Ch.3: Vectors

Physics 103: Classical Mechanics

Dr. Abdulaziz Alqasem

Physics and Astronomy Department
King Saud University

2025

Outline

1. Vector and Scalar Quantities

2. Coordinate Systems

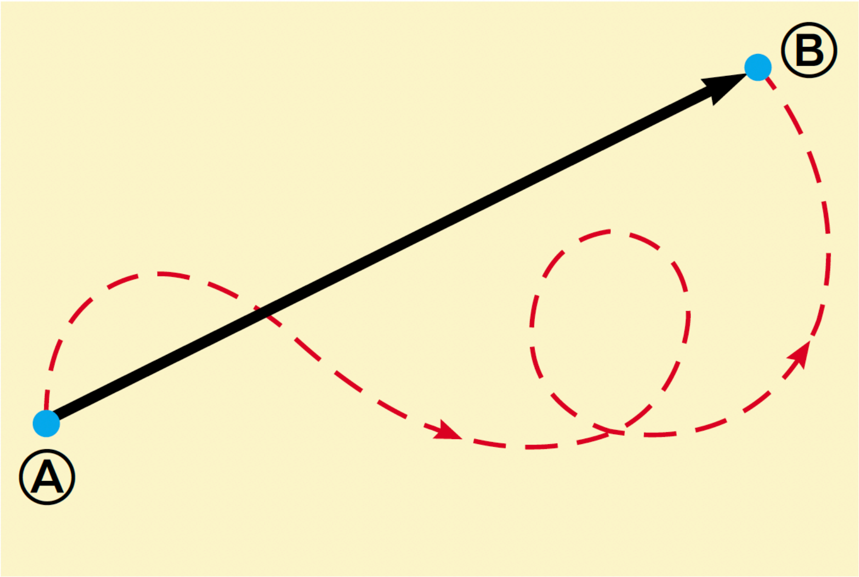
3. Some Properties of Vectors

4. Components of a Vector and Unit Vectors

5. Suggested Problems

1.1 Definitions

- A **scalar** quantity is specified by a single value with an appropriate unit and has *no* direction, such as time t and temperature T.
- A **vector** quantity is specified by a number and an appropriate unit plus a *direction*, such as displacement $\vec{x}_{\mathrm{AB}}$ and force $\vec{\mathbf{F}}$.



جامعة الملك سعود
King Saud University

1. Vector and Scalar Quantities

2. **Coordinate Systems**

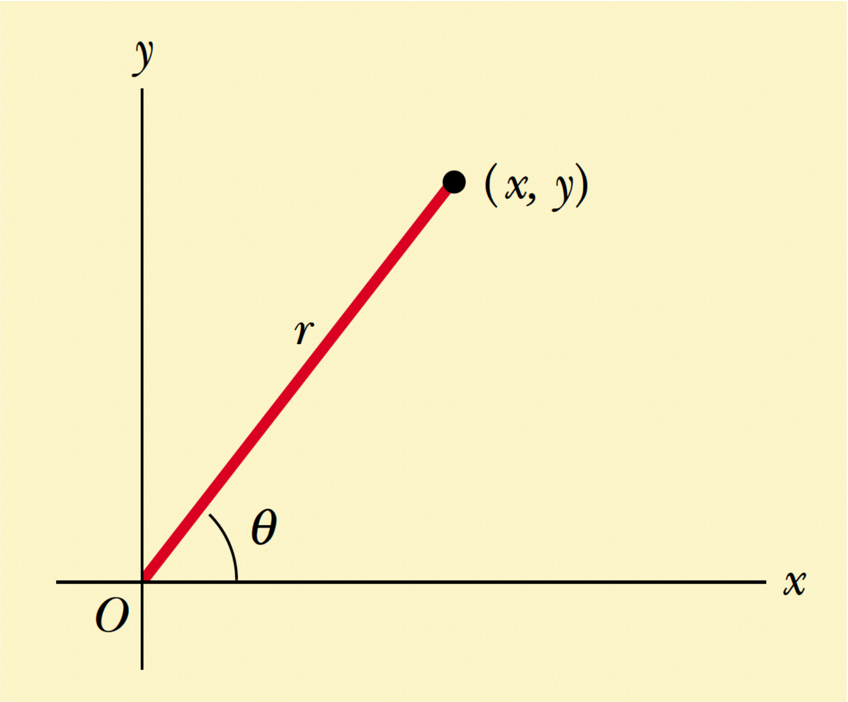
3. Some Properties of Vectors

4. Components of a Vector and Unit Vectors

5. Suggested Problems

2.1 Cartesian and Polar Coordinates

- A **point** in a plane (2D space) can be represented by *two* numbers.
- In the **Cartesian** (rectangular) coordinate system, the point is represented by its x and y coordinates, (x, y).
- In the **polar** coordinate system, the point is represented by its distance r from the origin and the angle θ that the line makes with the positive x-axis, (r, θ).
- The *choice* of a coordinate system depends on what simplifies the problem and analysis.



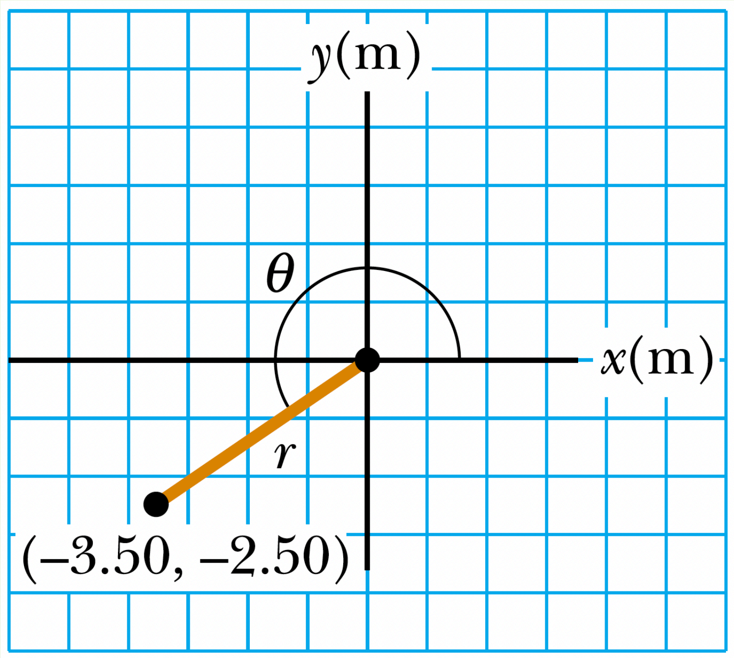
$$x = r\cos\theta \qquad r = \sqrt{x^2 + y^2}$$

$$y = r\sin\theta \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

2.1 Cartesian and Polar Coordinates

Example 2.1

The Cartesian coordinates of a point in the xy plane are $(x, y) = (-3.5, -2.5)$ m, as shown in the Figure. Find the polar coordinates of this point.



2.1 Cartesian and Polar Coordinates

Solution 2.1

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.5)^2 + (-2.5)^2} = 4.3 \text{ m}$$

$$\tan\theta = \frac{y}{x} = \frac{-2.5}{-3.5} = 0.714$$

$$\theta = \tan^{-1}(0.714) = 35.5°$$ ✗ Wrong

Notice that the point is in the **third** quadrant (from the signs of x and y); therefore, we add 180° to the angle:

$$\theta = 35.5° + 180° = 215.5°$$ ✓ Correct

Thus, the polar coordinates of the point are $(r, \theta) = (4.3 \text{ m}, 215.5°)$.

2.1 Cartesian and Polar Coordinates

King Saud University

Problem 2.1

The polar coordinates of a point are $r = 5.5$ m and $\theta = 240°$. What are the Cartesian coordinates of this point?

Answer 2.1

$$x = r\cos\theta = 5.5\cos(240°) = -2.75 \text{ m}$$
$$y = r\sin\theta = 5.5\sin(240°) = -4.75 \text{ m}$$

Thus, the Cartesian coordinates of the point are $(x, y) = (-2.75 \text{ m}, -4.75 \text{ m})$.

Problem 2.2

Two points in the xy plane have Cartesian coordinates $(2, -4)$ m and $(-3, 3)$ m. Determine (a) the distance between these points and (b) their polar coordinates.

2.1 Cartesian and Polar Coordinates

Solution 2.1

(a) The distance between the two points is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 8.6 \text{ m}$$

(b) The polar coordinates of the points are:

$$r_1 = \sqrt{2^2 + (-4)^2} = 4.47 \text{ m}, \quad \theta_1 = \tan^{-1}\left(-\frac{4}{2}\right) = -63.4°$$

$$r_2 = \sqrt{(-3)^2 + 3^2} = 4.24 \text{ m}, \quad \theta_2 = \tan^{-1}\left(\frac{3}{-3}\right) = 135°$$

3.1 Equality of Two Vectors

King Saud University

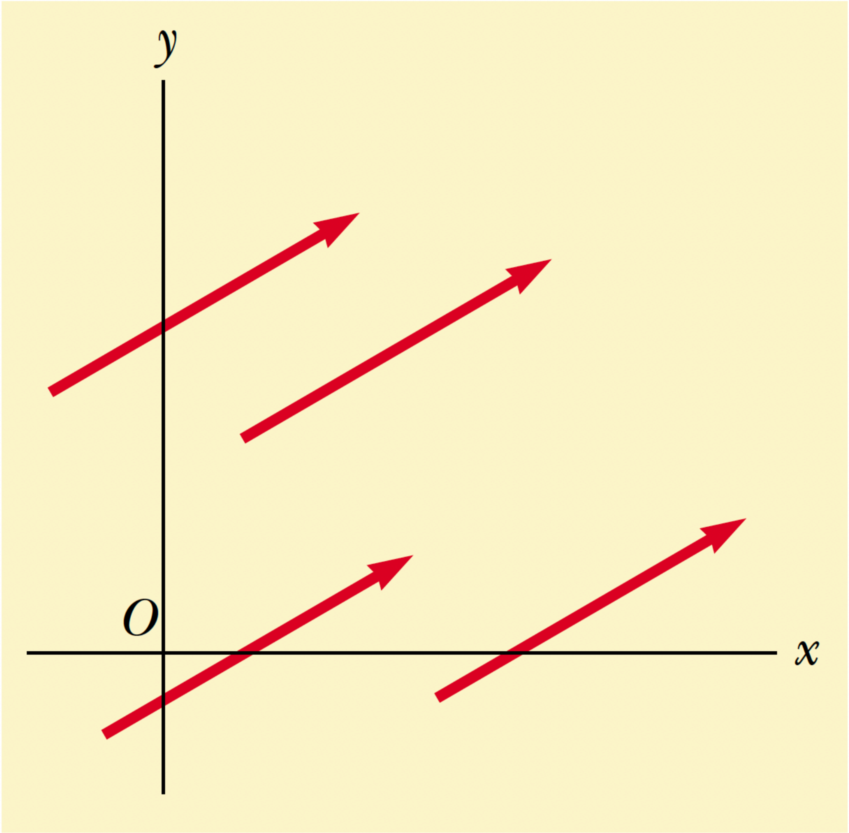
- Two vectors are equal if they have the same magnitude and direction:

$$\vec{A} = \vec{B} \qquad \text{Only if}$$

$$(1) \quad |\vec{A}| = |\vec{B}| \qquad \text{and,}$$

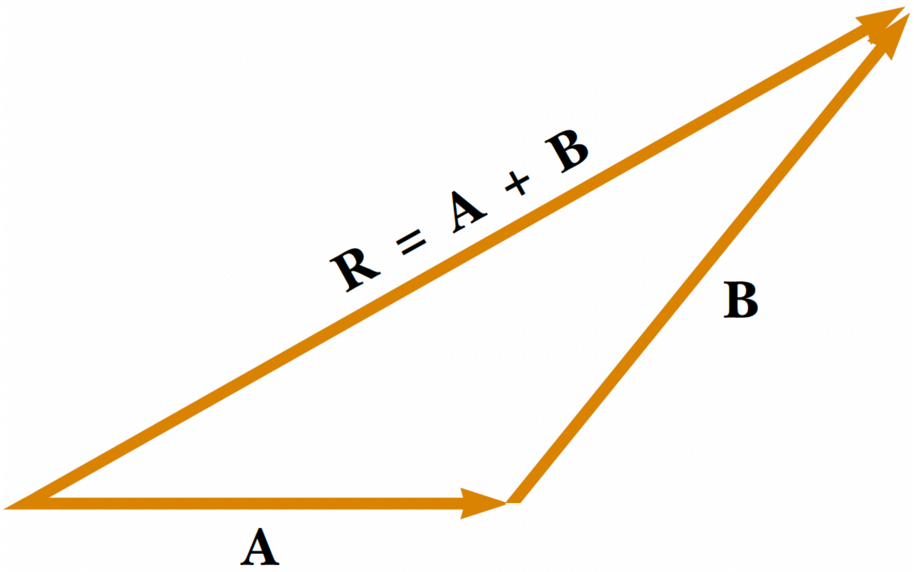
$$(2) \quad \theta_A = \theta_B$$

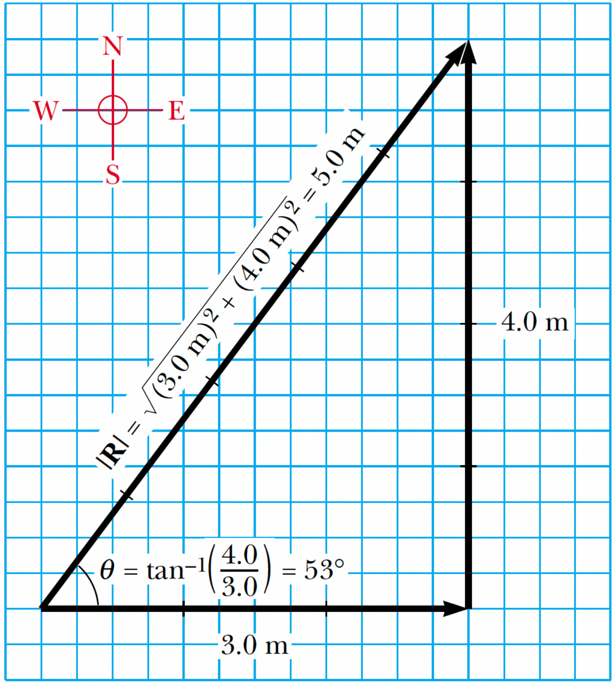
- Therefore, a vector can be *moved* to different locations parallel to itself *without* changing its properties.



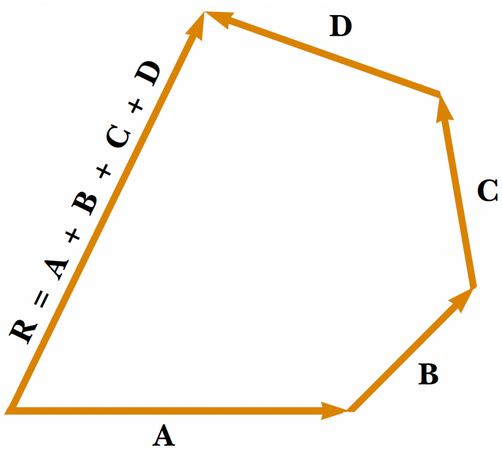
3.2 Adding Vectors

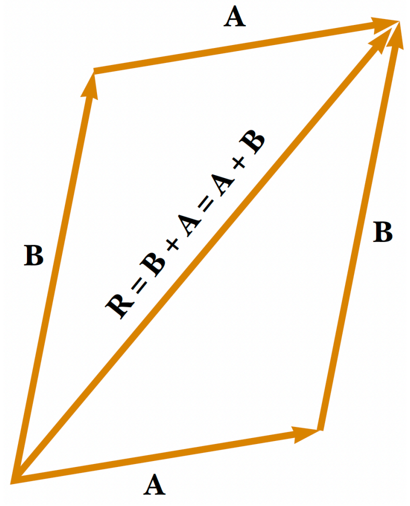
- **Graphical Method:** Vectors can be added graphically by placing them head-to-tail and drawing the resultant vector from the tail of the first vector to the head of the last vector.

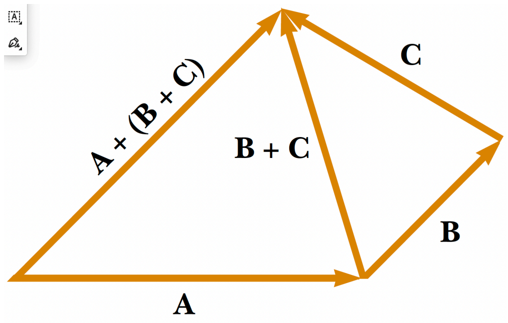


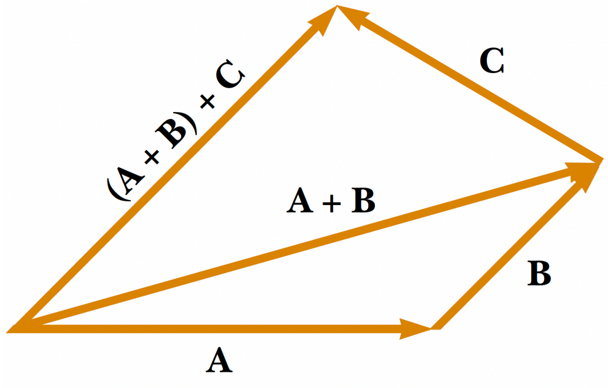


3.2 Adding Vectors



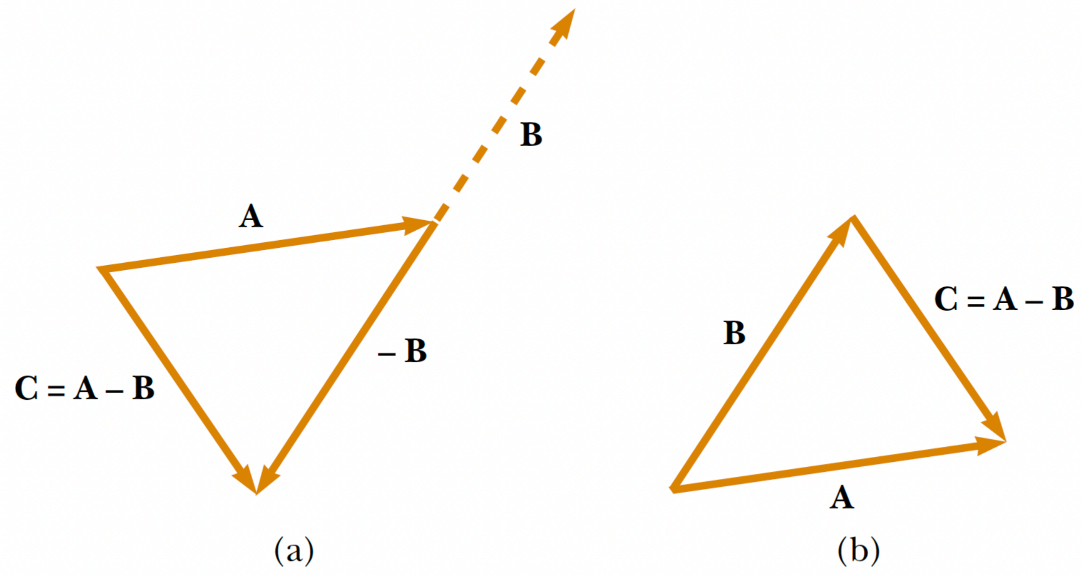






3.3 Subtracting Vectors

- The negative of a vector $-\vec{A}$ is a vector with the same magnitude as $\vec{A}$ but with the opposite direction, such that: $\vec{A} + \left(-\vec{A}\right) = 0$.
- Subtracting two vectors graphically is equivalent to adding a vector to the negative of the other vector: $\vec{C} = \vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$.



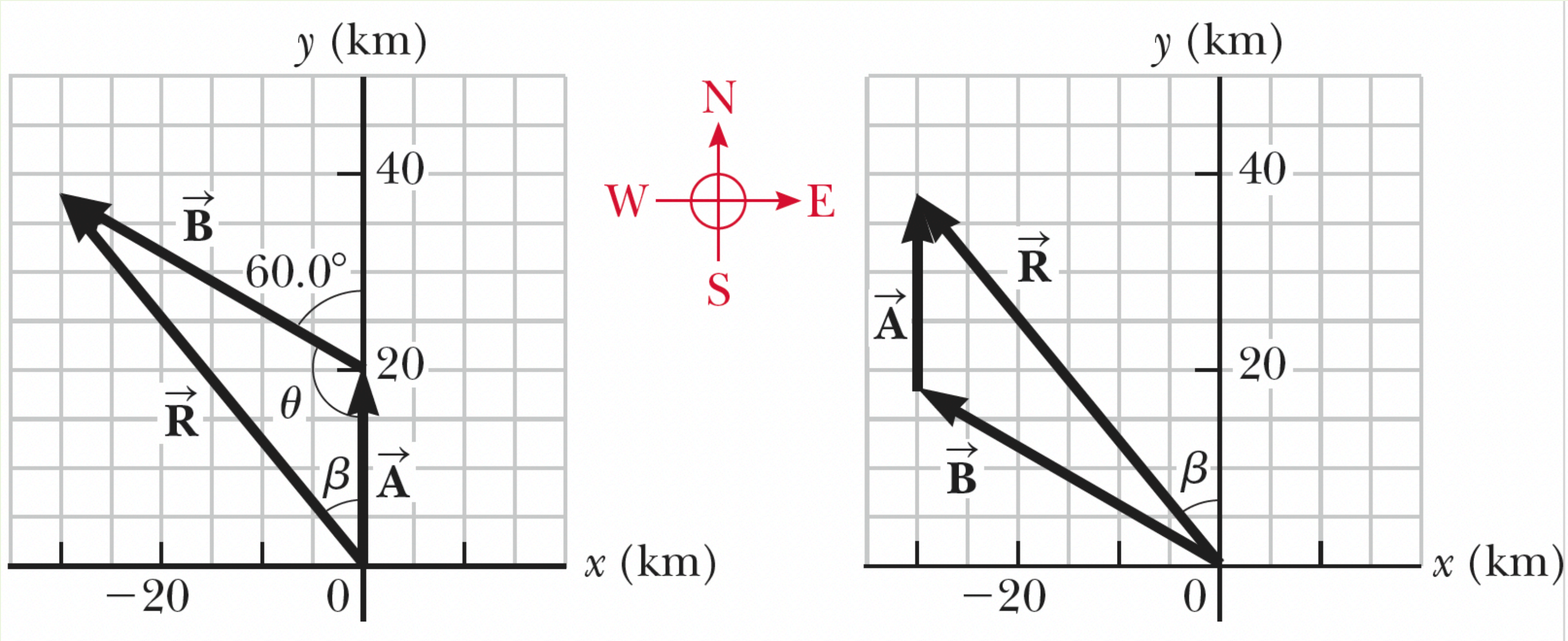
3.4 Multiplying a Vector by a Scalar

- If vector $\vec{A}$ is multiplied by a positive scalar quantity m, then the product $m\vec{A}$ is a vector that has the same direction as $\vec{A}$ and magnitude $m\ |\vec{A}|$.
- If vector $\vec{A}$ is multiplied by a negative scalar quantity $-m$, then the product $-m\vec{A}$ is directed opposite to $\vec{A}$.
- For example:
  - The vector $5\vec{A}$ is five times as long as $\vec{A}$ and points in the same direction as $\vec{A}$;
  - The vector $-\frac{1}{3}\vec{A}$ is one-third the length of $\vec{A}$ and points in the direction opposite to $\vec{A}$.

3.5 Example

Example 3.2

A car travels 20 km due north and then 35 km in a direction 60° west of north, as shown below. Find the magnitude and direction of the car's resultant displacement.



Solution 3.2

- First, notice that the angle between the two displacements is:

$$\theta = 180 - 60 = 120°$$

- Using the law of cosines, we find the magnitude of the resultant displacement R:

$$R^2 = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$
$$= \sqrt{20^2 + 35^2 - 2(20)(35)\cos(120°)} = 48.2 \text{ km}$$

- To find the direction of the resultant displacement, we can use the law of sines:

$$\frac{\sin\beta}{B} = \frac{\sin\theta}{R}$$

3.5 Example

$$\Rightarrow \sin\beta = \frac{B}{R}\sin\theta = \frac{35}{48.2}\sin 120^\circ = 0.629$$

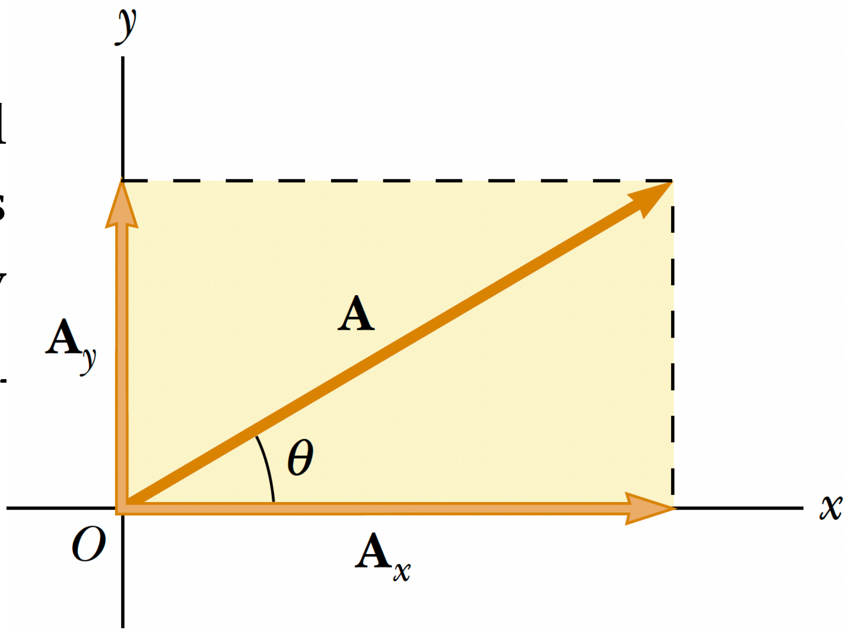
$$\Rightarrow \beta = \sin^{-1}(0.629) = 39^\circ$$

Therefore, the magnitude and direction of the car's resultant displacement are $48.2\ \text{km}$ and 39° west of north, respectively.

4.1 Components of a Vector

- To simplify analysis, a vector can be described by its components along the coordinate axes.
- For example, a vector $\vec{A}$ in two-dimensional space can be represented by its components along the x and y axes, A_x and A_y, respectively
- The components can be found using trigonometry:

$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$

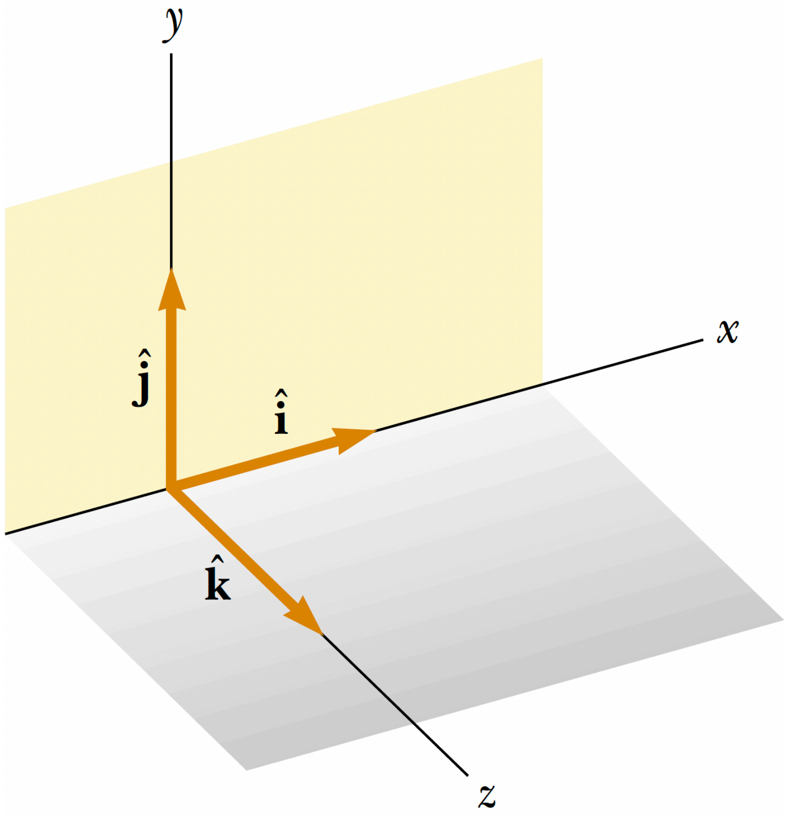


4.2 Unit Vectors

- A unit vector is a dimensionless vector that has a magnitude of one.

$$|\hat{\boldsymbol{\imath}}| = |\hat{\boldsymbol{\jmath}}| = \left|\hat{\boldsymbol{k}}\right| = 1,$$

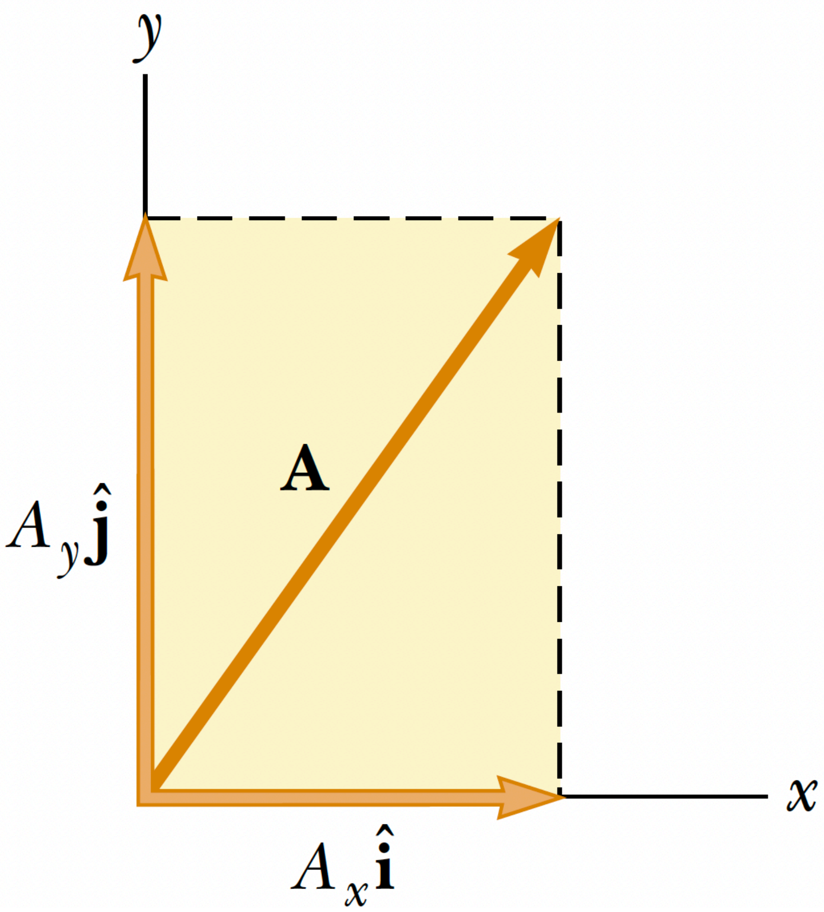
- $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, and $\hat{\boldsymbol{k}}$ are the unit vectors in the x, y, and z directions, respectively.
- Therefore, unit vectors are used to specify directions in space.



4.3 How to Express a Vector in Terms of Unit Vectors?

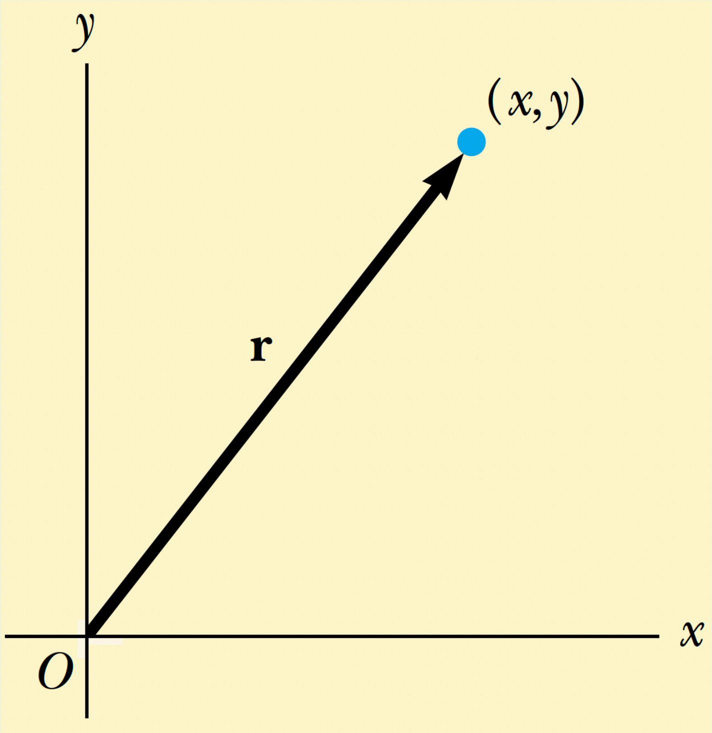
For any vector $\vec{\boldsymbol{A}}$, we can express it in terms of its components and unit vectors:

$$\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}}$$



Example 4.3

Express the position vector $\vec{r}$ of a point in the xy plane in terms of its components and unit vectors, given that its polar coordinates are $r = 5$ m and $\theta = 60°$.



4.3 How to Express a Vector in Terms of Unit Vectors?

Solution 4.3

- The components of the position vector $\vec{\boldsymbol{r}}$ can be found using the relationships:

$$x = r\cos\theta = 5\cos 60^\circ = 2.5 \text{ m}$$
$$y = r\sin\theta = 5\sin 60^\circ = 4.33 \text{ m}$$

- Therefore, we can express the position vector as:

$$\vec{\boldsymbol{r}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} = (2.5\hat{\boldsymbol{\imath}} + 4.33\hat{\boldsymbol{\jmath}}) \text{ m}$$

4.4 How to Add Vectors Using their Components?

- To add two vectors $\vec{A}$ and $\vec{B}$ using their components, we simply add their corresponding components:

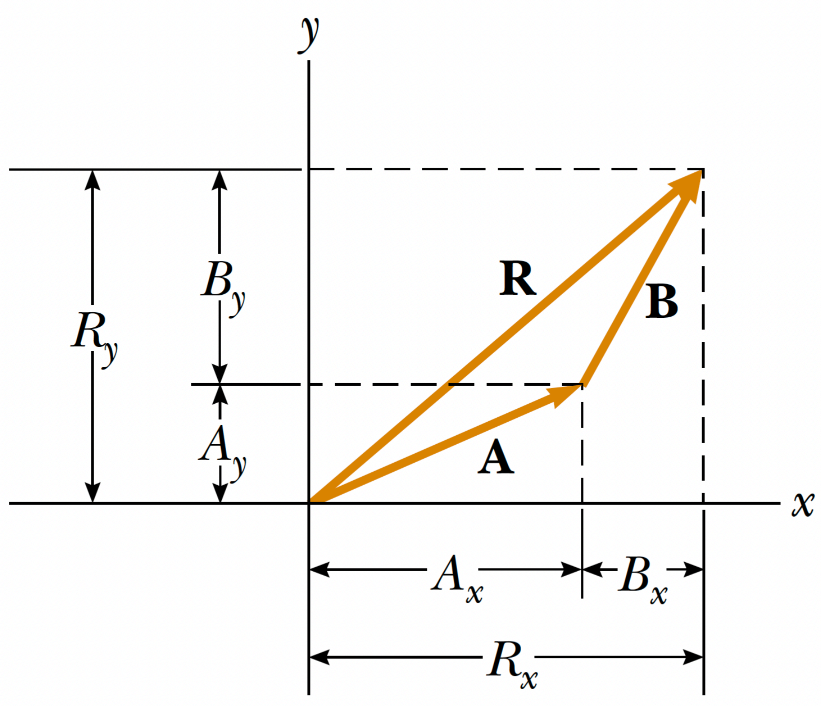
$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$

- Notice the components of the resultant vector

$$\vec{R} = R_x\hat{\imath} + R_y\hat{\jmath}$$

are:

$$R_x = A_x + B_x$$
$$R_y = A_y + B_y$$



4.4 How to Add Vectors Using their Components?

- The magnitude and direction of the resultant vector $\vec{\boldsymbol{R}}$ can be found using the Pythagorean theorem and trigonometry:

$$|\vec{\boldsymbol{R}}| = \sqrt{R_x^2 + R_y^2}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

4.5 Vectors in three Dimensions

King Saud University

- In three-dimensional space, a vector $\vec{\boldsymbol{A}}$ can be represented by its components along the x, y, and z axes, A_x, A_y, and A_z, respectively:

$$\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}} + A_z\hat{\boldsymbol{k}}$$

- The magnitude of the vector is given by:

$$|\vec{\boldsymbol{A}}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

  - The direction of the vector is given by the angles it makes with the coordinate axes:

$$\theta_x = \cos^{-1}\left(\frac{A_x}{|\vec{\boldsymbol{A}}|}\right), \quad \theta_y = \cos^{-1}\left(\frac{A_y}{|\vec{\boldsymbol{A}}|}\right), \quad \theta_z = \cos^{-1}\left(\frac{A_z}{|\vec{\boldsymbol{A}}|}\right)$$

4.5 Vectors in three Dimensions

- The sum of two vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ in three-dimensional space can be found by adding their corresponding components:

$$\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = (A_x + B_x)\hat{\boldsymbol{\imath}} + \left(A_y + B_y\right)\hat{\boldsymbol{\jmath}} + (A_z + B_z)\hat{\boldsymbol{k}}$$

King Saud University

Example 4.4

Find the sum of two vectors $\vec{\mathrm{A}}$ and $\vec{\mathrm{B}}$ lying in the xy plane and given by

$$\vec{A} = (2\hat{\imath} + 2\hat{\jmath})\ \mathrm{m}$$

$$\vec{B} = (2\hat{\imath} - 4\hat{\jmath})\ \mathrm{m}$$

4.6 Examples

Solution 4.4

- First, we find the resultant vector:

$$\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = (2\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}})\ \text{m} + (2\hat{\boldsymbol{\imath}} - 4\hat{\boldsymbol{\jmath}})\ \text{m}$$
$$= (4\hat{\boldsymbol{\imath}} - 2\hat{\boldsymbol{\jmath}})\ \text{m}$$

- The magnitude of the resultant vector is:

$$|\vec{\boldsymbol{R}}| = \sqrt{(4)^2 + (-2)^2} = \sqrt{16 + 4} = 4.5\ \text{m}$$

- The direction of the resultant vector is:

$$\theta = \tan^{-1}\left(-\frac{2}{4}\right) = -26.6^\circ \quad (\text{or } 360^\circ - 26.6^\circ = 333.4^\circ)$$

King Saud University

Example 4.5

A particle undergoes three consecutive displacements:

$$\vec{d}_1 = \left(15\hat{\imath} + 30\hat{\jmath} + 12\hat{k}\right)\ \text{cm}$$

$$\vec{d}_2 = \left(23\hat{\imath} - 14\hat{\jmath} + 5\hat{k}\right)\ \text{cm}$$

$$\vec{d}_3 = \left(-13\hat{\imath} + 15\hat{\jmath}\right)\ \text{cm}$$

Find the components of the resultant displacement and its magnitude.

4.6 Examples

Solution 4.5

- First, we find the components of the resultant displacement:

$$\begin{aligned}\vec{\boldsymbol{R}} &= \vec{\boldsymbol{d}}_1 + \vec{\boldsymbol{d}}_2 + \vec{\boldsymbol{d}}_3 \\ &= (15 + 23 - 13)\hat{\boldsymbol{\imath}} + (30 - 14 + 15)\hat{\boldsymbol{\jmath}} + (12 - 5 + 0)\hat{\boldsymbol{k}} \\ &= \left(25\hat{\boldsymbol{\imath}} + 31\hat{\boldsymbol{\jmath}} + 7\hat{\boldsymbol{k}}\right)\ \text{cm}\end{aligned}$$

- The magnitude of the resultant displacement is:

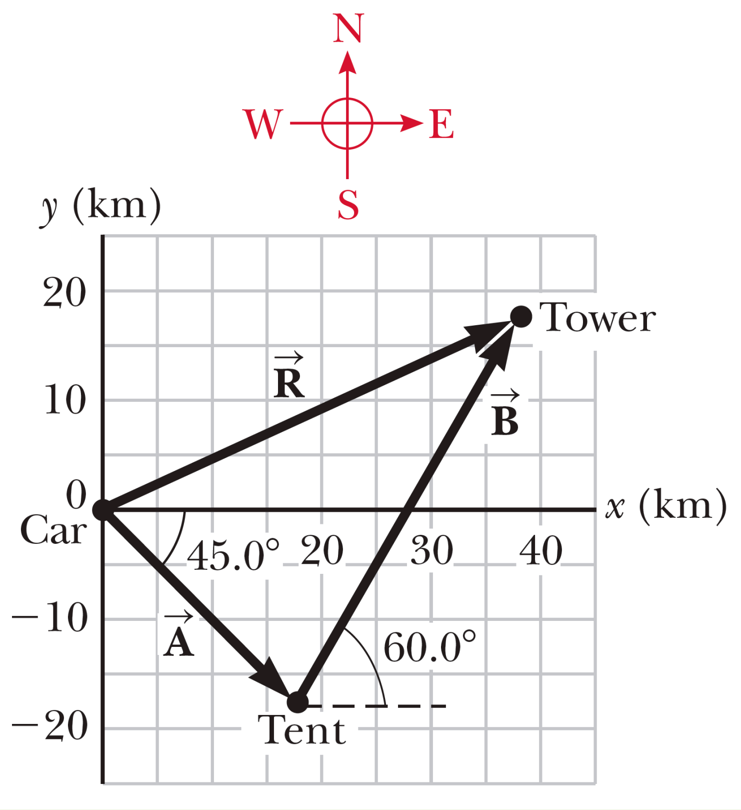
$$|\vec{\boldsymbol{R}}| = \sqrt{(25)^2 + (31)^2 + (7)^2} = 39.5\ \text{cm}$$

4.6 Examples

Example 4.6

A hiker begins a trip by first walking 25 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(A) Determine the components of the hiker's displacement for each day.



4.6 Examples

Solution 4.6

- The components of the hiker's displacement for the first day are:

$$A_x = 25\cos(-45°) = 17.7 \text{ km}$$
$$A_y = 25\sin(-45°) = -17.7 \text{ km}$$

- The components of the hiker's displacement for the second day are:

$$B_x = 40\cos(60°) = 20 \text{ km}$$
$$B_y = 40\sin(60°) = 34.6 \text{ km}$$

Example 4.6

(B) Determine the components of the hiker's resultant displacement $\vec{\mathbf{R}}$ for the trip. Find an expression for $\vec{\mathbf{R}}$ in terms of unit vectors.

4.6 Examples

King Saud University

Solution 4.6

- The resultant displacement for the trip,

$$\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}},$$

has components given by:

$$R_x = A_x + B_x = 17.7 \text{ km} + 20 \text{ km} = 37.7 \text{ km}$$

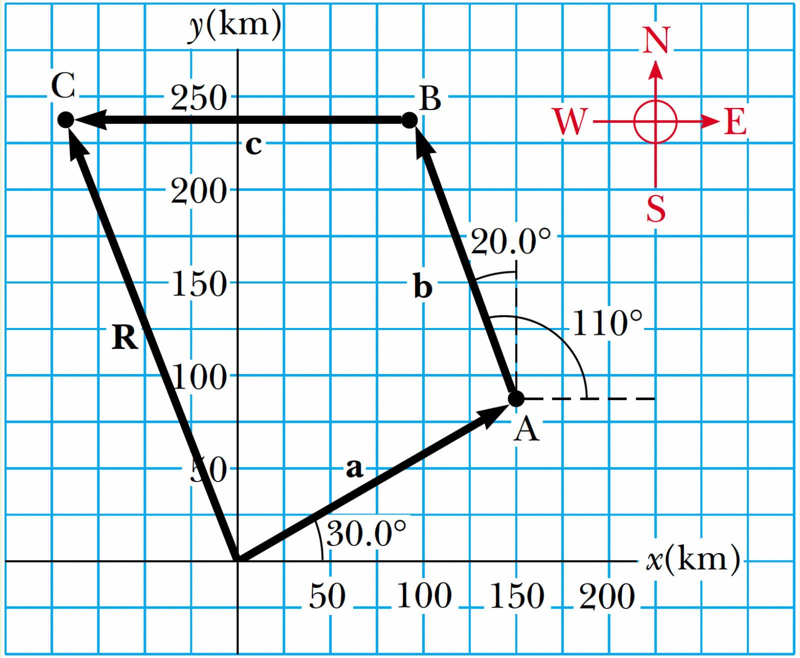
$$R_y = A_y + B_y = -17.7 \text{ km} + 34.6 \text{ km} = 16.9 \text{ km}$$

- In unit vector notation, we can write the resultant displacement as:

$$\vec{\boldsymbol{R}} = (37.7\hat{\boldsymbol{\imath}} + 16.9\hat{\boldsymbol{\jmath}}) \quad \text{km}$$

Example 4.7

A commuter airplane takes the route shown in the Figure. First, it flies from the origin of the coordinate system shown to city A, located 175 km in a direction 30.0° north of east. Next, it flies 153 km 20.0° west of north to city B. Finally, it flies 195 km due west to city C. Find the location of city C relative to the origin



Solution 4.7

- First, we find the components of the displacement to city A:

$$a_x = 175\cos(30°) = 151.6 \quad \text{km}$$
$$a_y = 175\sin(30°) = 87.5 \quad \text{km}$$

- Next, we find the components of the displacement to city B:

$$b_x = 153\cos(110°) = -52.3 \quad \text{km}$$
$$b_y = 153\sin(110°) = 144 \quad \text{km}$$

- Finally, we find the components of the displacement to city C:

$$c_x = -195 \quad \text{km}$$
$$c_y = 0 \quad \text{km}$$

4.6 Examples

- The total displacement components are:

$$R_x = a_x + b_x + c_x = 151.6 - 52.3 - 195 = -95.7 \ \text{km}$$

$$R_y = a_y + b_y + c_y = 87.5 + 144 + 0 = 231.5 \ \text{km}$$

- In unit vector notation, we can write the resultant displacement as:

$$\vec{\boldsymbol{R}} = (-95.7\hat{\boldsymbol{\imath}} + 231.5\hat{\boldsymbol{\jmath}}) \ \text{km}$$

5. Suggested Problems

1, 4, 19, 21, 27, 30, 31, 33, 39, 49, 50