



Ch.24: Electric Potential

Physics 104: Electricity and Magnetism

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Remember From Previous Chapters

Classical Mechanics

- Equations of motion:

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a} \cdot (\vec{r}_f - \vec{r}_i)$$

- Newton's Second Law:

$$\sum \vec{F} = m\vec{a}$$

- Work-Energy Theorem:

$$W = \Delta K$$

$$W = \vec{F} \cdot \vec{d}$$

$$K = \frac{1}{2}m\vec{v}^2$$

Electric Field

- Coulomb's Law:

$$\vec{F}_e = k_e \frac{q_1 q_2}{r^2} \hat{r}$$

- Electric Field:

$$\vec{E} = \frac{\vec{F}_e}{q_0} = k_e \frac{q}{r^2} \hat{r}$$

$$\vec{a} = \left(\frac{q}{m} \right) \vec{E}$$

Flux

- Gauss's Law:

$$\Phi_E = \oint \vec{E} \cdot \vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

1. Electric Potential and Potential Difference

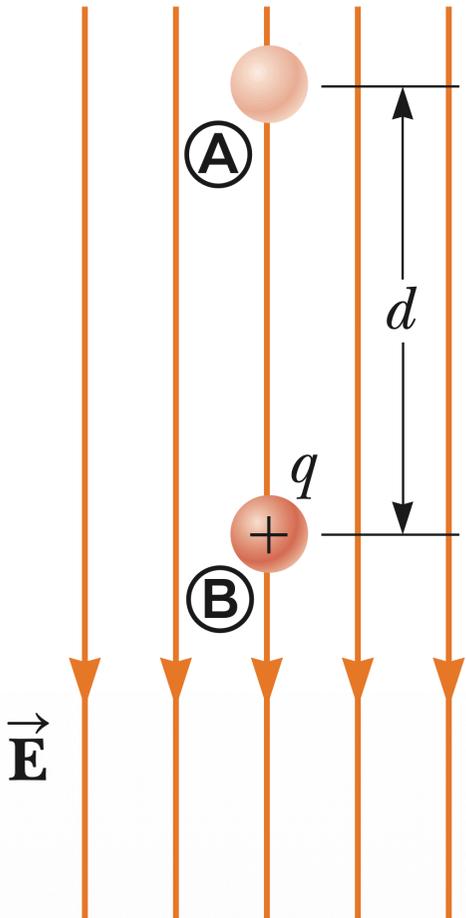
2. Potential Difference in a Uniform Electric Field

3. Electric Potential and Potential Energy Due to Point Charges

4. Conductors in Electrostatic Equilibrium

5. Problems

1.1 Electric Potential Energy



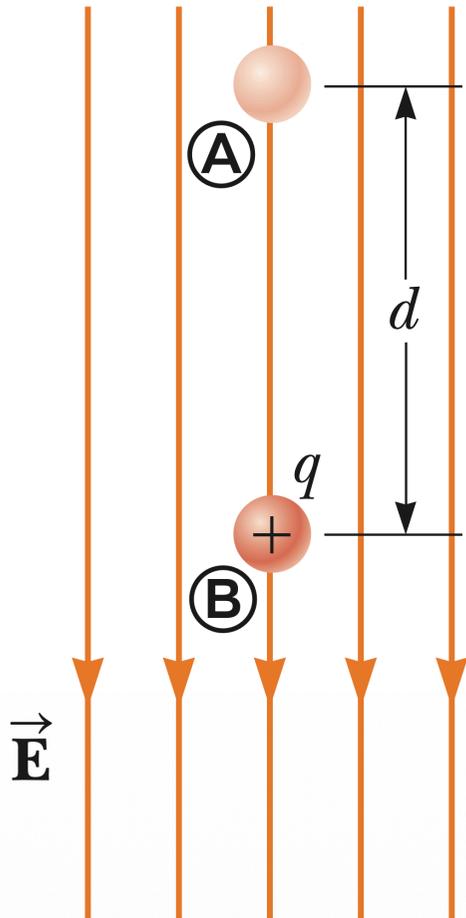
- When a charge q is placed in an electric field \vec{E} , the work done by the electric field on the charge to move it through a small displacement $d\vec{s}$ is given by: $W = q\vec{E} \cdot d\vec{s}$
- The **electric potential energy** (U_E) of the charge changes by an amount equal to the negative of the work done by the electric field:

$$dU_E = -W = -q\vec{E} \cdot d\vec{s}$$

- Integrating both sides from point A to point B, we get

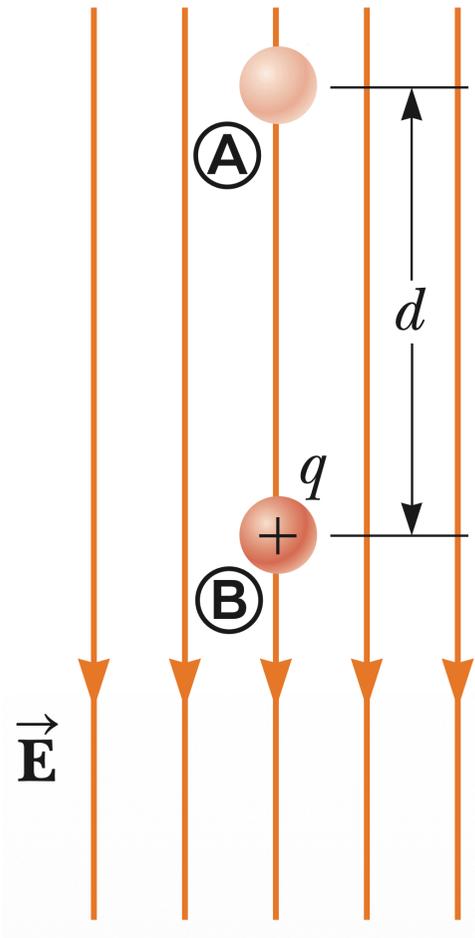
$$\Delta U_E = U_{E_B} - U_{E_A} = -q \int_A^B \vec{E} \cdot d\vec{s}$$

1.1 Electric Potential Energy



- The electric potential energy is a scalar quantity measured in joules (J).
- The electric potential energy depends on the position of the charge in the electric field.
- The electric potential energy increases when a positive charge moves against the direction of the electric field and decreases when it moves in the direction of the electric field.
- The opposite is true for a negative charge.
- The integral does not depend on the path taken but only on the initial and final positions A and B.
- U_A can be set to zero by choosing point A at infinity.

1.2 Electric Potential

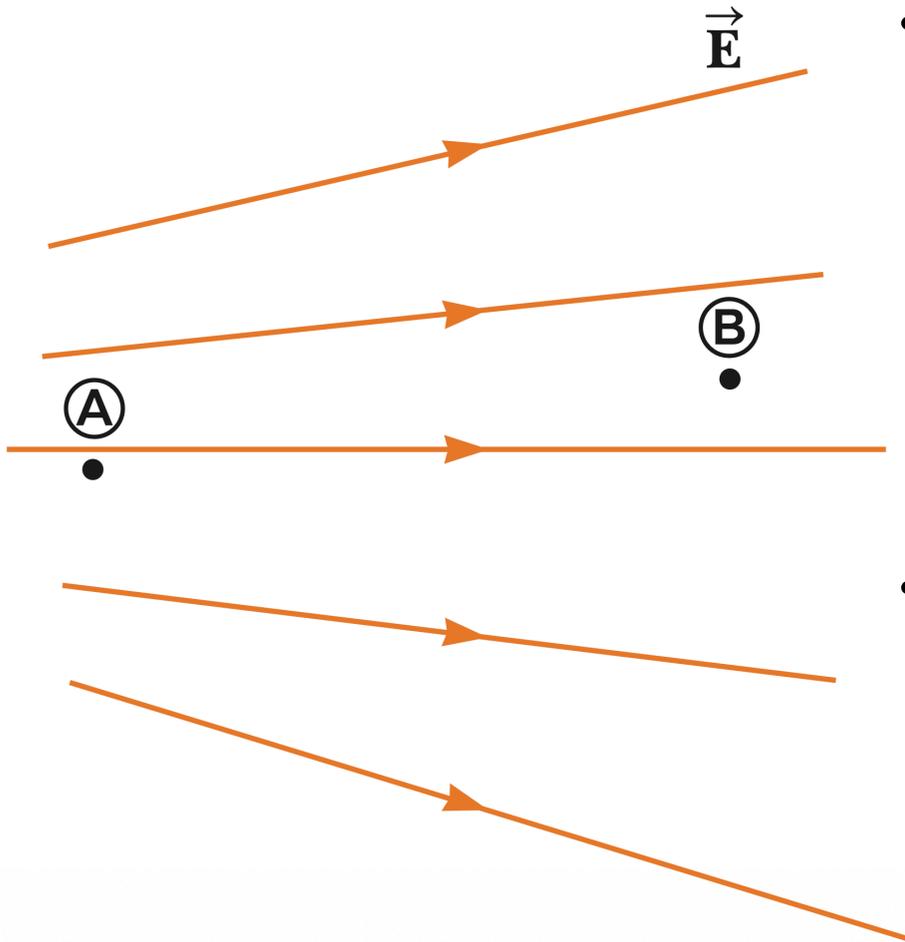


- Dividing the potential energy by the charge gives a physical quantity that depends only on the source charge distribution and has a value at every point in an electric field. This quantity is called the **electric potential** (or simply the **potential**) V :

$$V = \frac{U_E}{q}$$

- The electric potential is a scalar quantity measured in volts (V), where $1 \text{ V} = 1 \text{ J/C}$.

1.2 Electric Potential

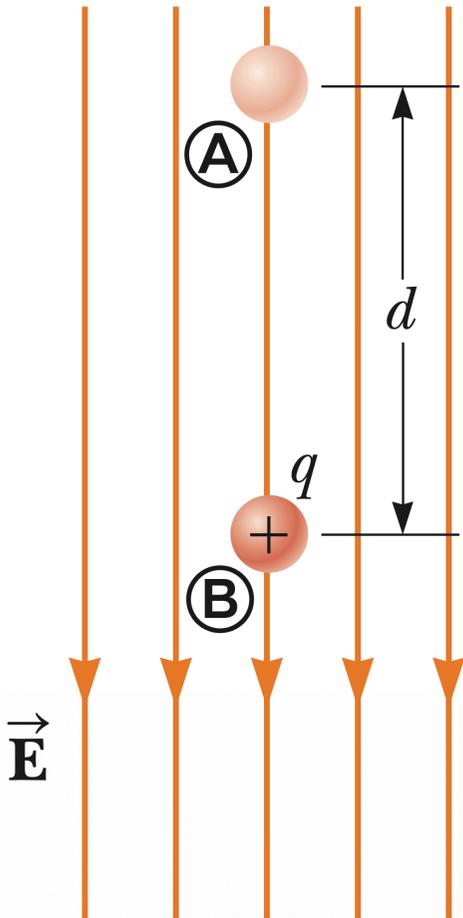


- The **potential difference** between two points A and B is:

$$\Delta V = \frac{\Delta U_E}{q} = - \int_A^B \vec{E} \cdot d\vec{s}$$

- Notice that the potential difference does not depend on the test charge q , but only on the electric field and the positions of points A and B.

1.3 External Work Done on a System of Charges

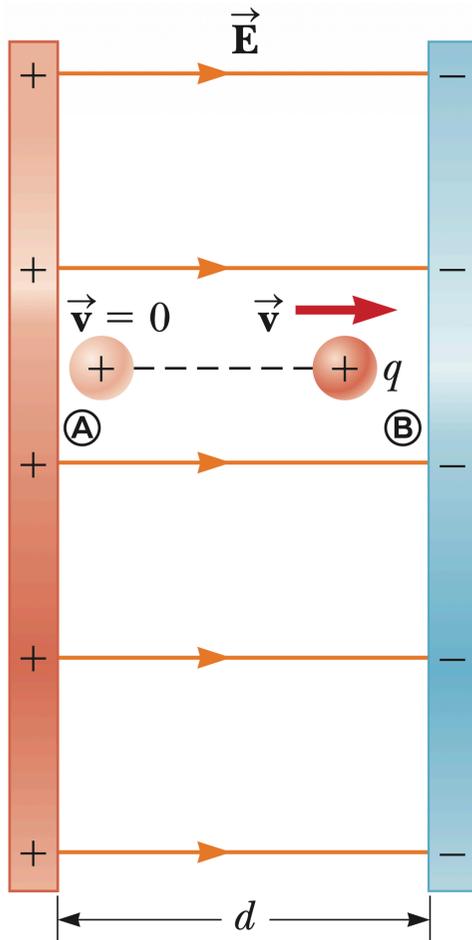


- If an external agent moves a charge q from point B *back* to point A **against** the electric field, the work done by the **external** agent is equal to the increase in the electric potential energy of the charge:

$$W = +q\Delta V$$

- The work done by the external agent is positive when it moves a positive charge against the electric field.
- The work done by the external agent is equal in magnitude but opposite in sign to the work done by the electric field when it moves the charge from point A to point B.

1.4 Electron Volt



- When an electron (charge q_e) is accelerated through a potential difference of 1 V, it gains an energy of

$$q_e \times 1 \text{ V} = 1.6 \times 10^{-19} \text{ J}$$

- This amount of energy is called an **electron volt** (eV):
- Typically, very small energies of particles are expressed in electron volts rather than in joules. For example, an energy of $3.2 \times 10^{-19} \text{ J}$ is more conveniently expressed as 2 eV, therefore,

$$\text{Energy in eV} = \frac{\text{Energy in J}}{1.6 \times 10^{-19} \text{ J/eV}}$$

1. Electric Potential and Potential Difference

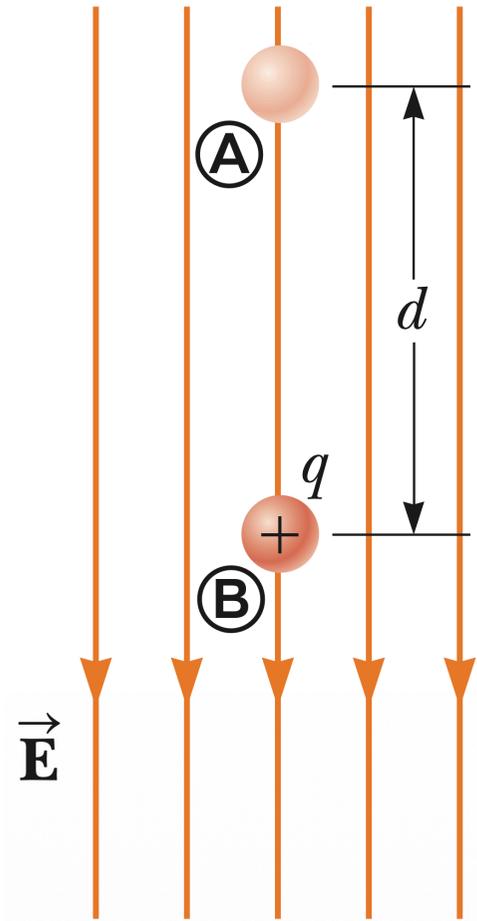
2. Potential Difference in a Uniform Electric Field

3. Electric Potential and Potential Energy Due to Point Charges

4. Conductors in Electrostatic Equilibrium

5. Problems

2.1 The relationship between Electric Field and Electric Potential



- For a uniform electric field \vec{E} , the potential difference between two points A and B separated by a distance d along the direction of the field is simplified to (no integral needed):

$$\Delta V = -Ed$$

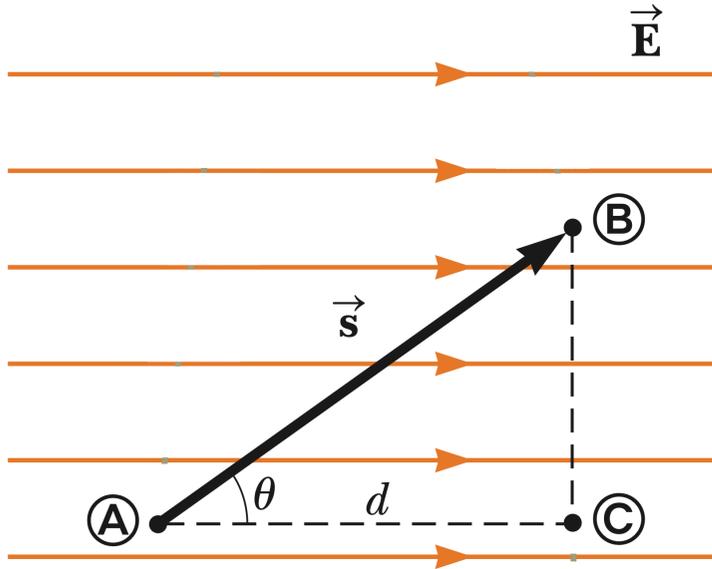
- The negative sign indicates that the potential at point B is lower than that at point A.
- The electric field can have a unit of volts per meter (V/m).

2.2 Electric Potential Energy in a Uniform Electric Field

- Now, if a charge q moves through this potential difference, the change in its electric potential energy is:

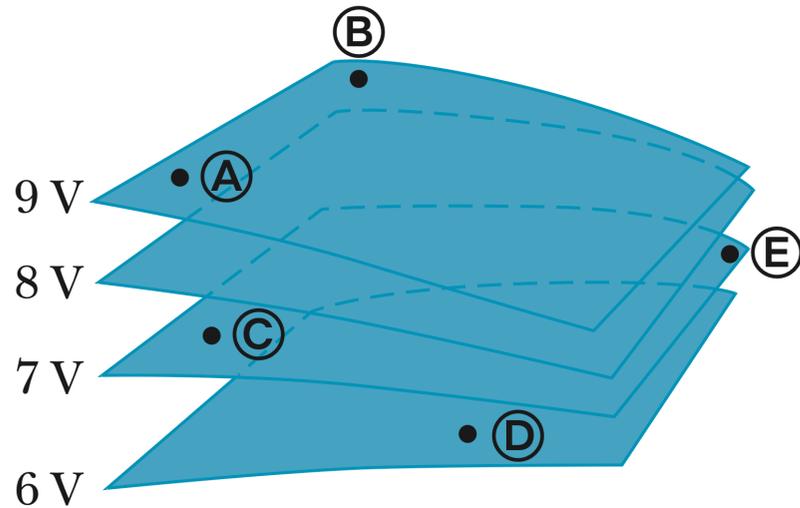
$$\Delta U_E = q\Delta V = -qEd$$

2.3 Equipotential Surfaces



- Point (B) and (C) are at the same potential; therefore, moving a charge from (B) to (C) requires no work against the electric field, since $\Delta V = 0$.
- The vertical line connecting points (B) and (C) has the same potential at every point along it.

2.3 Equipotential Surfaces



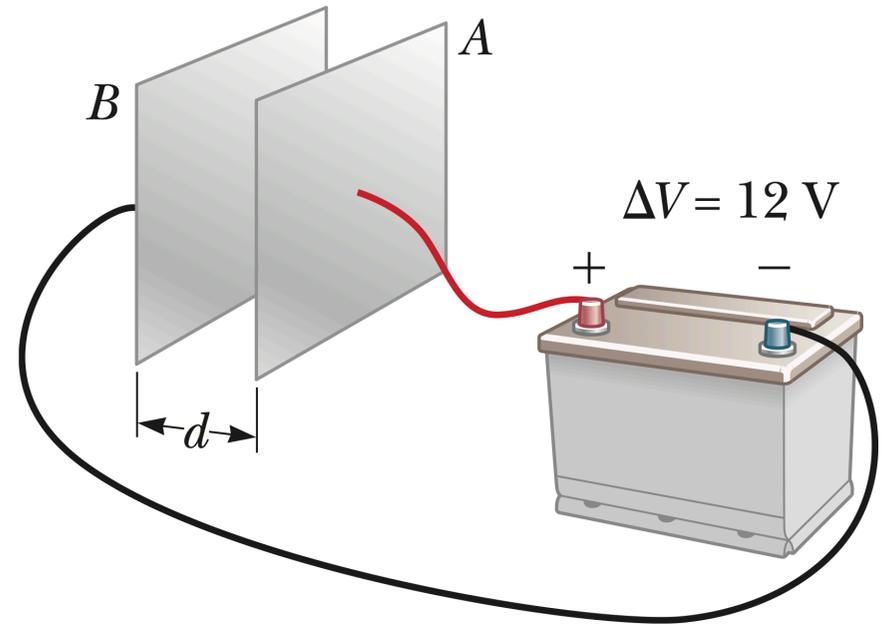
- An **equipotential surface** is a surface on which the electric potential is the same at every point.
- No work is done by the electric field when a charge moves along an equipotential surface because the potential difference is zero.
- Equipotential surfaces are always perpendicular to the electric field lines.
- In a uniform electric field, the equipotential surfaces are equally spaced parallel planes.

2.4 example

Example 2.1

A 12-V battery is connected between two parallel plates as shown in Figure. The separation between the plates is $d = 0.3$ cm, and we assume the electric field between the plates to be uniform.

Find the magnitude of the electric field between the plates.



2.4 example

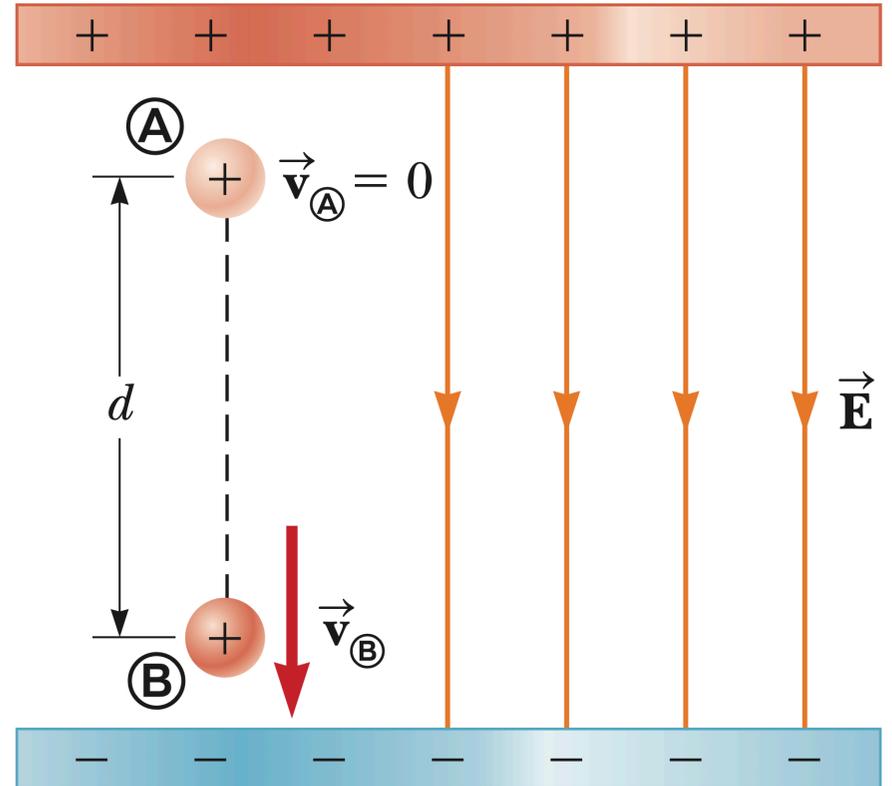
Solution 2.1

$$E = \frac{\Delta V}{d} = \frac{12}{0.3 \times 10^{-2}} = 4 \times 10^3 \text{ V/m}$$

2.4 example

Example 2.2

A proton is released from rest at point (A) in a uniform electric field that has a magnitude of 8×10^4 V/m . The proton undergoes a displacement of magnitude $d = 0.5$ m to point (B) in the direction of \vec{E} . Find the speed of the proton after completing the displacement.



2.4 example

Solution 2.2

$$\Delta K + \Delta U_E = 0$$

$$\left(\frac{1}{2}mv^2 - 0\right) + e\Delta V = 0$$

$$\Rightarrow v = \sqrt{\frac{-2e\Delta V}{m}} = \sqrt{\frac{-2e(-Ed)}{m}} = \frac{\sqrt{2eEd}}{m}$$

$$v = \sqrt{\frac{2(1.6 \times 10^{-19})(8 \times 10^4)(0.5)}{1.67 \times 10^{-27}}} = 2.8 \times 10^6 \text{ m/s}$$

1. Electric Potential and Potential Difference

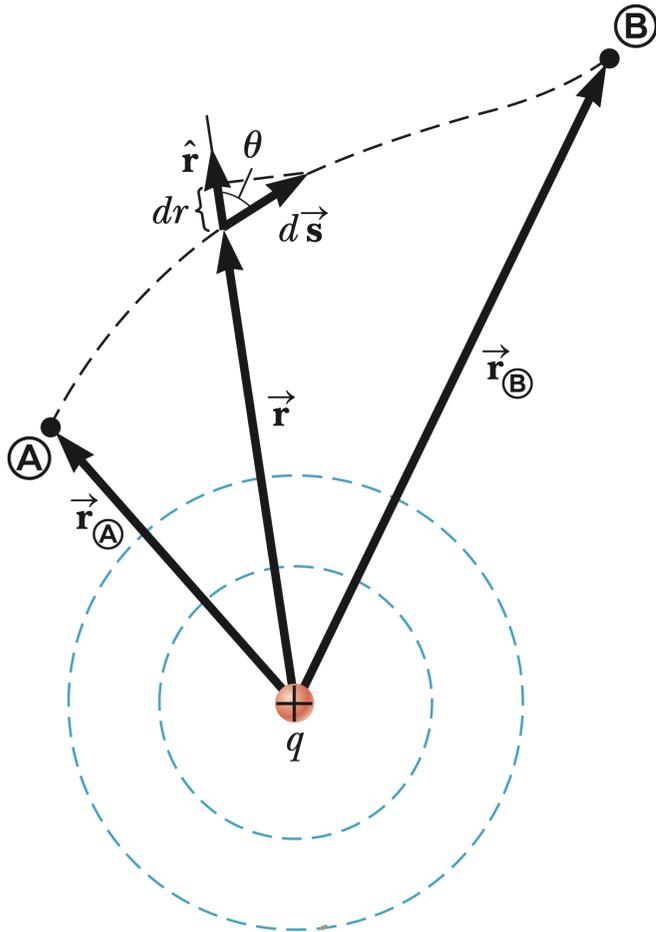
2. Potential Difference in a Uniform Electric Field

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4. Conductors in Electrostatic Equilibrium

5. Problems

3.1 Electric Potential Due to a Point Charge



- The electric potential difference between two points A and B due to a point charge q is given by:

$$V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{s} = \dots = k_e q \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

- Choosing $r_A = \infty$, we get $V_A = 0$, and the electric potential at point B due to the point charge q is:

$$V = k_e \frac{q}{r}$$

3.2 Electric Potential Due to Point Charges

- For a group of point charges, the total electric potential at a point is the sum of the potentials due to each charge:

$$V = k_e \sum \left(\frac{q_i}{r_i} \right)$$

- Notice that the electric potential is a scalar quantity (no vector addition); therefore, we can add the potentials algebraically, taking into account their signs.

3.3 Electric Potential Energy of two Point Charges

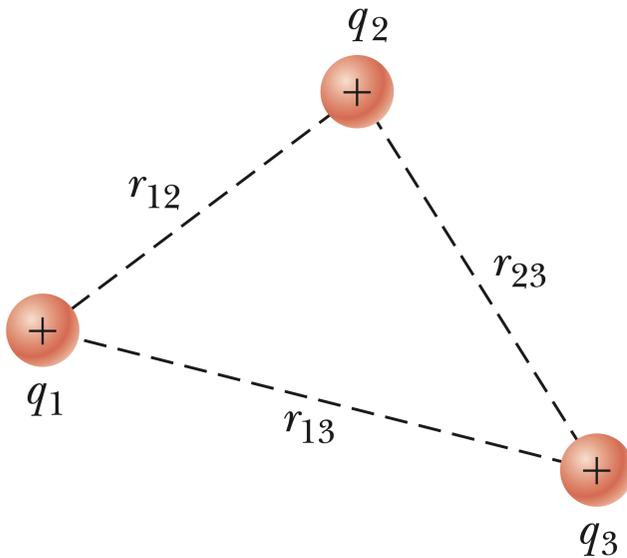
- The electric potential energy of a system of two point charges q_1 and q_2 separated by a distance r_{12} is given by:

$$\Delta U_E = q_2 \Delta V$$

therefore,

$$U_E = k_e \frac{q_1 q_2}{r_{12}}$$

3.4 Electric Potential Energy of a System of Point Charges



If the system consists of more than two charges, the total potential energy of the system is the sum of the potential energies for all pairs of charges:

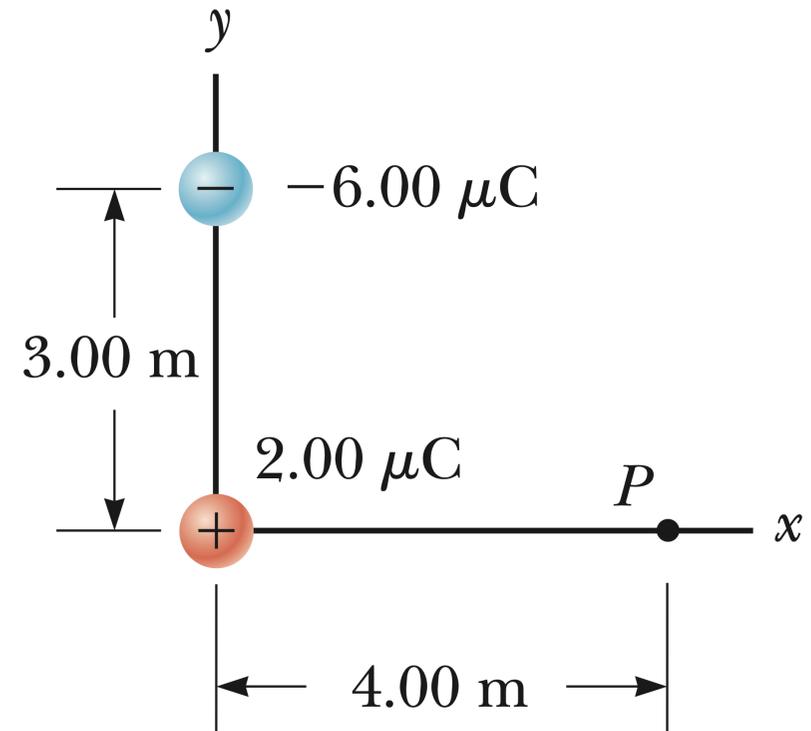
$$U_E = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} + \dots \right)$$

3.5 Example

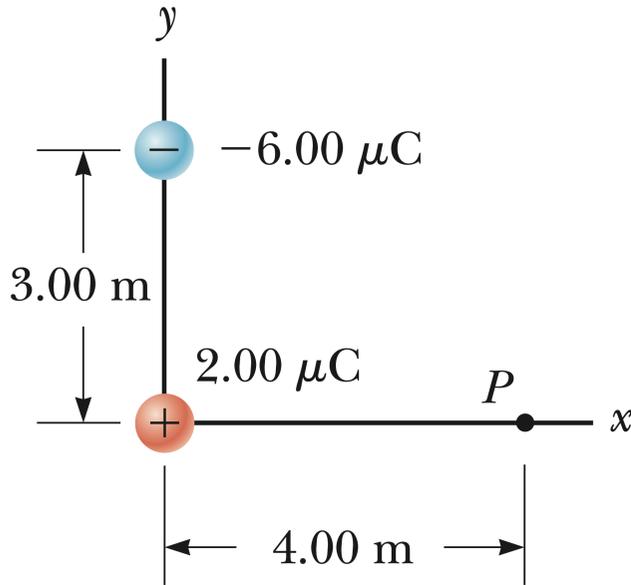
Example 3.3

As shown in the Figure, a charge $q_1 = 2\mu\text{C}$ is located at the origin and a charge $q_2 = -6\mu\text{C}$ is located at $(0, 3)$ m.

(A) Find the total electric potential due to these charges at the point P, whose coordinates are $(4, 0)$ m.



3.5 Example



Solution 3.3

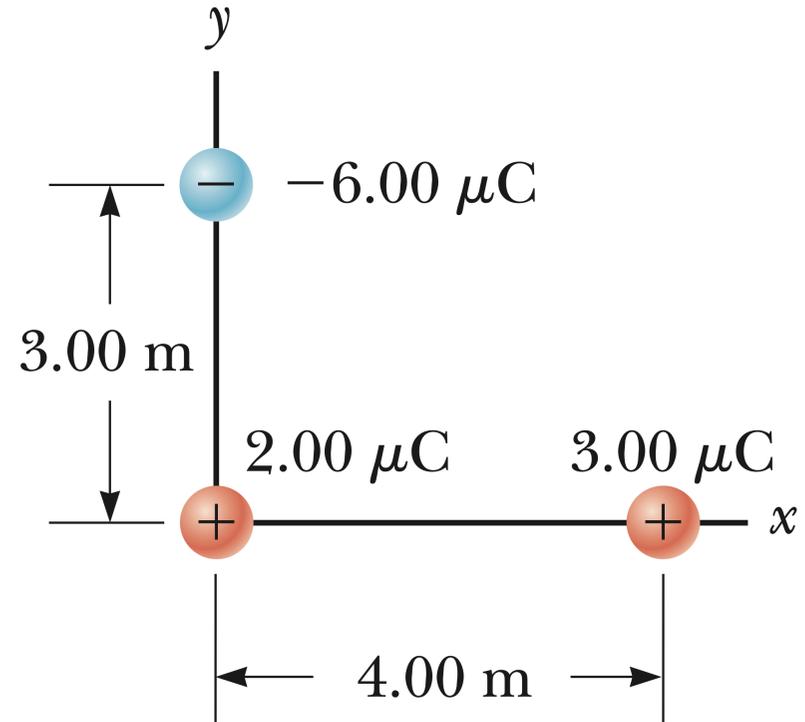
$$V_p = k_e \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} \right)$$

$$V_p = (9 \times 10^9) \left(\frac{2 \times 10^{-6}}{4} + \frac{-6 \times 10^{-6}}{5} \right)$$

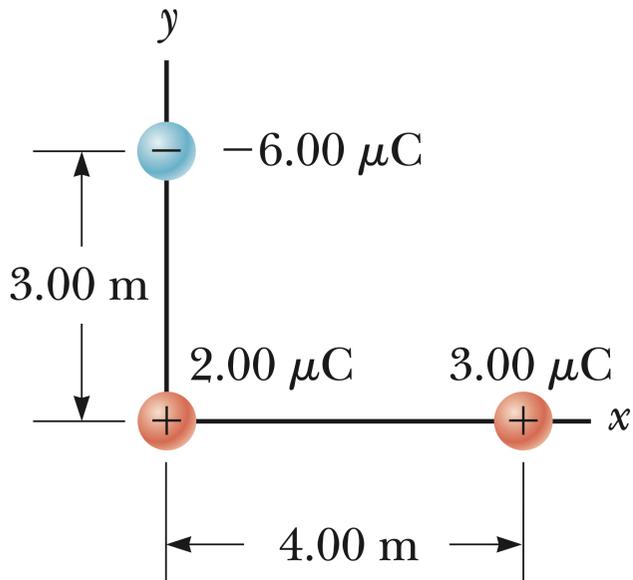
$$V_p = -6.29 \times 10^3 \text{ V}$$

3.5 Example

(B) Find the change in potential energy of the system of two charges plus a third charge $q_3 = 3\mu\text{C}$ as the later charge moves from infinity to point P.



3.5 Example



Solution 3.3

$$\Delta U_E = U_f - U_i = q_3 V_p - 0$$
$$\Delta U_E = (3 \times 10^{-6})(-6.29 \times 10^3)$$
$$\Delta U_E = -1.89 \times 10^{-2} \text{ J}$$

1. Electric Potential and Potential Difference

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4. Conductors in Electrostatic Equilibrium

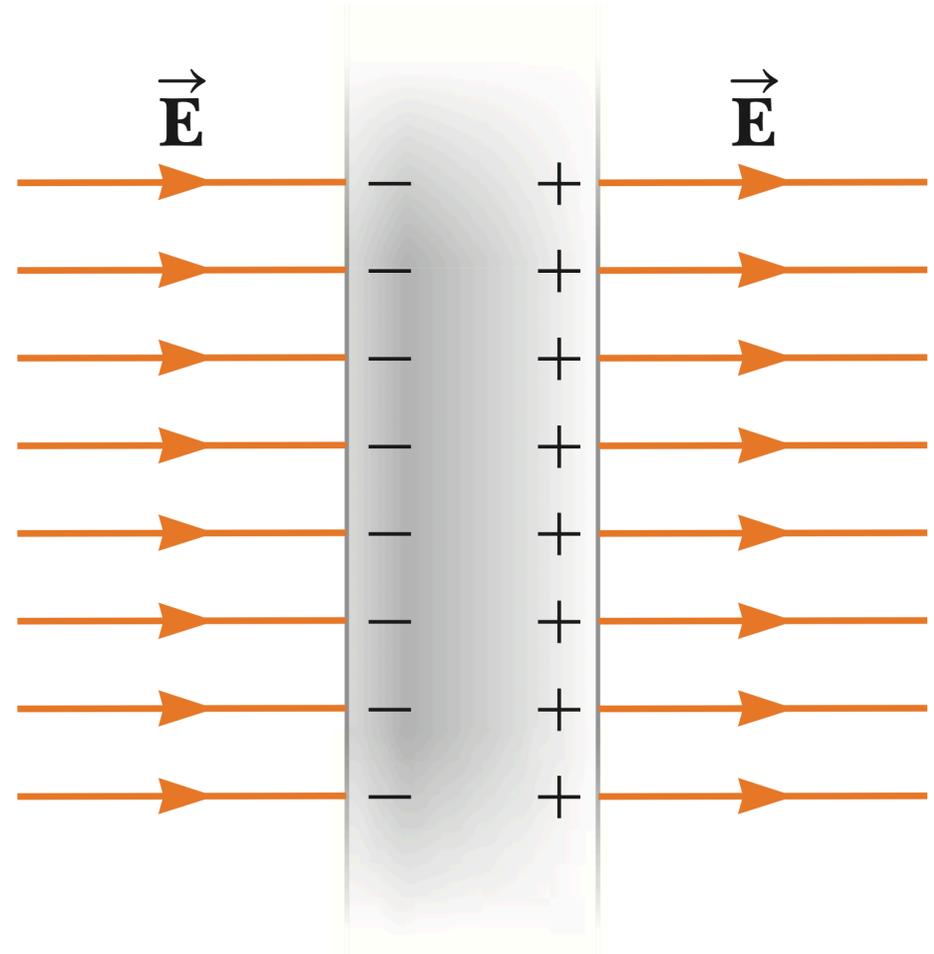
5. Problems

4.1 What is a Conductor?

A conductor is a material that contains free charges (electrons) that are unbound to any atom and therefore can move throughout the material.

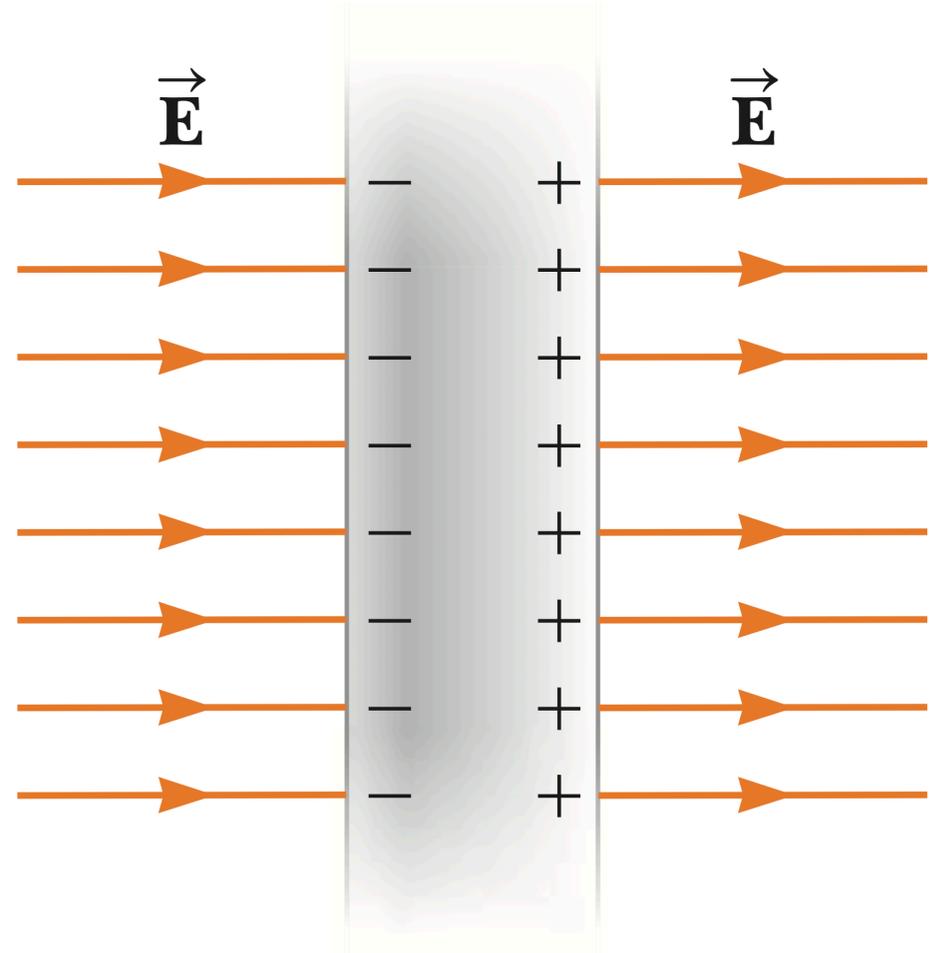
What is Electrostatic Equilibrium?

When a conductor is placed in an electric field, the free charges inside the conductor will move in response to the electric field until they reach a state of **electrostatic equilibrium**, where there is no net motion of charges within the conductor.



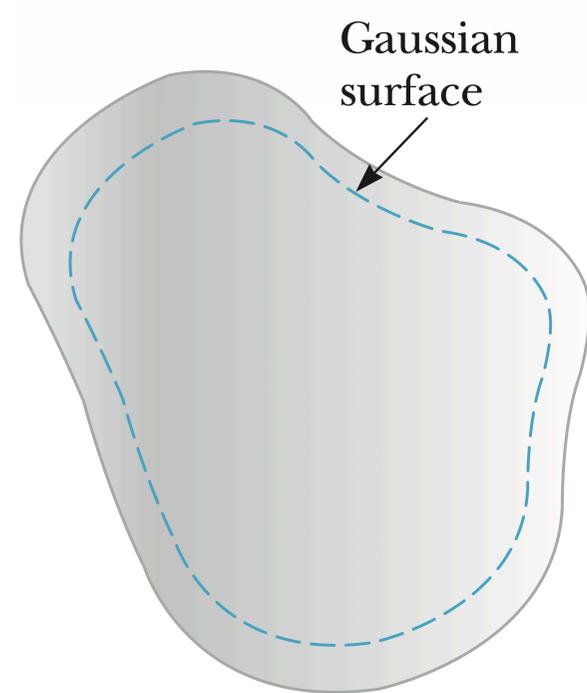
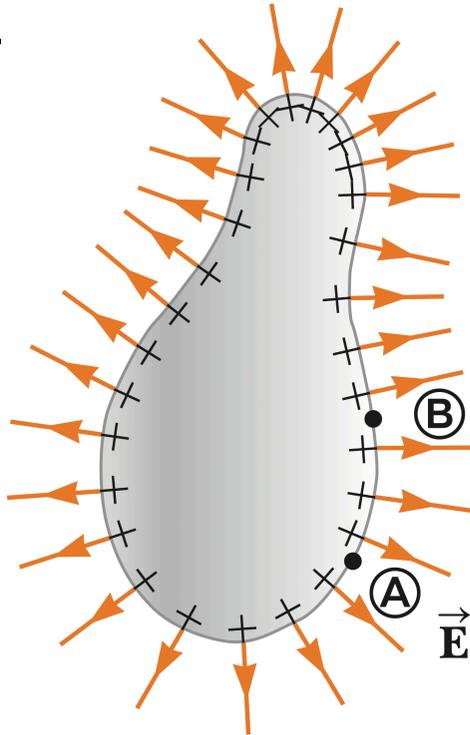
4.2 What are the Four Properties of a Conductor in Electrostatic Equilibrium?

1. The electric field inside the conductor is *zero*, whether the conductor is solid or hollow.



4.2 What are the Four Properties of a Conductor in Electrostatic Equilibrium?

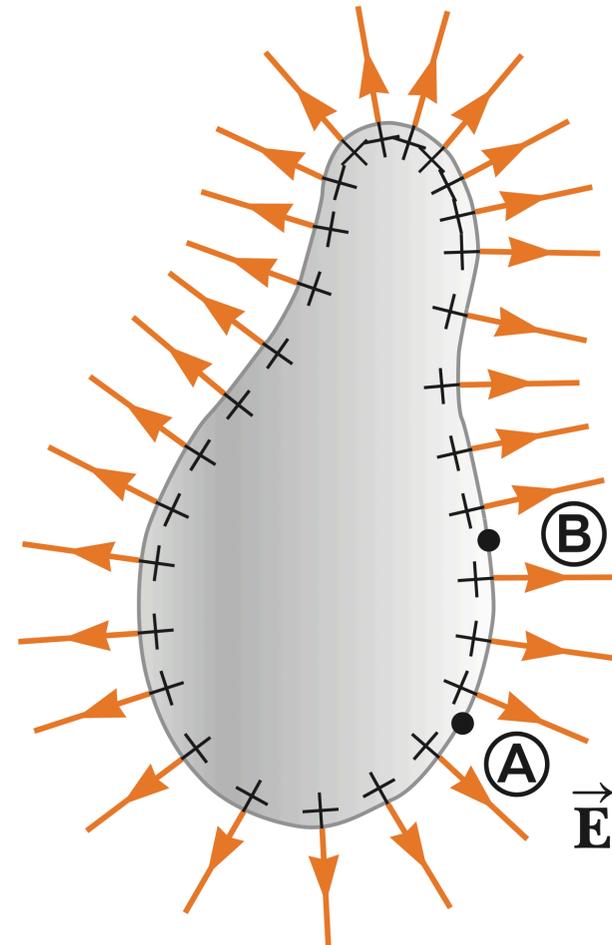
2. If the conductor is isolated and carries a charge, the charge resides on its *surface*.



- The net charge, electric field, and flux inside the gaussian surface is zero.

4.2 What are the Four Properties of a Conductor in Electrostatic Equilibrium?

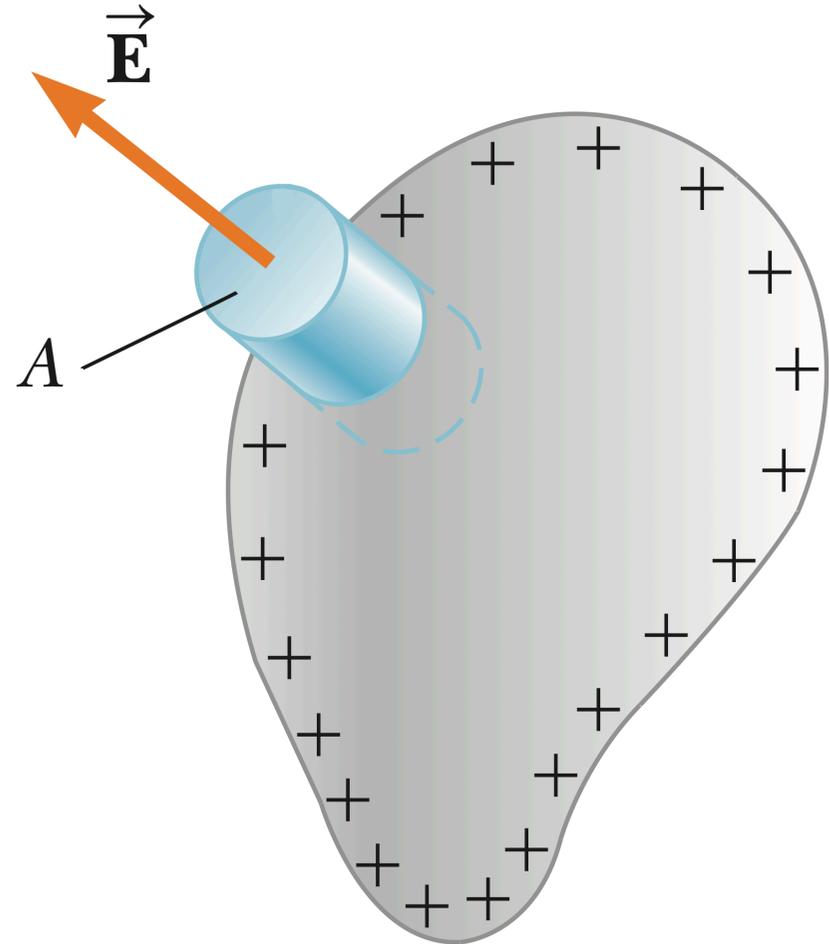
3. The electric field at a point just outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude σ/ϵ_0 , where σ is the surface charge density at that point.



4.2 What are the Four Properties of a Conductor in Electrostatic Equilibrium?

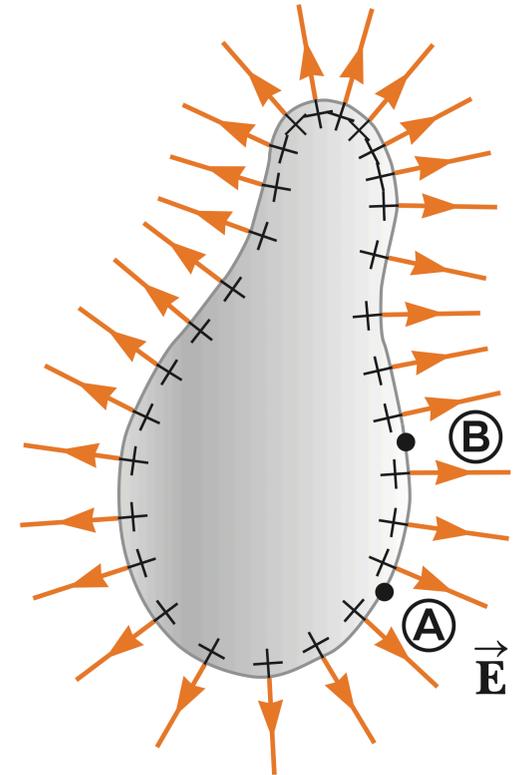
To find the electric field just outside a charged conductor, we use a small gaussian cylinder that extends just outside the conductor's surface. The flux through the curved surface is zero, but the flux through the flat surface (outside the conductor) is:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$
$$\implies E = \frac{\sigma}{\epsilon_0}$$



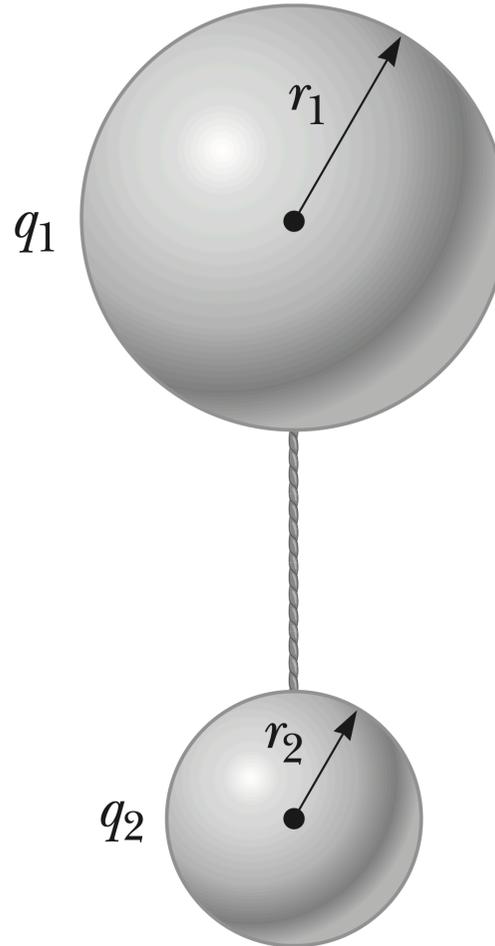
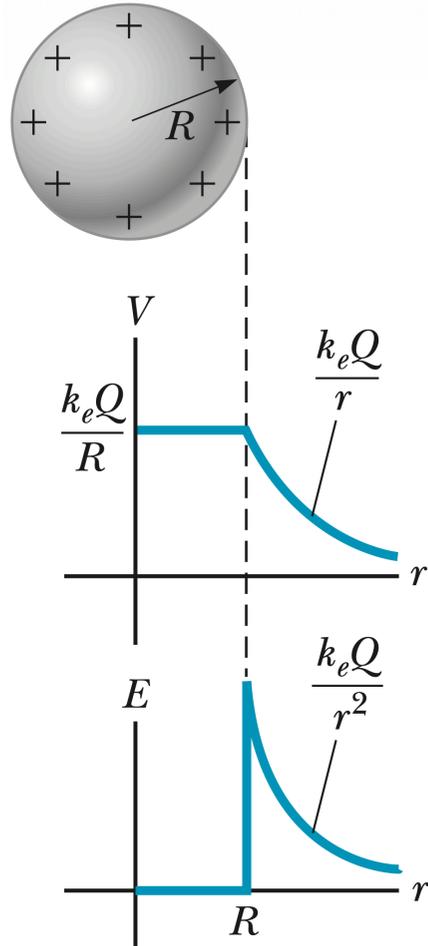
4.2 What are the Four Properties of a Conductor in Electrostatic Equilibrium?

4. On an irregularly shaped conductor, the surface charge density is greatest at locations where the radius of curvature of the surface is smallest.
- Even though charges are nonuniformly distributed on the surface, the electric potential is constant everywhere on the surface of the conductor.



$$V_A = V_B$$

4.3 Electric Field and Potential of a Conducting Sphere



A conducting wire is connected to two conducting spheres spaced far apart. The electric potential is the same on both spheres,

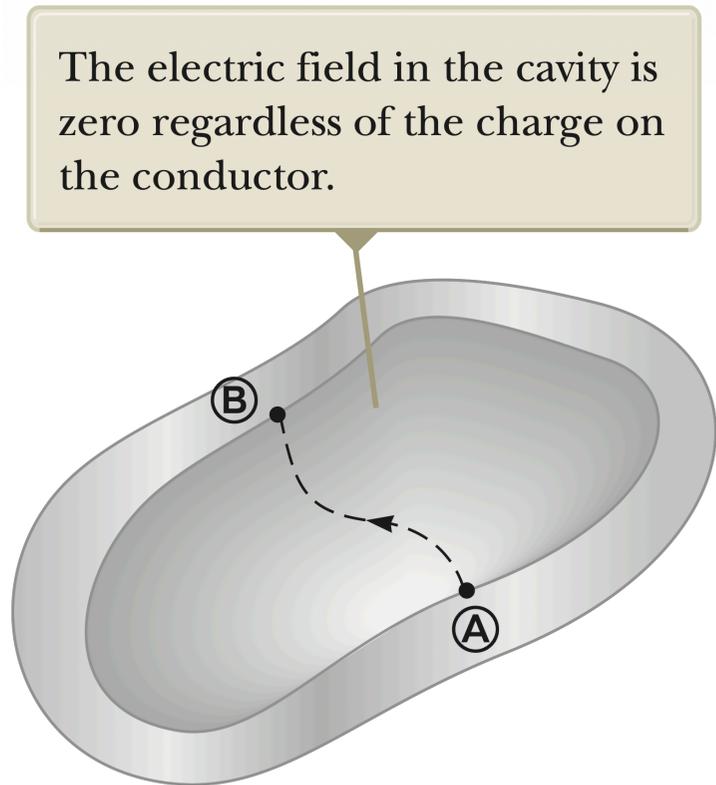
$$V_1 = V_2,$$

and the electric field just outside the surface of each sphere is:

$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$

4.4 A Cavity Inside a Conductor

- When there is no charge inside the cavity, the electric field inside the cavity is zero, even if there is an electric field outside the conductor!
- This cavity is also known as a **Faraday Cage** and is used to shield sensitive electronic equipment from external electric fields and electromagnetic radiation.



$$V_A = V_B$$

Suggested Problems

1, 2, 3, 8, 34, 35

Book: Serway, R. A., & Jewett, J. W. (2018). Physics for Scientists and Engineers (10th ed.)

Chapter: 24 - Electric Potential

1. Electric Potential and Potential Difference

2. Potential Difference in a Uniform Electric Field

3. Electric Potential and Potential Energy Due to Point Charges

4. Conductors in Electrostatic Equilibrium

5. Problems

5.1 Electric Potential and Potential Difference

Problem 5.1

- V** 1. How much work is done (by a battery, generator, or some other source of potential difference) in moving Avogadro's number of electrons from an initial point where the electric potential is 9.00 V to a point where the electric potential is -5.00 V ? (The potential in each case is measured relative to a common reference point.)

5.1 Electric Potential and Potential Difference

Answer 5.1

$$W = Q\Delta V$$

$$Q = -N_a q_e = -(6.02 \times 10^{23})(1.6 \times 10^{-19}) = -9.63 \times 10^4 \text{ C}$$

$$\Delta V = V_f - V_i = -5 - 9 = -14 \text{ V}$$

Therefore,

$$W = (-9.63 \times 10^4)(-14) = 1.35 \times 10^6 \text{ J}$$

5.1 Electric Potential and Potential Difference

Problem 5.2

- 2.** (a) Find the electric potential difference ΔV_e required to stop an electron (called a “stopping potential”) moving with an initial speed of 2.85×10^7 m/s. (b) Would a proton traveling at the same speed require a greater or lesser magnitude of electric potential difference? Explain. (c) Find a symbolic expression for the ratio of the proton stopping potential and the electron stopping potential, $\Delta V_p / \Delta V_e$.

5.1 Electric Potential and Potential Difference

Answer 5.2

a)

$$\begin{aligned}\Delta K + \Delta U_E &= 0 \\ \left(0 - \frac{1}{2}mv_i^2\right) + q\Delta V &= 0 \\ \Rightarrow \Delta V &= \frac{mv_i^2}{2q}\end{aligned}$$

Using $q = -e$ and $m = m_e$ for an electron, we get

$$\begin{aligned}\Delta V &= \frac{(9.11 \times 10^{-31})(2.85 \times 10^7)^2}{2 \times 1.6 \times 10^{-19}} \\ \Delta V &= -2.31 \times 10^3 \text{ V}\end{aligned}$$

5.1 Electric Potential and Potential Difference

b) The stopping potential for a proton is greater than that for an electron because the proton has a much larger mass than the electron, and therefore requires more energy (and thus a greater potential difference) to stop it.

c)

$$\frac{\Delta V_p}{\Delta V_e} = \frac{m_p}{m_e}$$

5.2 Potential Difference in a Uniform Electric Field

Problem 5.3

- T** 3. Oppositely charged parallel plates are separated by 5.33 mm. A potential difference of 600 V exists between the plates. (a) What is the magnitude of the electric field between the plates? (b) What is the magnitude of the force on an electron between the plates? (c) How much work must be done on the electron to move it to the negative plate if it is initially positioned 2.90 mm from the positive plate?

5.2 Potential Difference in a Uniform Electric Field

Answer 5.3

a)

$$E = \frac{\Delta V}{d} = \frac{600}{5.33 \times 10^{-3}} = 1.13 \times 10^5 \text{ N/C.}$$

b)

$$F = qE = (1.6 \times 10^{-19})(1.13 \times 10^5) = 1.8 \times 10^{-14} \text{ N}$$

c)

$$W = F \cdot s = (1.8 \times 10^{-14})((5.33 - 2.9) \times 10^{-3}) = 4.37 \times 10^{-17} \text{ J}$$

5.2 Potential Difference in a Uniform Electric Field

Problem 5.4

8. Two point charges $Q_1 = +5.00 \text{ nC}$ and $Q_2 = -3.00 \text{ nC}$ are separated by 35.0 cm. (a) What is the electric potential at a point midway between the charges? (b) What is the potential energy of the pair of charges? What is the significance of the algebraic sign of your answer?

5.2 Potential Difference in a Uniform Electric Field

Answer 5.4

a)

$$V = K_e \sum \frac{q_i}{r_i} = k_e \left(\frac{Q_1}{r} + \frac{Q_2}{r} \right) = 103 \text{ V}$$

where $r = \frac{d}{2} = 0.175 \text{ m}$

b)

$$U_E = k_e \frac{q_1 q_2}{r_{12}} = -3.85 \times 10^{-7} \text{ J}$$

The negative sign indicates that the potential energy of the system is attractive, and you would need to do positive work to separate the charges to infinity where the potential energy would be zero.

5.2 Potential Difference in a Uniform Electric Field

Problem 5.5

- 34.** A solid conducting sphere of radius 2.00 cm has a charge of $8.00 \mu\text{C}$. A conducting spherical shell of inner radius 4.00 cm and outer radius 5.00 cm is concentric with the solid sphere and has a charge of $-4.00 \mu\text{C}$. Find the electric field at (a) $r = 1.00$ cm, (b) $r = 3.00$ cm, (c) $r = 4.50$ cm, and (d) $r = 7.00$ cm from the center of this charge configuration.

5.2 Potential Difference in a Uniform Electric Field

Answer 5.5

a) $E = 0$

b) $E = K_e Q / r^2 = 7.99 \times 10^7 \text{ N/C}$

c) $E = 0$

d) $E = K_e Q_{\text{inside}} / r^2 = 7.34 \times 10^6 \text{ N/C}$

where $Q_{\text{inside}} = (8 + -4)\mu\text{C} = 4\mu\text{C}$ is the charge enclosed by the gaussian surface at radius r .

5.2 Potential Difference in a Uniform Electric Field

Problem 5.6

- 35.** A spherical conductor has a radius of 14.0 cm and a charge of $26.0 \mu\text{C}$. Calculate the electric field and the electric potential at (a) $r = 10.0$ cm, (b) $r = 20.0$ cm, and (c) $r = 14.0$ cm from the center.
- T**

5.2 Potential Difference in a Uniform Electric Field

Answer 5.6

- a) $E = 0$
 $V = k_e q/R = 1.67 \text{ MV}$
- b) $E = K_e q/r^2 = 5.84 \times 10^6 \text{ N/C}$
 $V = k_e q/r = 1.17 \text{ MV}$
- c) $E = K_e q/R^2 = 11.9 \times 10^6 \text{ N/C}$
 $V = k_e q/R = 1.67 \text{ MV}$