Ch.29: Sources of the Magnetic Field

Physics 104: Electricity and Magnetism

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Ref.: Serway, R. A., & Jewett, J. W. (2018). Physics for Scientists and Engineers (10th ed.)

Version 1

Outline

Remember From Previous Chapters

Classical Mechanics

- Equations of motion:

$$\vec{\boldsymbol{v}}_f = \vec{\boldsymbol{v}}_i + \vec{\boldsymbol{a}}t$$

$$\vec{\boldsymbol{r}}_f = \vec{\boldsymbol{r}}_i + \vec{\boldsymbol{v}}_i t + \frac{1}{2}\vec{\boldsymbol{a}}t^2$$

$$\vec{\boldsymbol{v}}_f^2 = \vec{\boldsymbol{v}}_i^2 + 2\vec{\boldsymbol{a}} \cdot \left(\vec{\boldsymbol{r}}_f - \vec{\boldsymbol{r}}_i\right)$$

- Newton's Second Law:

$$\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$$

- Work-Energy Theorem:

$$W = \Delta K$$

$$W = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{d}}$$

$$K = \frac{1}{2}m\vec{\boldsymbol{v}}^2$$

Electric Field

- Coulomb's Law:

$$\vec{\boldsymbol{F}}_e = k_e \frac{q_1 q_2}{r^2}\hat{\boldsymbol{r}}$$

- Electric Field:

$$\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}_e}{q_0} = k_e \frac{q}{r^2}\hat{\boldsymbol{r}}$$

$$\vec{\boldsymbol{a}} = \left(\frac{q}{m}\right)\vec{\boldsymbol{E}}$$

Flux

- Gauss's Law:

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot \mathrm{d}\vec{\boldsymbol{A}} = \frac{q_{\text{in}}}{\varepsilon_0}$$

Remember From Previous Chapters

Electric Potential and Energy

- Electric Potential:

$$V = k_e \frac{q}{r}$$

- Potential Energy:

$$U_E = k_e \frac{q_1 q_2}{r}$$

- Relation to Electric Field:

$$\Delta V = -\vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{d}}$$

- Potential and Energy:

$$\Delta U_E = q \Delta V$$

Capacitance and Dielectrics

- Capacitance:

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$

- Capacitors in Parallel:

$$C_{\text{eq}} = \sum C_i$$

- Capacitors in Series:

$$\frac{1}{C_{\text{eq}}} = \sum \left(\frac{1}{C_i} \right)$$

- Energy Stored in Capacitor:

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V$$

$$= \frac{1}{2} C (\Delta V)^2$$

Remember From Previous Chapters

- Energy Density of Electric Field:

$$u_E = \frac{U_E}{V} = \frac{1}{2}\varepsilon_0 E^2$$

- Dielectric Constant:

$$\Delta V = \Delta V_0/\kappa$$

$$C = \kappa C_0$$

$$Q = \kappa Q_0$$

$$U_E = U_0/\kappa$$

Current and Resistance

- Current:

$$I = \frac{\Delta Q}{\Delta t}$$

$$I_{\mathsf{avg}} = nAv_d q$$

- Ohm's Relation:

$$\Delta V = IR$$

- Resistance:

$$R = \rho\frac{L}{A}$$

- conductivity:

$$\sigma = \frac{1}{\rho}$$

- Temperature Effect

$$R = R_0[1 + \alpha(T - T_0)]$$

- Electrical Power:

$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

$$\text{Energy} = P\Delta t$$

Remember From Previous Chapters

Direct-Current Circuits

- Electromotive Force:

$$\Delta V = \varepsilon - Ir = IR,$$

- Resistors in Series:

$$R_{\text{eq}} = R_1 + R_2 + \ldots + R_n$$

- Resistors in Parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

- Kirchhoff's Rules:

1. Junction Rule:

$$\sum_{\text{node}} I = 0$$

2. Loop Rule:

$$\sum_{\text{loop}} \Delta V = 0$$

Magnetic Fields

- Magnetic Force on a Moving Charge:

$$\vec{\boldsymbol{F}}_B = q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$$

$$\left|\vec{\boldsymbol{F}}_B\right| = qvB\sin\theta$$

- Charges in a circular path under a magnetic field:

$$r = \frac{mv}{qB}$$

Remember From Previous Chapters

- Angular Velocity:

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

- Period of Revolution:

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

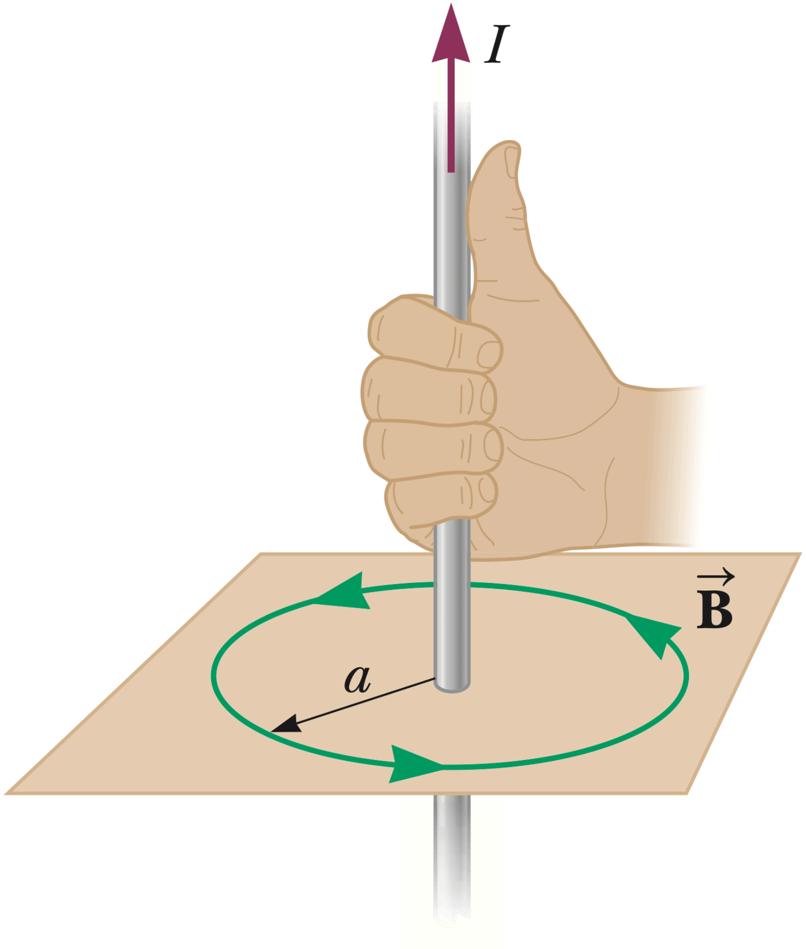
- Magnetic Force on a Current-Carrying Wire:

$$\vec{\boldsymbol{F}}_B = I\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}}$$

$$\left|\vec{\boldsymbol{F}}_B\right| = ILB\sin\theta$$

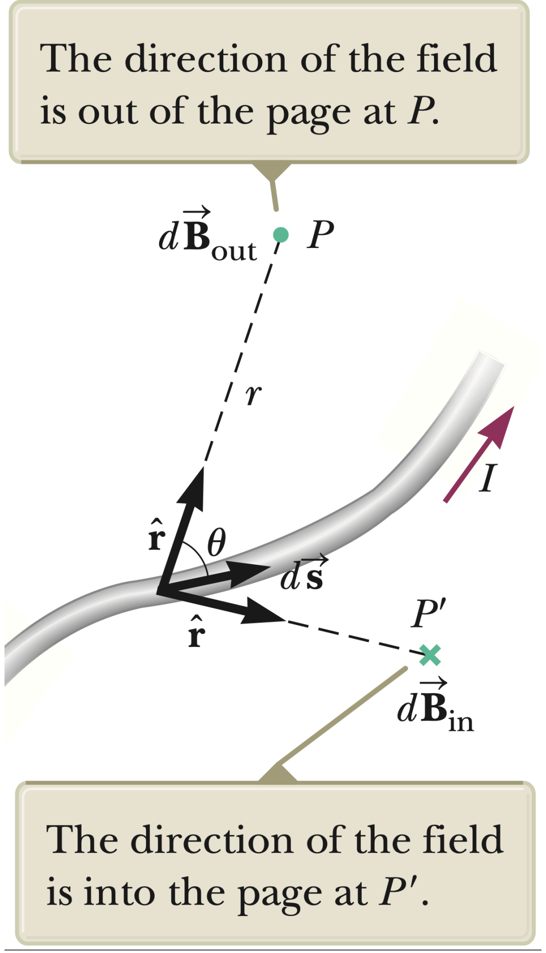
1. The Biot-Savart Law
2. The Magnetic Force Between Two Parallel Conductors
3. Ampère’s Law
4. The Magnetic Field of a Solenoid
5. Gauss’s Law in Magnetism

1.1 Magnetic Field from a Current Element



- When a current *flows* through a conductor, it *creates* a magnetic field that circulates around it.
- The created magnetic field is proportional to the current (I) and inversely proportional to the distance from the conductor (r).
- The direction of the magnetic field is given by the right-hand rule, where the thumb points in the direction of the current and the curled fingers indicate the direction of the magnetic field.

1.2 The Biot-Savart Law



- The Biot-Savart Law provides a mathematical expression for calculating the magnetic field ($\vec{B}$) produced by a small segment of current-carrying conductor ($\mathrm{d}\vec{s}$).
- The magnetic field at position r from the current element and direction $\hat{\boldsymbol{r}}$ is given by the formula:

$$\boxed{\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{r^2}\,\mathrm{d}\vec{s} \times \hat{\boldsymbol{r}}}$$

where μ_0 is the permeability of free space,

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\cdot m/A}$$

1. The Biot-Savart Law

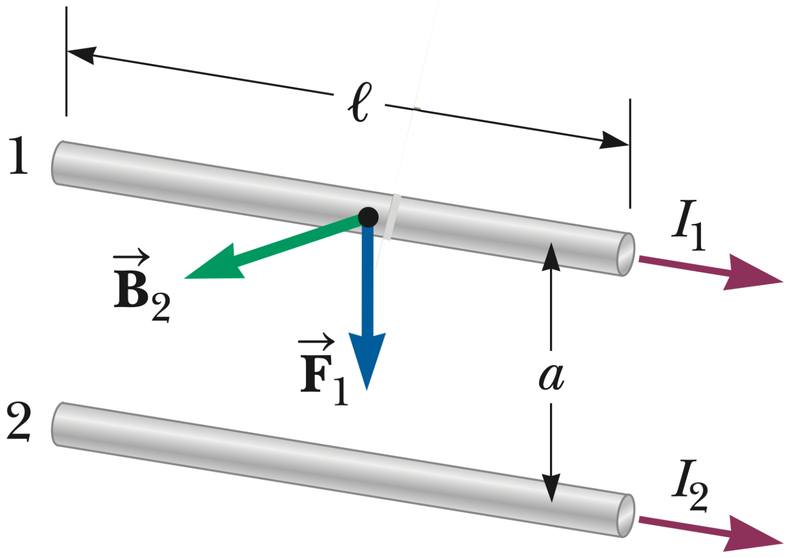
2. **The Magnetic Force Between Two Parallel Conductors**

3. Ampère’s Law

4. The Magnetic Field of a Solenoid

5. Gauss’s Law in Magnetism

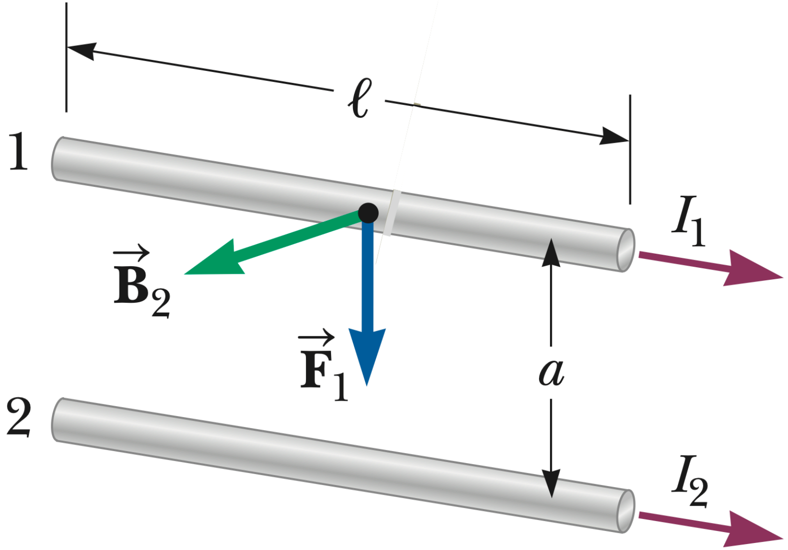
2.1 Magnetic Force Between Two Parallel Conductors



When two parallel conductors carry currents, they exert magnetic forces on each other. The force per unit length (F/ℓ) between two conductors separated by a distance a and carrying currents I_1 and I_2 is given by:

$$\boxed{\frac{F}{\ell} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{a}}$$

2.1 Magnetic Force Between Two Parallel Conductors



The direction of the force depends on the direction of the currents:

1. If the currents are in the **same** direction, the force is **attractive**.
2. If the currents are in **opposite** directions, the force is **repulsive**.

1. The Biot-Savart Law

2. The Magnetic Force Between Two Parallel Conductors

3. **Ampère’s Law**

4. The Magnetic Field of a Solenoid

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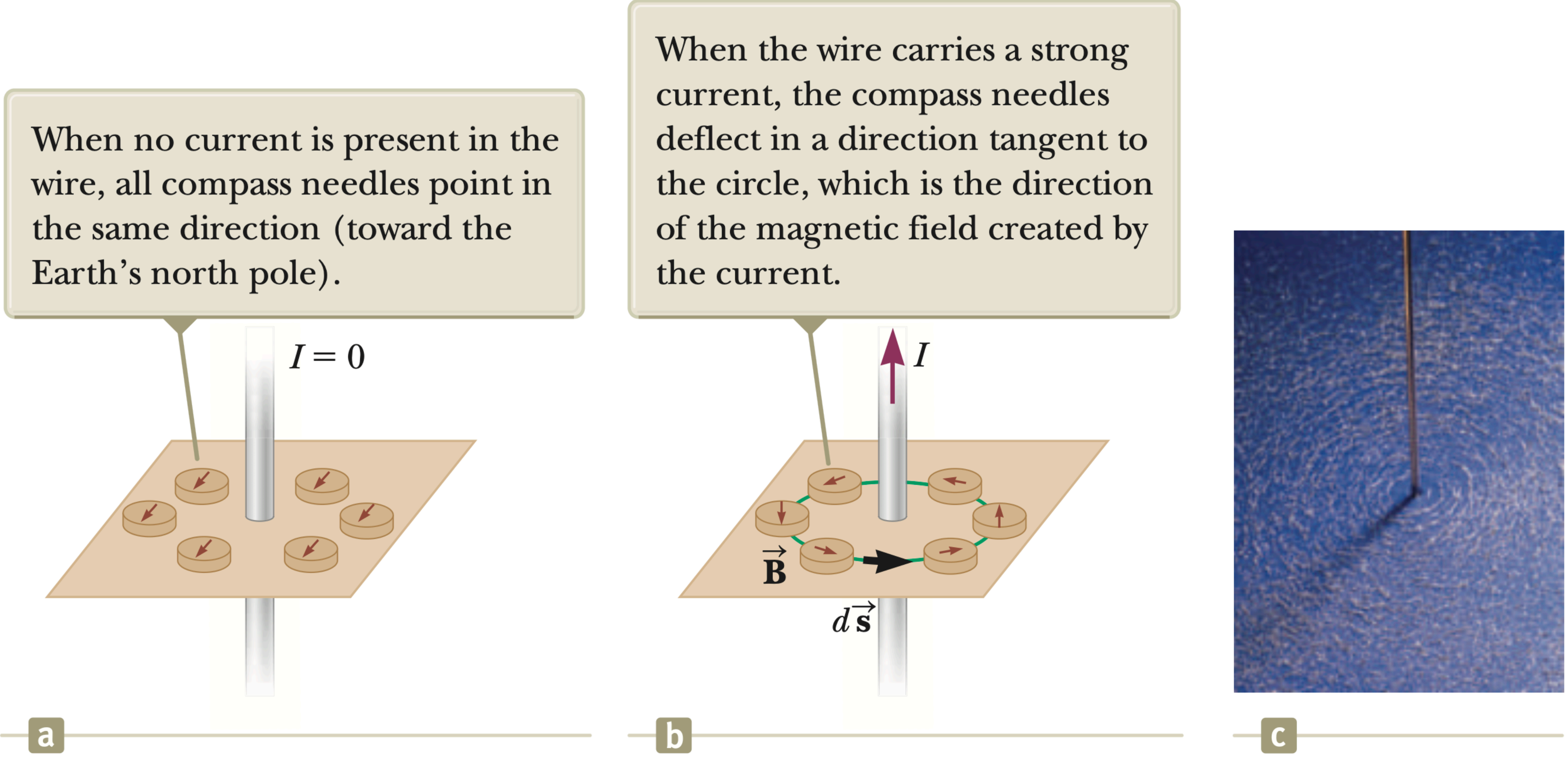
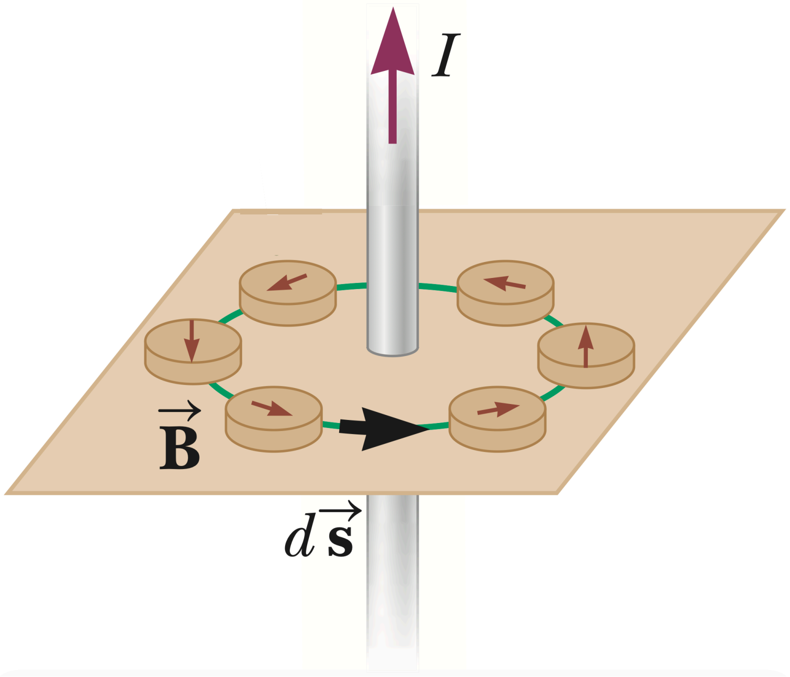
3.1 Ampère’s Law



When no current is present in the wire, all compass needles point in the same direction (toward the Earth’s north pole).

When the wire carries a strong current, the compass needles deflect in a direction tangent to the circle, which is the direction of the magnetic field created by the current.

3.1 Ampère's Law



The line integral of $\vec{B}\cdot \mathrm{d}\vec{s}$ around *any* closed path is equal to $\mu_0 I$.

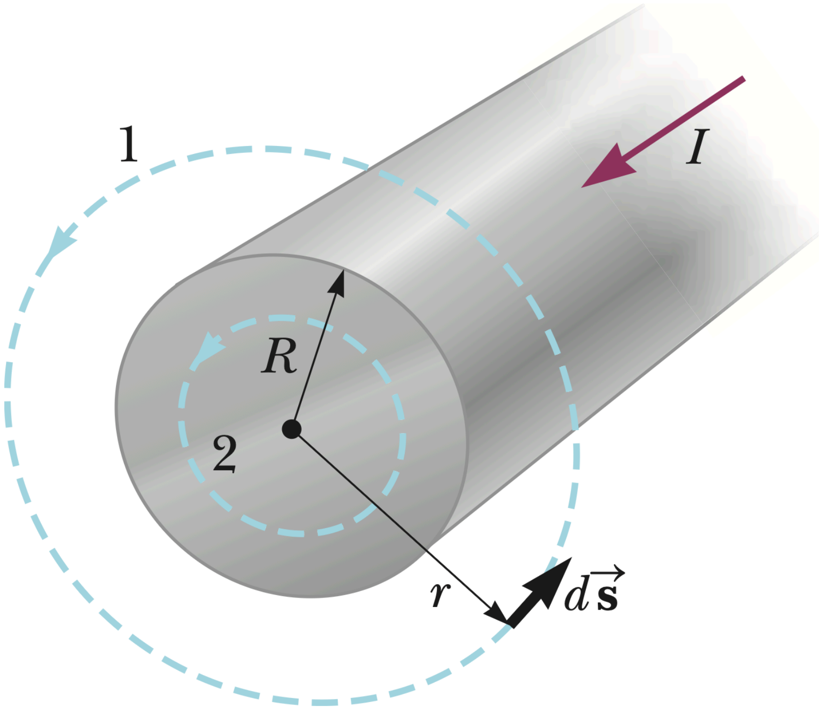
$$\boxed{\oint \vec{B}\cdot \mathrm{d}\vec{s} = \mu_0 I}$$

3.1 Ampère's Law

Example 3.1

A long, straight wire of radius R carries a steady current I that is uniformly distributed through the cross section of the wire.

Calculate the magnetic field a distance r from the center of the wire in the regions $r > R$ and $r < R$.



3.1 Ampère's Law

Solution 3.1

- For $r > R$, we can use Ampère's Law directly, and use a circular path of radius r centered on the wire as our Amperian loop. The magnetic field will be tangential to this loop and have the same magnitude at every point on it, so we can simplify the line integral:

$$\oint \vec{\boldsymbol{B}} \cdot \mathrm{d}\vec{\boldsymbol{s}} = B \oint \mathrm{d}s = B(2\pi r) = \mu_0 I$$

$$\implies \quad B = \frac{\mu_0 I}{2\pi r} \qquad (\text{for } r \geq R)$$

3.1 Ampère's Law

Solution 3.1

For $r < R$, we need to consider the current enclosed by the circular path (I'), which is a fraction of the total current (I). To find this fraction, we can use the fact that the current is uniformly distributed across the cross section of the wire. Therefore, the current density (J) is constant, and we can express I' as:

$$\frac{I'}{I} = \frac{JA'}{JA} = \frac{\pi r^2}{\pi R^2} = \frac{r^2}{R^2}$$

$$\implies \quad I' = \frac{r^2}{R^2} I$$

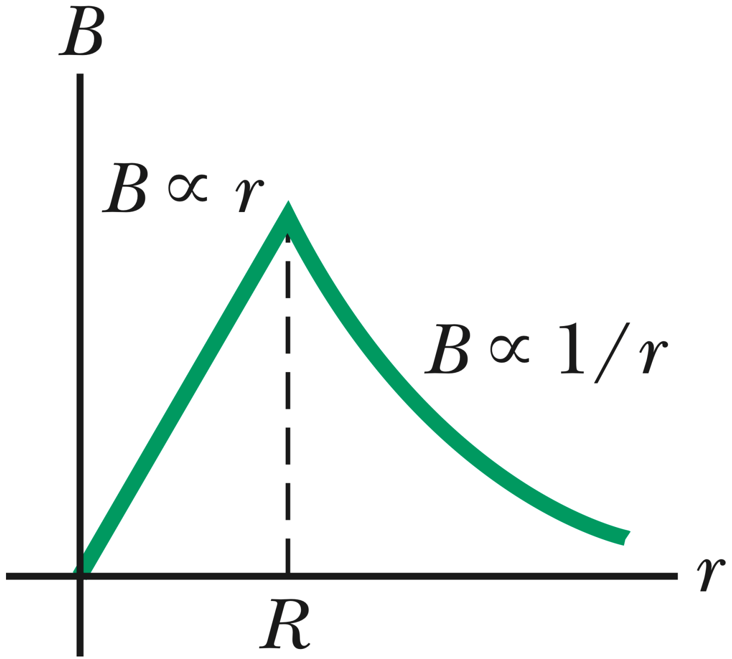
3.1 Ampère's Law

Solution 3.1

Then, we can apply Ampère's Law again, using the same circular path of radius r:

$$\oint \vec{\boldsymbol{B}} \cdot \mathrm{d}\vec{\boldsymbol{s}} = B(2\pi r) = \mu_0 I'$$

$$\implies \quad B = \frac{\mu_0}{2\pi r} I' = \frac{\mu_0 I}{2\pi R^2}\, r \qquad (\text{for } r < R)$$



1. The Biot-Savart Law

2. The Magnetic Force Between Two Parallel Conductors

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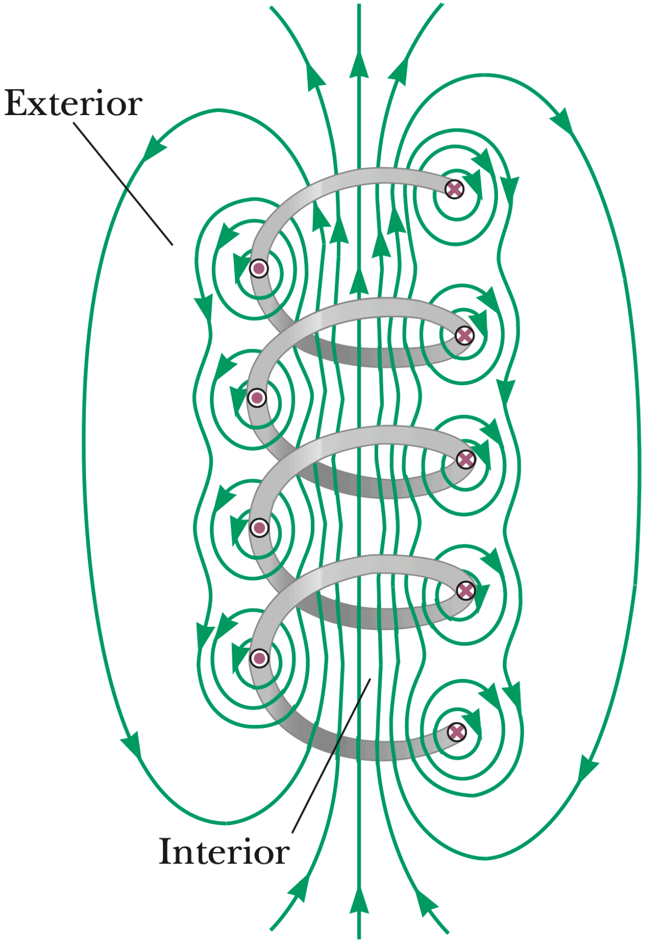
4. **The Magnetic Field of a Solenoid**

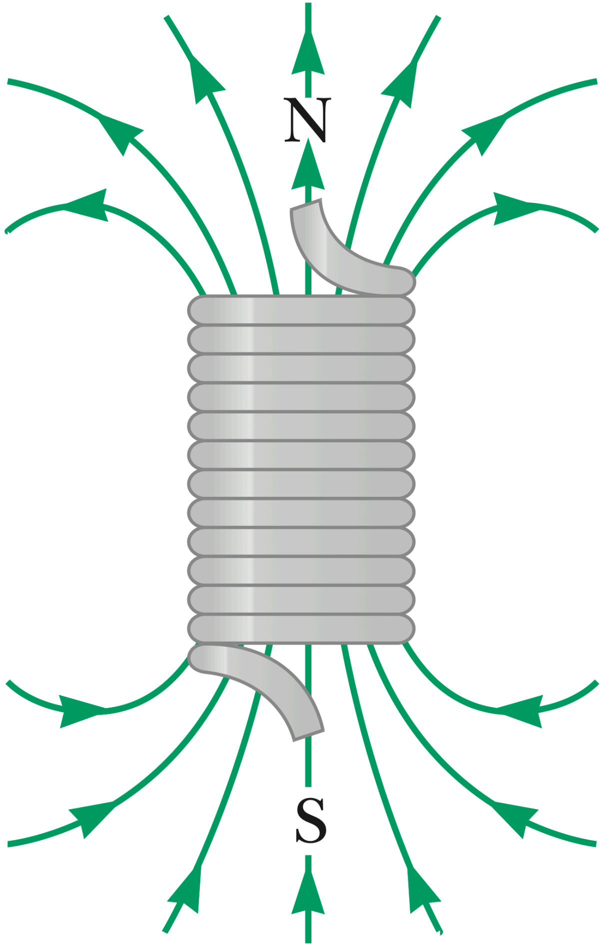
5. Gauss’s Law in Magnetism

4.1 The Magnetic Field of a Solenoid



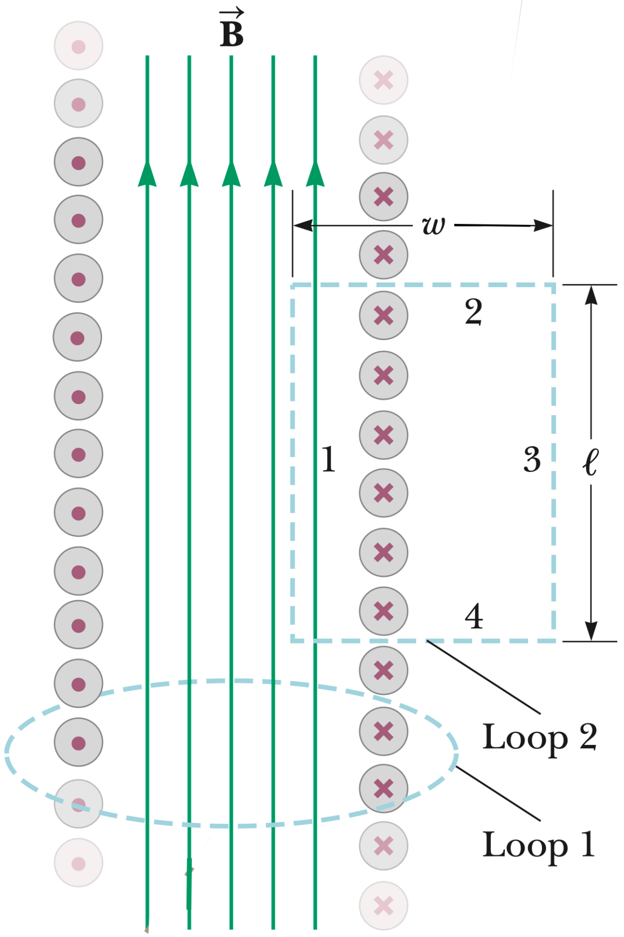
- A solenoid is a long wire wound in the form of a helix. When a current flows through the solenoid, it creates a magnetic field that is strong and uniform inside it and weak outside it.

4.1 The Magnetic Field of a Solenoid



- If the turns are closely spaced and the solenoid is of finite length, the external magnetic field lines are similar to a bar magnet.
- If the length is much greater than the radius, the magnetic field inside the solenoid is approximately uniform and parallel to the axis of the solenoid, while the magnetic field outside is negligible. This type of solenoid is called an *ideal solenoid.*

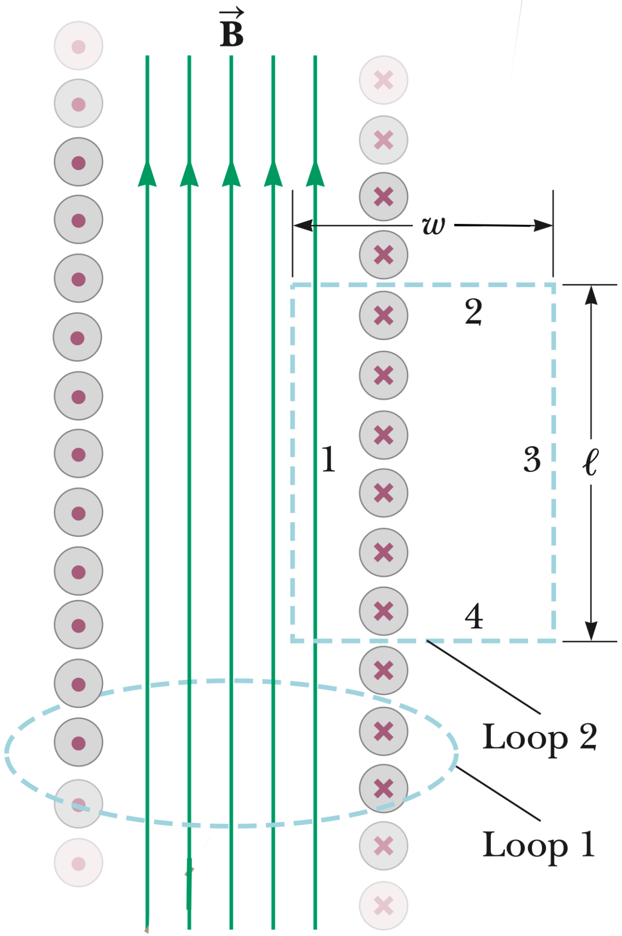
4.2 Calculating the Magnetic Field of a Solenoid



We can use Ampère’s Law to calculate the magnetic field inside an ideal solenoid. We choose a rectangular Amperian loop that extends inside and outside the solenoid, with one side parallel to the axis of the solenoid and the other side perpendicular to it. The line integral of $\vec{B} \cdot \mathrm{d}\vec{s}$ around this loop will only have a contribution from the side that is inside the solenoid, since the magnetic field outside is negligible. Therefore, we can express Ampère’s Law as (loop 2):

$$\oint \vec{B} \cdot \mathrm{d}\vec{s} = \oint_{\text{path 1}} \vec{B} \cdot \mathrm{d}\vec{s} = B \oint_{\text{path 1}} \mathrm{d}s = B\ell$$

4.2 Calculating the Magnetic Field of a Solenoid



Therefore, using Ampère's Law, we can relate this line integral to the current enclosed by the loop, which is the current in the solenoid (I) multiplied by the number of turns (N):

$$\oint \vec{\boldsymbol{B}} \cdot \mathrm{d}\vec{\boldsymbol{s}} = B\ell = \mu_0 N I$$

Solving for B, the magnetic field inside a solenoid is:

$$\boxed{B = \mu_0 \left(\frac{N}{\ell} \right) I = \mu_0 n I}$$

where n is the number of turns per unit length.

1. The Biot-Savart Law

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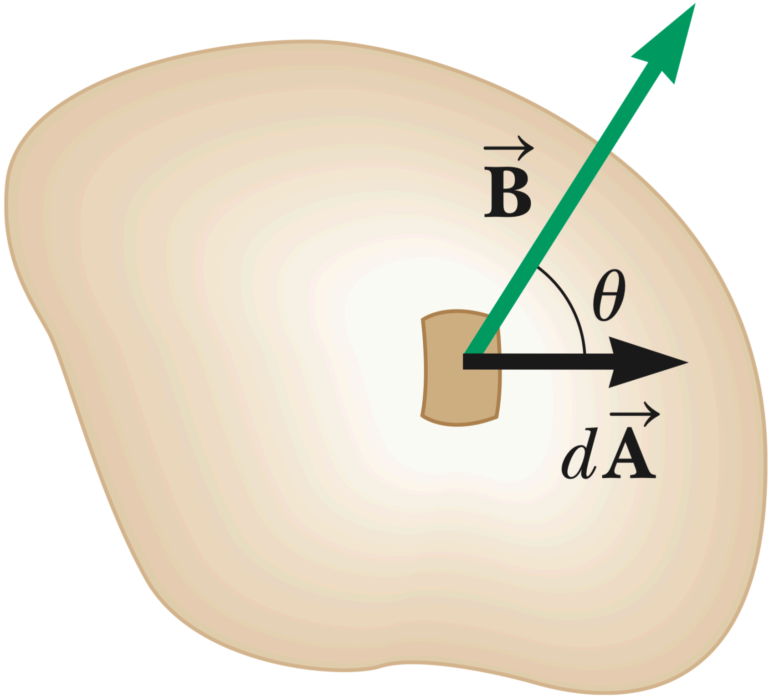
5. **Gauss’s Law in Magnetism**

5.1 Magnetic Flux and Gauss's Law in Magnetism



- The magnetic flux (Φ_B) through a surface is defined as the surface integral of the magnetic field ($\vec{B}$) over that surface:

$$\boxed{\Phi_B = \int \vec{B} \cdot \mathrm{d}\vec{A}}$$

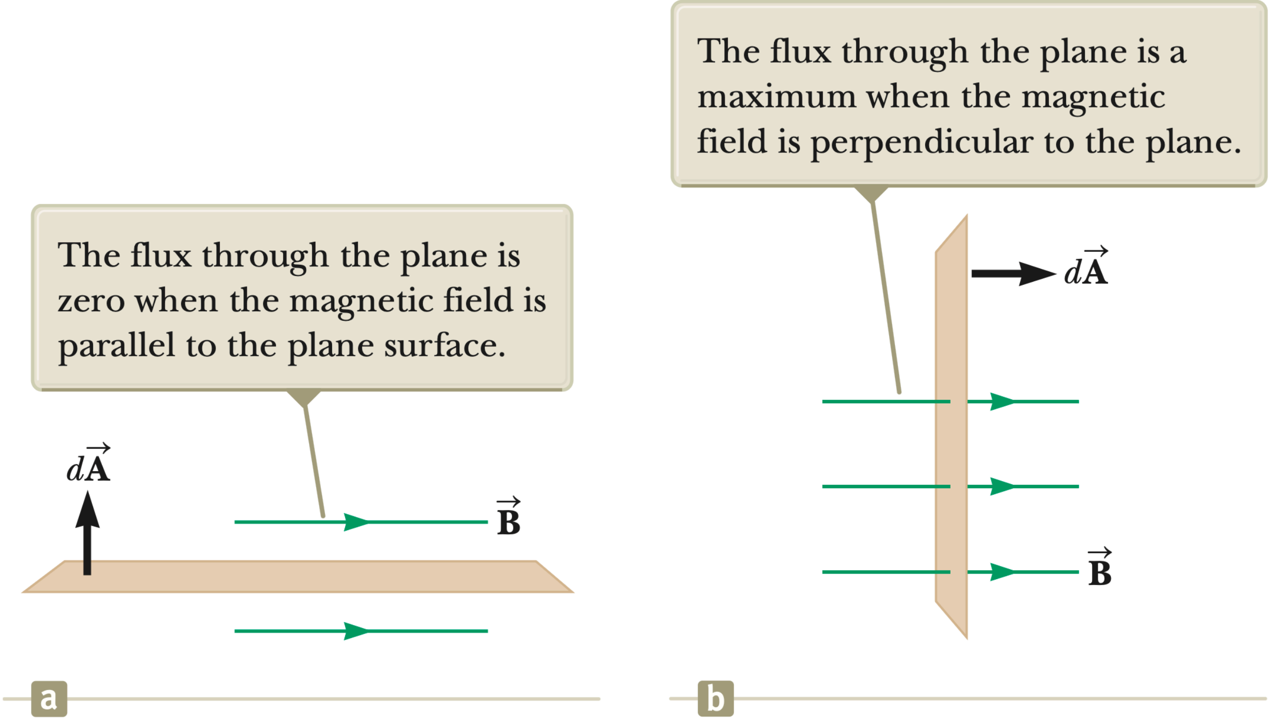
- The unit of magnetic flux is the weber (Wb), where

$$1\ \mathrm{Wb} = 1\ \mathrm{T} \cdot \mathrm{m}^2$$

5.2 Special Case of Magnetic Flux

The special case of a plane surface with a uniform magnetic field that makes an angle θ with the normal to the surface can be simplified to:

$$\Phi_B = BA\cos\theta$$



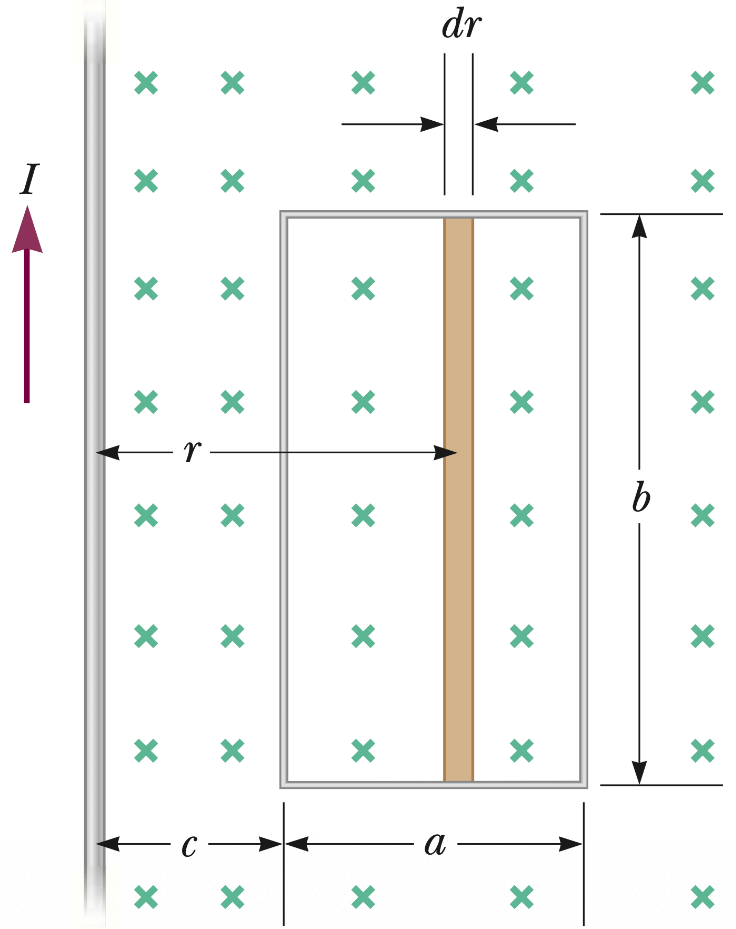
The flux through the plane is zero when the magnetic field is parallel to the plane surface.

The flux through the plane is a maximum when the magnetic field is perpendicular to the plane.

5.3 Example

Example 5.2

A rectangular loop of width a and length b is located near a long wire carrying a current I. The distance between the wire and the closest side of the loop is c. The wire is parallel to the long side of the loop. Find the total magnetic flux through the loop due to the current in the wire.



5.3 Example

Solution 5.2

To find the total magnetic flux through the loop, we need to calculate the magnetic field at each point on the loop due to the current in the wire and then integrate this field over the area of the loop. The magnetic field at a distance r from the wire is given by the Ampère's Law for a long straight conductor (that we derived earlier):

$$B = \frac{\mu_0 I}{2\pi r}$$

Next, using the magnetic flux formula, we find:

$$\Phi_B = \int \vec{\boldsymbol{B}} \cdot \mathrm{d}\vec{\boldsymbol{A}} = \int B \, \mathrm{d}A = \int \frac{\mu_0 I}{2\pi r} \, \mathrm{d}A = \frac{\mu_0 I}{2\pi} \int \left(\frac{1}{r}\right) \mathrm{d}A$$

5.3 Example

Since $\mathrm{d}A = b\,\mathrm{d}r$, and b is a constant, we can simplify the integral:

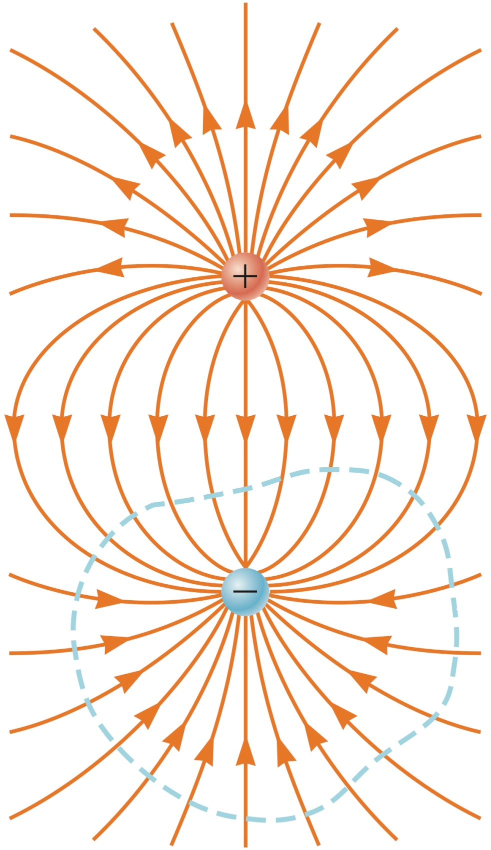
$$\Phi_B = \frac{\mu_0 I b}{2\pi} \int \left(\frac{\mathrm{d}r}{r} \right)$$

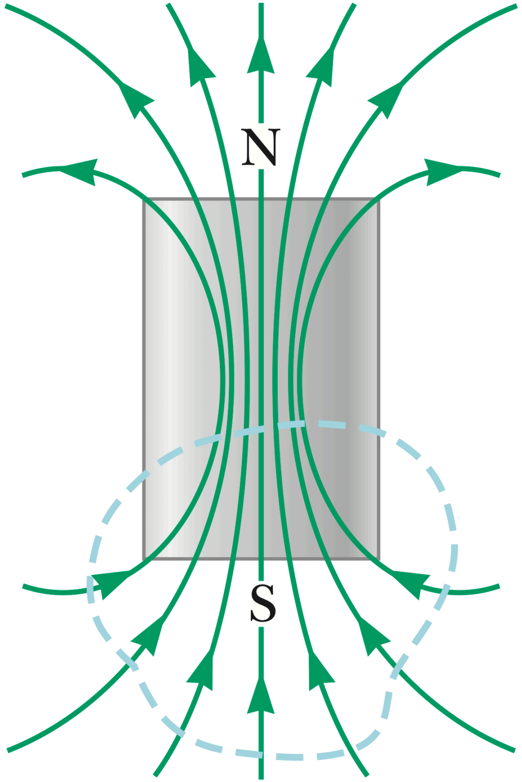
Next, we choose the limits of integration over r, which is from c to $a + c$:

$$\Phi_B = \frac{\mu_0 I b}{2\pi} \int_{\mathrm{c}}^{\mathrm{a+c}} \left(\frac{\mathrm{d}r}{r} \right) = \frac{\mu_0 I b}{2\pi} \ln\left(\frac{a+c}{c} \right) = \frac{\mu_0 I b}{2\pi} \ln\left(1 + \frac{a}{c} \right)$$

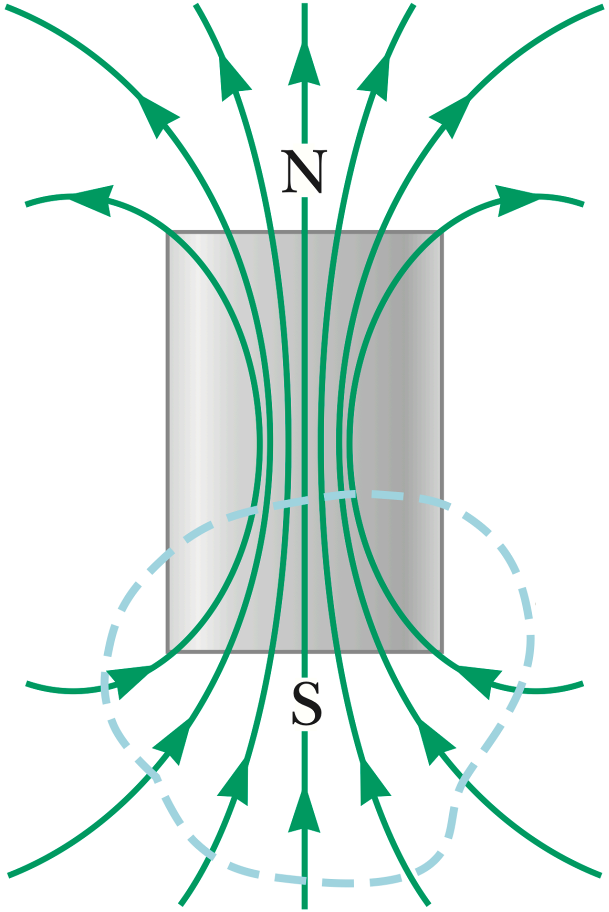
5.4 Gauss’s Law in Magnetism

Recall that electric filed lines and magnetic field lines have different properties.





5.4 Gauss’s Law in Magnetism



Gauss’s law in magnetism states that:

The net magnetic flux through any closed surface is always zero

$$\boxed{\oint \vec{B} \cdot \mathrm{d}\vec{A} = 0}$$

This implies that magnetic monopoles do not exist.

Suggested Problems

1, 12, 13, 23, 27

Book: Serway, R. A., & Jewett, J. W. (2018). Physics for Scientists and Engineers (10th ed.)

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