Find the inductance of the solenoid

Solution 1.1

$$\Phi_B = BA = \mu_0 n i A$$

$$L = N\frac{\Phi_B}{i} = N\mu_0 n A = \mu_0 N^2 \frac{A}{\ell}$$

1.2 Example

(B) Calculate the inductance of the solenoid if it contains 300 turns, its length is 25 cm, and its cross-sectional area is 4 cm^2.

$$L = \mu_0 N^2 \frac{A}{\ell} = (4\pi \times 10^{-7})(300)^2 \frac{4 \times 10^{-4}}{0.25} = 0.181 \text{ mH}$$

(C) Calculate the self-induced emf in the solenoid if the current it carries decreases at the rate of 50 A/s.

$$\varepsilon_L = -L \frac{\mathrm{d}i}{\mathrm{d}t} = -(0.181 \times 10^{-3})(-50) = 9.05 \text{ mV}$$

1.3 Inductance of a Solenoid

As discussed in the previous example, the formulas for inductance of a solenoid are given by:

$$\boxed{L = \frac{\mu_0 N^2 A}{\ell} = \mu_0 n^2 A\ell = \mu_0 n^2 V} \tag{4}$$

- where $n = N/\ell$ is the number of turns per unit length, and V is the volume of the solenoid.

1. Self-Induction and Inductance

2. Energy in a Magnetic Field

2.1 Energy Stored in an Inductor

- In general, a battery in a circuit containing an inductor must provide more energy than one in a circuit without the inductor.
- The extra energy is stored in the inductor as magnetic potential energy, denoted as U_B.

$$\boxed{U_B = \frac{1}{2}Li^2} \tag{5}$$

2.2 Energy Density Stored in a Solenoid

- For a solenoid, $L = \mu_0 n^2 V$ (Equation 4), and the magnetic field inside the solenoid is $B = \mu_0 n i \implies i = B/(\mu_0 n)$, therefore, the energy stored in the solenoid (Equation 5) can be expressed as:

$$U_B = \frac{1}{2}(\mu_0 n^2 V)\left(\frac{B^2}{\mu_0^2 n^2}\right) = \left(\frac{B^2}{2\mu_0}\right)V$$

- The energy density, denoted as u_B, is the energy stored per unit volume, therefore:

$$\boxed{u_B = \frac{B^2}{2\mu_0}} \tag{6}$$

Suggested Problems

3, 4, 20, 21

Book: Serway, R. A., & Jewett, J. W. (2018). Physics for Scientists and Engineers (10th ed.)

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