Ch.28: Magnetic Fields

Physics 104: Electricity and Magnetism

Dr. Abdulaziz Alqasem

Physics and Astronomy Department
King Saud University

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Outline

Remember From Previous Chapters

Classical Mechanics

- Equations of motion:

$$\vec{\boldsymbol{v}}_f = \vec{\boldsymbol{v}}_i + \vec{\boldsymbol{a}}t$$

$$\vec{\boldsymbol{r}}_f = \vec{\boldsymbol{r}}_i + \vec{\boldsymbol{v}}_i t + \frac{1}{2}\vec{\boldsymbol{a}}t^2$$

$$\vec{\boldsymbol{v}}_f^2 = \vec{\boldsymbol{v}}_i^2 + 2\vec{\boldsymbol{a}} \cdot \left(\vec{\boldsymbol{r}}_f - \vec{\boldsymbol{r}}_i\right)$$

- Newton's Second Law:

$$\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$$

- Work-Energy Theorem:

$$W = \Delta K$$

$$W = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{d}}$$

$$K = \frac{1}{2}m\vec{\boldsymbol{v}}^2$$

Electric Field

- Coulomb's Law:

$$\vec{\boldsymbol{F}}_e = k_e \frac{q_1 q_2}{r^2}\hat{\boldsymbol{r}}$$

- Electric Field:

$$\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}_e}{q_0} = k_e \frac{q}{r^2}\hat{\boldsymbol{r}}$$

$$\vec{\boldsymbol{a}} = \left(\frac{q}{m}\right)\vec{\boldsymbol{E}}$$

Flux

- Gauss's Law:

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot \mathrm{d}\vec{\boldsymbol{A}} = \frac{q_{\text{in}}}{\varepsilon_0}$$

Remember From Previous Chapters

Electric Potential and Energy

- Electric Potential:

$$V = k_e \frac{q}{r}$$

- Potential Energy:

$$U_E = k_e \frac{q_1 q_2}{r}$$

- Relation to Electric Field:

$$\Delta V = -\vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{d}}$$

- Potential and Energy:

$$\Delta U_E = q\Delta V$$

Capacitance and Dielectrics

- Capacitance:

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$

- Capacitors in Parallel:

$$C_{\text{eq}} = \sum C_i$$

- Capacitors in Series:

$$\frac{1}{C_{\text{eq}}} = \sum \left(\frac{1}{C_i} \right)$$

- Energy Stored in Capacitor:

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$$

Remember From Previous Chapters

- Energy Density of Electric Field:

$$u_E = \frac{U_E}{V} = \frac{1}{2}\varepsilon_0 E^2$$

- Dielectric Constant:

$$\Delta V = \Delta V_0/\kappa$$

$$C = \kappa C_0$$

$$Q = \kappa Q_0$$

$$U_E = U_0/\kappa$$

Current and Resistance

- Current:

$$I = \frac{\Delta Q}{\Delta t}$$

$$I_{\mathsf{avg}} = nAv_d q$$

- Ohm's Relation:

$$\Delta V = IR$$

- Resistance:

$$R = \rho\frac{L}{A}$$

- conductivity:

$$\sigma = \frac{1}{\rho}$$

- Temperature Effect

$$R = R_0[1 + \alpha(T - T_0)]$$

- Electrical Power:

$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

$$\text{Energy} = P\Delta t$$

Remember From Previous Chapters

Direct-Current Circuits

- Electromotive Force:

$$\Delta V = \varepsilon - Ir = IR,$$

- Resistors in Series:

$$R_{\text{eq}} = R_1 + R_2 + \ldots + R_n$$

- Resistors in Parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

- Kirchhoff's Rules:

1. Junction Rule:

$$\sum_{\text{node}} I = 0$$

2. Loop Rule:

$$\sum_{\text{loop}} \Delta V = 0$$

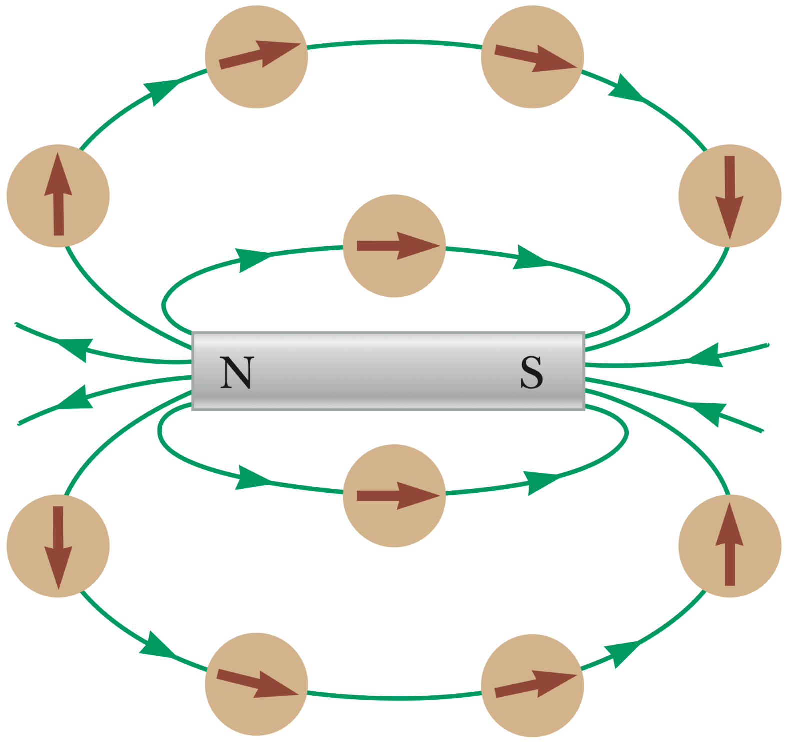
1. Analysis Model: Particle in a Field (Magnetic)

2. Motion of a Charged Particle in a Uniform Magnetic Field

3. Applications Involving Charged Particles Moving in a Magnetic Field

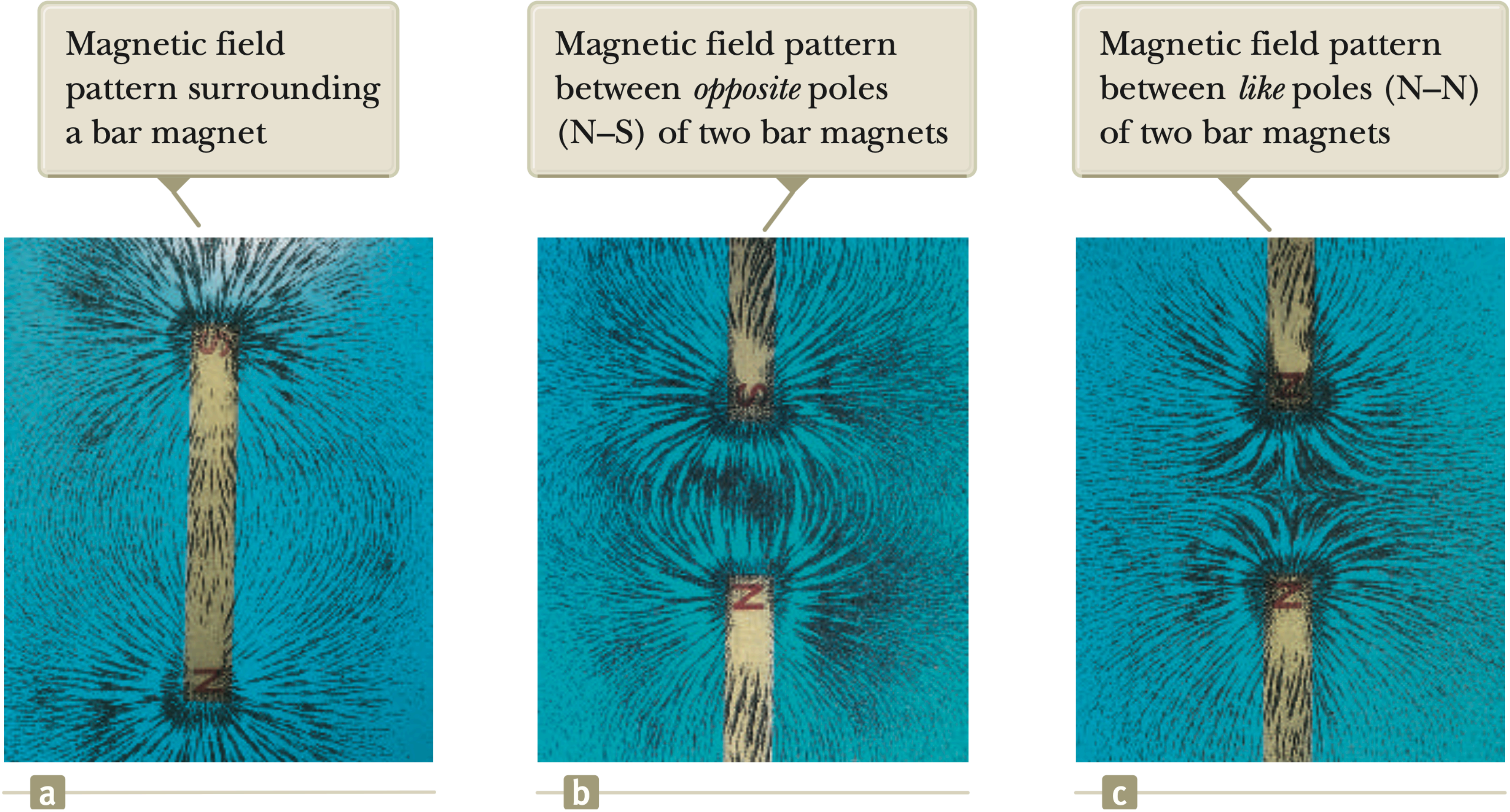
4. Magnetic Force Acting on a Current-Carrying Conductor

1.1 Magnetic Field



- Magnetic field has **two poles**: North and South, with field lines from N to S.
- Some differences from electric field: (1) magnetic field does not start or end on charges, but forms **closed loops**. (2) magnetic field is generated by **moving charges**, not by static charges (more on this later). (3) magnetic field **exerts a force** on other magnets or moving charges, not on stationary charges.
- Magnetic field (B) is a **vector** field, with units of **Tesla** (T).

1.1 Magnetic Field



1.1 Magnetic Field

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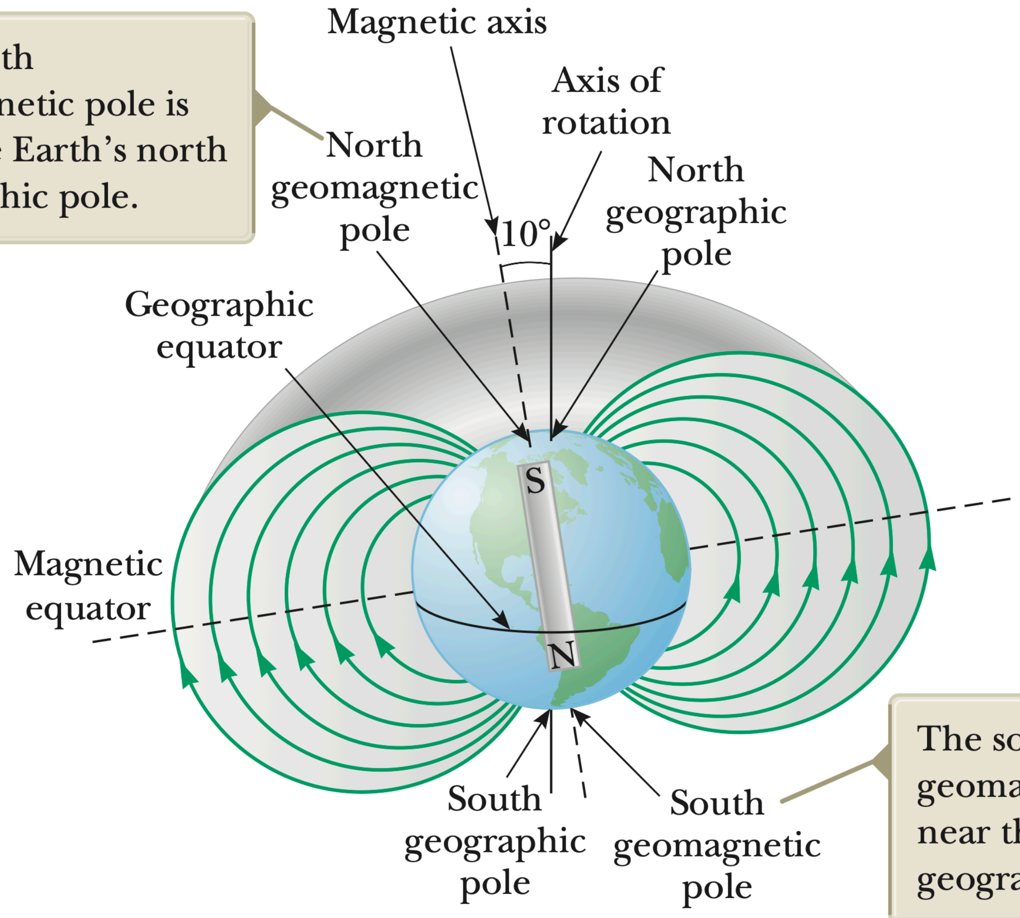
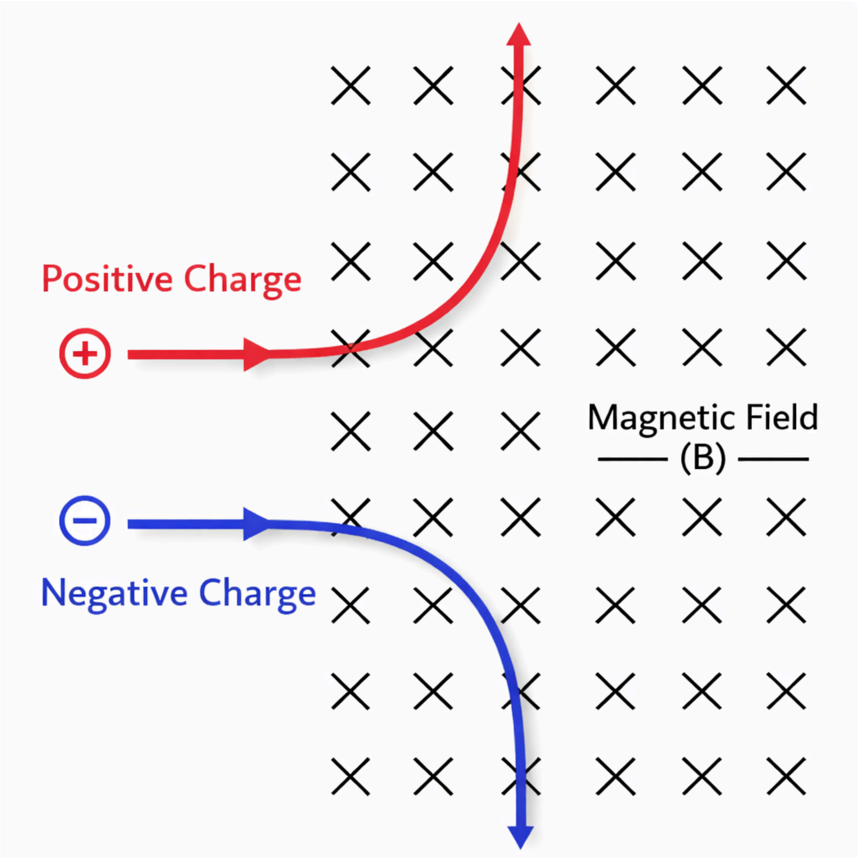
The north geomagnetic pole is near the Earth's north geographic pole.



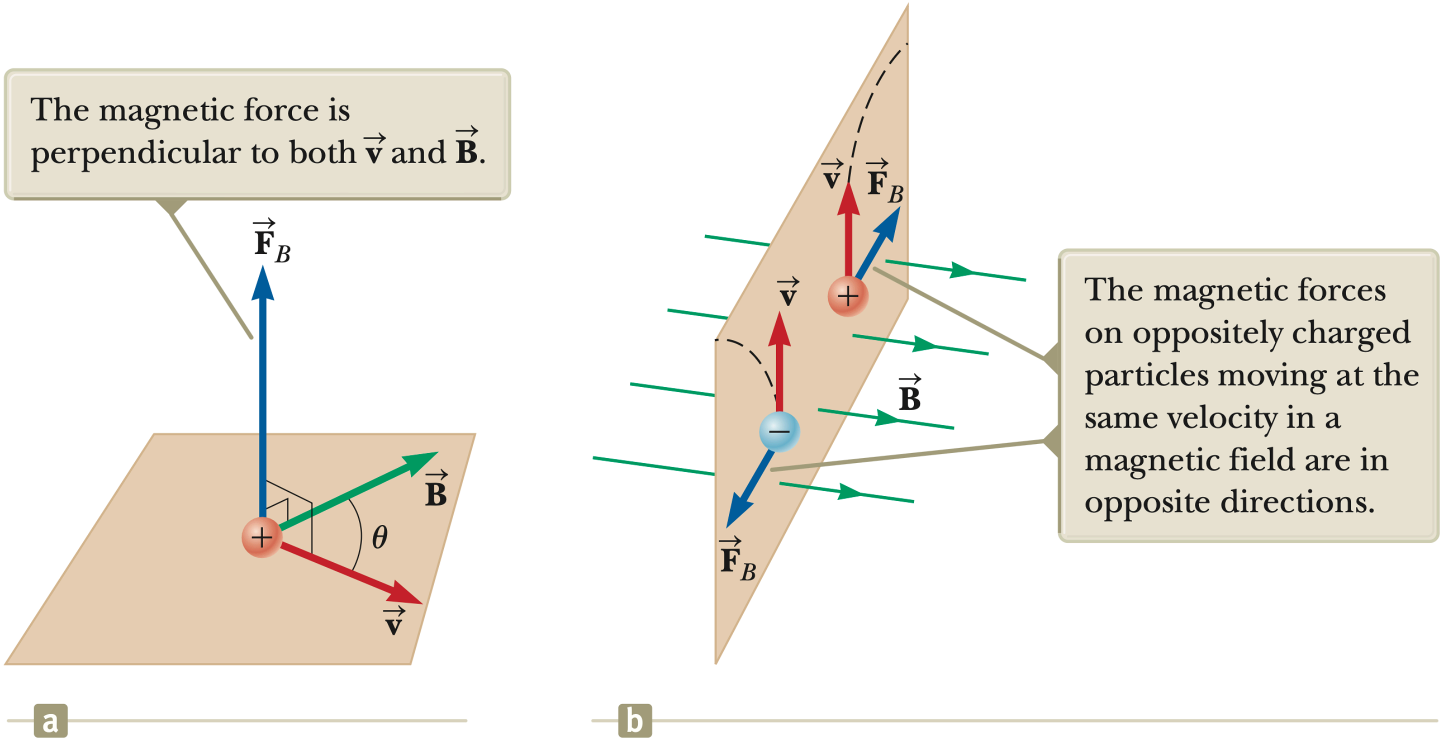
The south geomagnetic pole is near the Earth's south geographic pole.

1.2 Magnetic Force on a Moving Charge



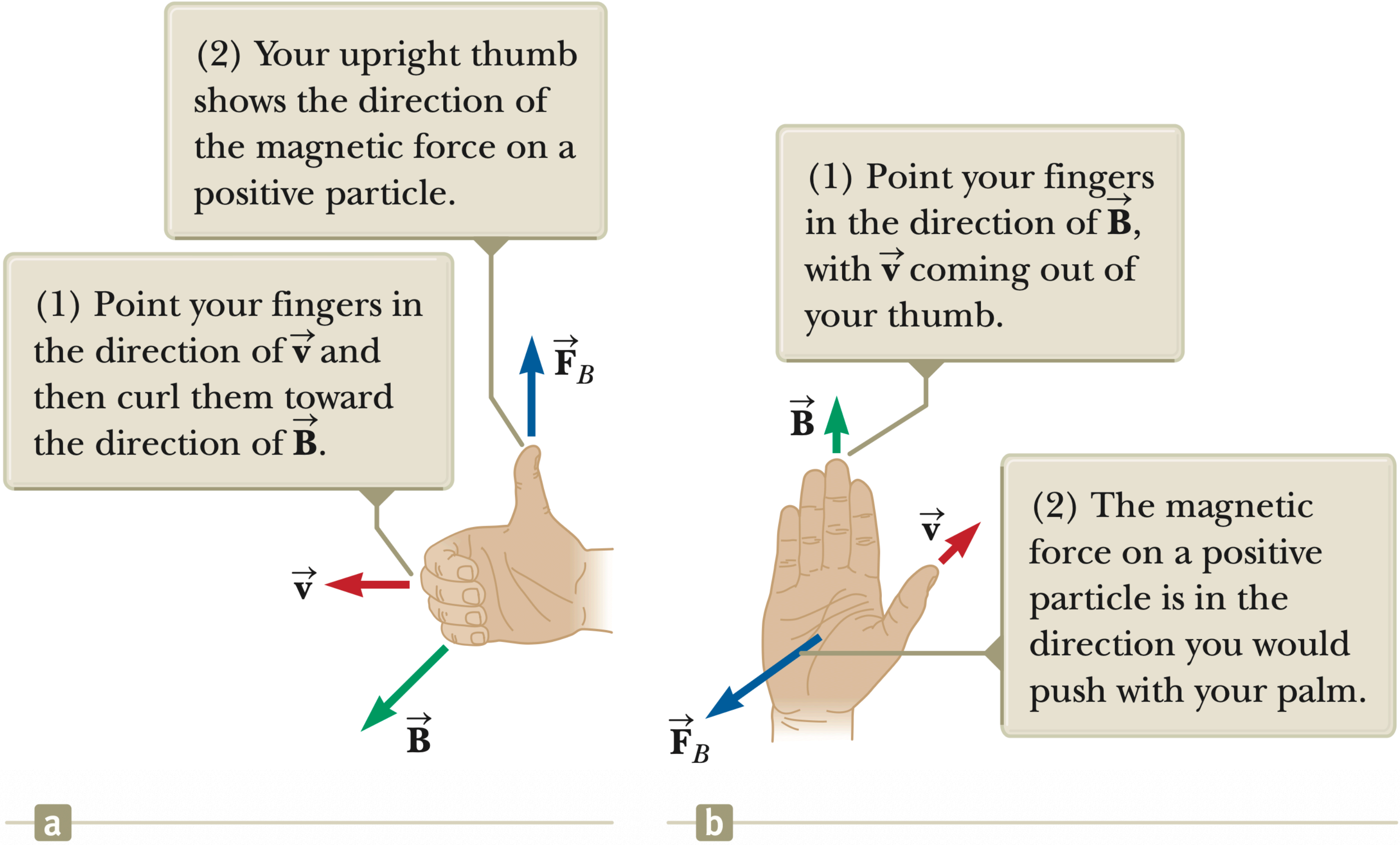
- When a charged particle moves in a magnetic field, it experiences a **magnetic force** ($\vec{F}_B$) that is perpendicular to both the velocity of the particle and the magnetic field.
- The force is **proportional** to the charge (q), the speed (v), the magnetic field strength (B), and the sine of the angle (θ) between the velocity and the magnetic field
- The **direction** of the force of a negatively charged particle is *opposite* to that of a positively charged particle, but the magnitude is the same for both.

1.2 Magnetic Force on a Moving Charge



$$\vec{F}_B = q\vec{v} \times \vec{B} = |q|vB\sin\theta$$

1.3 Direction of the Magnetic Force



1.4 Differences Between Electric and Magnetic Forces

	Electric Force	**Magnetic Force**
Source	charges	*Only* moving charges
Force on Charges	Acts on stationary and moving charges, in the direction of the electric field for positive charges	Acts only on moving charges, perpendicular to both velocity and magnetic field
Work in Displacement	Can do work on charges, changing their kinetic energy	Does *no* work on charges, only changes their direction of motion
Kinetic Energy	Can change kinetic energy	Magnetic forces cannot change the kinetic energy of charges

1.5 Magnetic Field Unit

The SI unit of magnetic field is the Tesla (T),

$$1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{C} \cdot \mathrm{m/s}}$$

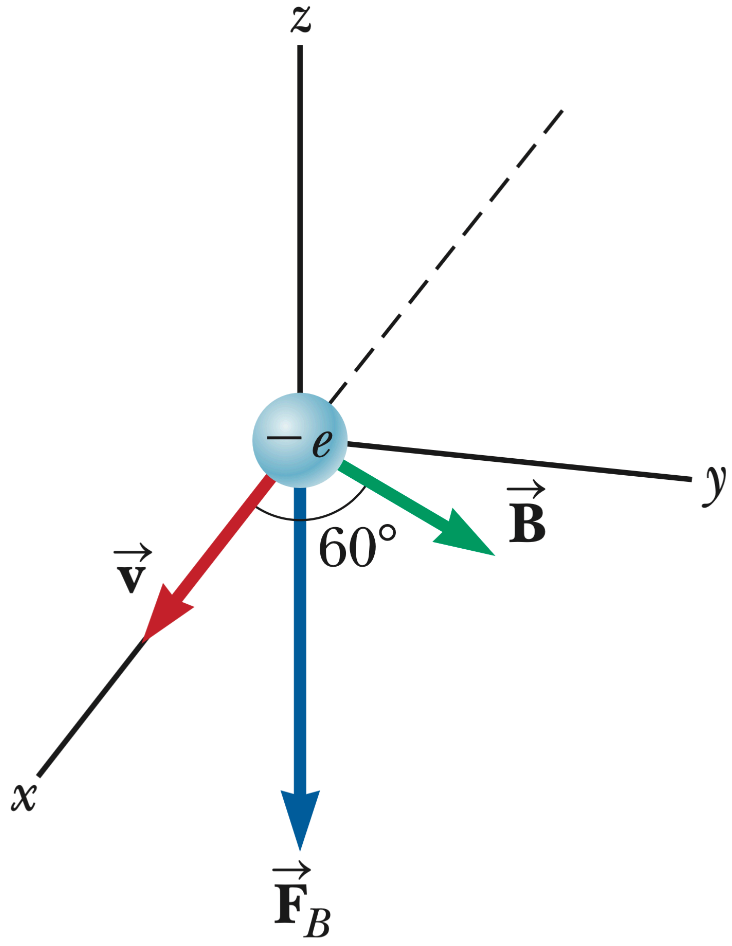
TABLE 28.1 Some Approximate Magnetic Field Magnitudes

Source of Field	Field Magnitude (T)
Strong superconducting laboratory magnet	30
Strong conventional laboratory magnet	2
Medical MRI unit	1.5
Bar magnet	10^{-2}
Surface of the Sun	10^{-2}
Surface of the Earth	5×10^{-5}
Inside human brain (due to nerve impulses)	10^{-13}

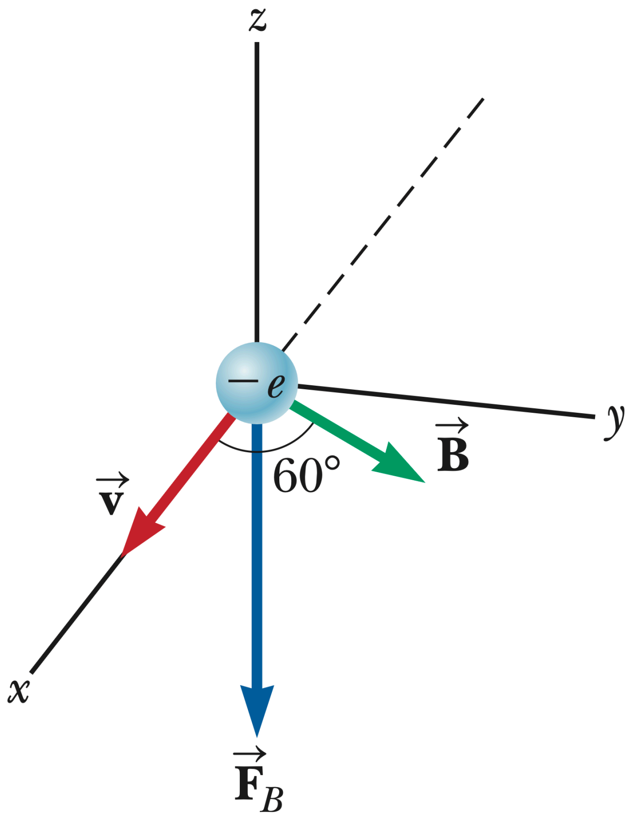
1.5 Magnetic Field Unit

Example 1.1

An electron moves through space as a cosmic ray with a speed of 8×10^6 m/s along the x-axis. At its location, the magnetic field of the Earth has a magnitude of 0.05 mT, and is directed at an angle of 60° to the x-axis, lying in the xy plane. Calculate the magnetic force on the electron.



1.5 Magnetic Field Unit



Solution 1.1

$$
\begin{aligned}
F_B &= |q| v B \sin\theta \\
&= (1.6 \times 10^{-19})(8 \times 10^{6})(5 \times 10^{-5}) \sin 60^\circ \\
&= 5.5 \times 10^{-17} \quad \text{N}
\end{aligned}
$$

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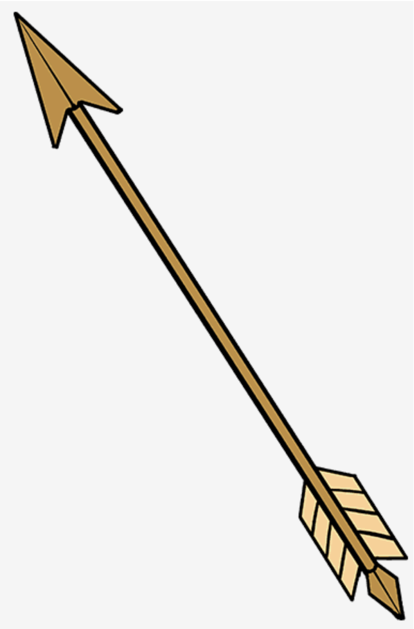
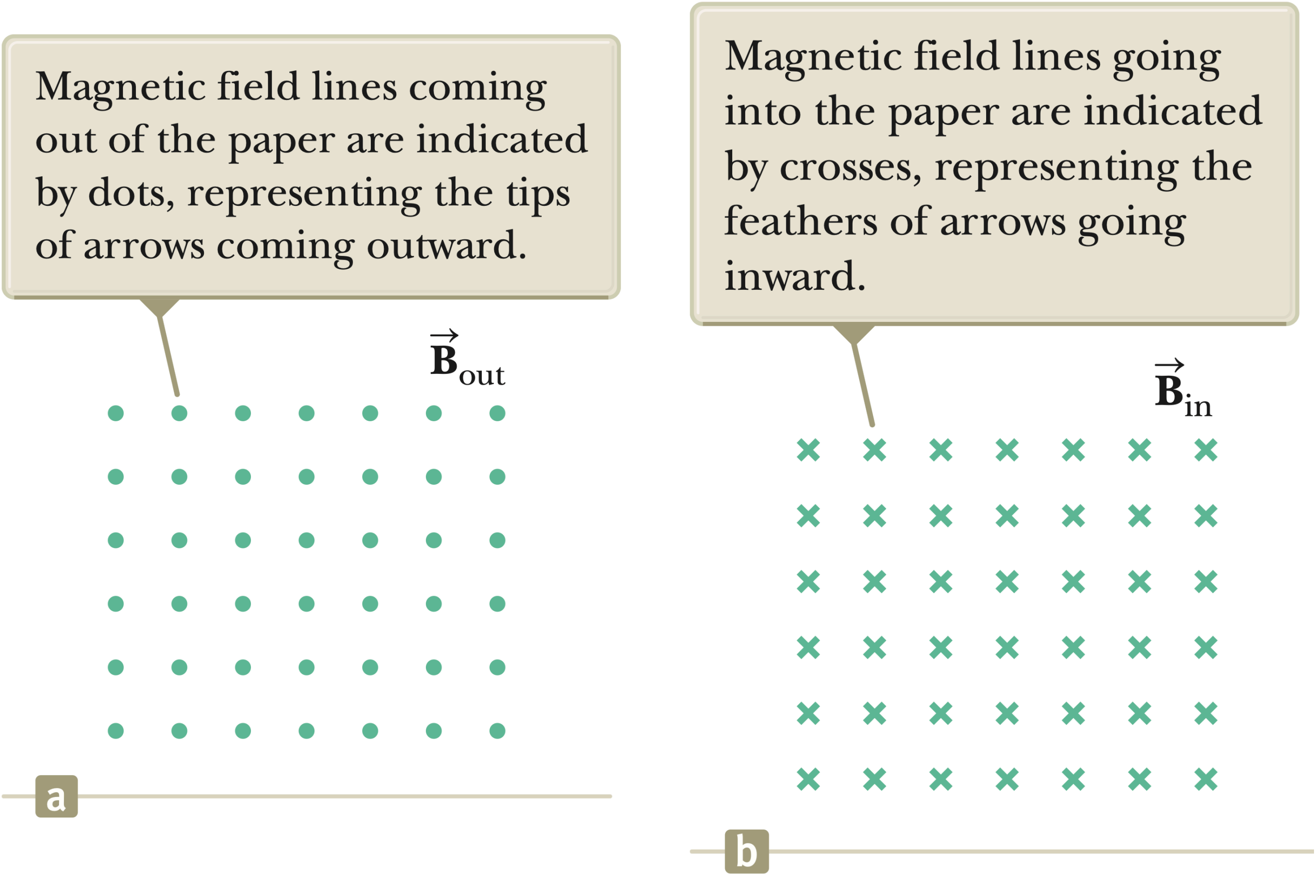
2.1 Representation of the Magnetic Field





2.2 Charged Particle in a Uniform Magnetic Field

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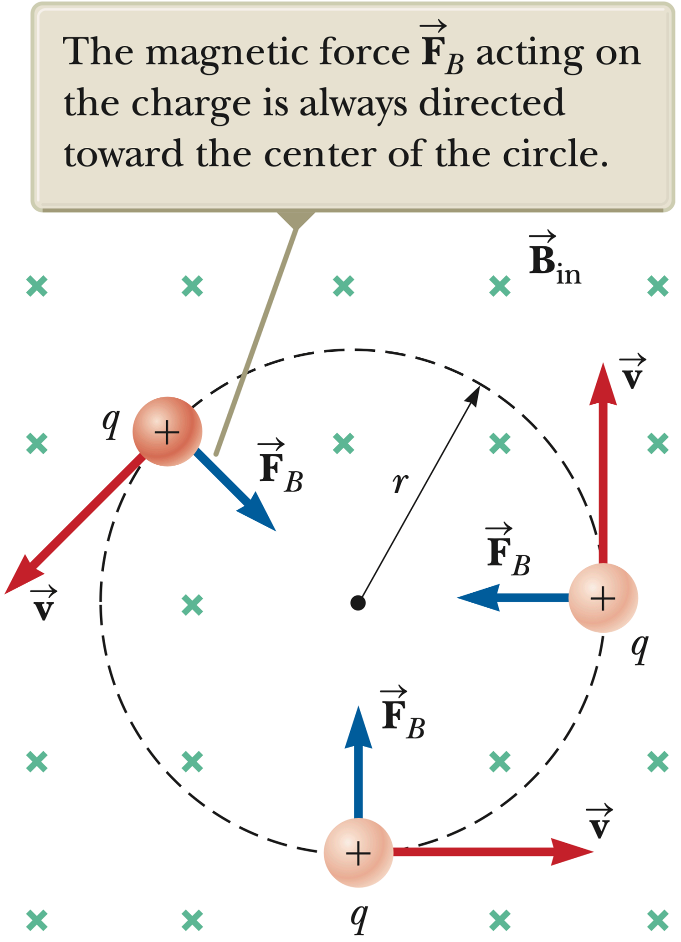
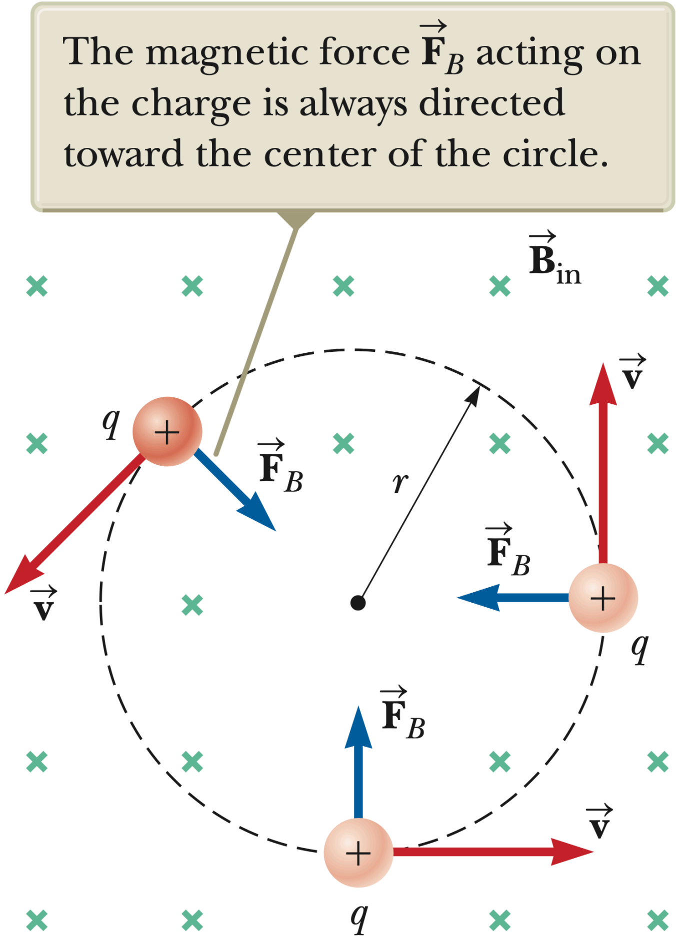


- If $\vec{v}$ of the particle is perpendicular to $\vec{B}$, it will move in a **circular path** with a radius (r) determined by the balance between $\vec{F}_B$ and the moment of inertia.
- If q were **negative**, the particle would move in the **opposite** direction, but the radius of the path would be the same.
- Using newton's second law, we find:

$$F_B = qvB = m\frac{v^2}{r}$$

2.2 Charged Particle in a Uniform Magnetic Field



- The radius of the circular path is:

$$r = \frac{mv}{qB}$$

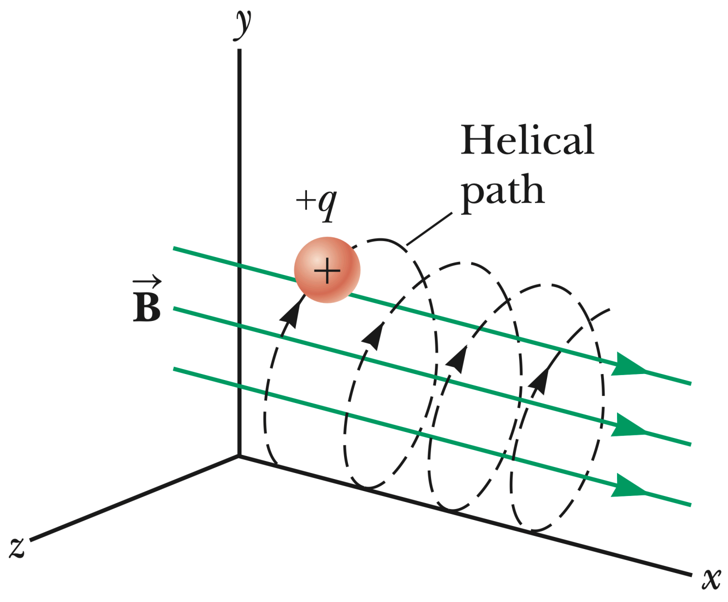
- The angular speed of the particle is:

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

- The period of the circular motion is:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

2.3 Helical Motion



- If $\vec{v}$ of the particle has a component parallel to $\vec{B}$, it will move in a **helical path**, with the same radius and angular speed as before, but with a *constant* velocity along the direction of $\vec{B}$,

$$a_x = 0$$

- The perpendicular component of the velocity $v_\perp$ is:

$$v_\perp = \sqrt{v_y^2 + v_z^2} = \text{constant}$$

2.3 Helical Motion

Example 2.2

A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35 T magnetic field perpendicular to the velocity of the proton. Find the speed of the proton.

Solution 2.2

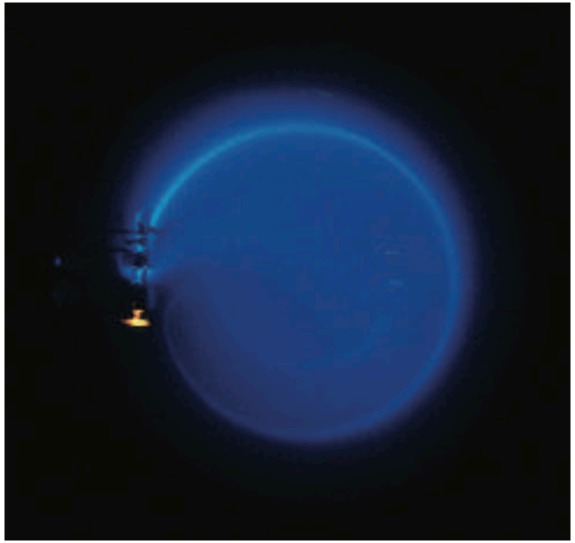
$$v = \frac{qBr}{m_p} = \frac{(1.6 \times 10^{-19})(0.35)(0.14)}{1.67 \times 10^{-27}} = 4.7 \times 10^{6} \text{ m/s}$$

2.3 Helical Motion

Example 2.3

In an experiment designed to measure the magnitude of a uniform magnetic field, electrons are accelerated from rest through a potential difference of 350 V and then enter a uniform magnetic field that is perpendicular to the velocity vector of the electrons. The electrons travel along a curved path because of the magnetic force exerted on them, and the radius of the path is measured to be 7.5 cm.

(A) What is the magnitude of the magnetic field?



2.3 Helical Motion

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Solution 2.3

First, we can find the speed of the electrons after being accelerated through the potential difference using the work-energy theorem:

$$\Delta K + \Delta U_E = 0$$

$$\left(\frac{1}{2}m_e v^2 - 0\right) + (q\Delta V) = 0$$

$$\Rightarrow v = \sqrt{\frac{-2q\Delta V}{m_e}} = 1.11 \times 10^7 \text{ m/s}$$

Now we can use the formula derived earlier to find the magnetic field:

$$B = \frac{m_e v}{er} = 8.4 \times 10^{-4} \text{ T}$$

2.3 Helical Motion

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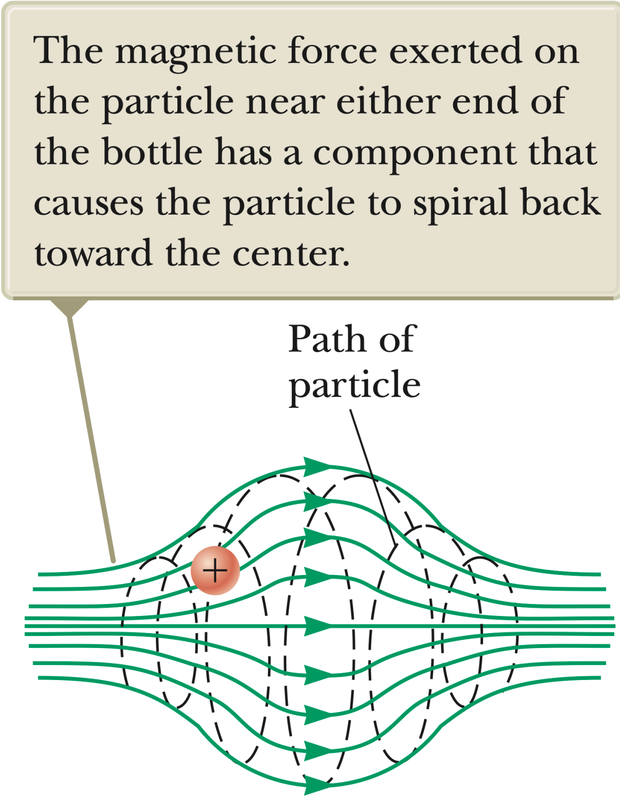
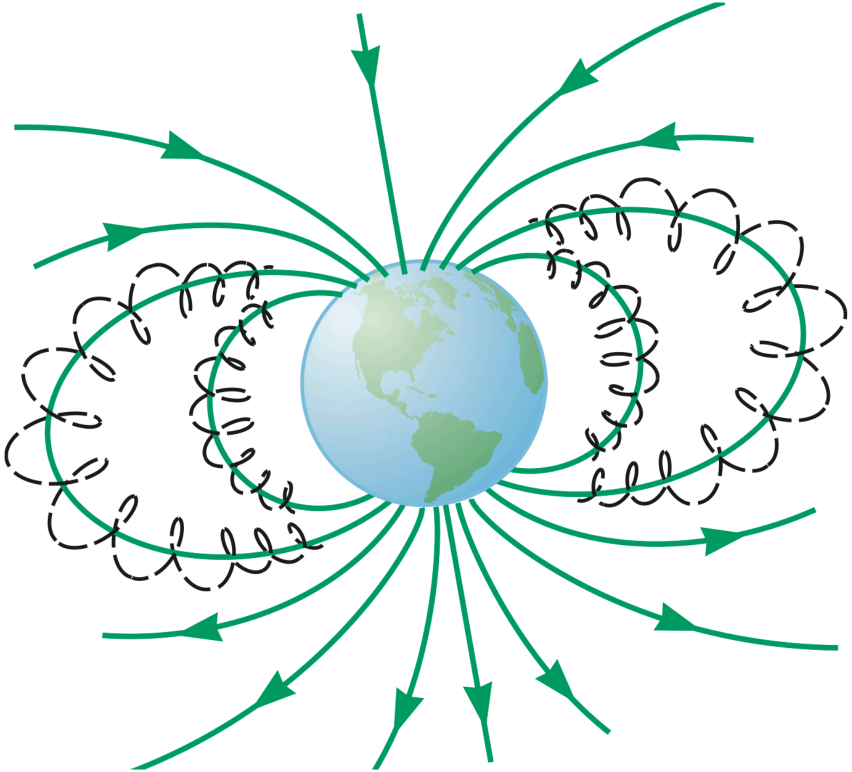
(B) What is the angular speed of the electrons?

$$\omega = \frac{v}{r} = 1.5 \times 10^8 \frac{\text{rad}}{s}$$

2.4 Magnetic Trap



Magnetic Bottle



Earth Magnetic Field

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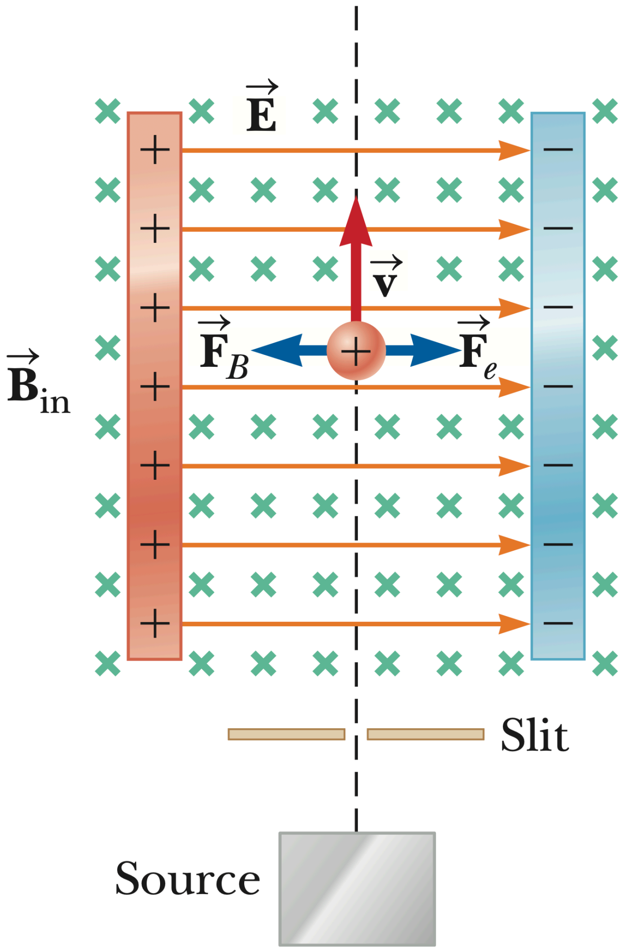
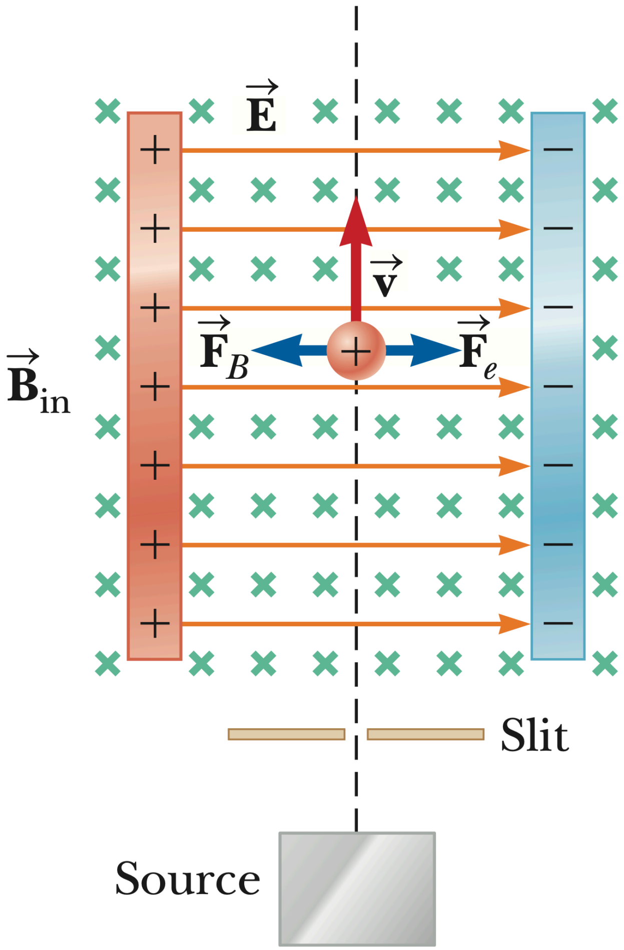
3.1 Total Electromagnetic Force (Lorentz Force)



When a charged particle moves in both electric and magnetic fields, it experiences a **total electromagnetic force** (also called the **Lorentz force**) that is the vector sum of the electric and magnetic forces:

$$\vec{F} = \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B}$$

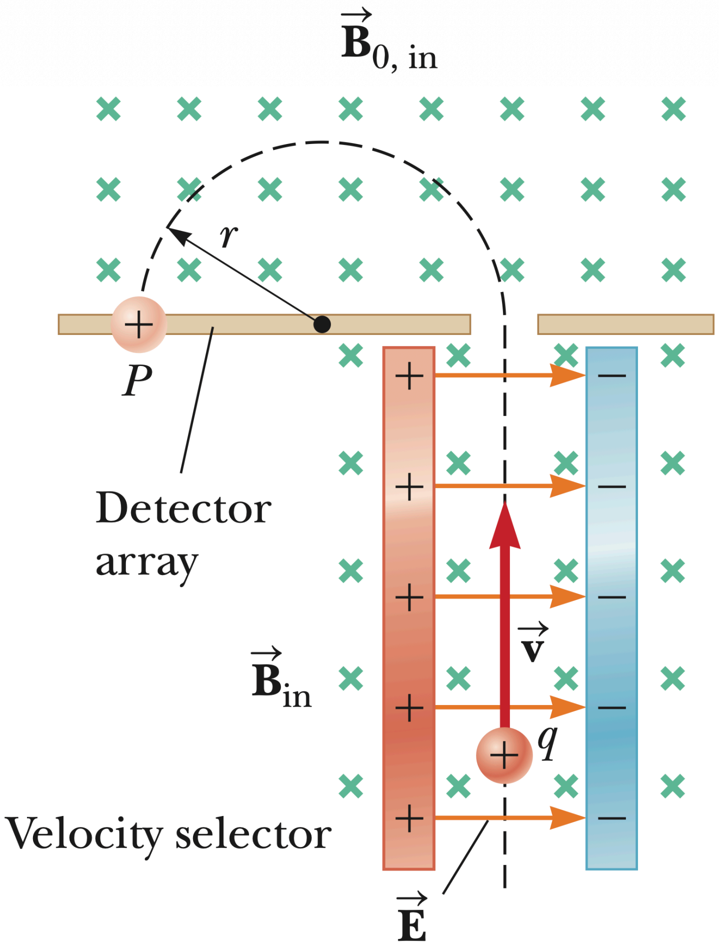
3.2 Application: Velocity Selector



- A **velocity selector** is a device that uses perpendicular electric and magnetic fields to select charged particles with a specific velocity.
- If the electric and magnetic forces are equal in magnitude and opposite in direction ($qE = qvB$), the particle will pass through the selector undeflected, which means it has a velocity of:

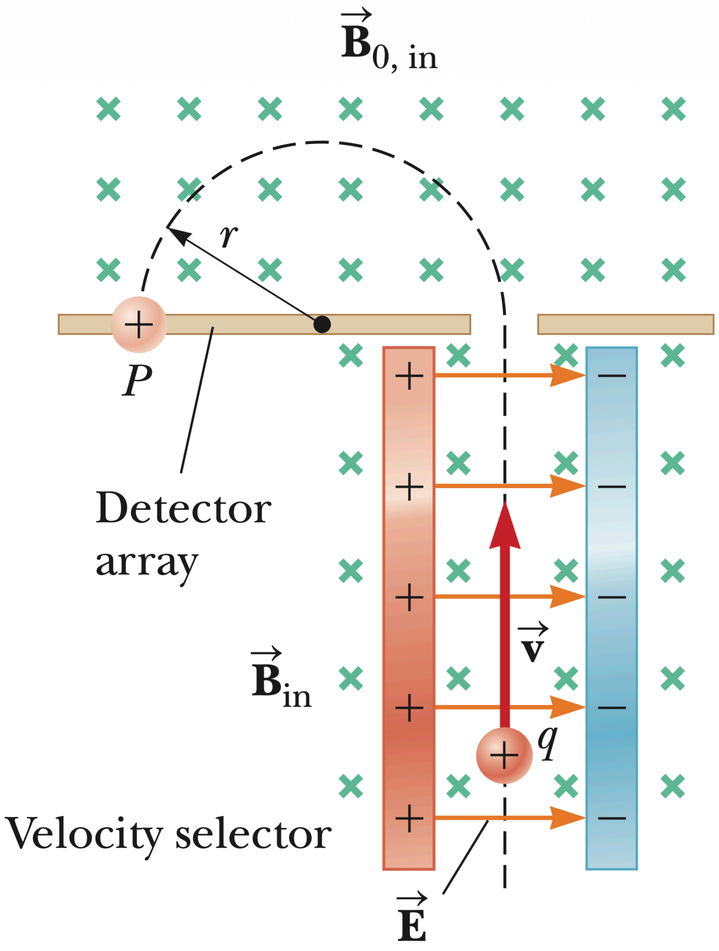
$$v = \frac{E}{B}$$

3.3 The Mass Spectrometer



- A **mass spectrometer** is a device that uses magnetic fields to separate charged particles of equal speed based on their mass-to-charge ratio.
- The particles enters the mass spectrometer at a known speed (from the velocity selector), and then they enter a magnetic field B_0 that causes them to move in circular paths with different radii depending on their mass-to-charge ratio.

3.3 The Mass Spectrometer



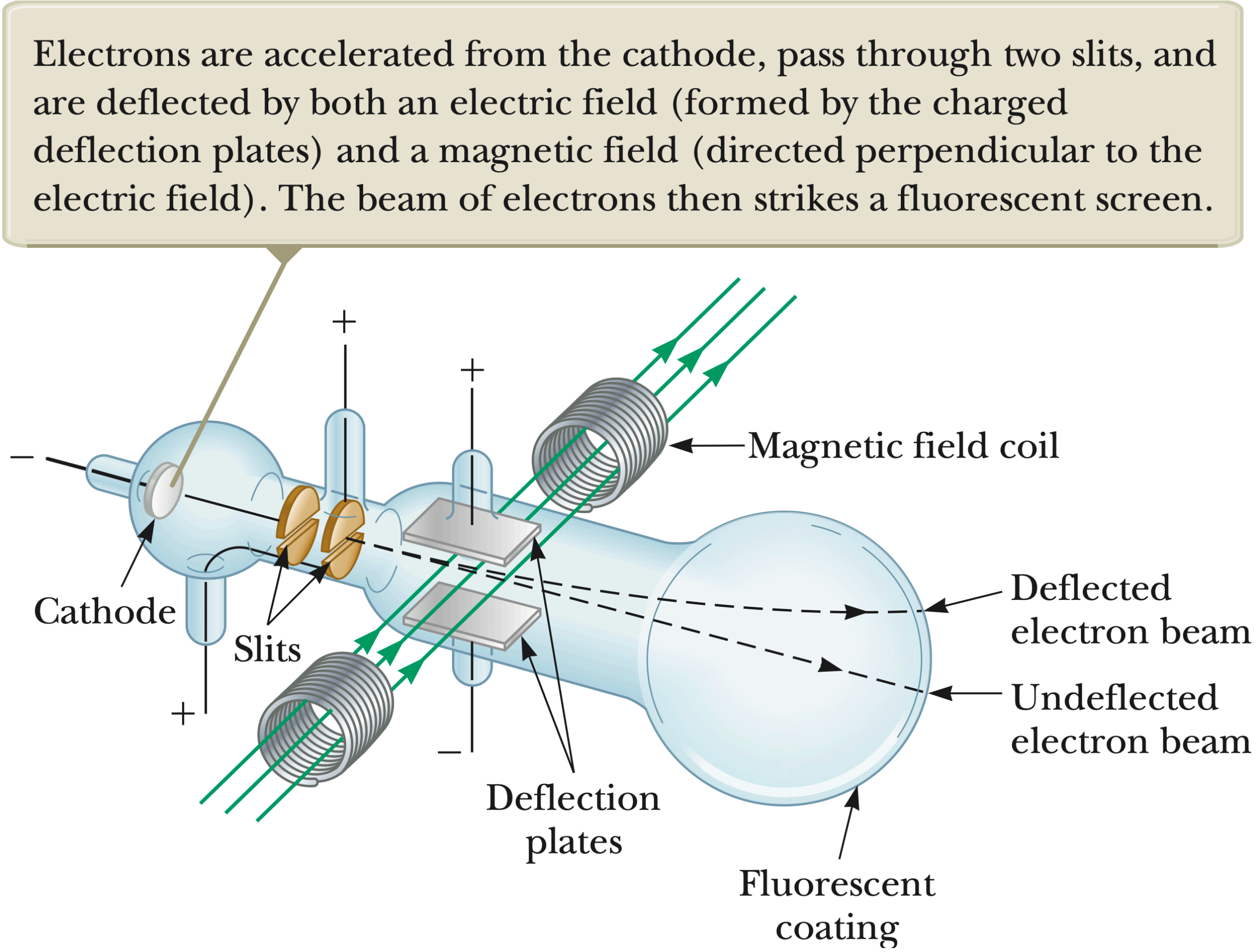
- By measuring the radius of the path, we can determine the mass-to-charge ratio of the particle:

$$\frac{m}{q} = \frac{B_0 r}{v}$$

- Since the velocity is known from the velocity selector as $v = E/B$, we can also express the mass-to-charge ratio as:

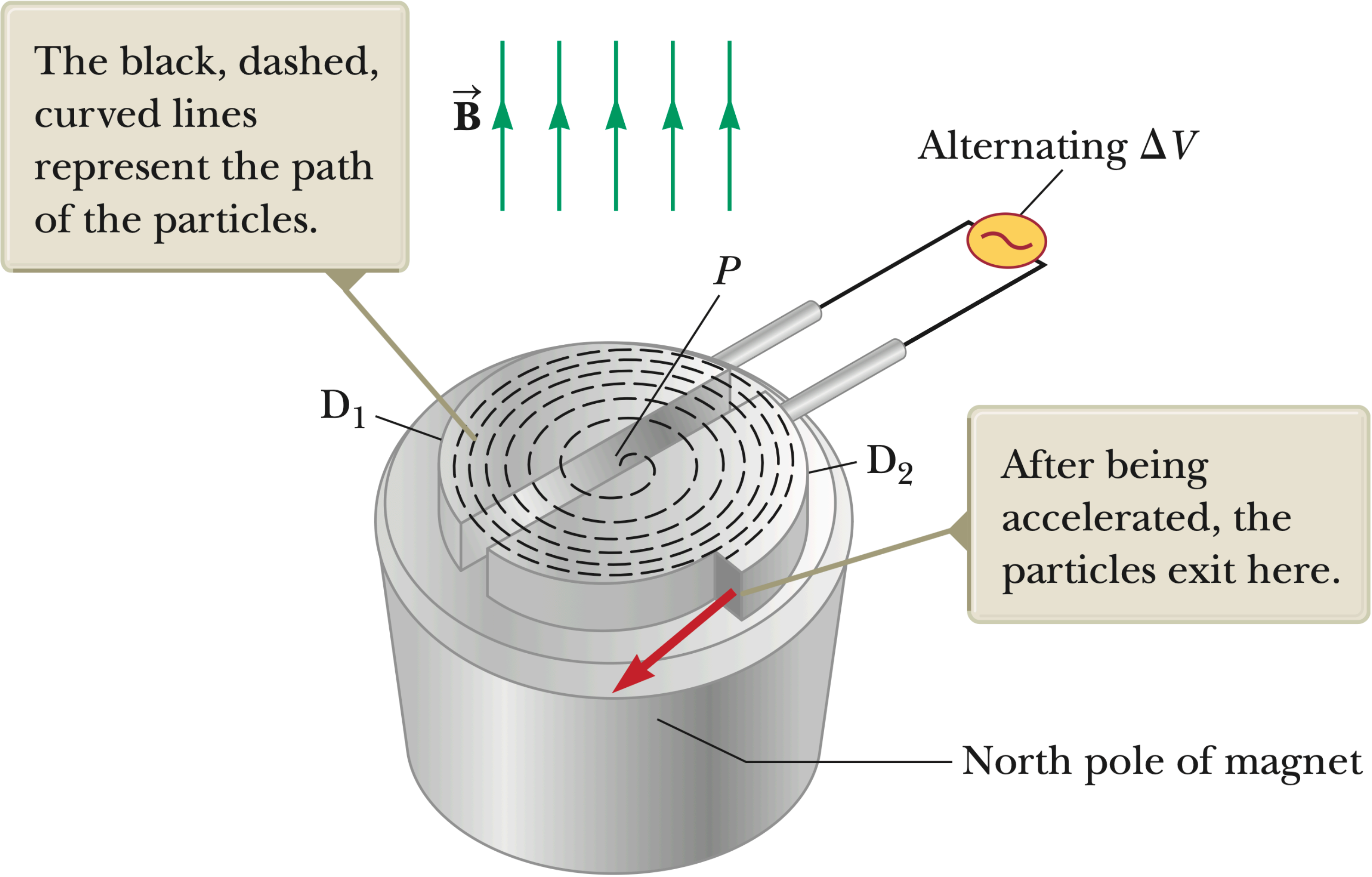
$$\frac{m}{q} = \frac{B_0 B}{E} r$$

3.4 Thomson's Apparatus



Thomson used a cathode ray tube with perpendicular electric and magnetic fields to measure the charge-to-mass ratio of the electron.

3.5 Application: The Cyclotron



3.5 Application: The Cyclotron

- A **cyclotron** is a type of particle accelerator that uses a combination of a constant magnetic field and an oscillating electric field to accelerate charged particles to high speeds.
- The particles are injected into the cyclotron at the center, and they move in a spiral path due to the magnetic field.
- Each time the particles cross the gap between the two "dees" (the D-shaped electrodes), they are accelerated by the electric field, which increases their speed and the radius of their path.

3.5 Application: The Cyclotron

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- The cyclotron can accelerate particles to high energies, which can be used for various applications such as medical treatments (e.g., cancer therapy) and nuclear physics research.
- The maximum kinetic energy that a particle can achieve in a cyclotron is given by:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{qBr}{m}\right)^2 = \frac{q^2B^2r^2}{2m}$$

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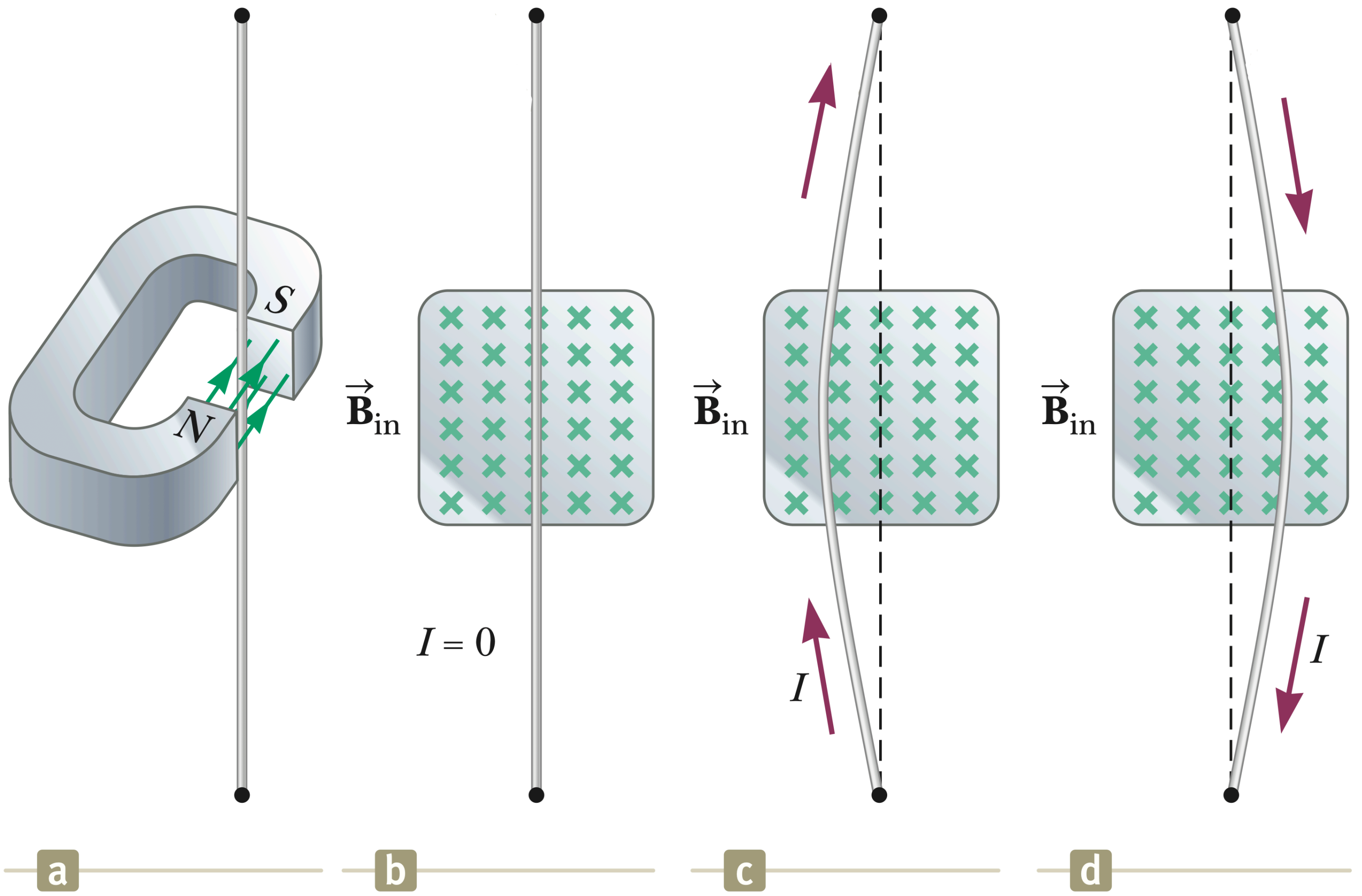
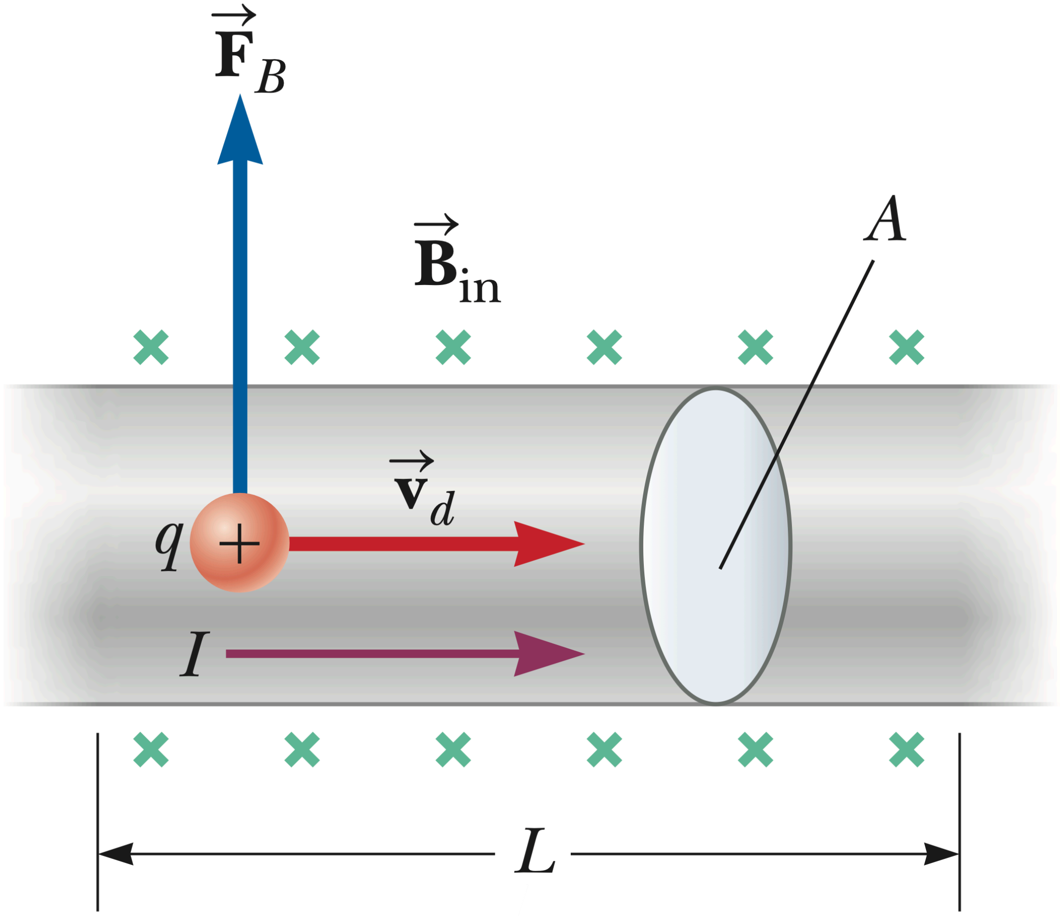
4.1 Magnetic Force on a Current-Carrying Wire



4.1 Magnetic Force on a Current-Carrying Wire



4.1 Magnetic Force on a Current-Carrying Wire

- When a current-carrying conductor is placed in a magnetic field, it experiences a **magnetic force** that is proportional to the current (I), the length of the conductor (L), the magnetic field strength (B), and the sine of the angle (θ) between the conductor and the magnetic field:

$$\vec{\boldsymbol{F}}_B = I\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}} = ILB\sin\theta$$

4.1 Magnetic Force on a Current-Carrying Wire

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Example 4.4

A wire carries a steady current of 2.4 A. A straight section of the wire is 0.75 m long and lies along the x-axis within a uniform magnetic field, $\vec{B} = 1.6\hat{k}$ T. If the current is in the positive x direction, what is the magnetic force on the section of wire?

Solution 4.4

$$\vec{F}_B = I\vec{L} \times \vec{B}$$

$$= (2.4)(0.75\ \hat{\imath})\left(1.6\ \hat{k}\right) = 2.88\ \left(\hat{\imath} \times \hat{k}\right)$$

The cross product of $\hat{\imath}$ and $\hat{k}$ is $-\hat{\jmath}$, so:

$$\vec{F}_B = -2.88\ \hat{\jmath}\ \ \text{N}$$

4.2 Cross Product

The cross product cycles of the unit vectors is:

$$\hat{\imath} \rightarrow \hat{\jmath} \rightarrow \hat{k}$$

A positive cross product follows the cycle, while a negative cross product goes in the opposite direction. So:

$$\hat{\imath} \times \hat{\jmath} = \hat{k}$$

$$\hat{\jmath} \times \hat{k} = \hat{\imath}$$

$$\hat{k} \times \hat{\imath} = \hat{\jmath}$$

$$\hat{\jmath} \times \hat{\imath} = -\hat{k}$$

$$\hat{k} \times \hat{\jmath} = -\hat{\imath}$$

$$\hat{\imath} \times \hat{k} = -\hat{\jmath}$$

Suggested Problems

3, 4, 9, 16, 21, 25

Book: Serway, R. A., & Jewett, J. W. (2018). Physics for Scientists and Engineers (10th ed.)

Chapter: 28 - Magnetic Fields