Ch.25: Capacitance and Dielectrics

Physics 104: Electricity and Magnetism

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Outline

Remember From Previous Chapters

Classical Mechanics

- Equations of motion:

$$\vec{\boldsymbol{v}}_f = \vec{\boldsymbol{v}}_i + \vec{\boldsymbol{a}}t$$

$$\vec{\boldsymbol{r}}_f = \vec{\boldsymbol{r}}_i + \vec{\boldsymbol{v}}_i t + \frac{1}{2}\vec{\boldsymbol{a}}t^2$$

$$\vec{\boldsymbol{v}}_f^2 = \vec{\boldsymbol{v}}_i^2 + 2\vec{\boldsymbol{a}} \cdot \left(\vec{\boldsymbol{r}}_f - \vec{\boldsymbol{r}}_i\right)$$

- Newton's Second Law:

$$\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$$

- Work-Energy Theorem:

$$W = \Delta K$$

$$W = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{d}}$$

$$K = \frac{1}{2}m\vec{\boldsymbol{v}}^2$$

Electric Field

- Coulomb's Law:

$$\vec{\boldsymbol{F}}_e = k_e \frac{q_1 q_2}{r^2}\hat{\boldsymbol{r}}$$

- Electric Field:

$$\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}_e}{q_0} = k_e \frac{q}{r^2}\hat{\boldsymbol{r}}$$

$$\vec{\boldsymbol{a}} = \left(\frac{q}{m}\right)\vec{\boldsymbol{E}}$$

Flux

- Gauss's Law:

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{A}} = \frac{q_{\text{in}}}{\varepsilon_0}$$

Remember From Previous Chapters

Electric Potential and Energy

- Electric Potential:

$$V = k_e \frac{q}{r}$$

- Potential Energy:

$$U_E = k_e \frac{q_1 q_2}{r}$$

- Relation to Electric Field:

$$\Delta V = -\vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{d}}$$

- Potential and Energy:

$$\Delta U_E = q \Delta V$$

1. Definition of Capacitance

2. Calculating Capacitance

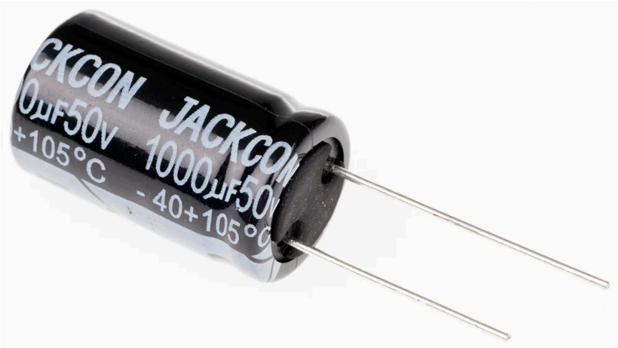
3. Combinations of Capacitors

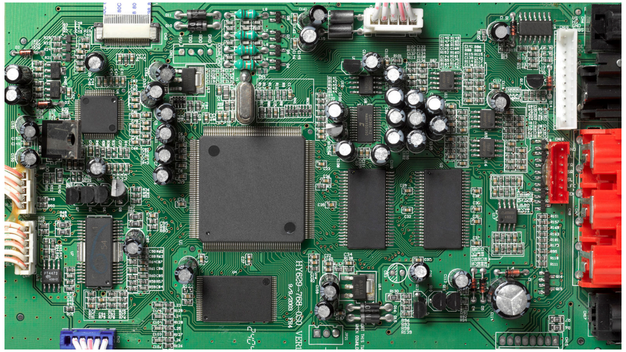
4. Energy Stored in a Charged Capacitor

5. Capacitors with Dielectrics

6. Problems

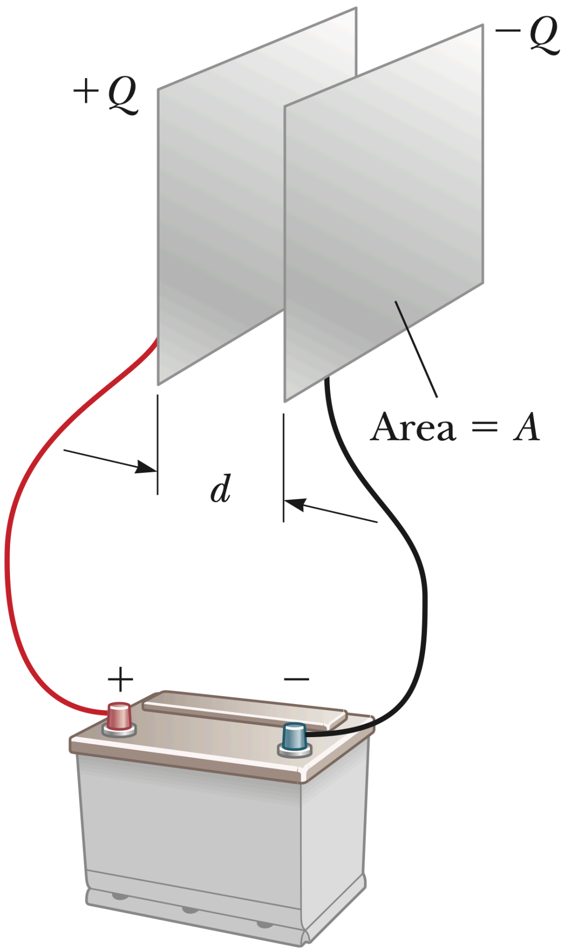
1.1 Have you seen this part before?





This is a capacitor. A capacitor is a device that stores electric charge and energy.

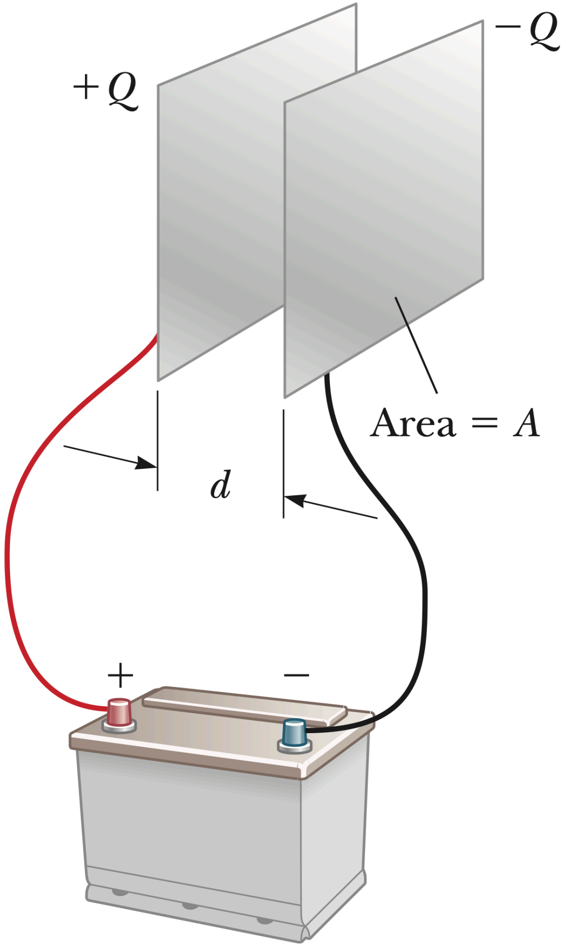
1.2 Definition of Capacitance



- A capacitor consists of two conductors (plates) separated by an insulating material (dielectric) such as air, glass, or plastic.
- When a voltage is applied across the plates, an electric field is created, and charge accumulates on the plates.
- The capacitance (C) of a capacitor is defined as the ratio of the magnitude of the charge (Q) on one plate to the magnitude of the voltage (ΔV) across the plates:

$$C = \frac{Q}{\Delta V}$$

1.2 Definition of Capacitance



- The capacitance is always a positive quantity and is measured in farads (F).

$$1\ \text{F} = 1\ \text{C/V}$$

- Typical values of capacitance range from picofarads (pF) to microfarads (μF).
- The larger the capacitance, the more charge a capacitor can store for a given voltage.
- After disconnecting the voltage source, the capacitor can retain its charge for a long time, making it useful for storing energy in electronic circuits.

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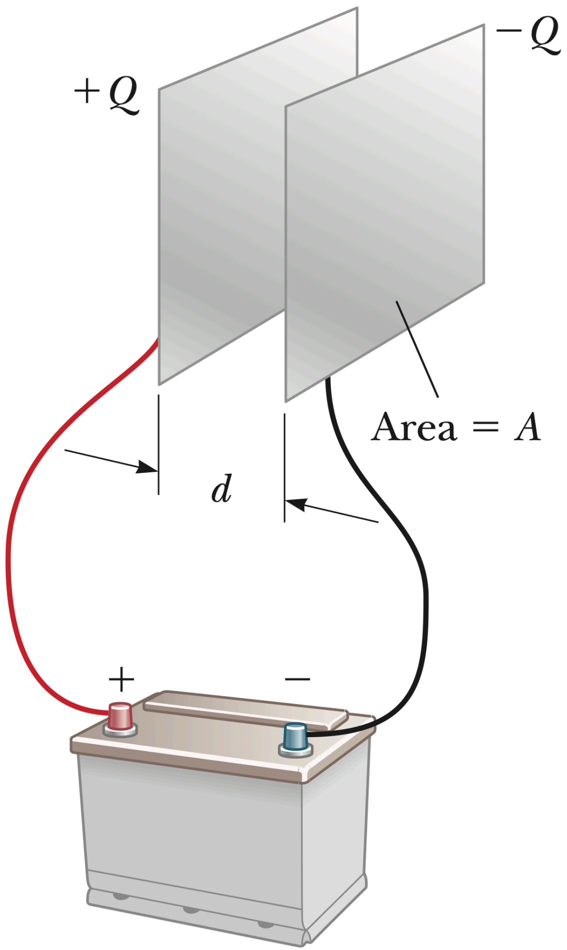
2.1 Capacitance of a Parallel-Plate Capacitor



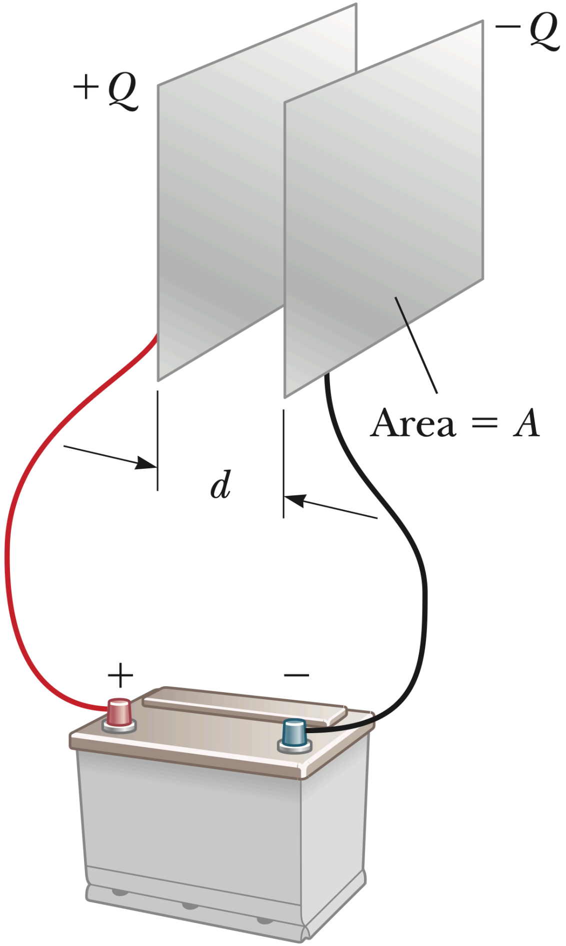
For a parallel-plate capacitor with plate area A and plate separation d, the electric field between the plates is uniform and given by:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

Since the field is uniform, the voltage across the plates can be calculated as:

$$\Delta V = Ed = \frac{Qd}{\varepsilon_0 A}$$

2.1 Capacitance of a Parallel-Plate Capacitor



Substituting this into the definition of capacitance gives:

$$C = \frac{Q}{\Delta V} = \frac{Q}{(Qd)/(\varepsilon_0 A)}$$

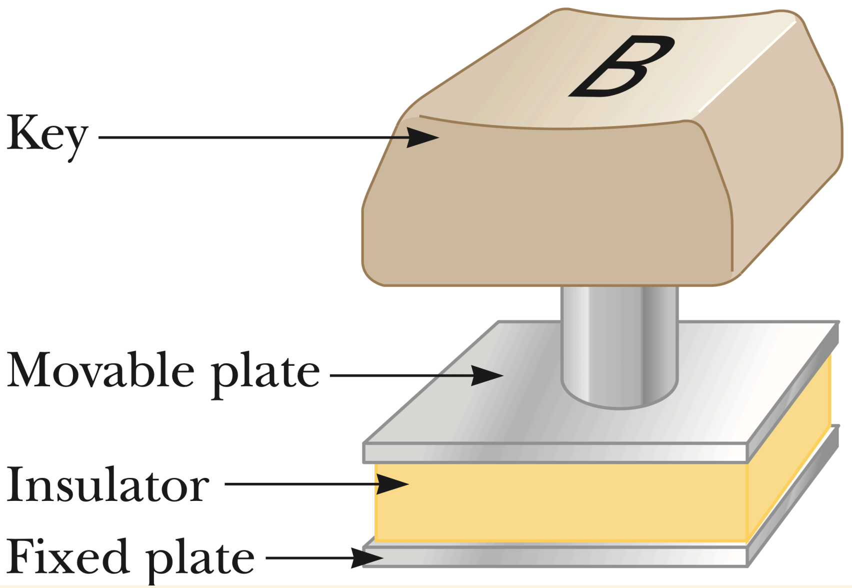
Simplifying this expression gives:

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

The capacitance depends on the geometry of the capacitor and the properties of the dielectric material between the plates.

2.1 Capacitance of a Parallel-Plate Capacitor

Quiz



What happens when you push down a keyboard key?

Answer: The capacitance increases and the voltage decreases.

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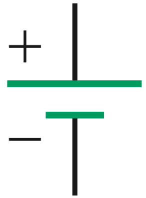
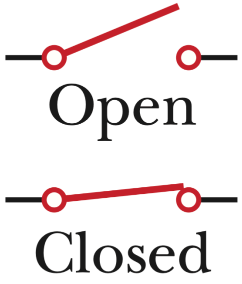
3.1 Electric Circuits

Capacitor symbol

Battery symbol

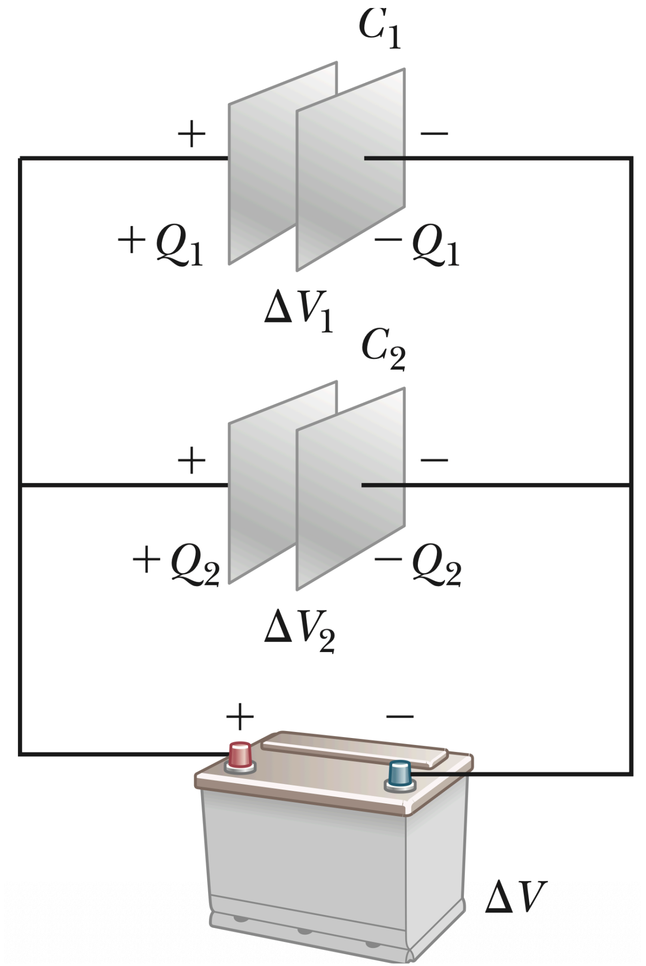


Switch symbol



- An electric circuit is a closed loop that allows electric charge to flow through its components, such as a battery, capacitor and switch.
- Typically we use *circuit diagrams* to represent *electric circuits*, where different *circuit symbols* are used for different components.
- The capacitor is represented by two parallel lines, and the battery is represented by a pair of lines of different lengths.

3.2 Capacitors in Parallel



- When two capacitors are connected in parallel, the voltage across each capacitor is the same as the voltage of the source (battery).

$$\Delta V_1 = \Delta V_2 = \Delta V$$

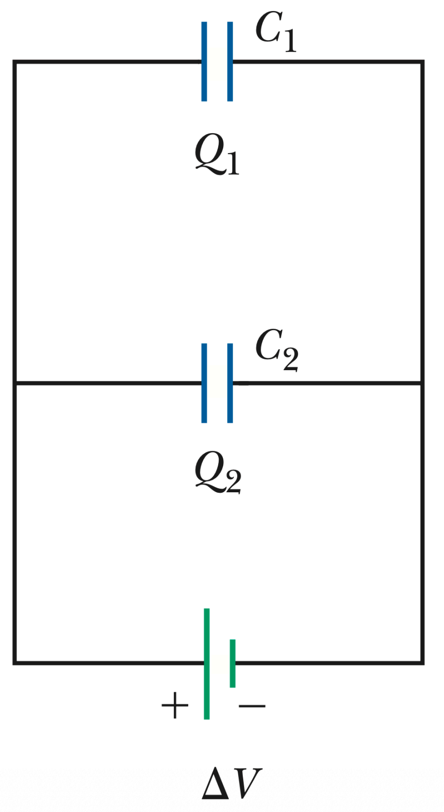
- The total charge stored in the system is the sum of the charges on each capacitor:

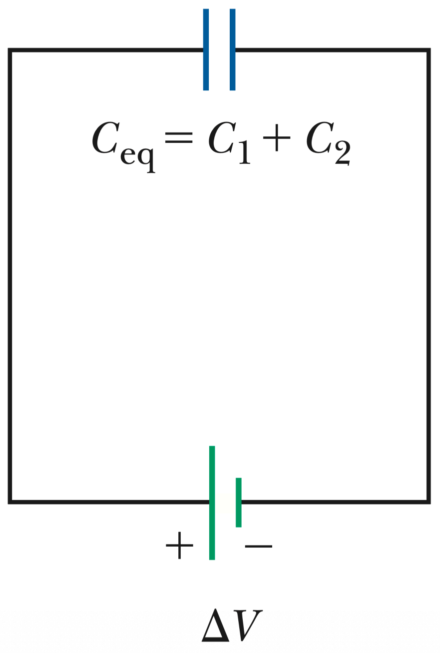
$$Q_{\text{tot}} = Q_1 + Q_2 = C_1 \Delta V_1 + C_2 \Delta V_2$$

- Since the voltages are the same across both capacitors, we can factor it out:

$$Q_{\text{tot}} = (C_1 + C_2)\Delta V$$

3.2 Capacitors in Parallel

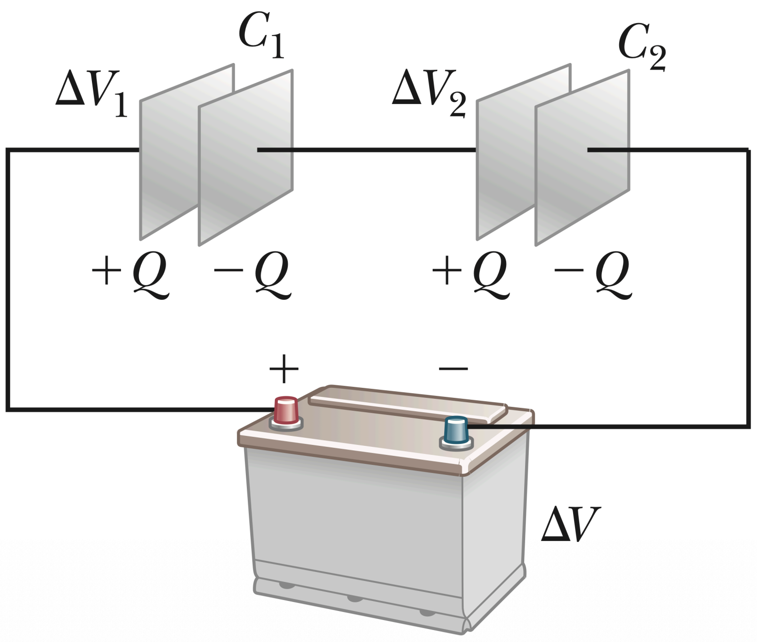




The equivalent capacitance for capacitors in parallel is given by:

$$C_{eq} = \sum C_i$$

3.3 Capacitors in Series



- When two capacitors are connected in series, their charges are the same, but the voltage across each capacitor can be different.

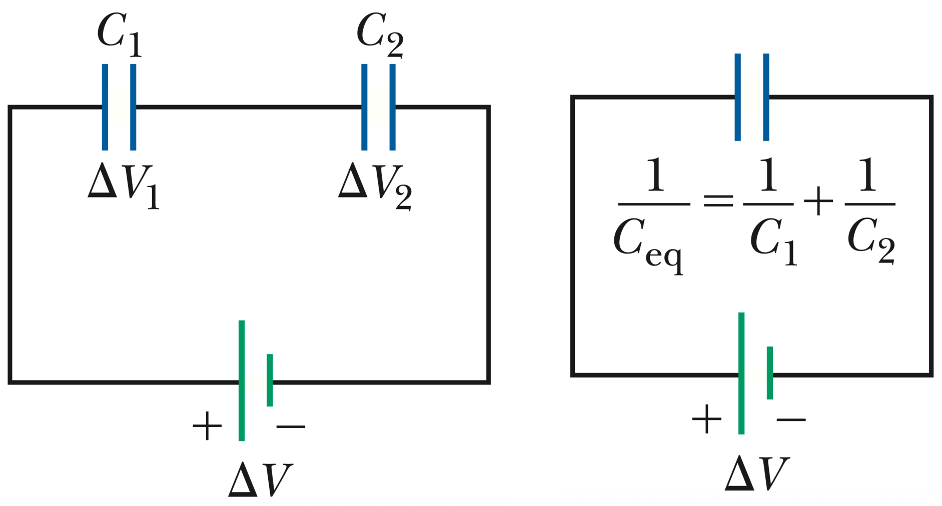
$$Q_1 = Q_2 = Q$$

- The total voltage across the system is the sum of the voltages across each capacitor:

$$\Delta V_{\mathrm{tot}} = \Delta V_1 + \Delta V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$\Delta V_{\mathrm{tot}} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = Q\left(\frac{1}{C_{\mathrm{eq}}}\right)$$

3.3 Capacitors in Series



The equivalent capacitance for capacitors in series is given by:

$$\frac{1}{C_{eq}} = \sum \frac{1}{C_i}$$

Notice that the equivalent capacitance for capacitors in series is always **less** than the smallest individual capacitance, while the equivalent capacitance for capacitors in parallel is always **greater** than the largest individual capacitance.

3.4 Example

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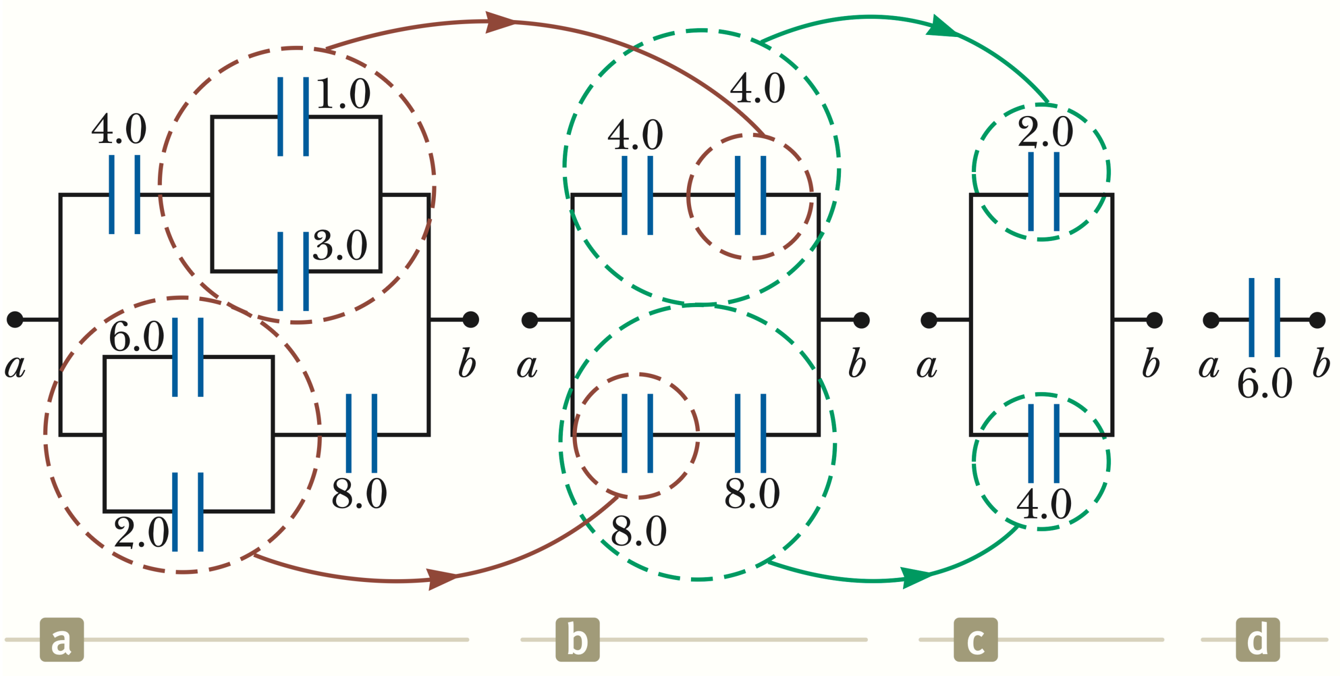
Example 3.1

Find the equivalent capacitance between a and b for the combination of capacitors shown in the Figure. All capacitances are in microfarads.



3.4 Example

Solution 3.1

$$C_{eq} = C_1 + C_2 = 1 + 3 = 4\mu\text{F}$$

$$C_{eq} = C_1 + C_2 = 6 + 2 = 8\mu\text{F}$$

$$\frac{1}{C_{eq}} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \implies C_{eq} = 2\mu\text{F}$$

$$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4} \implies C_{eq} = 4\mu\text{F}$$

$$C_{eq} = C_1 + C_2 = 2 + 4 = 6\mu\text{F}$$

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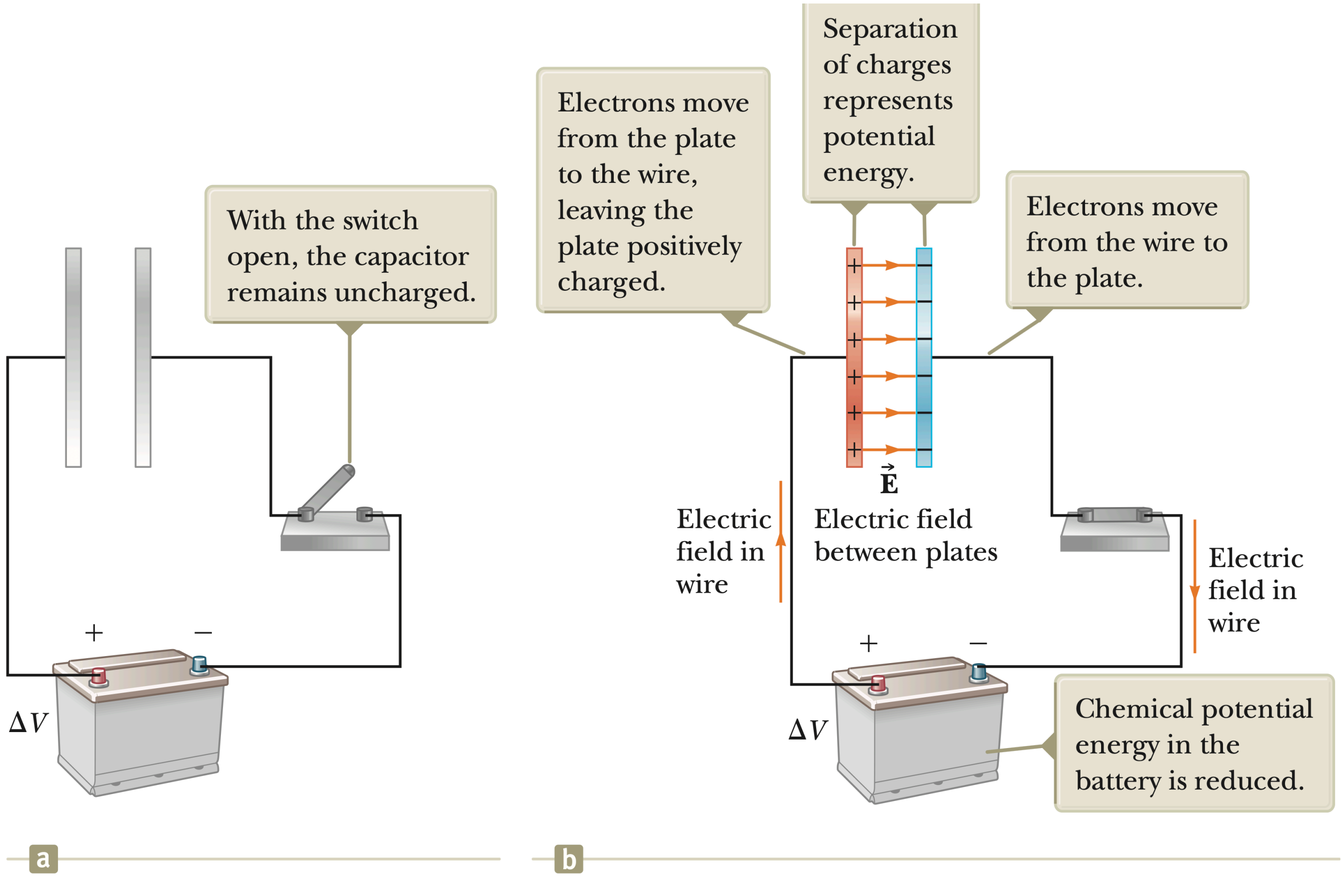
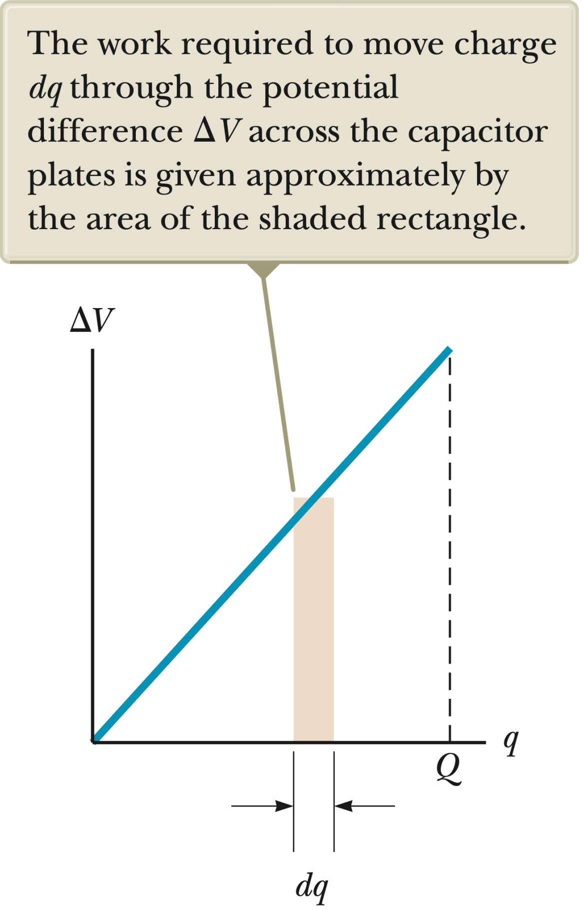
4.1 Energy Stored in a Charged Capacitor





4.2 Derivation of the Energy Stored in a Charged Capacitor

- The work done to move a small amount of charge dq from the battery to the capacitor is given by:

$$\mathrm{d}W = \Delta V\,\mathrm{d}q = \left(\frac{q}{C}\right)\mathrm{d}q$$

Therefore, the total work done to charge the capacitor from 0 to Q is given by:

$$W = \int_0^Q \left(\frac{q}{C}\right)\mathrm{d}q = \frac{1}{2}\frac{Q^2}{C}.$$

Since the work done to charge the capacitor is stored as potential energy,:

$$U_E = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

4.3 Energy Density

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- Since $\Delta V = Ed$ and $C = \varepsilon_0 A/d$, we can express the energy stored in the capacitor in terms of the electric field:

$$U_E = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\varepsilon_0\frac{A}{d}(Ed)^2 = \frac{1}{2}\varepsilon_0(Ad)E^2$$

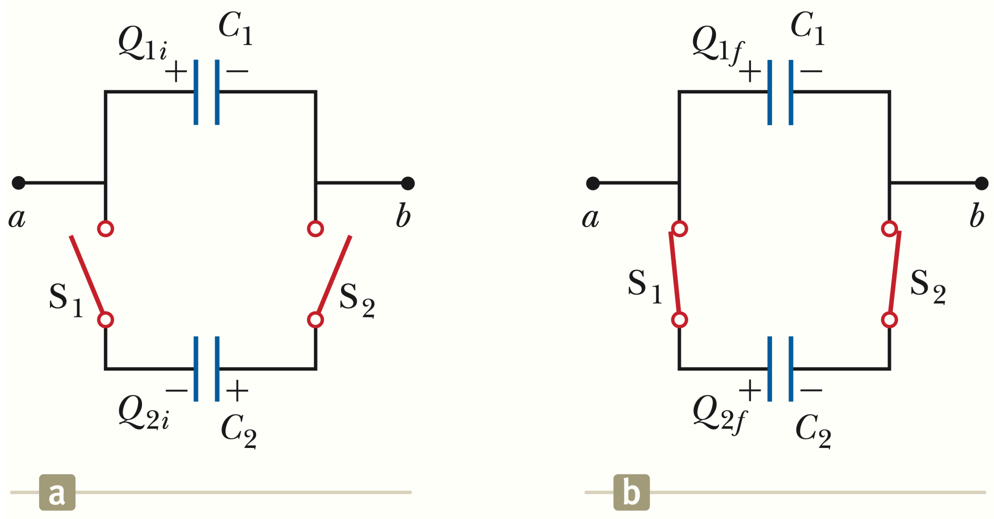
Therefore, the energy density (energy per unit volume) in the electric field is given by:

$$u_E = \frac{U_E}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

Example 4.2

Two capacitors C_1 and C_2 (where $C_1 > C_2$) are charged to the same initial potential difference ΔV_i. The charged capacitors are removed from the battery, and their plates are connected with opposite polarity as in Figure (a). The switches S_1 and S_2 are then closed as in Figure (b).

(A) Find the final potential difference ΔV_f between a and b after the switches are closed.



4.3 Energy Density

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Solution 4.2

- The capacitor is a device that stores electric charges to an amount defined by the voltage across its plates and its capacitance.

- Before closing the switches, the total charge on the left capacitor plate is:
$$Q_i = Q_{1i} + Q_{2i} = C_1 \Delta V_i - C_2 \Delta V_i = (C_1 - C_2)\Delta V_i$$

- After closing the switches, the total charge on the left capacitor plate is:
$$Q_f = Q_{1f} + Q_{2f} = C_1 \Delta V_f + C_2 \Delta V_f = (C_1 + C_2)\Delta V_f$$

4.3 Energy Density

- Since the total charge is conserved before and after closing the switches,

$$Q_i = Q_f$$

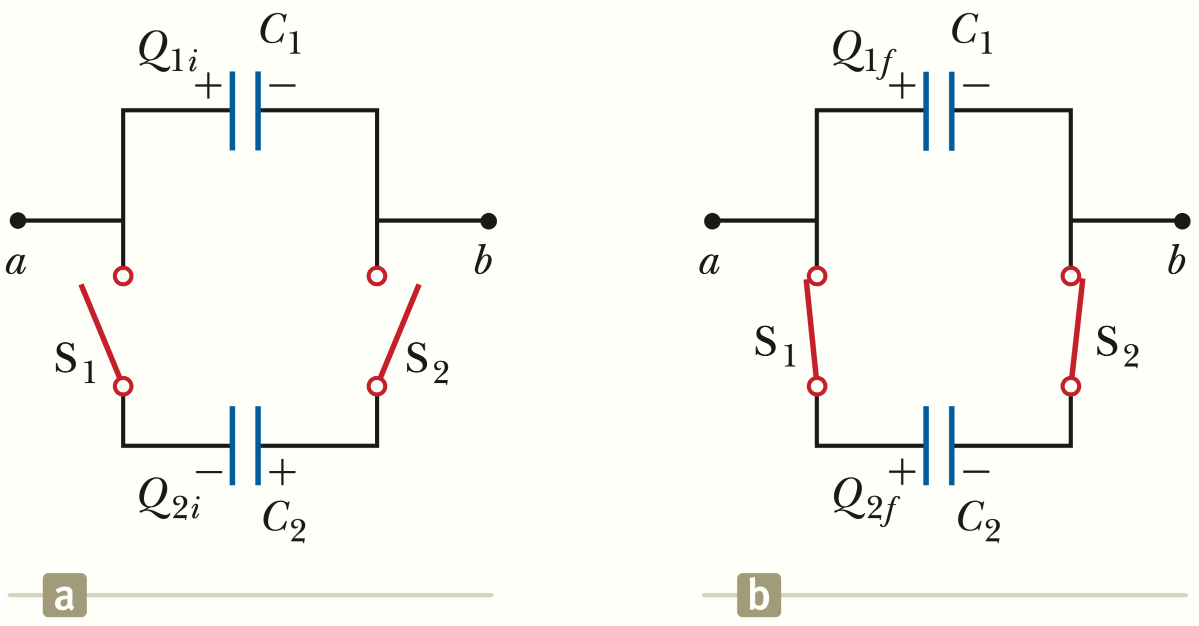
$$(C_1 - C_2)\Delta V_i = (C_1 + C_2)\Delta V_f$$

Therefore, the final potential difference between a and b is given by:

$$\Delta V_f = \frac{C_1 - C_2}{C_1 + C_2}\Delta V_i$$

4.3 Energy Density

(B) Find the total energy stored in the capacitors before and after the switches are closed and determine the ratio of the final energy to the initial energy.



4.3 Energy Density

$$U_i = \frac{1}{2}C_1(\Delta V_i)^2 + \frac{1}{2}C_2(\Delta V_i)^2 = \frac{1}{2}(C_1 + C_2)(\Delta V_i)^2$$

$$U_f = \frac{1}{2}(C_1 + C_2)(\Delta V_f)^2$$

Using the expression for ΔV_f from part (A), we can express the final energy in terms of ΔV_i:

$$U_f = \frac{1}{2}(C_1 + C_2)\left[\frac{C_1 - C_2}{C_1 + C_2}\Delta V_i\right]^2 = \frac{1}{2}\frac{(C_1 - C_2)^2}{C_1 + C_2}(\Delta V_i)^2$$

Therefore, the ratio of the final energy to the initial energy is:

$$\frac{U_f}{U_i} = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2$$

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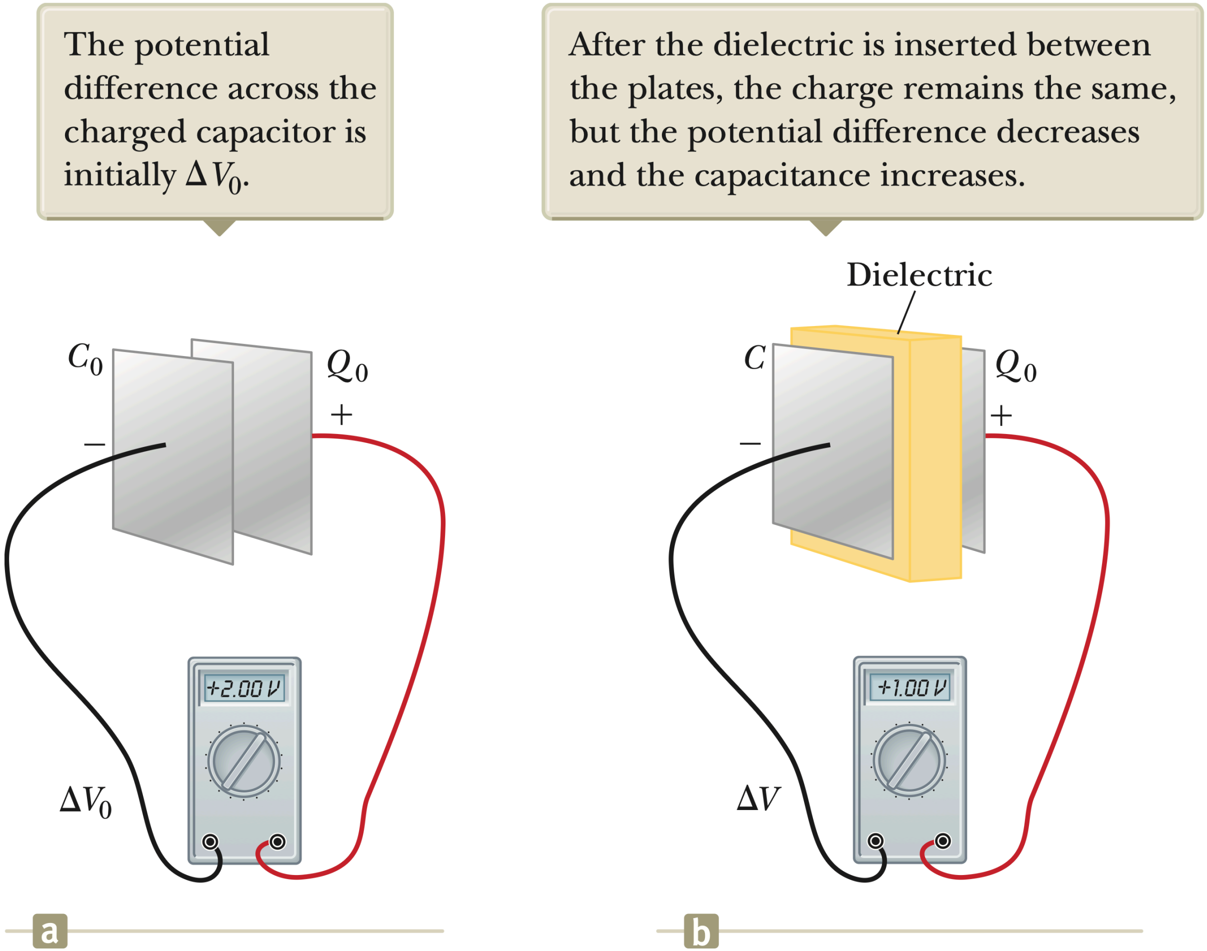
5.1 Effect of a Dielectric on Capacitance



$$\Delta V = \Delta V_0/\kappa$$

$$C = \kappa C_0$$

$$Q = \kappa Q_0$$

$$U_E = U_0/\kappa$$

where κ is the **dielectric constant** of the material.

5.2 Advantages of Using Dielectrics

1. Dielectrics increase the capacitance of a capacitor, allowing it to store more charge for a given voltage.
2. Dielectrics can increase the maximum operating voltage of a capacitor, allowing it to operate at higher voltages without breaking down.
3. They provide mechanical support between the plates, which allows the plates to be close together without touching, thereby decreasing d and increasing C

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polyethylene	2.30	18
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—

Suggested Problems

1, 3, 4, 7, 9, 11, 12, 17, 25, 35

Book: Serway, R. A., & Jewett, J. W. (2018). Physics for Scientists and Engineers (10th ed.)

Chapter: 25 - Capacitance and Dielectrics

1. Definition of Capacitance

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Problem 6.1

1. (a) When a battery is connected to the plates of a 3.00-μF capacitor, it stores a charge of 27.0 μC. What is the voltage of the battery? (b) If the same capacitor is connected to another battery and 36.0 μC of charge is stored on the capacitor, what is the voltage of the battery?

6.1 Definition of Capacitance

Answer 6.1

a)

$$\Delta V = \frac{Q}{C} = \frac{27\ \mu\text{C}}{3\ \mu\text{F}} = 9V$$

b)

$$\Delta V = \frac{Q}{C} = \frac{36\ \mu\text{C}}{3\ \mu\text{F}} = 12V$$

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Problem 6.2

3. When a potential difference of 150 V is applied to the plates of a parallel-plate capacitor, the plates carry a surface charge density of 30.0 nC/cm^2. What is the spacing between the plates?

6.2 Calculating Capacitance

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Answer 6.2

$$C = \varepsilon_0 \frac{A}{d}$$

$$\Rightarrow d = \frac{\varepsilon_0 A}{C} = \frac{\varepsilon_0 A}{\frac{Q}{\Delta V}} = \frac{\varepsilon_0 A}{\frac{\sigma A}{\Delta V}}$$

$$\Longrightarrow d = \varepsilon_0 \frac{\Delta V}{\sigma} = 4.43 \mu\text{m}$$

* Note: σ has to be converted to SI units before substituting into the formula.

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Problem 6.3

4. An air-filled parallel-plate capacitor has plates of area $2.30\ \text{cm}^2$ separated by 1.50 mm. (a) Find the value of its capacitance. The capacitor is connected to a 12.0-V battery. (b) What is the charge on the capacitor? (c) What is the magnitude of the uniform electric field between the plates?

6.2 Calculating Capacitance

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Answer 6.3

a)

$$C = \varepsilon_0 \frac{A}{d} = 1.36 \times 10^{-12}\ \mathrm{F} = 1.36\ \mathrm{pF}$$

b)

$$Q = C\Delta V = 16.3 \times 10^{-12}\ \mathrm{C} = 16.3\ \mathrm{pC}$$

c)

$$E = \frac{\Delta V}{d} = 8\ \ \mathrm{KV/m}$$

6.3 Combinations of Capacitors

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Problem 6.4

7. Find the equivalent capacitance of a 4.20-μF capacitor and an 8.50-μF capacitor when they are connected (a) in series and (b) in parallel.

6.3 Combinations of Capacitors

Answer 6.4

a) When connected in series:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{\text{eq}} = 2.81\mu\text{F}$$

b) When connected in parallel:

$$C_{\text{eq}} = C_1 + C_2$$

$$C_{\text{eq}} = 12.7\mu\text{F}$$

6.3 Combinations of Capacitors

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Problem 6.5

9. T A group of identical capacitors is connected first in series and then in parallel. The combined capacitance in parallel is 100 times larger than for the series connection. How many capacitors are in the group?

6.3 Combinations of Capacitors

Answer 6.5

$$C_{\text{parallel}} = C + C + C + \ldots = NC$$

$$\frac{1}{C_{\text{series}}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \ldots = \frac{N}{C} \implies C_{\text{series}} = \frac{C}{N}$$

The ration then is:

$$\frac{C_{\text{parallel}}}{C_{\text{series}}} = \frac{NC}{C/N} = N^2$$

$$\implies n = \sqrt{\frac{C_{\text{parallel}}}{C_{\text{series}}}} = \sqrt{100} = 10$$

6.3 Combinations of Capacitors

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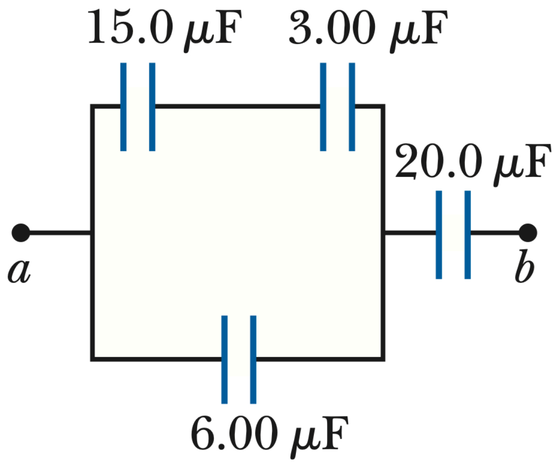
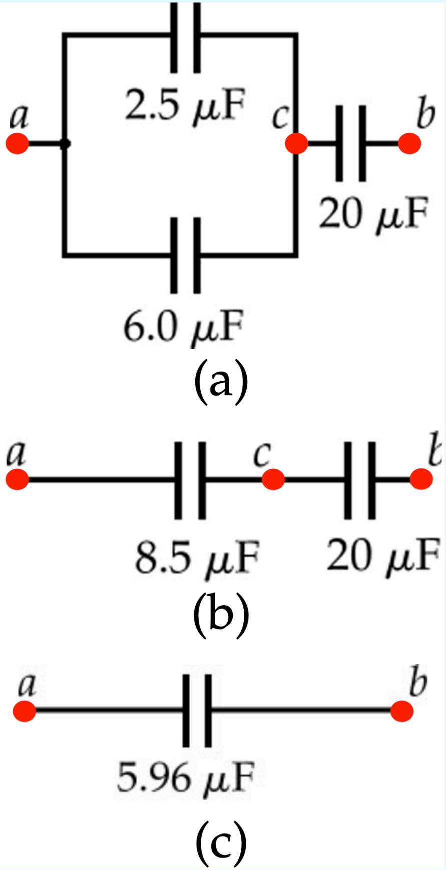
Problem 6.6

11. T Four capacitors are connected as shown in Figure P25.11. (a) Find the equivalent capacitance between points *a* and *b*. (b) Calculate the charge on each capacitor, taking $\Delta V_{ab} = 15.0$ V.



6.3 Combinations of Capacitors



To find the charge in each capacitor, we start from the final single capacitor at (c):

$$Q = C\Delta V = (5.96\mu\text{F})(15V) = 89.5\mu\text{C}$$

The two capacitors in series at (b) have the same charge as in (c), but with different voltages:

$$V_{\text{ac}} = \frac{Q}{C} = \frac{89.5\mu\text{C}}{8.5\mu\text{F}} = 10.5V$$

$$V_{\text{cb}} = \frac{Q}{C} = \frac{89.5\mu\text{C}}{20\mu\text{F}} = 4.74V$$

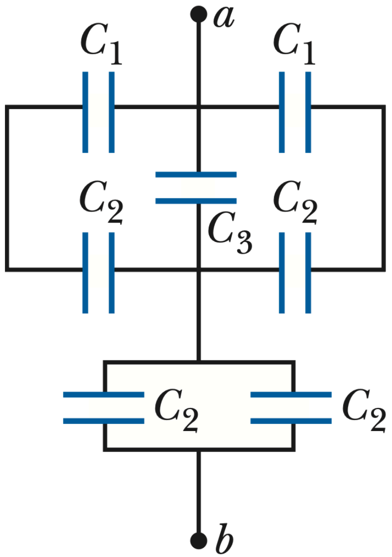
Finally at (a), we find the charge Q using the V_{ac}

$$Q = C\Delta V_{\text{ac}} = 26.3\mu\text{C} \quad \text{and} \quad 63.2\mu\text{C}$$

6.3 Combinations of Capacitors

Problem 6.7

12. V (a) Find the equivalent capacitance between points a and b for the group of capacitors connected as shown in Figure P25.12 (page 686). Take $C_1 = 5.00\ \mu\text{F}$, $C_2 = 10.0\ \mu\text{F}$, and $C_3 = 2.00\ \mu\text{F}$. (b) What charge is stored on C_3 if the potential difference between points a and b is 60.0 V?



6.3 Combinations of Capacitors

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Answer 6.7

- (a) First the upper section, left and right sides (C_1 and C_2) are in series, with equivalent capacitance:
$$C_{\text{Upper-sides}} = 3.33\mu\text{F}$$
- C_3 is in series with the two upper sides, with equivalent capacitance:
$$C_{\text{Upper}} = 2(3.33) + 2 = 8.67\mu\text{F}$$
- The lower section (C_2-C_2) is in parallel, with equivalent capacitance:
$$C_{\text{Lower}} = 20\mu\text{F}$$
- Finally, the upper and lower sections are in series, with equivalent capacitance:
$$C_{\text{eq}} = 6.05\mu\text{F}$$

6.3 Combinations of Capacitors

(b)

$$Q_{\text{upper}} = Q_{\text{eq}} = C_{\text{eq}} \Delta V = 363 \mu\text{C}$$

$$\Delta V_{\text{upper}} = \frac{Q_{\text{upper}}}{C_{\text{Upper}}} = 41.9V$$

The charge on C_3 is:

$$Q_3 = C_3 \Delta V_{\text{upper}} = 83.7 \mu\text{C}$$

6.4 Energy Stored in a Charged Capacitor

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Problem 6.8

17. **V** A 3.00-μF capacitor is connected to a 12.0-V battery. How much energy is stored in the capacitor? (b) Had the capacitor been connected to a 6.00-V battery, how much energy would have been stored?

6.4 Energy Stored in a Charged Capacitor

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Answer 6.8

(a)

$$U_E = \frac{1}{2}C(\Delta V)^2 = 216\mu\text{J}$$

(b)

$$U_E = 54\mu\text{J}$$

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Problem 6.9

25. **V** Determine (a) the capacitance and (b) the maximum potential difference that can be applied to a Teflon-filled parallel-plate capacitor having a plate area of 1.75 cm^2 and a plate separation of 0.040 0 mm.

6.5 Capacitors with Dielectrics

Answer 6.9

(a)

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

Using $\kappa = 2.1$ for Teflon, we get:

$$C = 81.3 \text{ pF}$$

(b)

$$\Delta V_{\text{max}} = E_{\text{max}} d$$

From the table of dielectric materials, we find that the maximum electric field for Teflon is $E_{\text{max}} = 60 \text{ MV/m}$, therefore:

$$\Delta V_{\text{max}} = 2.4 \text{ kV}$$

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Problem 6.10

35. A uniform electric field $E = 3\,000$ V/m exists within a certain region. What volume of space contains an energy equal to 1.00×10^{-7} J? Express your answer in cubic meters and in liters.

6.5 Capacitors with Dielectrics

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Answer 6.10

The energy density of an electric field is given by:

$$u_E = \frac{U_E}{V} = \frac{1}{2}\varepsilon_0 E^2$$

$$\Longrightarrow V = \frac{U_E}{\frac{1}{2}\varepsilon_0 E^2} = 2.51 \times 10^{-3}\ \text{m}^3 = 2.51\ \text{L}$$