Ch.30: Faraday's Law

Physics 104: Electricity and Magnetism

Dr. Abdulaziz Alqasem

Physics and Astronomy Department
King Saud University

2026

Outline

Remember From Previous Chapters

Classical Mechanics

- Equations of motion:

$$\vec{\boldsymbol{v}}_f = \vec{\boldsymbol{v}}_i + \vec{\boldsymbol{a}}t$$

$$\vec{\boldsymbol{r}}_f = \vec{\boldsymbol{r}}_i + \vec{\boldsymbol{v}}_i t + \frac{1}{2}\vec{\boldsymbol{a}}t^2$$

$$\vec{\boldsymbol{v}}_f^2 = \vec{\boldsymbol{v}}_i^2 + 2\vec{\boldsymbol{a}} \cdot \left(\vec{\boldsymbol{r}}_f - \vec{\boldsymbol{r}}_i\right)$$

- Newton's Second Law:

$$\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$$

- Work-Energy Theorem:

$$W = \Delta K$$

$$W = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{d}}$$

$$K = \frac{1}{2}m\vec{\boldsymbol{v}}^2$$

Electric Field

- Coulomb's Law:

$$\vec{\boldsymbol{F}}_e = k_e \frac{q_1 q_2}{r^2}\hat{\boldsymbol{r}}$$

- Electric Field:

$$\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}_e}{q_0} = k_e \frac{q}{r^2}\hat{\boldsymbol{r}}$$

$$\vec{\boldsymbol{a}} = \left(\frac{q}{m}\right)\vec{\boldsymbol{E}}$$

Flux

- Gauss's Law:

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot \mathrm{d}\vec{\boldsymbol{A}} = \frac{q_{\text{in}}}{\varepsilon_0}$$

Remember From Previous Chapters

Electric Potential and Energy

- Electric Potential:

$$V = k_e \frac{q}{r}$$

- Potential Energy:

$$U_E = k_e \frac{q_1 q_2}{r}$$

- Relation to Electric Field:

$$\Delta V = -\vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{d}}$$

- Potential and Energy:

$$\Delta U_E = q \Delta V$$

Capacitance and Dielectrics

- Capacitance:

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$

- Capacitors in Parallel:

$$C_{\text{eq}} = \sum C_i$$

- Capacitors in Series:

$$\frac{1}{C_{\text{eq}}} = \sum \left(\frac{1}{C_i} \right)$$

- Energy Stored in Capacitor:

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V$$
$$= \frac{1}{2} C (\Delta V)^2$$

Remember From Previous Chapters

- Energy Density of Electric Field:

$$u_E = \frac{U_E}{V} = \frac{1}{2}\varepsilon_0 E^2$$

- Dielectric Constant:

$$\Delta V = \Delta V_0/\kappa$$

$$C = \kappa C_0$$

$$Q = \kappa Q_0$$

$$U_E = U_0/\kappa$$

Current and Resistance

- Current:

$$I = \frac{\Delta Q}{\Delta t}$$

$$I_{\text{avg}} = nAv_d q$$

- Ohm's Relation:

$$\Delta V = IR$$

- Resistance:

$$R = \rho\frac{L}{A}$$

- conductivity:

$$\sigma = \frac{1}{\rho}$$

- Temperature Effect

$$R = R_0[1 + \alpha(T - T_0)]$$

- Electrical Power:

$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

$$\text{Energy} = P\Delta t$$

Remember From Previous Chapters

Direct-Current Circuits

- Electromotive Force:

$$\Delta V = \varepsilon - Ir = IR,$$

- Resistors in Series:

$$R_{\text{eq}} = R_1 + R_2 + \ldots + R_n$$

- Resistors in Parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

- Kirchhoff's Rules:

1. Junction Rule:

$$\sum_{\text{node}} I = 0$$

2. Loop Rule:

$$\sum_{\text{loop}} \Delta V = 0$$

Magnetic Fields

- Magnetic Force on a Moving Charge:

$$\vec{\boldsymbol{F}}_B = q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$$

$$\left|\vec{\boldsymbol{F}}_B\right| = qvB\sin\theta$$

- Charges in a circular path under a magnetic field:

$$r = \frac{mv}{qB}$$

Remember From Previous Chapters

- Angular Velocity:

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

- Period of Revolution:

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

- Magnetic Force on a Current-Carrying Wire:

$$\vec{\boldsymbol{F}}_B = I\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}}$$

$$\left|\vec{\boldsymbol{F}}_B\right| = ILB\sin\theta$$

Sources of the Magnetic Field

- Biot-Savart Law:

$$\mathrm{d}\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{I}{r^2}(\mathrm{d}\vec{\boldsymbol{s}} \times \hat{\boldsymbol{r}})$$

- Magnetic Force Between Two Parallel Currents:

$$\frac{F}{\ell} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{a}$$

- Ampere's Law:

$$\oint \vec{\boldsymbol{B}} \cdot \mathrm{d}\vec{\boldsymbol{s}} = \mu_0 I$$

- Magnetic Field of a solenoid

$$B = \mu_0 n I$$

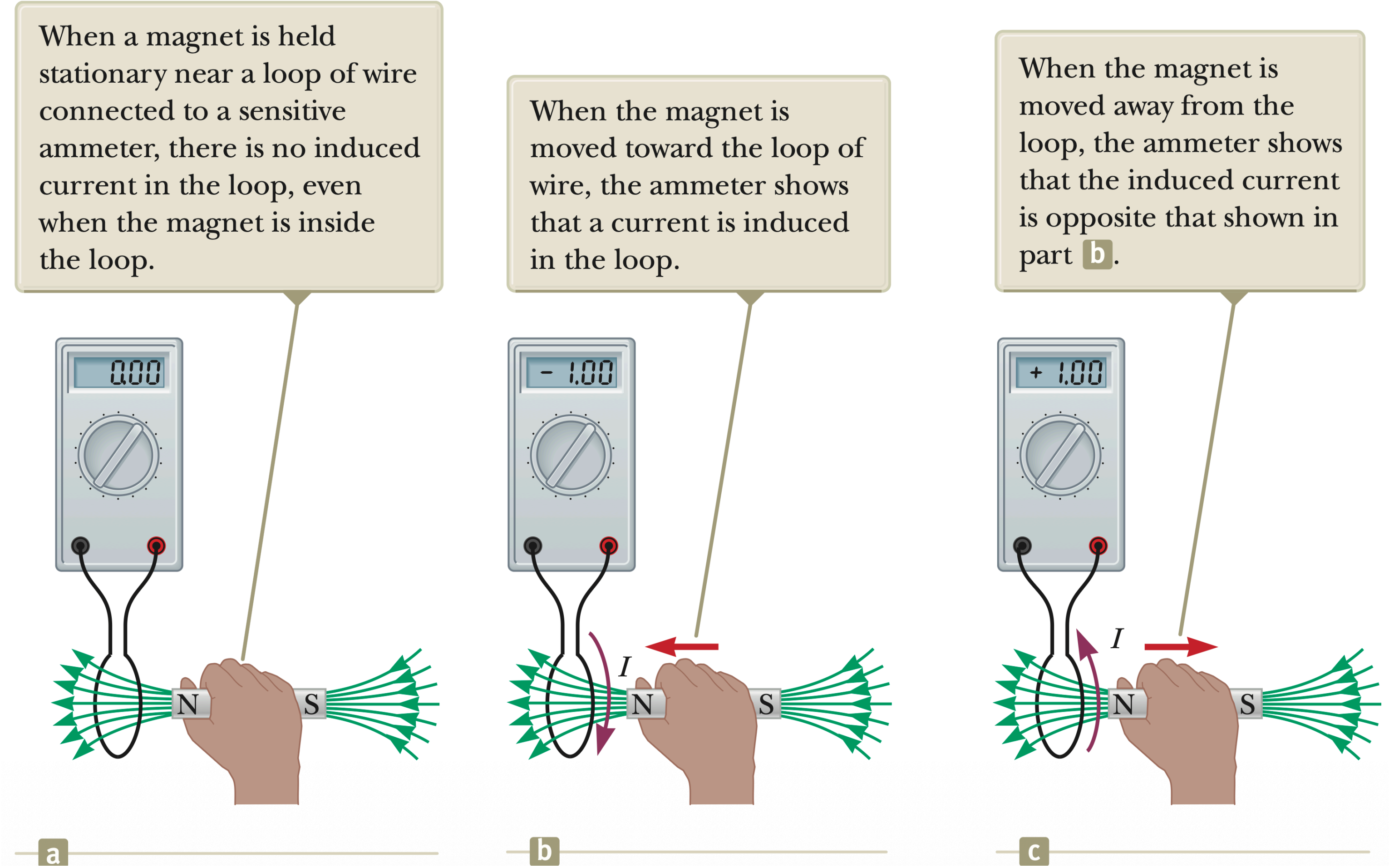
- Magnetic Flux and Gauss's Law:

$$\Phi_B = \oint \vec{\boldsymbol{B}} \cdot \mathrm{d}\vec{\boldsymbol{A}} = 0$$

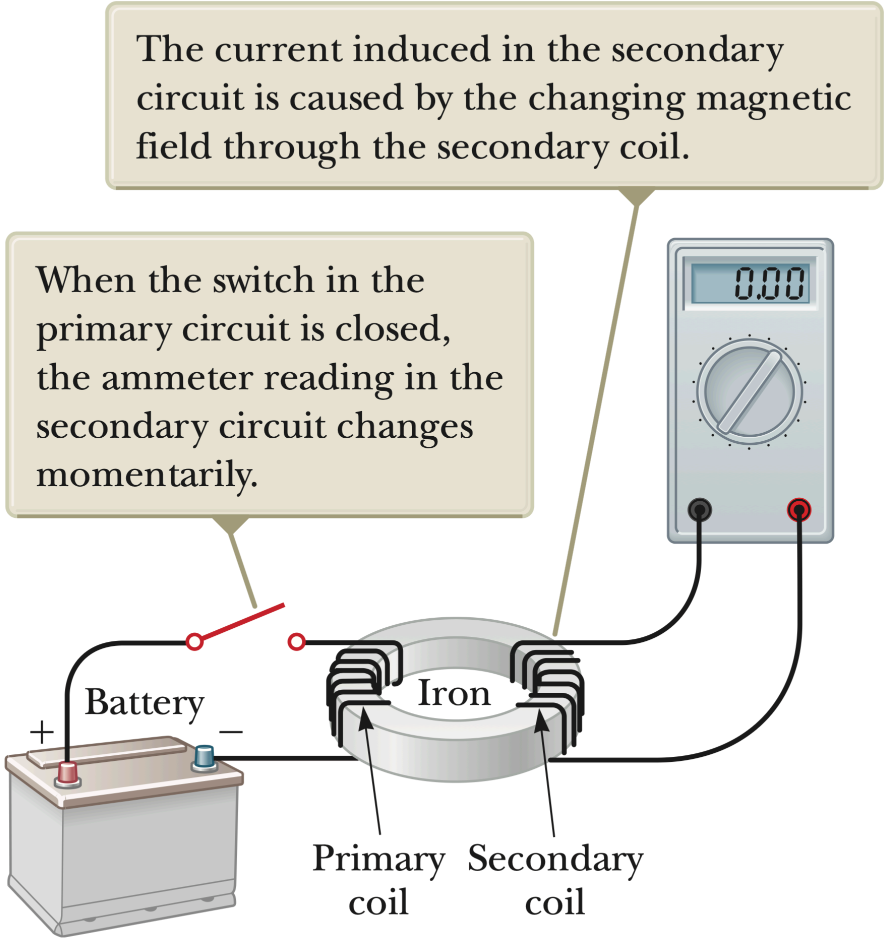
1. Faraday's Law of Induction

2. Motional EMF

1.1 What is Faraday's Law?



1.1 What is Faraday's Law?



Faraday's Law states that changing the magnetic flux Φ_B through a closed loop induces an electromotive force (emf ε) in the loop.

1.2 Fraday's Law of Induction

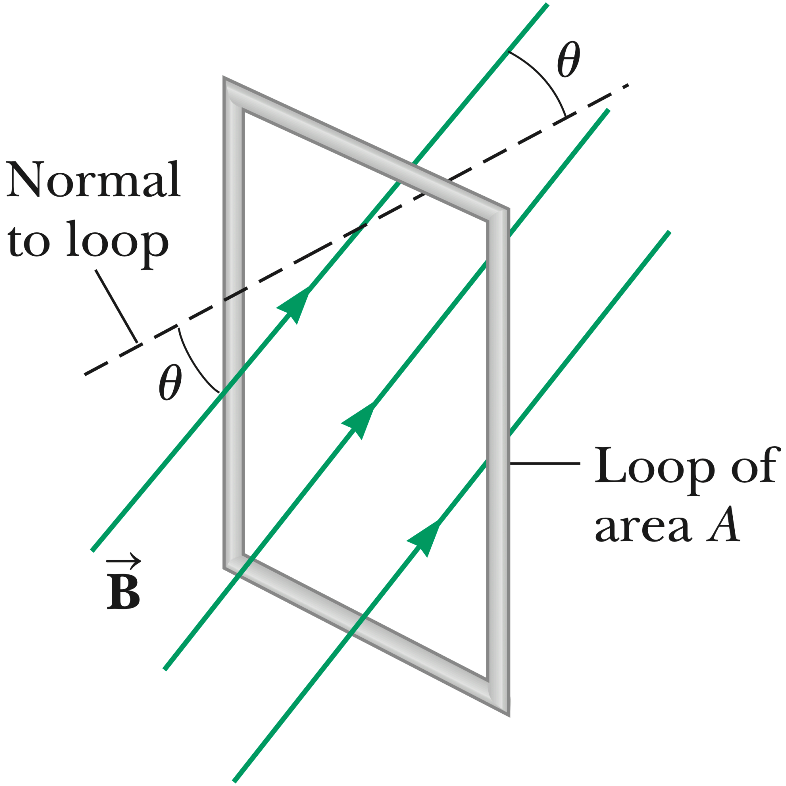
- The induced emf is proportional to the rate of change of the magnetic flux, therefore,

$$\varepsilon = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

- The negative sign indicates that the induced emf opposes the change in magnetic flux, as stated by Lenz's Law.
- If the loop has N turns, the total emf is:

$$\varepsilon = -N\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

1.3 Induction Through a Rectangular Loop



- With a rectangular loop of area A in a uniform magnetic field B, the magnetic flux is:

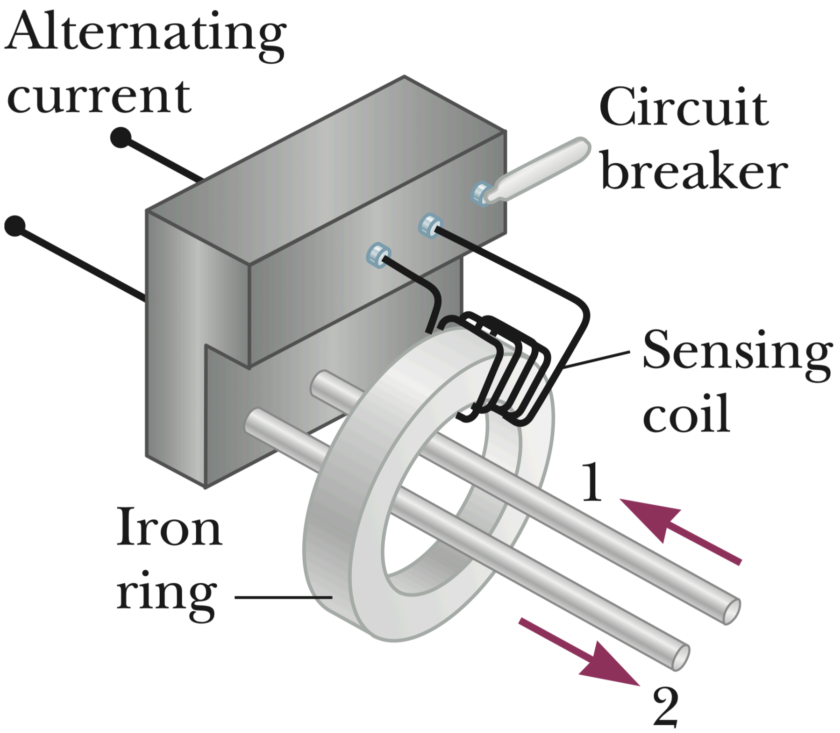
$$\Phi_B = BA\cos\theta$$

- If the magnetic flux changes with time, the induced emf is:

$$\varepsilon = -\frac{\mathrm{d}}{\mathrm{d}t}(BA\cos\theta)$$

- ε is non-zero if B, A, or θ changes with time.

1.4 Application: Safety Breaker



A safety breaker uses Faraday's Law to detect changes in magnetic flux caused by a current surge. If the generated current exceeds a certain threshold, the breaker opens the circuit to prevent damage to the electrical system.

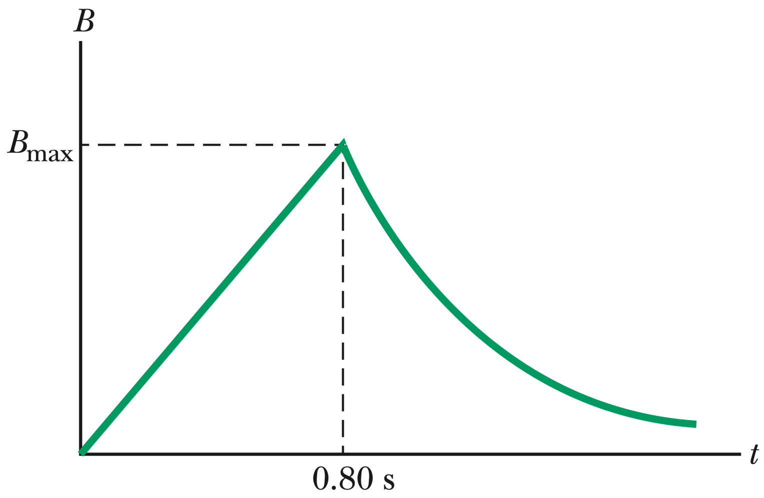
1.5 Example

King Saud University

Example 1.1

A coil consists of 200 turns of wire. Each turn is a square of side $d =$ 18cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. The figure shows the behavior of the magnitude of the magnetic field with time. From $t = 0$ to $t = 0.8$ s, the field changes linearly from 0 to 0.5 T. After $t = 0.8$ s, the magnitude of the field decays in time according to the expression $B = B_{\max} e^{-at}$, where a is some constant and $B_{\max} = 0.5$ T.

(A) What is the magnitude of the induced emf in the coil between $t = 0$ and $t = 0.8$ s?



1.5 Example

King Saud University

Solution 1.1

The magnetic flux through each turn of the coil is given by:

$$\Phi_B = BA$$

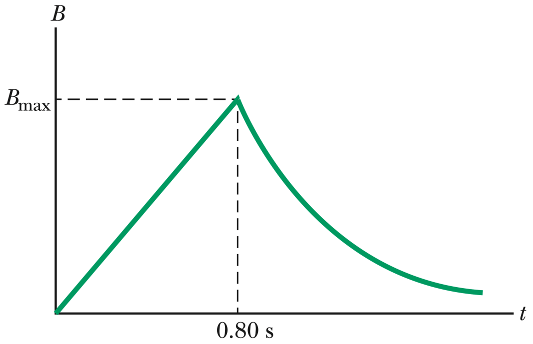
where A is the area of each turn, which is d^2. Therefore,

$$\Phi_B = Bd^2$$

The induced emf can be calculated using Faraday's Law:

$$|\varepsilon| = N\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = Nd^2\,\frac{\mathrm{d}B(t)}{\mathrm{d}t}$$

1.5 Example



For t between 0 and 0.8 s, the magnetic field changes linearly, so we can express it as:

$$B(t) = \text{slope} \times t = \left(\frac{B_{\text{max}} - 0}{t_{\text{max}} - 0}\right) t = \left(\frac{0.5}{0.8}\right) t = 0.625t$$

Therefore, the time derivative of the magnetic field is:

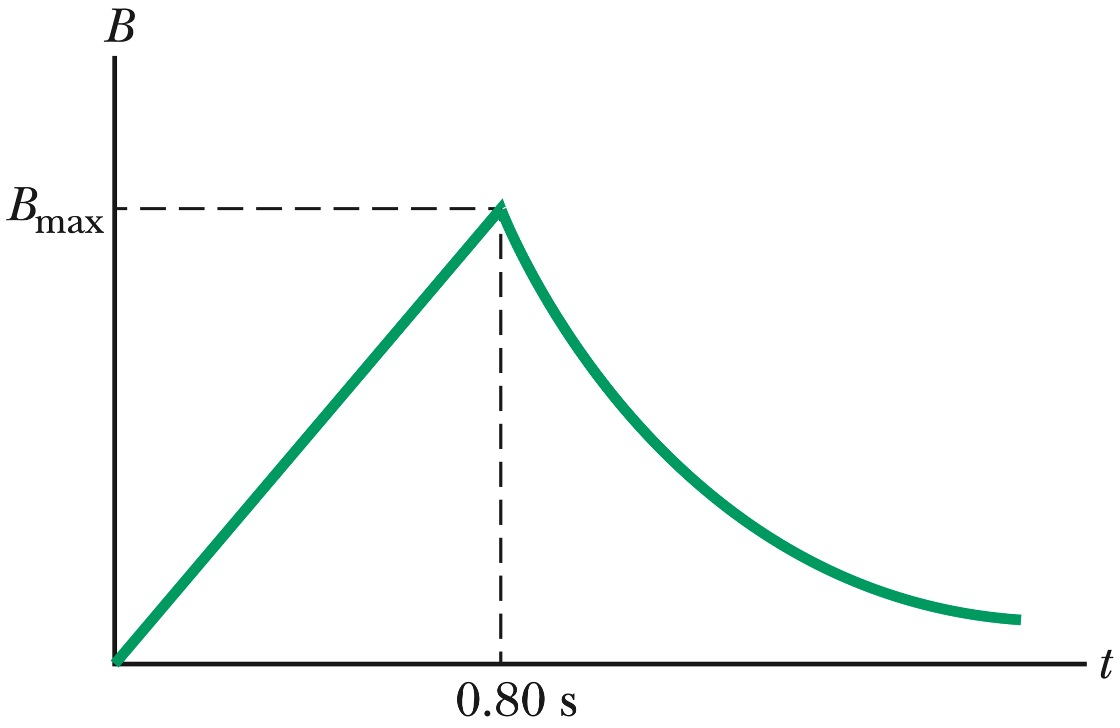
$$\frac{\mathrm{d}B(t)}{\mathrm{d}t} = 0.625$$

Finally, the induced emf is:

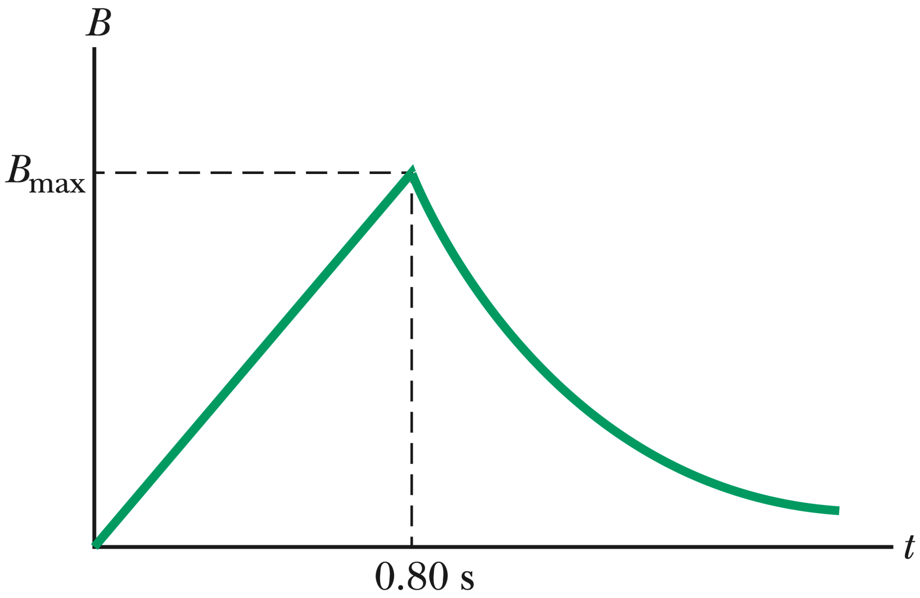
$$|\varepsilon| = Nd^2(0.625) = 200(0.18)^2(0.625) = 4 \text{ V}$$

1.5 Example

(B) What is the magnitude of the induced ε in the coil after $t = 0.8$ s?



1.5 Example



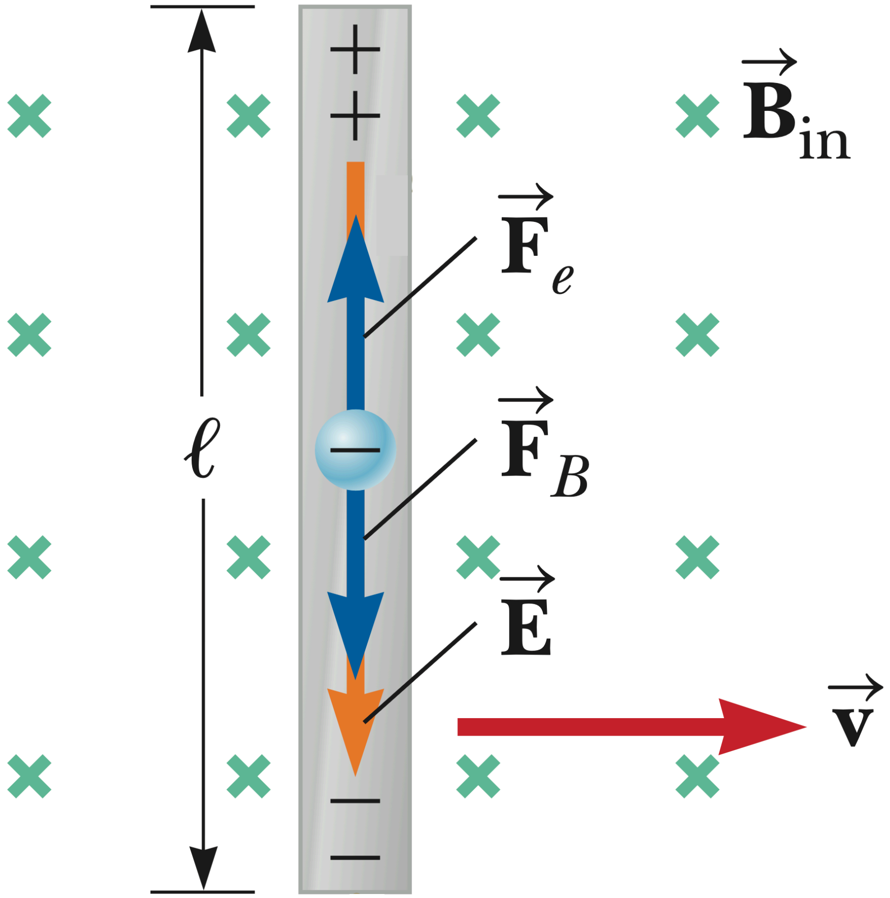
$$\varepsilon = -N\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = -N\frac{\mathrm{d}}{\mathrm{d}t}(d^2 B_{\mathrm{max}} e^{-at}) = -Nd^2 B_{\mathrm{max}}\frac{\mathrm{d}}{\mathrm{d}t}e^{-at} = aNd^2 B_{\mathrm{max}} e^{-at}$$

$$\varepsilon = a(200)(0.18m)^2(0.5T)e^{-at} = 3.2ae^{-at}$$

1. Faraday’s Law of Induction

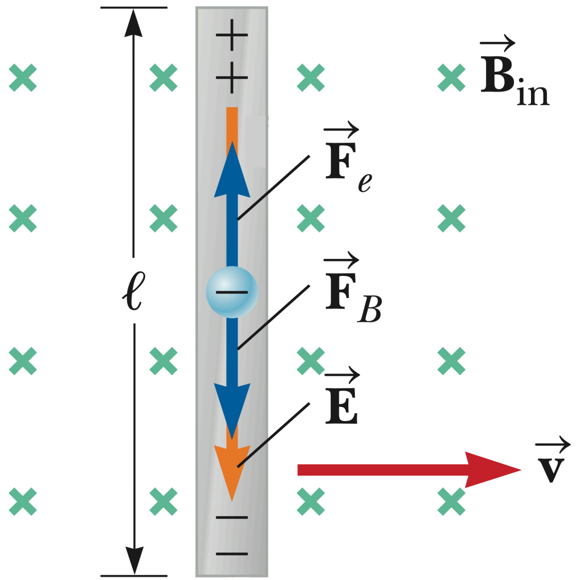
2. Motional EMF

2.1 EMF Induced by a Moving Conductor



When a conductor moves through a magnetic field, an emf (ε) is induced across the conductor due to the magnetic force acting on the charges in the conductor. This is known as motional emf.

2.2 Motional EMF From Newton's Second Law



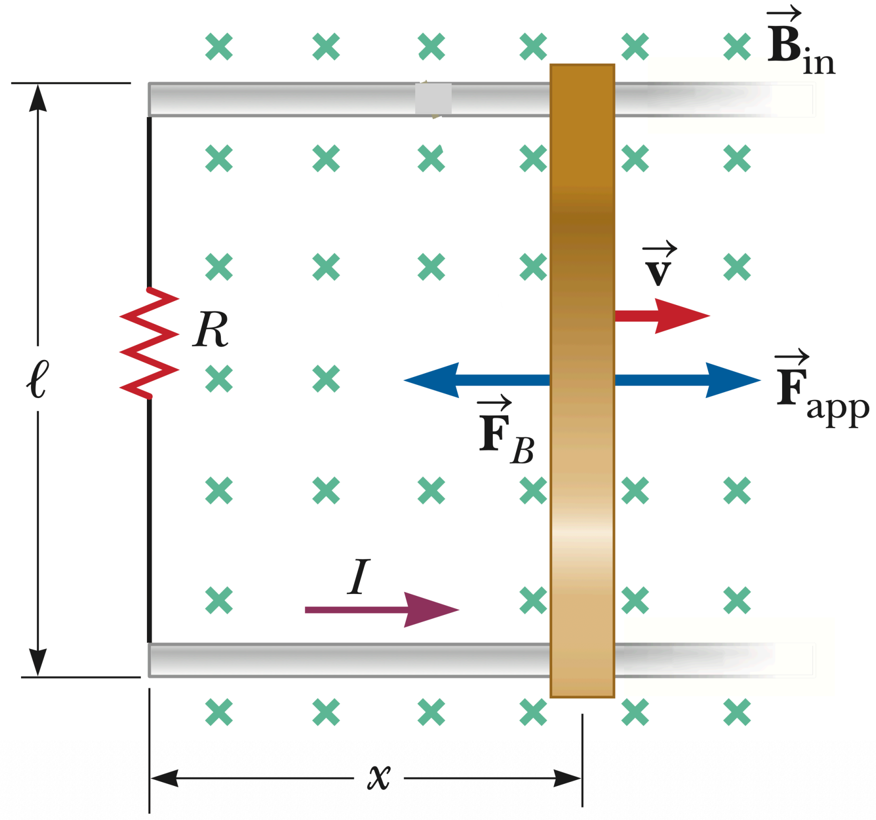
Consider a conductor moving with velocity v perpendicular to a magnetic field B. The magnetic force on an electron in the conductor is given by (qvB). This force causes the electrons to accumulate on one side of the conductor, creating an electric field $E = qE$ that opposes the magnetic force. At equilibrium, the electric force balances the magnetic force, therefore,

$$qE = qvB \implies E = vB$$

The potential difference (emf) across the conductor of length ℓ is then:

$$|\varepsilon| = E\ell = B\ell v$$

2.3 Motional EMF From Faraday's Law



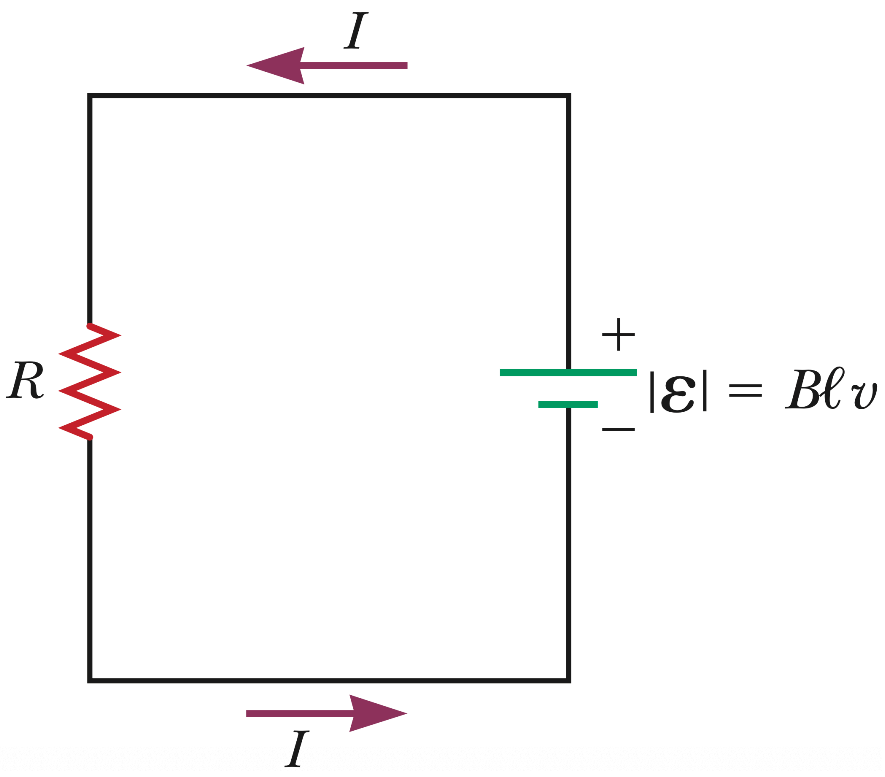
Consider a rectangular conductor loop with one side of length ℓ moving with velocity v perpendicular to a uniform magnetic field B. The magnetic flux through the loop is:

$$\Phi_B = BA = B(\ell x)$$

where x is the distance that the right side has moved. Therefore, the induced emf is:

$$\varepsilon = -\frac{\mathrm{d}}{\mathrm{d}t}(B\ell x) = -B\ell\frac{\mathrm{d}x}{\mathrm{d}t} = -B\ell v$$

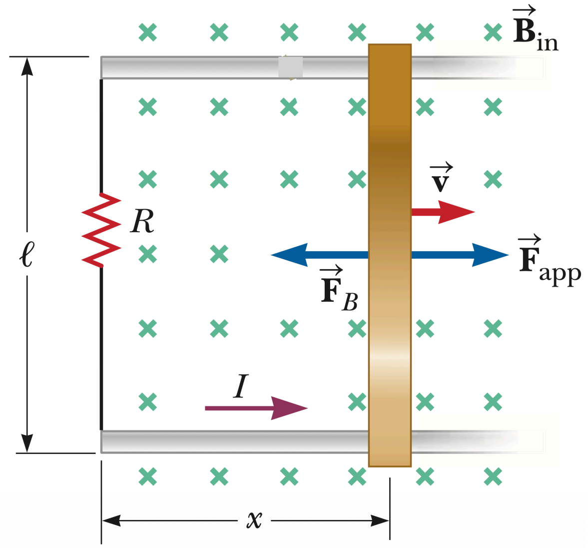
2.4 Current Induced by Motional EMF



If the moving conductor is part of a closed circuit, the induced emf will cause a current to flow. The magnitude of the current can be calculated using Ohm's Law:

$$I = \frac{|\varepsilon|}{R} = \frac{B\ell v}{R}$$

2.5 Conservation of Energy



- The force applied by the external agent F_{app} to keep the conductor moving at a constant velocity is equal to the magnetic force $F_B = I\ell B$ that opposes the motion ($F_{app} = F_B$).
- Therefore, the work done by the external agent W_{app} is equal to the electrical energy T_E transferred in the circuit to the resistor R. Therefore, the power delivered by the external agent is equal to the power transferred to the resistor:

$$P_{app} = F_{app}v = F_B v = (I\ell B)v = I(B\ell v) = I|\varepsilon| = I^2R = P_{elec}$$

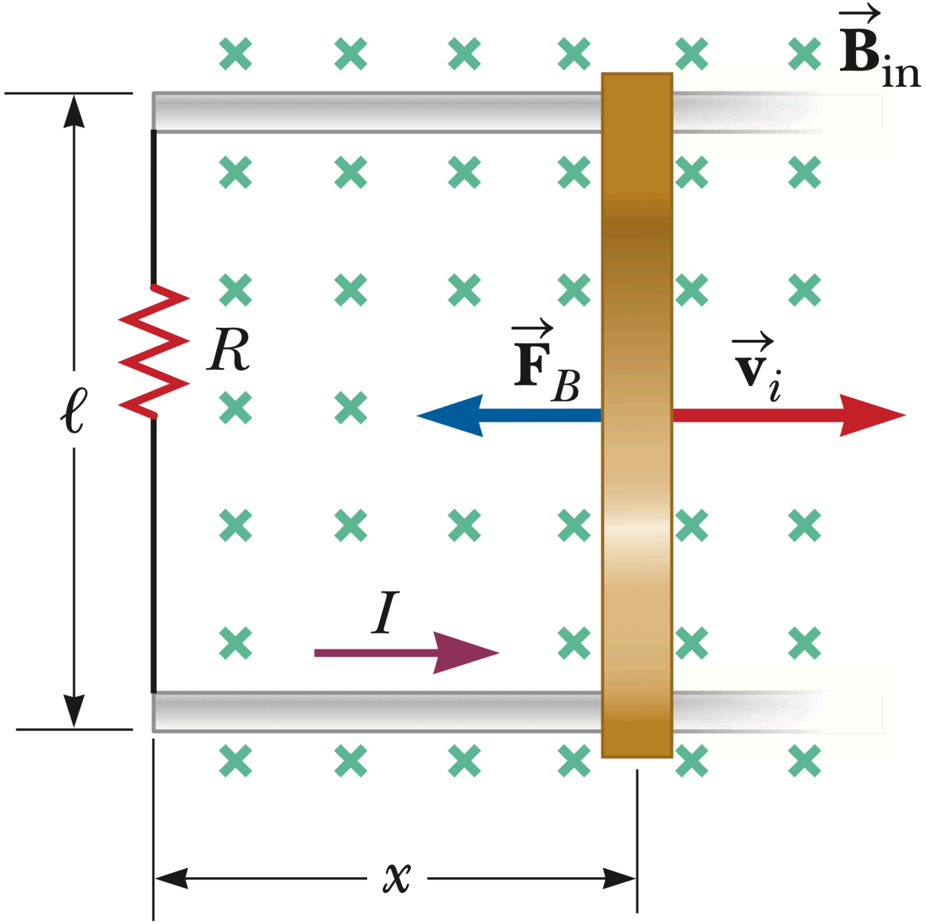
2.6 Example

King Saud University

Example 2.2

The conducting bar illustrated in the Figure moves on two frictionless, parallel rails in the presence of a uniform magnetic field directed into the page. The bar has mass m, and its length is ℓ, The bar is given an initial velocity v_i to the right and is released at $t = 0$.

(A) Using Newton's laws, find the speed of the bar as a function of time after it is released.



2.6 Example

$$\sum F_x = ma \longrightarrow -I\ell B = m\frac{\mathrm{d}v}{\mathrm{d}t}$$

rearranging gives:

$$\mathrm{d}v = -\left(\frac{I\ell B}{m}\right)\mathrm{d}t$$

Since $I = (B\ell v)/R$, we have:

$$\frac{\mathrm{d}v}{v} = -\frac{(B\ell)^2}{mR}\,\mathrm{d}t$$

Integrating both sides with respect to time gives:

$$\int_{v_i}^{v}\frac{\mathrm{d}v}{v} = -\left(\frac{(B\ell)^2}{mR}\right)\int_0^t \mathrm{d}t$$

$$\ln\left(\frac{v}{v_i}\right) = -\left(\frac{(B\ell)^2}{mR}\right)t$$

Therefore, the speed of the bar as a function of time is:

$$v(t) = v_i e^{-t/\tau}$$

where $\tau = (mR)/(B\ell)^2$ is the time constant of the system.

2.6 Example

(B) Show that the same result is found by using an energy approach

$$\frac{\mathrm{d}K}{\mathrm{d}t} = \frac{\mathrm{d}T_{\mathrm{ET}}}{\mathrm{d}t} = P_{\mathrm{elec}} = -I^2R$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}mv^2\right) = -\left(\frac{B\ell v}{R}\right)^2 R$$

$$mv\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{(B\ell v)^2}{R}$$

Finally, we get the same result:

$$\frac{\mathrm{d}v}{v} = -\frac{(B\ell)^2}{mR}\,\mathrm{d}t$$

Suggested Problems

1, 4, 15

Book: Serway, R. A., & Jewett, J. W. (2018). Physics for Scientists and Engineers (10th ed.)

Chapter: 30 - Faraday's Law