# Ch.27: Direct-Current Circuits

## Physics 104: Electricity and Magnetism

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Ref.: Serway, R. A., & Jewett, J. W. (2018). Physics for Scientists and Engineers (10th ed.)

# Outline

# Remember From Previous Chapters

## Classical Mechanics

- Equations of motion:

$$\vec{\boldsymbol{v}}_f = \vec{\boldsymbol{v}}_i + \vec{\boldsymbol{a}}t$$

$$\vec{\boldsymbol{r}}_f = \vec{\boldsymbol{r}}_i + \vec{\boldsymbol{v}}_i t + \frac{1}{2}\vec{\boldsymbol{a}}t^2$$

$$\vec{\boldsymbol{v}}_f^2 = \vec{\boldsymbol{v}}_i^2 + 2\vec{\boldsymbol{a}} \cdot \left(\vec{\boldsymbol{r}}_f - \vec{\boldsymbol{r}}_i\right)$$

- Newton's Second Law:

$$\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$$

- Work-Energy Theorem:

$$W = \Delta K$$

$$W = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{d}}$$

$$K = \frac{1}{2}m\vec{\boldsymbol{v}}^2$$

## Electric Field

- Coulomb's Law:

$$\vec{\boldsymbol{F}}_e = k_e \frac{q_1 q_2}{r^2}\hat{\boldsymbol{r}}$$

- Electric Field:

$$\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}_e}{q_0} = k_e \frac{q}{r^2}\hat{\boldsymbol{r}}$$

$$\vec{\boldsymbol{a}} = \left(\frac{q}{m}\right)\vec{\boldsymbol{E}}$$

## Flux

- Gauss's Law:

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot \mathrm{d}\vec{\boldsymbol{A}} = \frac{q_{\text{in}}}{\varepsilon_0}$$

# Remember From Previous Chapters

## Electric Potential and Energy

- Electric Potential:

$$V = k_e \frac{q}{r}$$

- Potential Energy:

$$U_E = k_e \frac{q_1 q_2}{r}$$

- Relation to Electric Field:

$$\Delta V = -\vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{d}}$$

- Potential and Energy:

$$\Delta U_E = q \Delta V$$

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## Capacitance and Dielectrics

- Capacitance:

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$

- Capacitors in Parallel:

$$C_{\text{eq}} = \sum C_i$$

- Capacitors in Series:

$$\frac{1}{C_{\text{eq}}} = \sum \left( \frac{1}{C_i} \right)$$

- Energy Stored in Capacitor:

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V$$
$$= \frac{1}{2} C (\Delta V)^2$$

# Remember From Previous Chapters

- Energy Density of Electric Field:

$$u_E = \frac{U_E}{V} = \frac{1}{2}\varepsilon_0 E^2$$

- Dielectric Constant:

$$\Delta V = \Delta V_0/\kappa$$

$$C = \kappa C_0$$

$$Q = \kappa Q_0$$

$$U_E = U_0/\kappa$$

## Current and Resistance

- Current:

$$I = \frac{\Delta Q}{\Delta t}$$

$$I_{\mathsf{avg}} = nAv_d q$$

- Ohm's Relation:

$$\Delta V = IR$$

- Resistance:

$$R = \rho\frac{L}{A}$$

- conductivity:

$$\sigma = \frac{1}{\rho}$$

- Temperature Effect

$$R = R_0[1 + \alpha(T - T_0)]$$

- Electrical Power:

$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

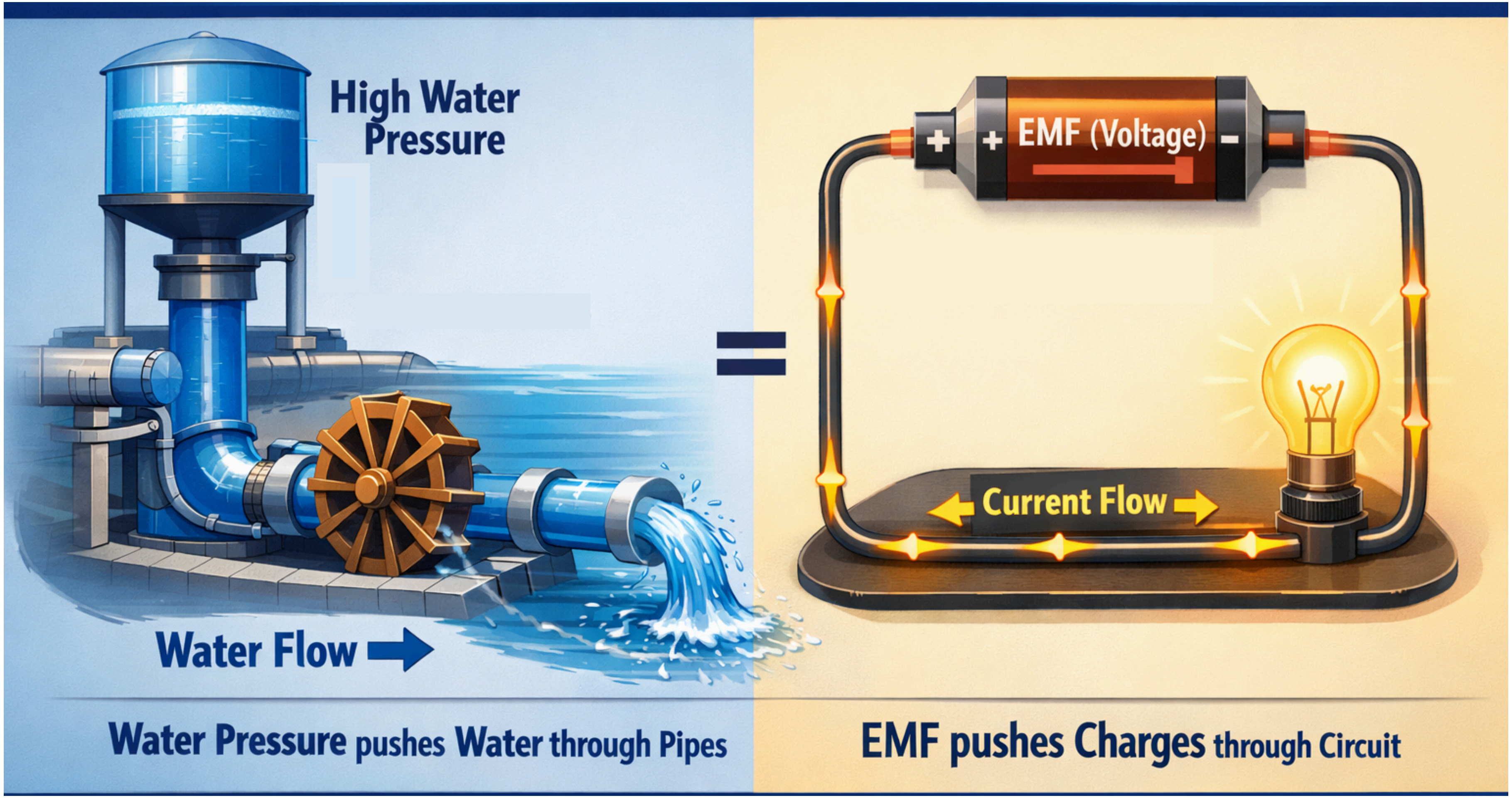
$$\text{Energy} = P\Delta t$$

# 1. Electromotive Force

# 2. Resistors in Series and Parallel

# 3. Kirchhoff’s Rules

# 1.1 Why using Batteries

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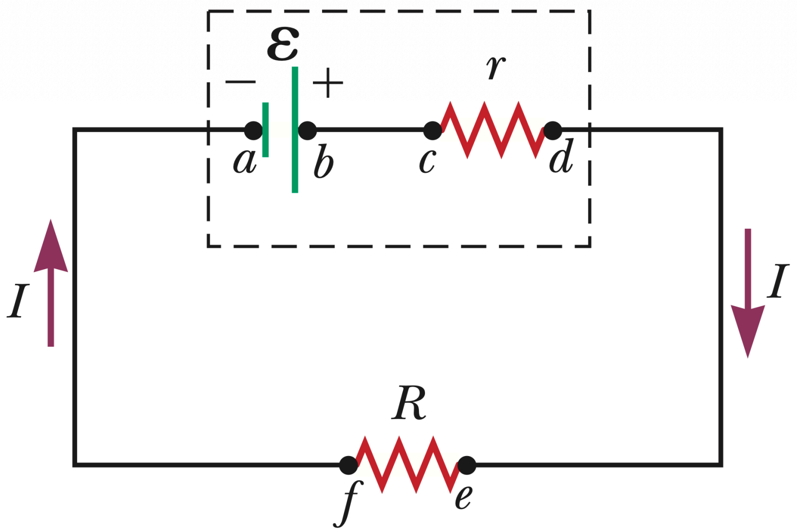
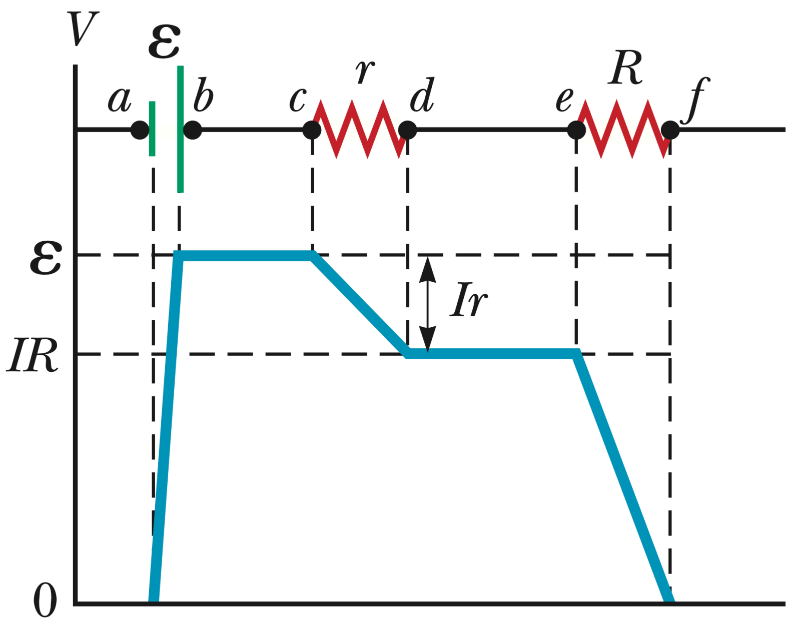
# 1.2 Electromotive Force of a Battery



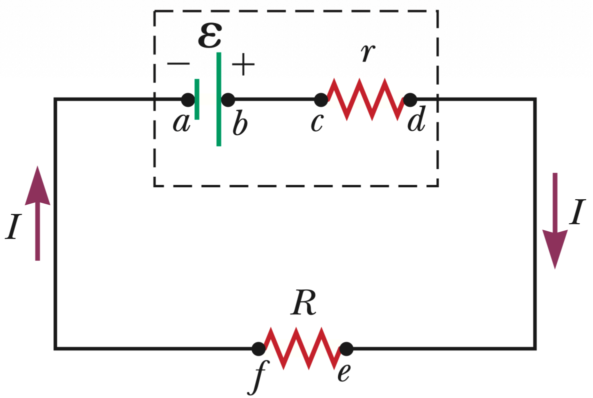

- A **battery** is a device that converts chemical energy into electrical energy.
- The chemical reactions create a *potential difference* known as **electromotive force (emf)** that drives the flow of electric charge when the circuit is closed.
- The emf of a battery is denoted by $\varepsilon$ and is measured in volts (V).
- The emf represents the maximum potential difference the battery can provide when no current is flowing (open circuit condition).

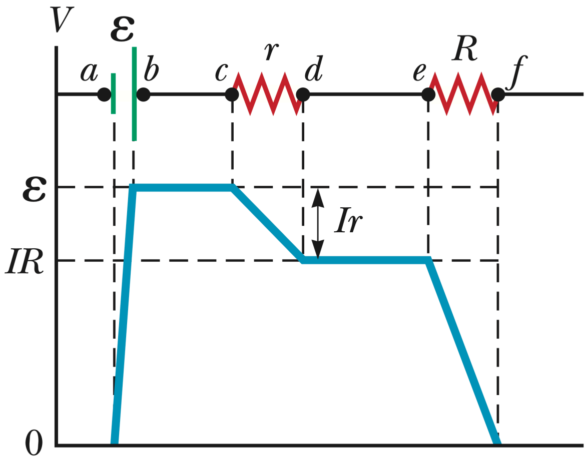
# 1.2 Electromotive Force of a Battery




- You can think of the emf as the "pressure" that pushes the electric charge through the circuit, similar to how water pressure pushes water through pipes.
- The current that flows through the battery-circuit is constant in magnitude and flow-direction and therefore is called **direct current (DC)**.

# 1.3 Internal Resistance of a Battery




- In reality, a battery is not ideal and has an **internal resistance** denoted by $r$.
- The internal resistance arises from the materials and chemical processes inside the battery that impede the flow of electric charge.
- When a current flows through the battery, some of the energy is lost as heat due to the internal resistance, which reduces the **terminal voltage** $\Delta V$ to the external circuit.

## 1.3 Internal Resistance of a Battery

- The terminal voltage $\Delta V$ of the battery when a current $I$ is flowing can be calculated using the formula:

$$\boxed{\Delta V = \varepsilon - Ir = IR,}$$

where $R$ is known as the **load resistance** connected to the battery.

- This equation shows that the terminal voltage decreases as the current increases due to the voltage drop across the internal resistance.
- We can rearrange the equation to find the current in the circuit:

$$I = \frac{\varepsilon}{R + r}.$$

## 1.4 Power Delivered by a Battery

- The power delivered by the battery to the external circuit can be calculated using the formula:

$$P = I\varepsilon = I^2R + I^2r = \text{Load Power} + \text{Internal Power}$$

- The power delivered to the load resistor is given by $P_R = I^2R$, while the power lost due to the internal resistance is given by $P_r = I^2r$.

## 1.5 Example

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**Example 1.1**

A battery has an emf of 12 V and an internal resistance of 0.05Ω. Its terminals are connected to a load resistance of 3Ω.
(A) Find the current in the circuit and the terminal voltage of the battery.

**Solution 1.1**

$$I = \frac{\varepsilon}{R+r} = \frac{12}{3+0.05} = 3.93 \text{ A}$$

$$\Delta V = \varepsilon - Ir = 12 - (3.93)(0.05) = 11.8 \text{ V}$$

we can also calculate the terminal voltage using:

$$\Delta V = IR = (3.93)(3) = 11.8 \text{ V}$$

# 1.5 Example

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(B) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

**Solution 1.1**

$$P_R = I^2 R = (3.93)^2 (3) = 46.4 \text{ W}$$

$$P_r = I^2 r = (3.93)^2 (0.05) = 0.77 \text{ W}$$

$$P = P_R + P_r = 47.1 \text{ W}$$

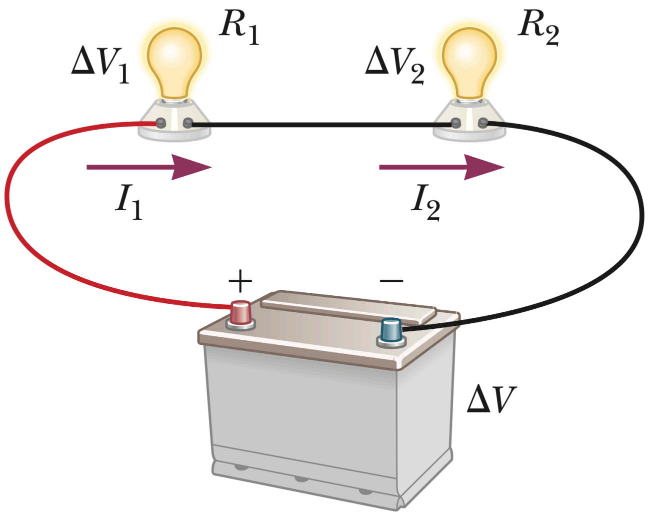
1. Electromotive Force

## 2. Resistors in Series and Parallel

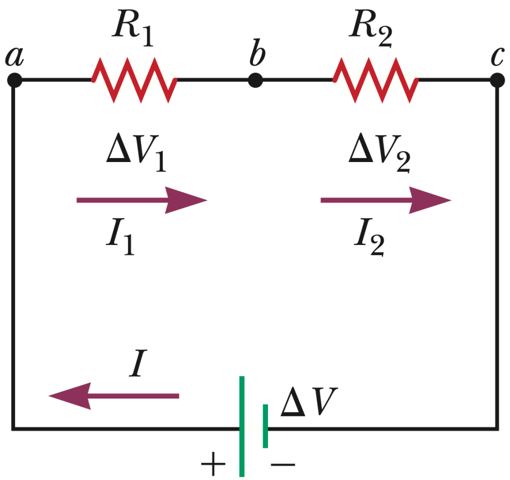
3. Kirchhoff’s Rules

# 2.1 Resistors in Series

A pictorial representation of two resistors connected in series to a battery

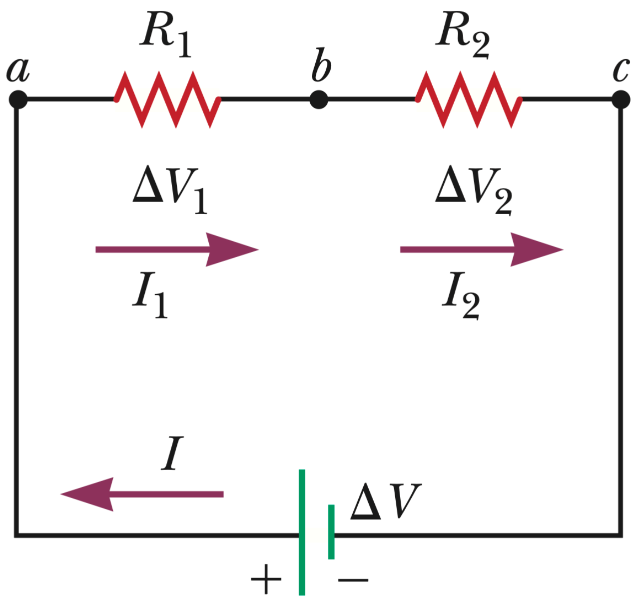



A circuit diagram showing the two resistors connected in series to a battery




When resistors are connected end-to-end in a single path for the current to flow, they are said to be in **series**.

# 2.1 Resistors in Series




- The current $I$ flowing through each resistor in series is **constant**

$$I = I_1 = I_2 = I_3 = \ldots = I_n$$

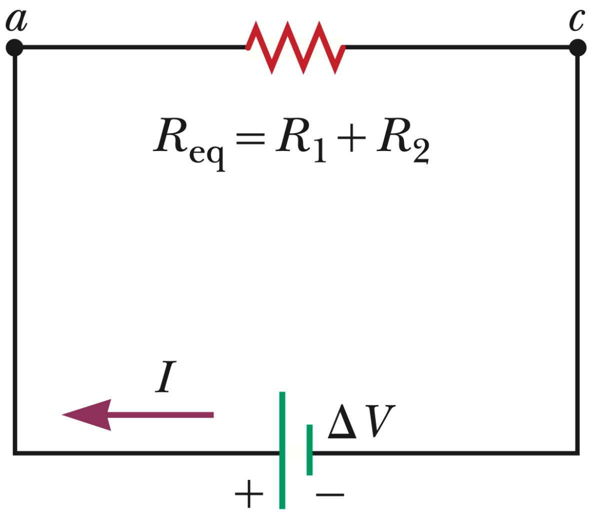
- The total voltage drop across the resistors is the sum of the individual voltage drops:

$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots + \Delta V_n$$

- Using Ohm's relation, we express the voltages in terms of current and resistances:

$$IR_1 + IR_2 + \ldots + IR_n = I(R_1 + R_2 + \ldots + R_n)$$

## 2.1 Resistors in Series



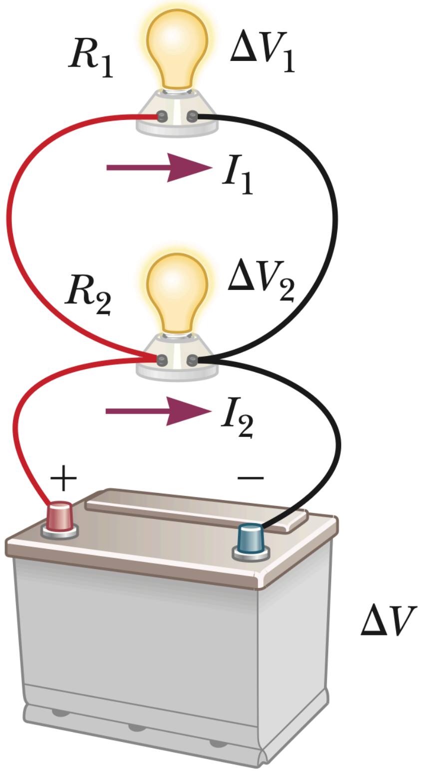

- Therefore the **equivalent resistance** $R_{eq}$ of resistors in **series** is the sum of their individual resistances:

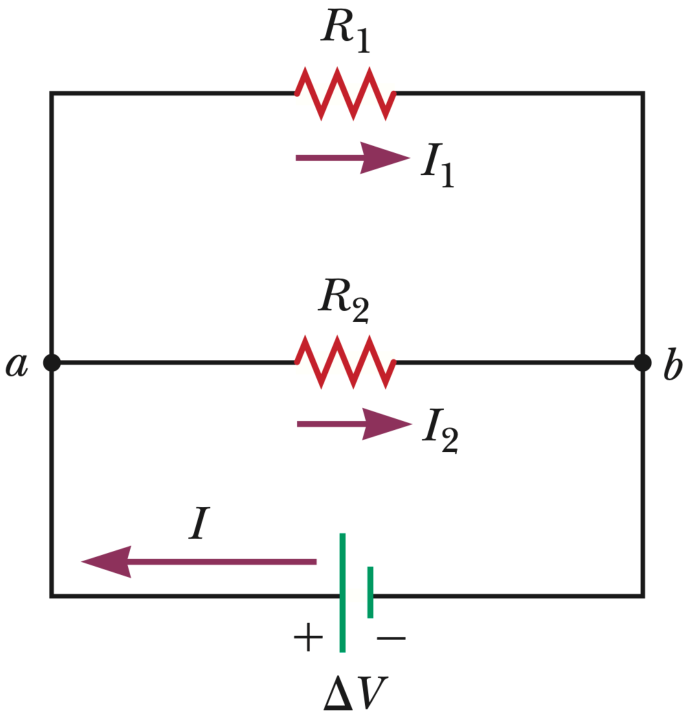
$$R_{eq} = R_1 + R_2 + R_3 + \ldots + R_n$$

- Recall that the voltage drops after each resistor $R_i$, and it is given by Ohm's relation as:

$$\Delta V_i = I R_i, \qquad \text{where } i = 1, 2, 3, \ldots, n$$

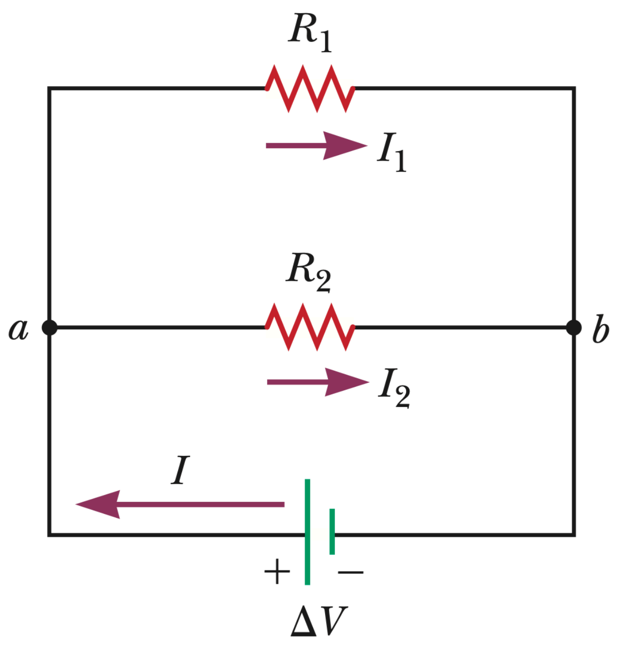
# 2.2 Resistors in Parallel




When resistors are connected such that they share both their start and end points, they are said to be in **parallel**.

## 2.2 Resistors in Parallel




- The voltage drop $\Delta V$ across each resistor in parallel is **constant**:

$$\Delta V = \Delta V_1 = \Delta V_2 = \ldots = \Delta V_n$$

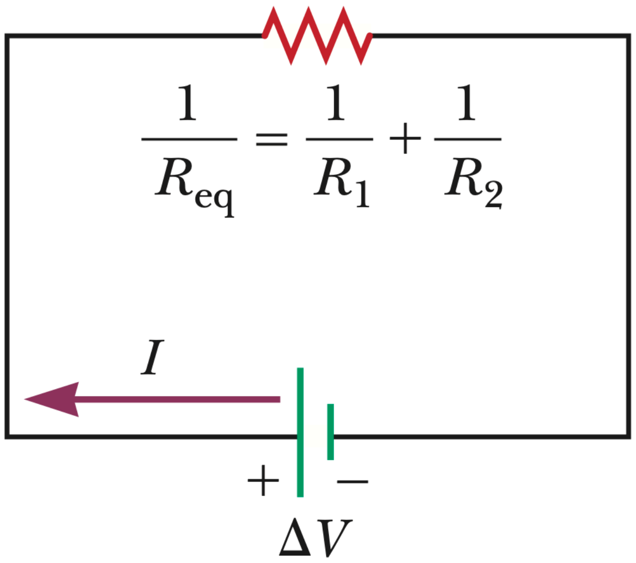
- The total current $I$ flowing into the parallel combination is the sum of the currents through each resistor:

$$I = I_1 + I_2 + \ldots + I_n .$$

- Using Ohm's relation, we express the currents in terms of voltage and resistances:

$$\frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} + \ldots + \frac{\Delta V}{R_n} = \Delta V \left( \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \right) = \frac{\Delta V}{R_{eq}}$$

# 2.2 Resistors in Parallel




- Therefore the **equivalent resistance** $R_{\text{eq}}$ of resistors in **parallel** is given by:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}$$

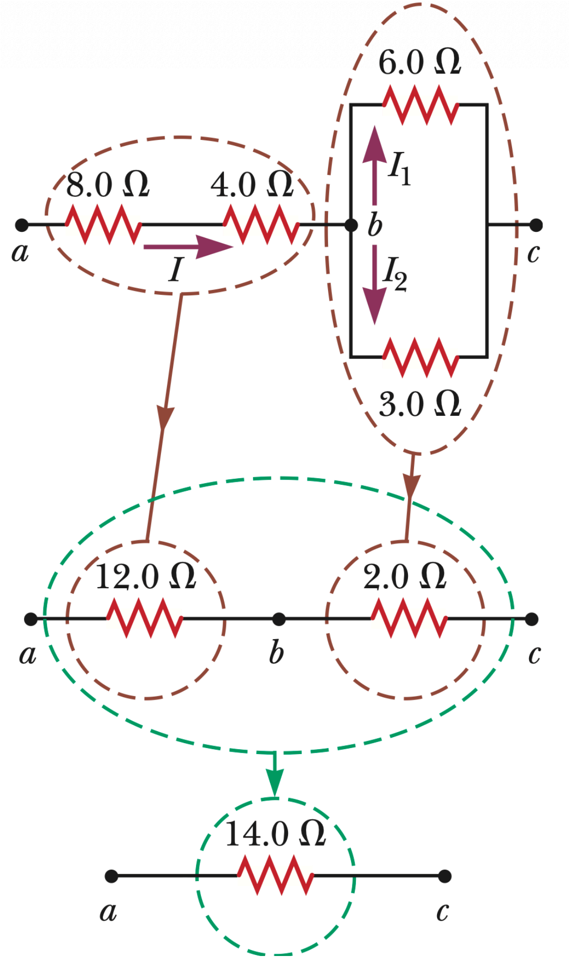
- The equivalent resistance of resistors in parallel is always less than the smallest individual resistance, which means that adding more resistors in parallel decreases the overall resistance of the circuit.

# 2.3 Example

**Example 2.2**

Four resistors are connected as shown in the Figure.

(A) Find the equivalent resistance between points a and c.



# 2.3 Example

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## Solution 2.2

First, we find the equivalent resistance of the two resistors in series:

$$R_{\text{eq}} = 8\ \Omega + 4\ \Omega = 12\ \Omega$$

Next, we find the equivalent resistance of the parallel combination:

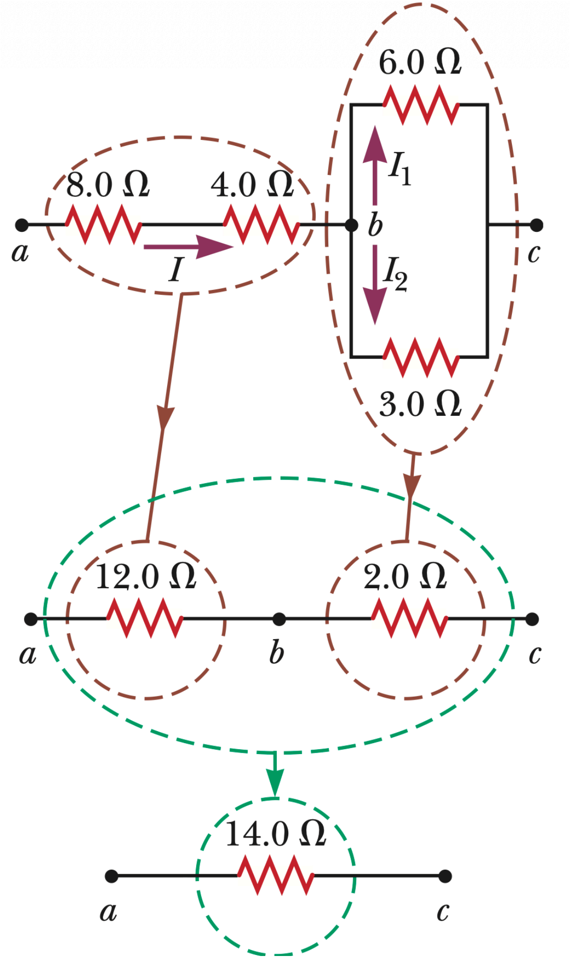
$$\frac{1}{R_{\text{eq}}} = \frac{1}{6} + \frac{1}{3} = \frac{3}{6\ \Omega} \implies R_{\text{eq}} = 2\ \Omega$$

Finally, we find the total equivalent resistance between points a and c:

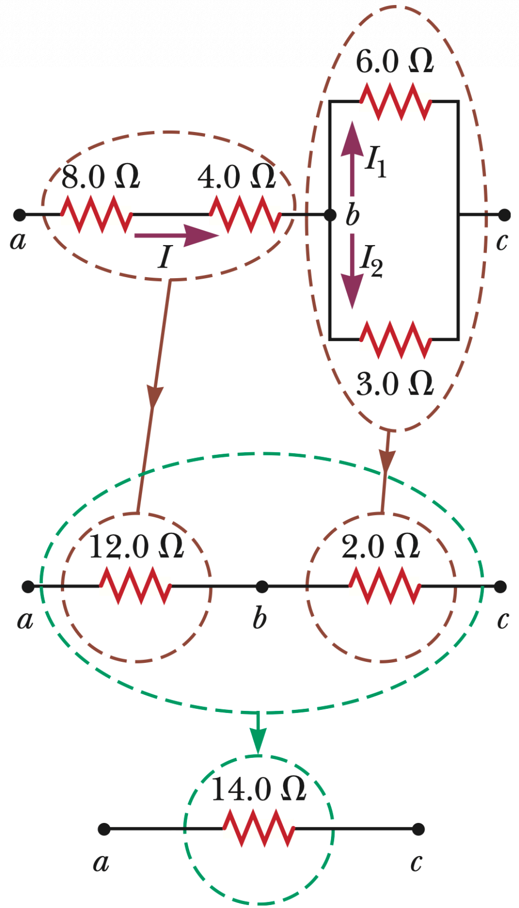
$$R_{\text{eq}} = 12\ \Omega + 2\ \Omega = 14\ \Omega$$

# 2.3 Example

(B) What is the current in each resistor if a potential difference of 42 V is maintained between a and c?



# 2.3 Example



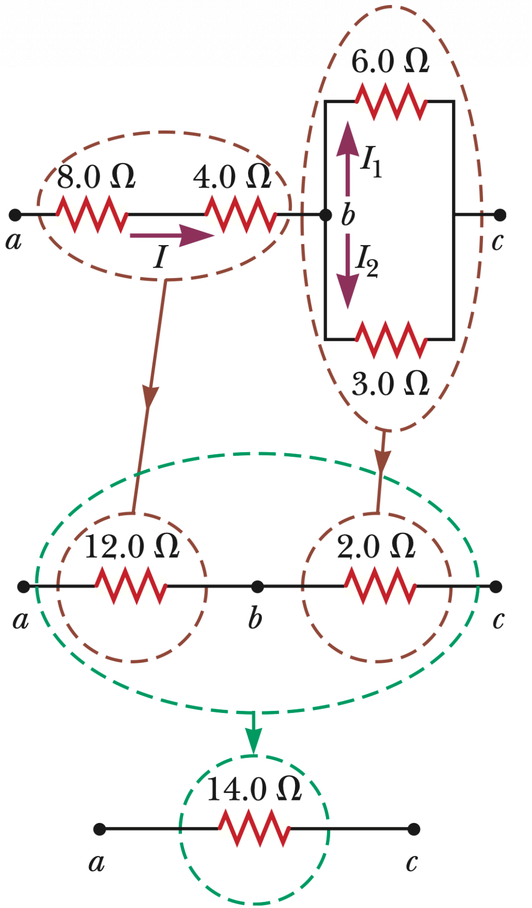

To find the current in each resistor, we start from the final equivalent resistance and the total voltage:

$$I = \frac{\Delta V}{R_{eq}} = \frac{42 \text{ V}}{14 \ \Omega} = 3 \text{ A}$$

Now that we know the total current, we can find the voltage across the parallel combination b to c:

$$\Delta V_{bc} = I R_{eq} = (3 \text{ A})(2 \ \Omega) = 6 \text{ V}$$

# 2.3 Example




Finally, the current through each resistor can be found using Ohm's law:

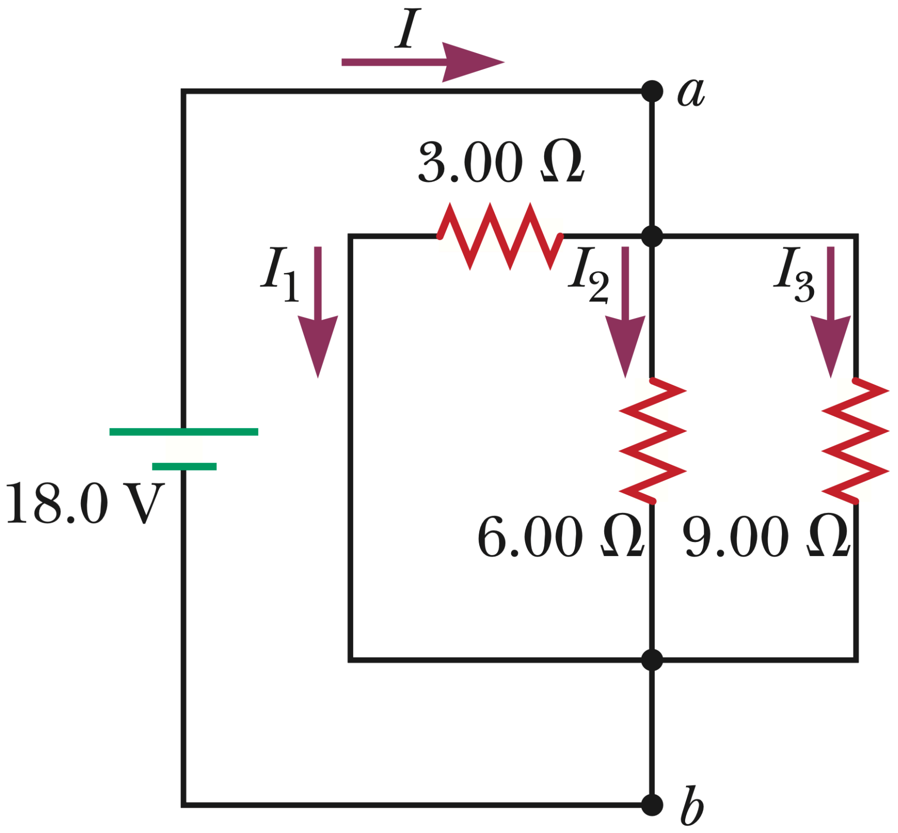
$$I_1 = \frac{\Delta V_{bc}}{R_1} = \frac{6\ \text{V}}{6\ \Omega} = 1\ \text{A}$$

$$I_2 = \frac{\Delta V_{bc}}{R_2} = \frac{6\ \text{V}}{3\ \Omega} = 2\ \text{A}$$

# 2.3 Example

**Example 2.3**

Three resistors are connected as shown in the Figure. A potential difference of 18 V is maintained between points a and b. (A) Calculate the equivalent resistance of the circuit.



# 2.3 Example

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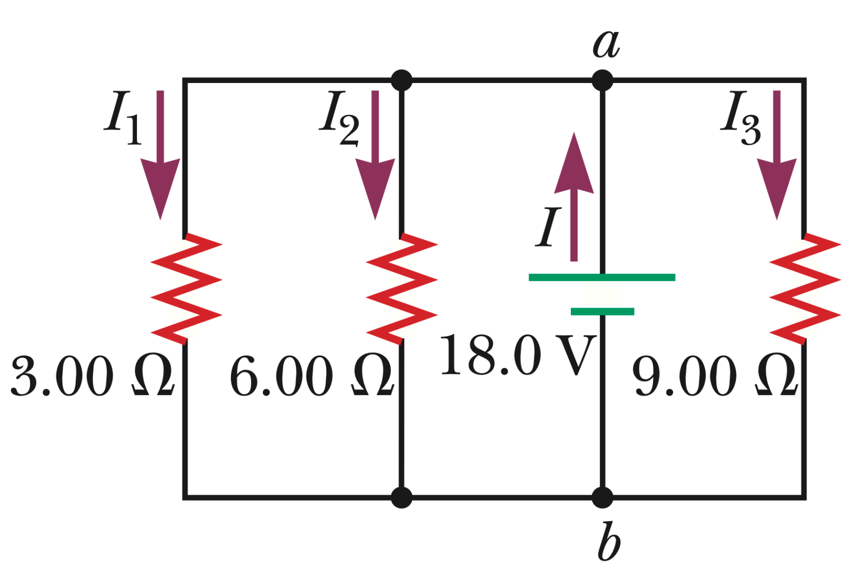
## Solution 2.3




$$\frac{1}{R_{\text{eq}}} = \frac{1}{3} + \frac{1}{6} + \frac{1}{9} = \frac{11}{18}\ \Omega^{-1} \implies R_{\text{eq}} = 1.64\ \Omega$$

## 2.3 Example

(B) Find the current in each resistor.

$$I_1 = \frac{\Delta V}{R_1} = \frac{18\ \text{V}}{3\ \Omega} = 6\ \text{A}$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18\ \text{V}}{6\ \Omega} = 3\ \text{A}$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18\ \text{V}}{9\ \Omega} = 2\ \text{A}$$

# 2.3 Example

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(C) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.

$$P_1 = I_1^2 R_1 = (6\ \text{A})^2 (3\ \Omega) = 108\ \text{W}$$
$$P_2 = I_2^2 R_2 = (3\ \text{A})^2 (6\ \Omega) = 54\ \text{W}$$
$$P_3 = I_3^2 R_3 = (2\ \text{A})^2 (9\ \Omega) = 36\ \text{W}$$

$$P = P_1 + P_2 + P_3 = 198\ \text{W}$$

Alternatively, we can calculate the total power using the equivalent resistance:

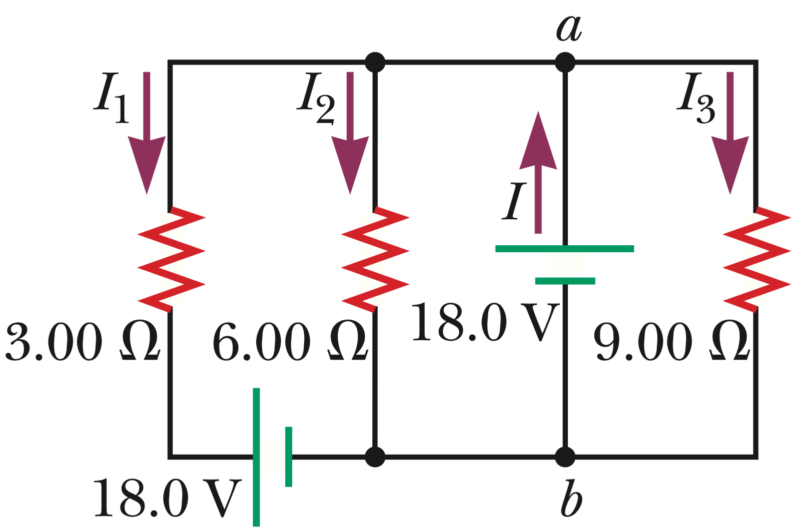
$$P = \frac{(\Delta V)^2}{R_{\text{eq}}} = \frac{(18\ \text{V})^2}{1.64\ \Omega} = 198\ \text{W}$$

1. Electromotive Force

2. Resistors in Series and Parallel

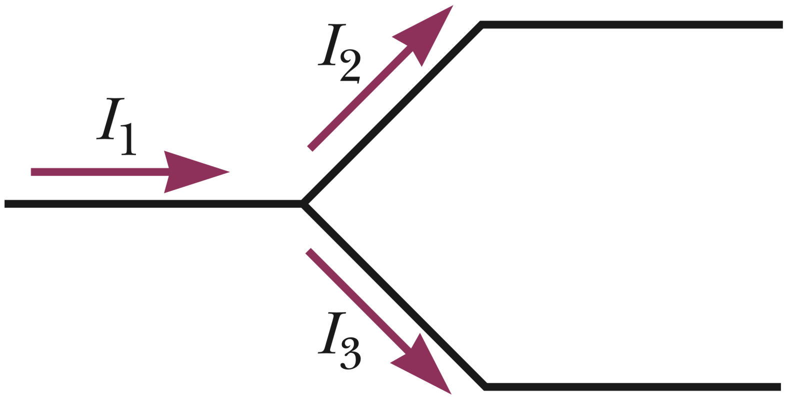
3. Kirchhoff's Rules

# 3.1 How to Simplify Complex Circuits when having multiple batteries and resistors?




- For simple circuits with only one battery and a few resistors, we can use the series and parallel rules to find the equivalent resistance and current.
- However, for more complex circuits with multiple batteries and resistors, we need a different systematic method to analyze the circuit.
- This is where **Kirchhoff's rules** come into play to help us solve for the unknown currents and voltages in such complex circuits.

# 3.2 Kirchhoff's Current Junction Rule




$$I_1 \longrightarrow I_2 + I_3$$

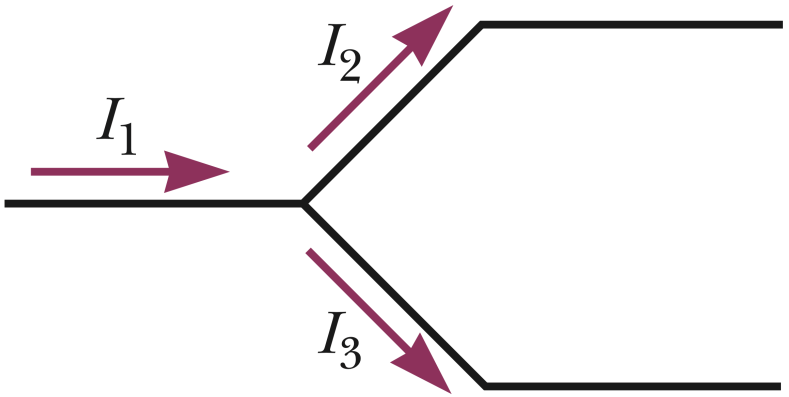
$$\Longrightarrow \quad I_1 = I_2 + I_3$$

- Kirchhoff's Current Law states that the total current entering a junction (or node) in a circuit must equal the total current leaving the junction,

$$I_{\text{in}} = I_{\text{out}},$$

- This law is based on the principle of conservation of electric charge, which implies that charge cannot accumulate at a junction; it must flow in and out at the same rate.

# 3.2 Kirchhoff's Current Junction Rule




equivalently we can write the current junction rule as:

$$\sum I = I_1 + I_2 + \cdots + I_n = 0$$

where we assign a direction to each current: positive for entering and negative for leaving the junction.

$$\implies \quad I_1 - I_2 - I_3 = 0$$

# 3.3 Kirchhoff's Voltage Loop Rule

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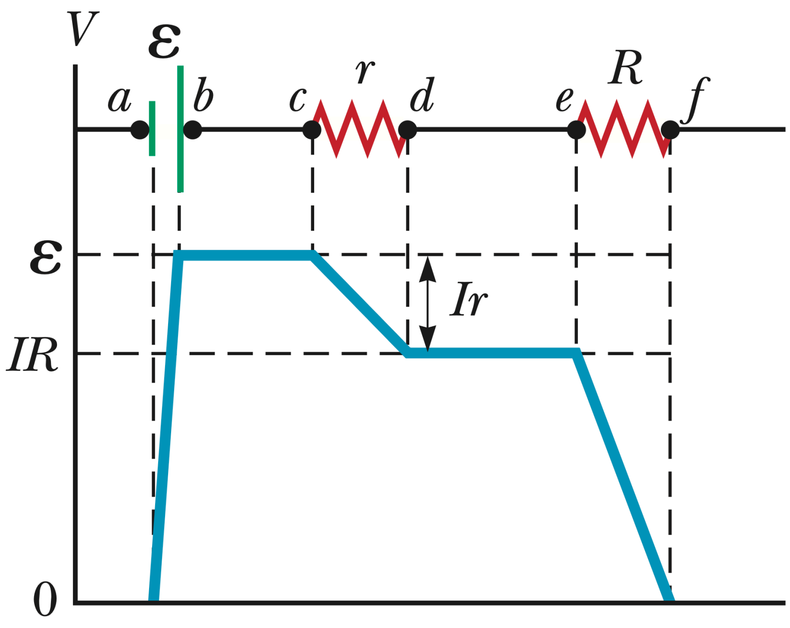
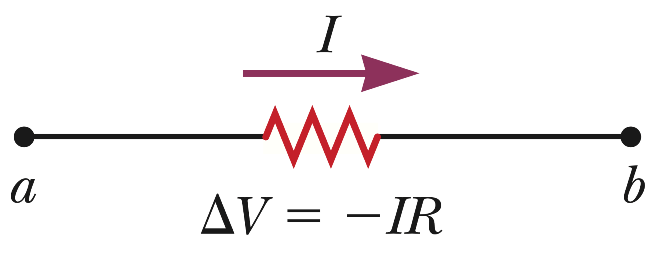


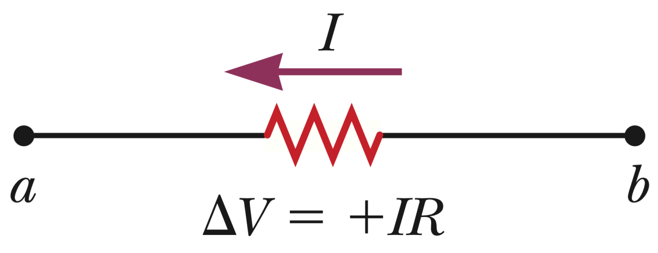

$$\Delta V_{a \longrightarrow f} = 0$$

- The sum of the potential differences across all elements around any closed circuit loop must be zero.

$$\sum_{\text{closed loop}} \Delta V = 0$$

# 3.4 How voltage changes when moving from a to b for the following 4 cases?

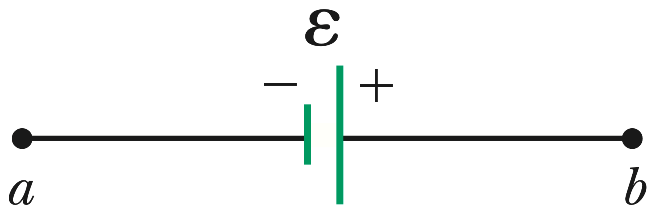



When moving across a resistor **in the direction** of current, the potential **decreases** by $IR$.

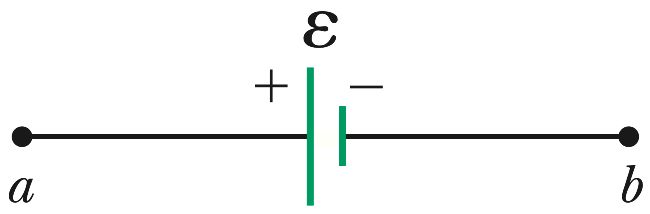
When moving across a resistor **against the direction** of current, the potential **increases** by $IR$.

# 3.4 How voltage changes when moving from a to b for the following 4 cases?




$$\Delta V = +\mathcal{E}$$

When moving across a battery **from the negative to the positive terminal**, the potential **increases** by the emf $\varepsilon$ of the battery.



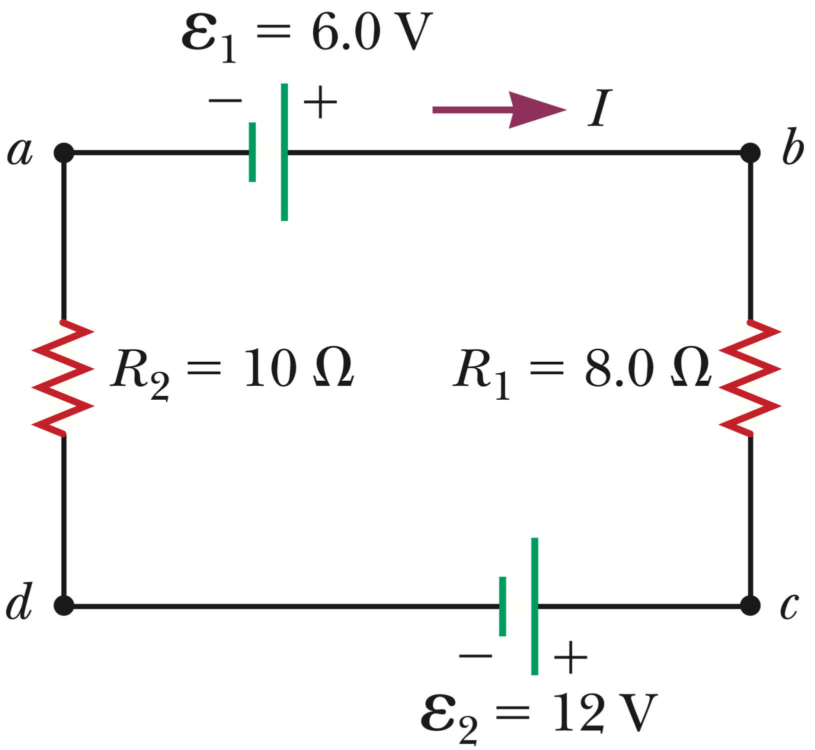

$$\Delta V = -\mathcal{E}$$

When moving across a battery **from the positive to the negative terminal**, the potential **decreases** by the emf $\varepsilon$ of the battery.

# 3.5 Example

**Example 3.4**

A single-loop circuit contains two resistors and two batteries as shown in the Figure. (Neglect the internal resistances of the batteries.) Find the current in the circuit.



# 3.5 Example

## Solution 3.4

Since there is no junction in the circuit, we can only apply Kirchhoff's voltage loop rule to find the current.

First let's assume the current is clockwise as shown in the Figure:

$$\sum \Delta V = 0 \quad \longrightarrow \quad \varepsilon_1 - IR_1 - \varepsilon_2 - IR_2 = 0$$

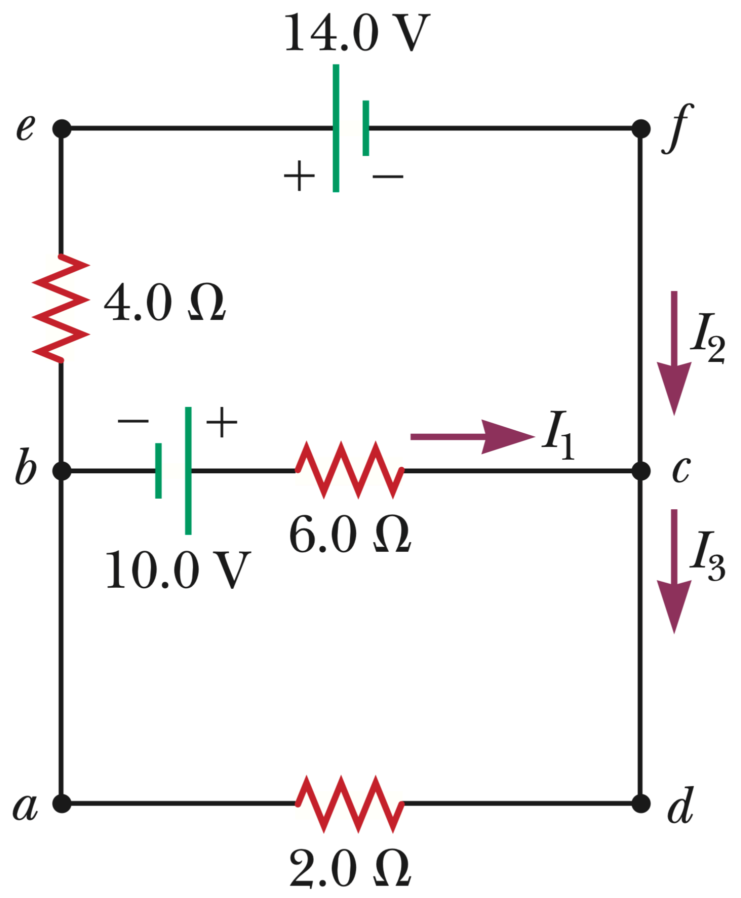
$$\Longrightarrow \quad I = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} = \frac{6\ \text{V} - 12\ \text{V}}{8\ \Omega + 19\ \Omega} = -0.33\ \text{A}$$

The negative sign for I indicates that the direction of the current is opposite the assumed direction

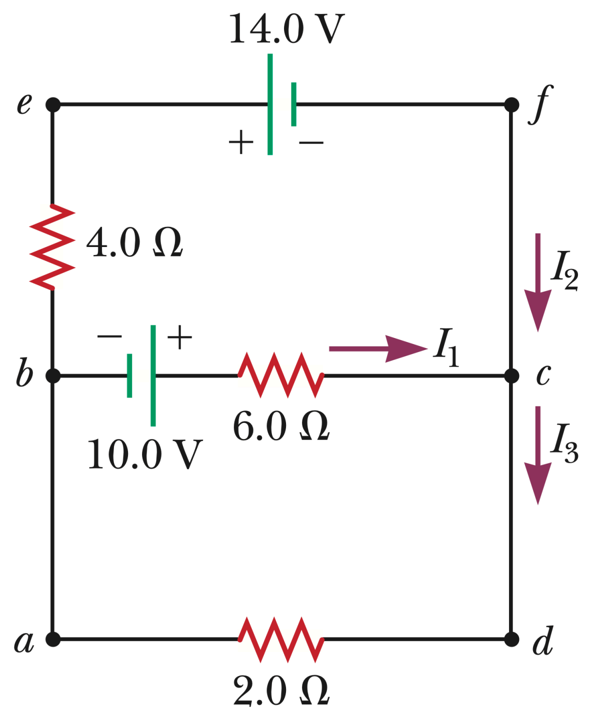
# 3.5 Example

**Example 3.5**

Find the currents $I_1$, $I_2$, and $I_3$ in the circuit shown in the figure.



# 3.5 Example




Apply Kirchhoff's junction rule to junction c, we get:

$$I_1 + I_2 - I_3 = 0 \tag{1}$$

There are three loops in the circuit: abcda, befcb, and aefda. We need only two loop equations to determine the unknown currents.

$$\text{abcda:} \quad 10\text{ V} - (6\ \Omega)I_1 - (2\ \Omega)I_3 = 0 \tag{2}$$

$$\text{befcb:} \quad -(4\ \Omega)I_2 - 14\text{ V} + (6\ \Omega)I_1 - 10\text{ V} = 0 \tag{3}$$

Rearranging Equation 3 equations gives:

$$\text{befcb:} \quad -24\text{ V} + (6\ \Omega)I_1 - (4\ \Omega)I_2 = 0 \tag{3}$$

# 3.5 Example

Now we have three equations with three unknowns, therefore we can solve for the currents using substitution and elimination method; from linear algebra, we find:

$$I_1 = 2 \text{ A}, \quad I_2 = -3 \text{ A}, \quad I_3 = -1 \text{ A}$$

## Suggested Problems

1, 9, 13, 21, 33, 37

**Book:** Serway, R. A., & Jewett, J. W. (2018). Physics for Scientists and Engineers (10th ed.)

**Chapter:** 27 - Direct-Current Circuits