Ch.22: Electric Fields

Physics 104: Electricity and Magnetism

Dr. Abdulaziz Alqasem

Physics and Astronomy Department
King Saud University

2026

Outline

جامعة الملك سعود
King Saud University

1. Coulomb’s Law

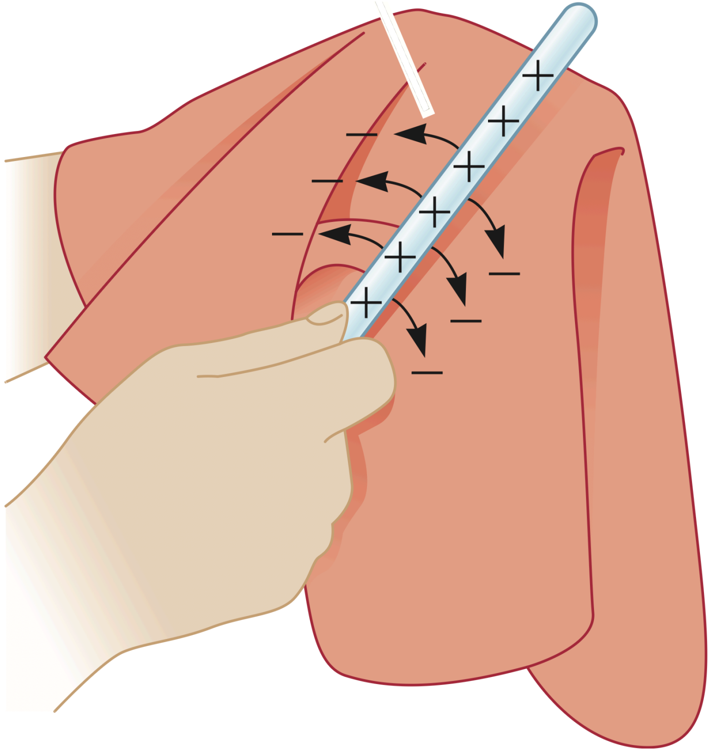
2. Analysis Model: Particle in a Field (Electric)

3. Electric Field Lines

4. Motion of a Charged Particle in a Uniform Electric Field

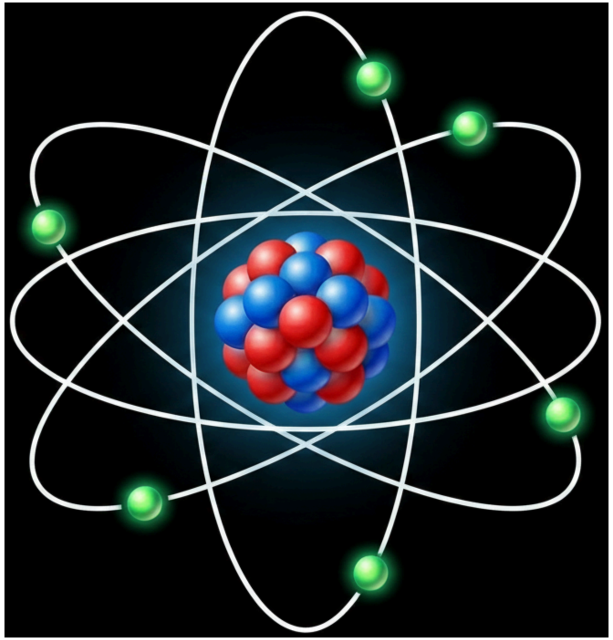
1.1 Electric Charges is everywhere!

1.2 Can you induce and transfer electric?



- Yes, one way to do it is by **friction** between two different materials.
- Electrons can be *transferred* from one material to another, resulting in one object becoming:
  - Positively charged (losing electrons) and,
  - The other becoming negatively charged (gaining electrons).

1.3 Origin of Electric Charges

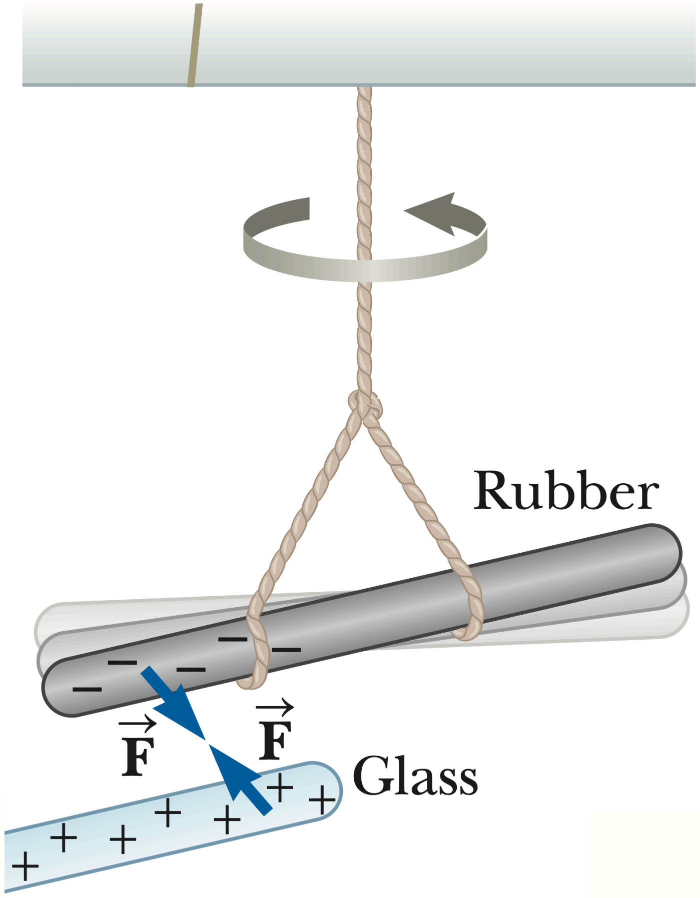


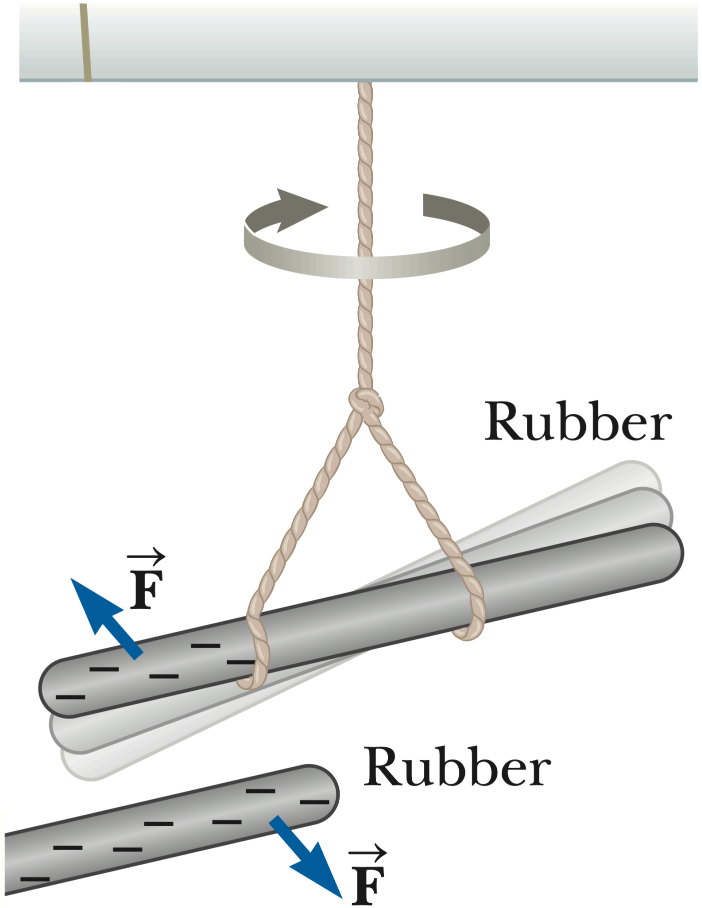
Cartoon of an atom
Electrons are shown in green

- The sources of electric charges are **electrons** and **protons** that make up atoms and molecules.

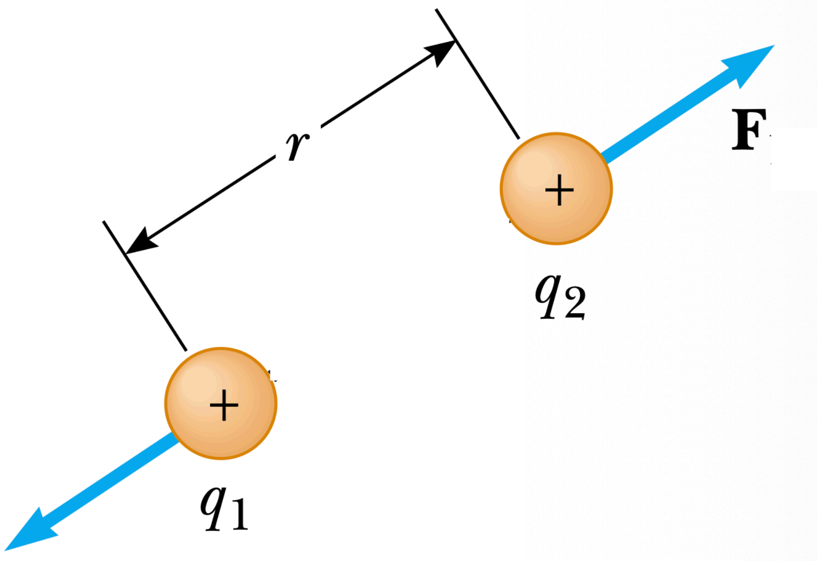
- **Point Charges** (like electrons) are objects that have electric charge and are small enough to be treated as points (of zero size!).

1.4 Electric Force between Electrically Charged Objects





1.5 Magnitude of the Electric Force



Coulomb's Law: The magnitude of the electric force F_e between two point charges q_1 and q_2 separated by a distance r is given by:

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

where k_e is the Coulomb's constant, given by:

$$k_e = \frac{1}{4\pi\varepsilon_0} \approx 8.988 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$$

and $\varepsilon_0 \approx 8.85 \times 10^{-12} \ \mathrm{C^2/\ N \cdot m^2}$ is the permittivity of free space.

1.5 Magnitude of the Electric Force

The smallest unit of charge is the charge of an electron (e) or a proton (p), which is

$$|e| = p = 1.602 \times 10^{-19}\ \text{C}.$$

Quiz

How many electrons in one coulomb ($Q = 1\ \text{C}$)?

Answer:

$$N_e = \frac{Q}{|e|} = \frac{1\ \text{C}}{1.6 \times 10^{-19}\ \text{C}} \approx 6.25 \times 10^{18} \quad \text{electrons.}$$

1.5 Magnitude of the Electric Force

Example 1.1

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately $\boldsymbol{d = 5.3 \times 10^{-11}\ \mathrm{m}}$.

Find the magnitudes of the electric force and the gravitational force between the two particles.

1.5 Magnitude of the Electric Force

Solution 1.1

We use Coulomb's law to find the electric force between the electron and proton.

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

Both the electron and proton have the same charge magnitude:

$$|q_1| = |q_2| = |e| \approx 1.6 \times 10^{-19}\ \text{C}$$

Therefore the electric force between them is:

$$F_e = k_e \frac{e^2}{r^2} = (9 \times 10^9) \frac{(1.6 \times 10^{-19})^2}{(5.3 \times 10^{-11})^2} = 8.2 \times 10^{-8}\ \text{N}$$

1.5 Magnitude of the Electric Force

For the gravitational force, we use Newton's law of gravitation:

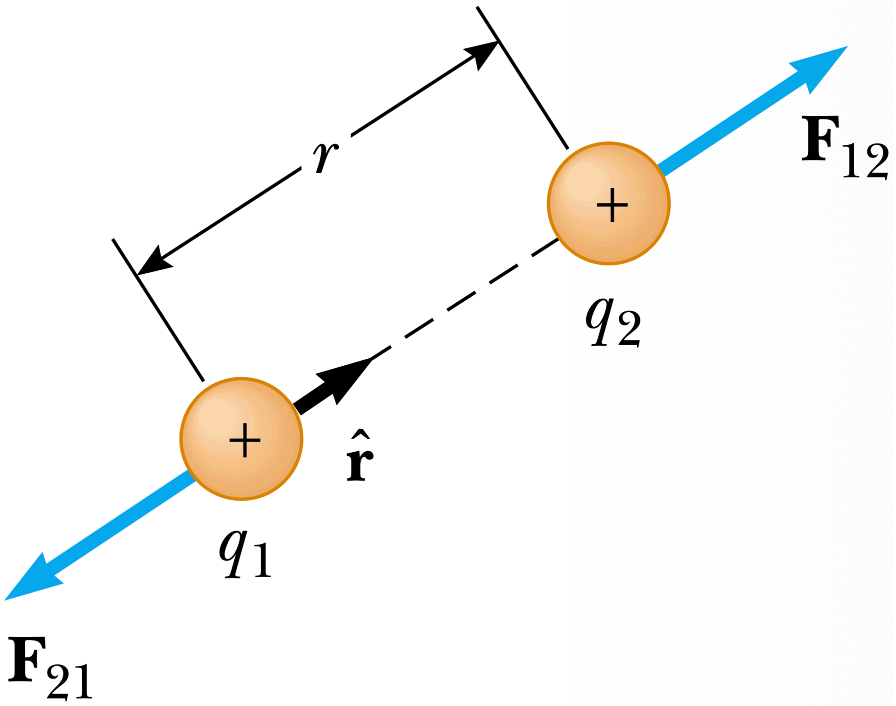
$$F_g = G\frac{m_e m_p}{r^2} \approx 3.6 \times 10^{-47}\ \text{N}$$

The ratio between the two forces is about

$$\frac{F_e}{F_g} \sim 10^{39}$$

indicating that the gravitational force is negligible compared to the electric force at atomic scale.

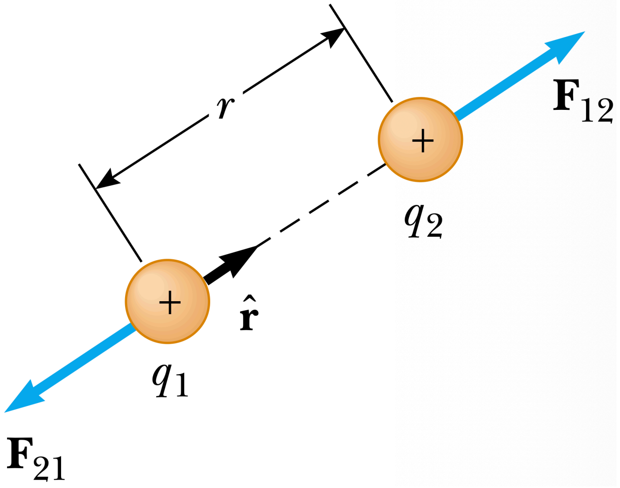
1.6 Direction of the Electric Force

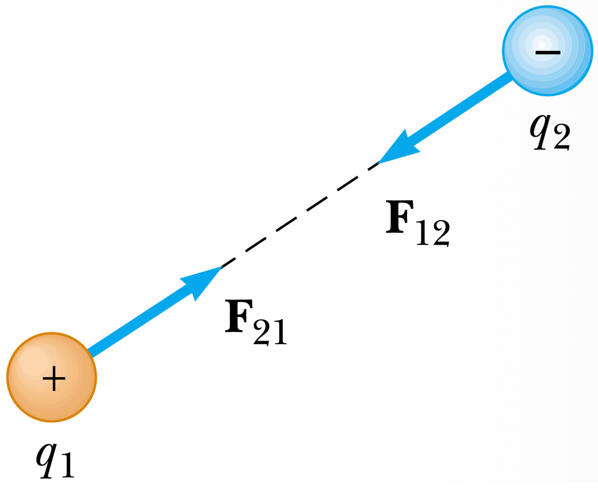


$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

- $\hat{r}_{12}$ is a unit vector pointing from charge q_1 to charge q_2.
- $\vec{F}_{12}$ reads as:
- **The force on charge 2 due to charge 1,** or
- **The force exerted by charge 1 on charge 2.**

1.6 Direction of the Electric Force





Repulsive Force: If q_1 and q_2 have the same sign, then $\vec{F}_{12}$ is along $\hat{r}_{12}$.

Attractive Force: If q_1 and q_2 have opposite signs, then $\vec{F}_{12}$ is opposite to $\hat{r}_{12}$.

$$\vec{F}_{21} = -\vec{F}_{12}$$

1.7 Electric Force due to Multiple Point Charges

The net electric force $\vec{\boldsymbol{F}}_1$ on a point charge q_1 due to **multiple** point charges is the vector sum of the individual forces exerted on q_1 by each of the other charges.

$$\vec{\boldsymbol{F}}_1 = \vec{\boldsymbol{F}}_{21} + \vec{\boldsymbol{F}}_{31} + \vec{\boldsymbol{F}}_{41} + \ldots + \vec{\boldsymbol{F}}_{n1}$$

1.7 Electric Force due to Multiple Point Charges

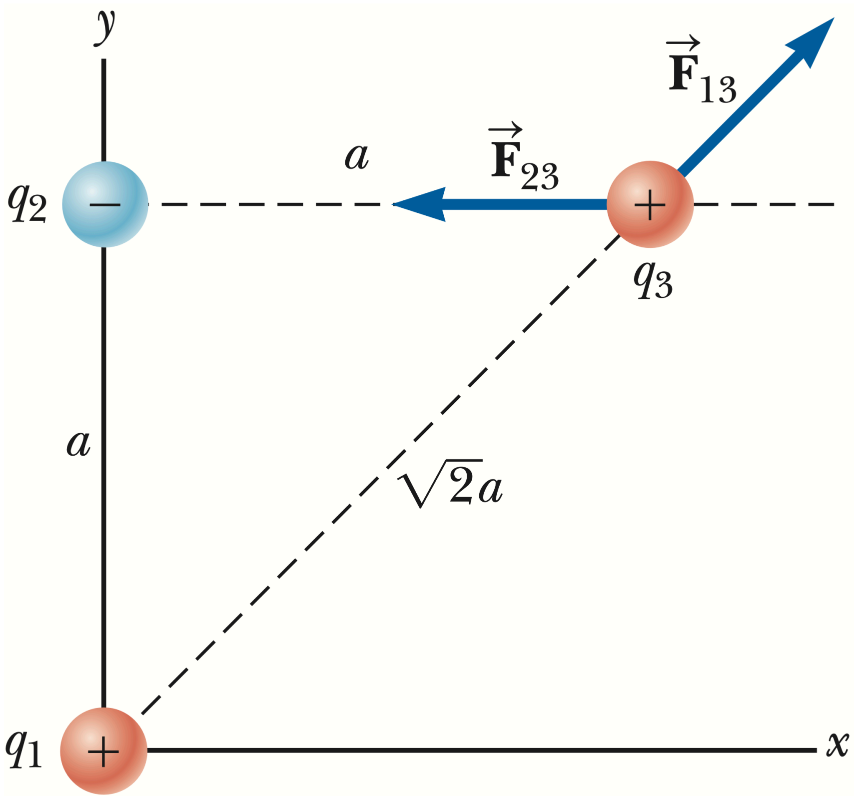
Example 1.2

Consider three point charges located at the corners of a right triangle as shown in the figure, where

$$q_1 = q_3 = 5\ \mu\text{C}$$
$$q_2 = -2\ \mu\text{C}$$
$$a = 0.1\ \text{m}$$

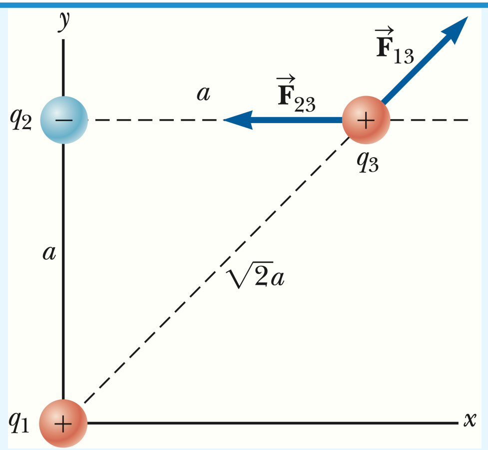
.

Find the resultant force exerted on q_3



1.7 Electric Force due to Multiple Point Charges

Solution 1.2



The net force exerted on q_3 is:

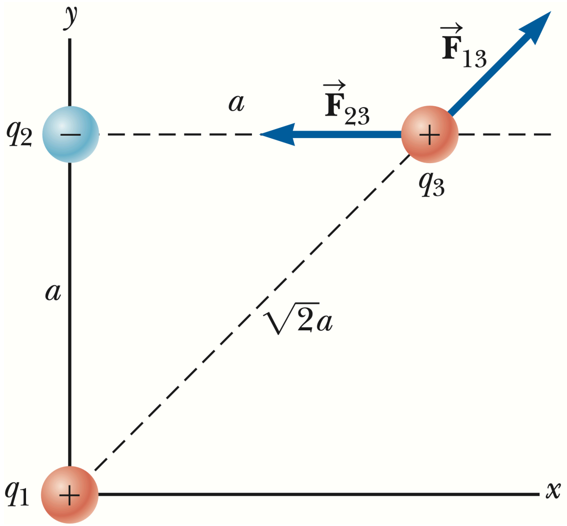
$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2} = (8.988 \times 10^9) \frac{(2 \times 10^{-6})(5 \times 10^{-6})}{(0.1)^2} = 8.99 \;\; \text{N}$$

from the figure, we see that the direction of $\vec{F}_{23}$ is along the negative x-axis. Therefore,

$$\vec{F}_{23} = -8.99 \; \hat{\imath} + 0 \; \hat{\jmath}$$

1.7 Electric Force due to Multiple Point Charges



$$F_{13} = k_e \frac{|q_1||q_3|}{\left(\sqrt{2}a\right)^2}$$

$$= \left(8.988 \times 10^9\right) \frac{\left(5 \times 10^{-6}\right)^2}{2 * (0.1)^2} = 11.2 \quad \text{N}$$

- The direction of $\vec{F}_{13}$ is at 45° above the positive x-axis, because

$$\tan\theta = \frac{a}{a} = 1 \quad \Longrightarrow \quad \theta = \tan^{-1}(1) = 45°$$

Therefore,

$$\vec{F}_{13} = F_{13,x} \quad \hat{\imath} \quad + \quad F_{13,y} \quad \hat{\jmath}$$
$$= F_{13} \cos 45° \; \hat{\imath} + F_{13} \sin 45° \; \hat{\jmath}$$

1.7 Electric Force due to Multiple Point Charges

King Saud University

$$\vec{F}_{13} = 11.2\cos 45^\circ\ \hat{\imath} + 11.2\sin 45^\circ\ \hat{\jmath}$$
$$= 7.94\ \hat{\imath} + 7.94\ \hat{\jmath}$$

Therefore, the net force on q_3 is:

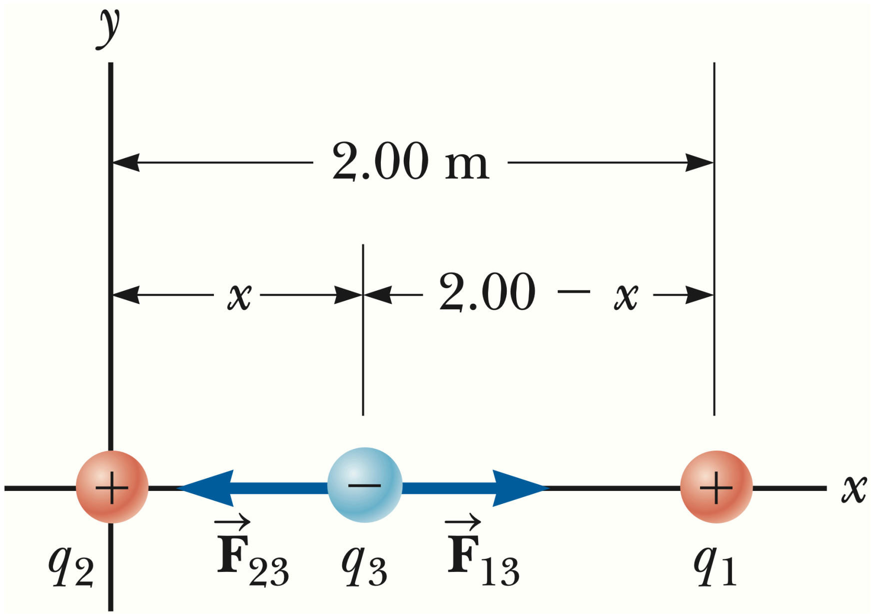
$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$
$$= \left[F_{13,x} + F_{23,x}\right]\ \hat{\imath} + \left[F_{13,y} + F_{23,y}\right]\ \hat{\jmath}$$
$$= (7.94 - 8.99)\ \hat{\imath} + (0 + 7.94)\ \hat{\jmath}$$
$$= (-1.04\ \hat{\imath} + 7.94\ \hat{\jmath})\quad \text{N}$$

1.7 Electric Force due to Multiple Point Charges

Example 1.3

Three point charges lie along the x-axis as shown in the Figure. The positive charge $q_1 = 15\mu\text{C}$ is at $x = 2$ m, the positive charge $q_2 = 6\mu\text{C}$ is at the origin, and the net force acting on q_3 is zero.

What is the x coordinate of q_3?



1.7 Electric Force due to Multiple Point Charges

King Saud University

Solution 1.3

$$\vec{\boldsymbol{F}}_3 = \vec{\boldsymbol{F}}_{13} + \vec{\boldsymbol{F}}_{23} = 0 \text{ N}$$

Therefore, the two forces on charge q_3 has to be equal in magnitude and opposite in direction,

$$F_{23} = F_{13}$$

$$k_e \frac{|q_2||q_3|}{r_{23}^2} = k_e \frac{|q_1||q_3|}{r_{13}^2}$$

$$\frac{|q_2|}{r_{23}^2} = \frac{|q_1|}{r_{13}^2}$$

$$\Rightarrow r_{23}^2 |q_1| = r_{13}^2 |q_2|$$

$$x^2 |q_1| = (2 - x)^2 |q_2|$$

1.7 Electric Force due to Multiple Point Charges

$$\pm x\sqrt{|q_1|} = (2-x)\sqrt{|q_2|}$$

$$\pm x\sqrt{|q_1|} + x\sqrt{|q_2|} = 2\sqrt{|q_2|}$$

$$x\left(\pm\sqrt{|q_1|} + \sqrt{|q_2|}\right) = 2\sqrt{|q_2|}$$

$$\Rightarrow x = 2\frac{\sqrt{|q_2|}}{\pm\sqrt{|q_1|} + \sqrt{|q_2|}}$$

$$x = 2\frac{\sqrt{6\times 10^{-6}\ \mathrm{C}}}{\pm\sqrt{15\times 10^{-6}\ \mathrm{C}} + \sqrt{6\times 10^{-6}\ \mathrm{C}}}$$

$$x = 0.774\ \mathrm{m} \quad \text{or} \quad x = -3.44\ \mathrm{m}$$

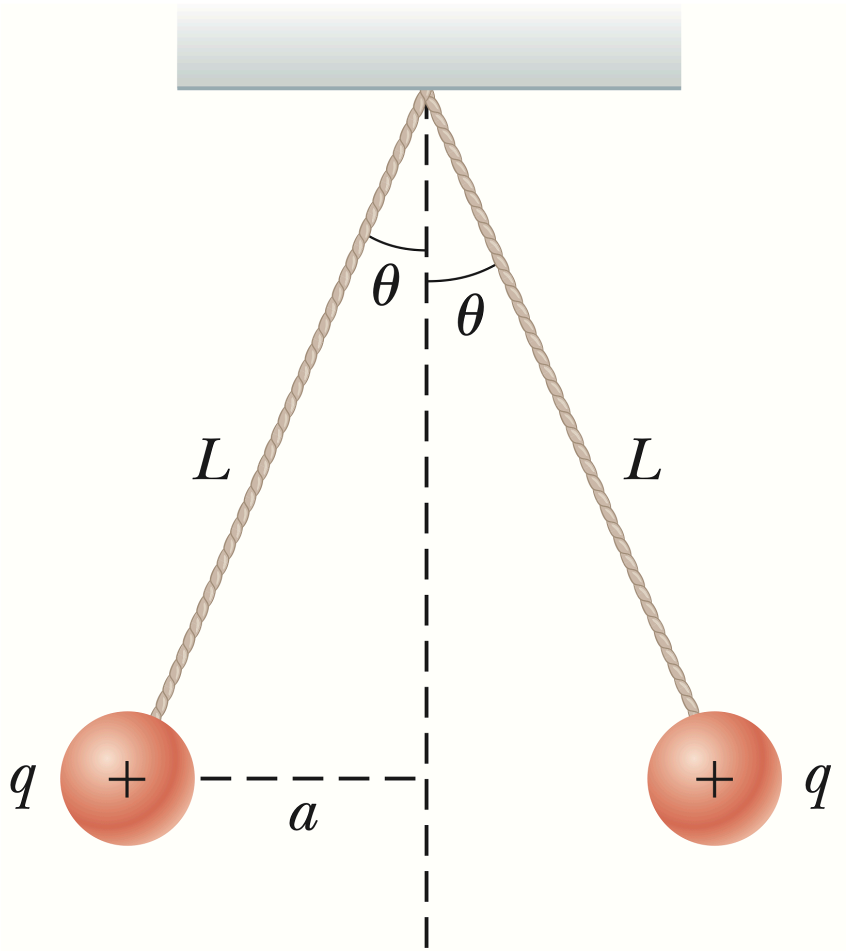
Only the positive x value is physical and acceptable here. The negative value is rejected because the net force will not be zero if q_3 is not between q_1 and q_2.

1.8 Electric Force and Weight of Charged Objects

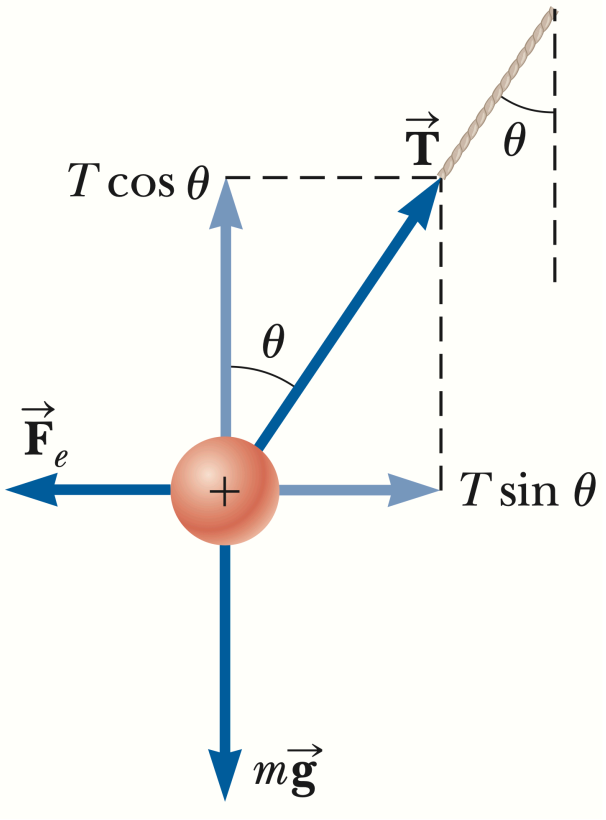
Example 1.4

Two identical small charged spheres, each having a mass of 3×10^{-2}kg, hang in equilibrium as shown in the Figure. The length L of each string is 0.15 m, and the angle θ is 5°.

Find the magnitude of the charge on each sphere.



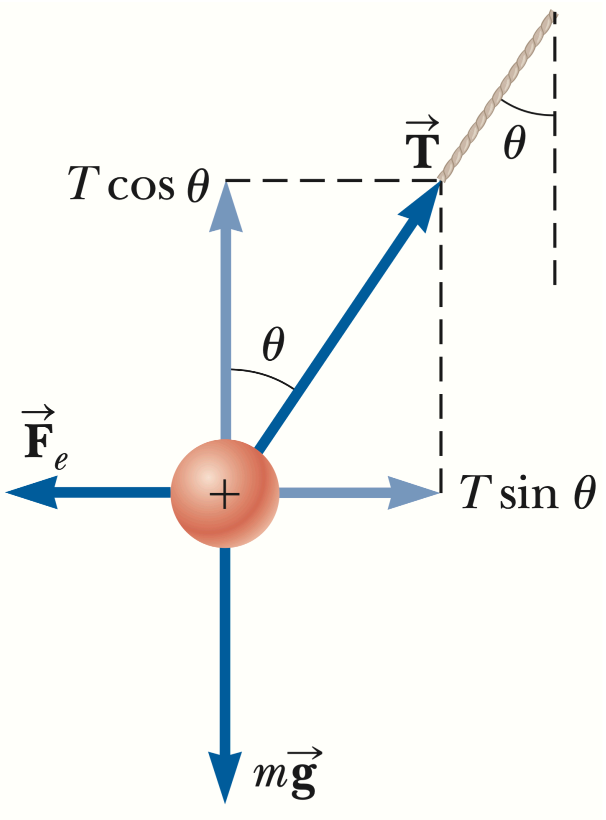
1.8 Electric Force and Weight of Charged Objects



Solution 1.4

- There are three forces exerted on each sphere:
  1. The gravitational force (downward),
  2. The tension in the string (towards the point of suspension),
  3. The electric force due to the other sphere.
- Since the spheres are in equilibrium, the net force on each sphere is **zero**.
- Using Newton second law for the left sphere, we have:

1.8 Electric Force and Weight of Charged Objects



$$\sum F_x = ma_x$$

$$+T\sin\theta - F_e = m(0)$$

$$\Rightarrow T\sin\theta = F_e \qquad (1)$$

$$\sum F_y = ma_y$$

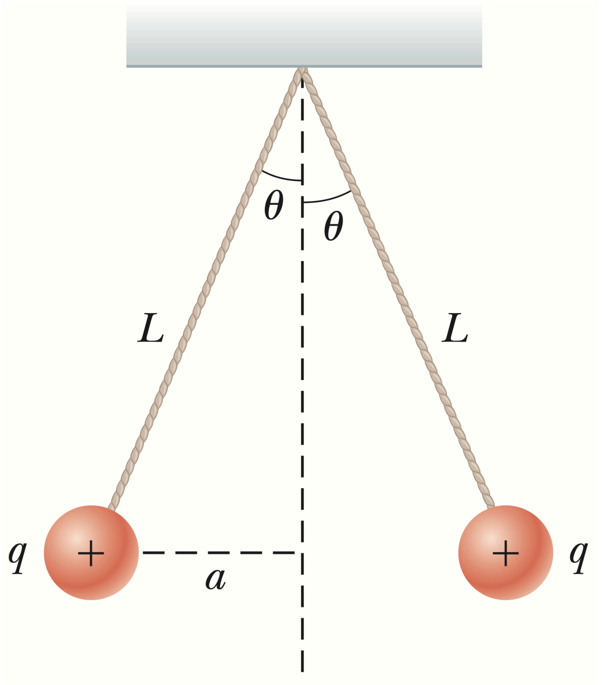
$$T\cos\theta - mg = m(0)$$

$$\Rightarrow T\cos\theta = mg \qquad (2)$$

- To eliminate T (since it is not given), we divide equation (1) by equation (2)

$$\tan\theta = \frac{F_e}{mg} \qquad \Rightarrow \quad F_e = mg\tan\theta$$

1.8 Electric Force and Weight of Charged Objects



- The spacing between the two spheres is

$$r = 2a = 2L\sin\theta$$

- From Coulomb's law, we have:

$$F_e = k_e \frac{q^2}{r^2}$$

$$\Rightarrow q^2 = F_e \frac{r^2}{k_e}$$

$$q = \sqrt{F_e \frac{r^2}{k_e}} = \sqrt{(mg\tan\theta)\frac{(2L\sin\theta)^2}{k_e}}$$

$$q = 4.42 \times 10^{-8} \quad \text{C}$$

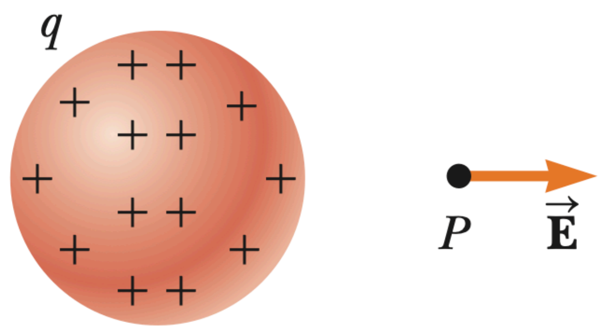
1. Coulomb’s Law

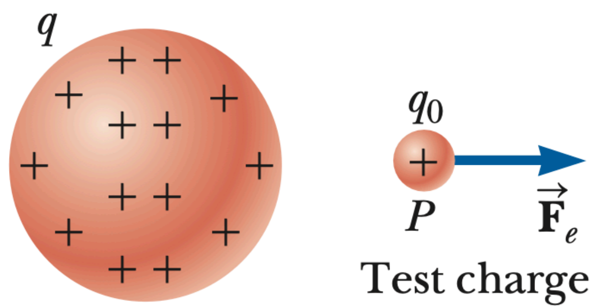
2. Analysis Model: Particle in a Field (Electric)

3. Electric Field Lines

4. Motion of a Charged Particle in a Uniform Electric Field

2.1 Electric Field Concept





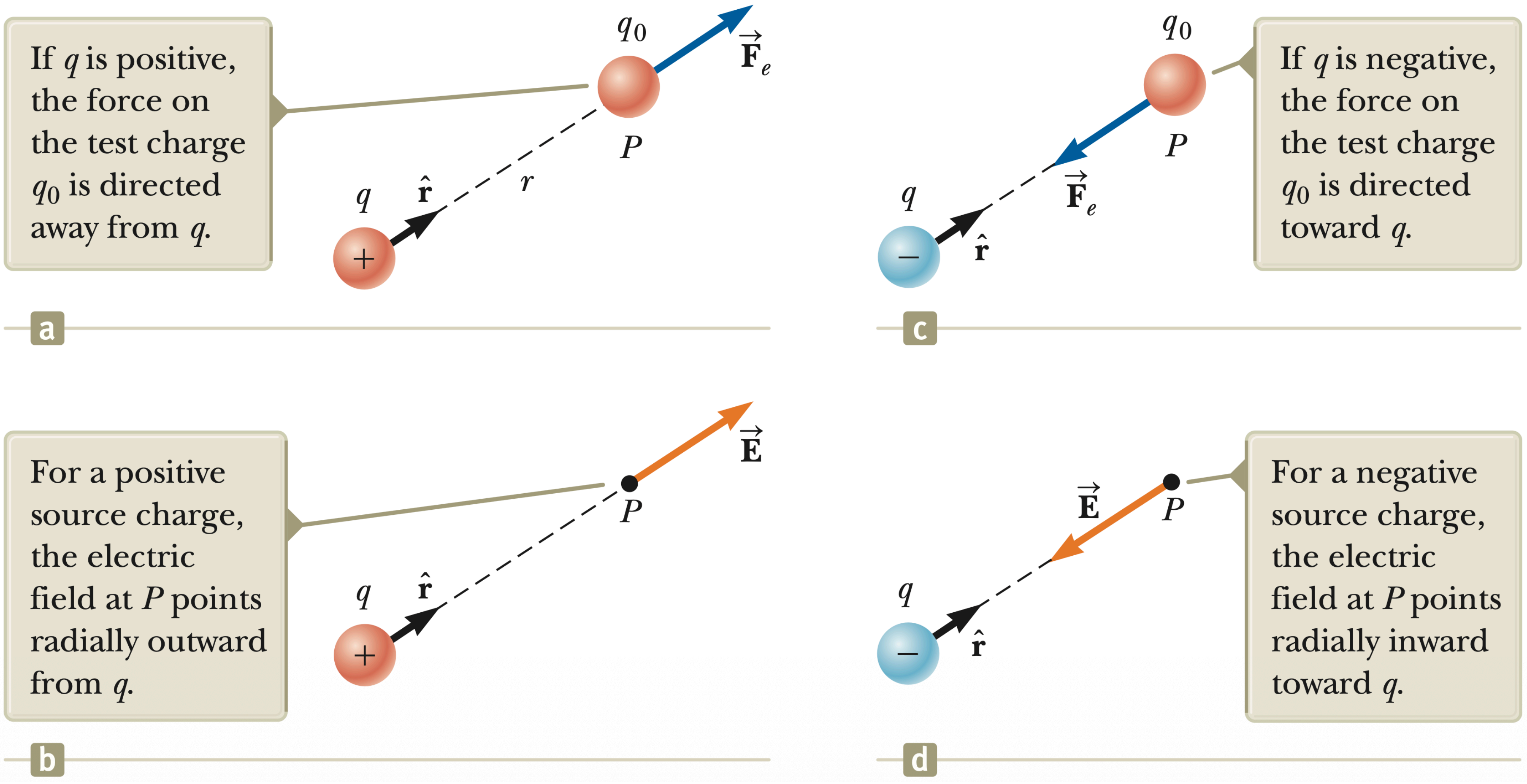
An electric field is a region of space around a charged object where other charged objects experience an electric force.

$$\vec{E} = \frac{\vec{F}_e}{q_0}$$

$$\vec{E} = k_e \frac{q}{r^2}\,\hat{r}$$

2.2 Direction of the Electric Field

King Saud University



2.3 Total Electric Field due to Multiple Point Charges

The net electric field at point p is the vector sum of the electric fields at p produced by each individual charge,

$$\vec{\boldsymbol{E}} = \vec{\boldsymbol{E}}_1 + \vec{\boldsymbol{E}}_2 + \vec{\boldsymbol{E}}_3 + \ldots$$

Since $\vec{\boldsymbol{E}}_i = k_e \frac{q_i}{r_i^2}\ \hat{\boldsymbol{r}}_i$, then

$$\vec{\boldsymbol{E}} = k_e \sum_{i=1}^{n} \frac{q_i}{r_i^2}\ \hat{\boldsymbol{r}}_i$$

2.4 Examples:

King Saud University

Example 2.5

A water droplet (قطرة ماء) of mass 3×10^{-12} kg is located in the air near the ground during a stormy day. An atmospheric electric field of magnitude 6×10^{3} N/C points vertically *downward* in the vicinity (بالقرب من) of the water droplet. The droplet remains suspended (معلقة) at rest in the air.

What is the electric charge on the droplet?

2.4 Examples:

King Saud University

Solution 2.5

- Gravity pulls the droplet *downward* with a force equal to its weight $\overrightarrow{\boldsymbol{W}} = m\vec{\boldsymbol{g}}$.
- The electric field exerts an electric force $\vec{\boldsymbol{F}}_e = q\vec{\boldsymbol{E}}$ on the droplet in the *upward* direction.
- Since the direction of the electric field is *downward*, then it is negative:

$$\vec{\boldsymbol{E}} = -E\ \hat{\boldsymbol{\jmath}}$$

- Since the droplet is at *rest*, the net force on it is *zero*:

$$F_e - W = 0$$

$$q(-E) - mg = 0$$

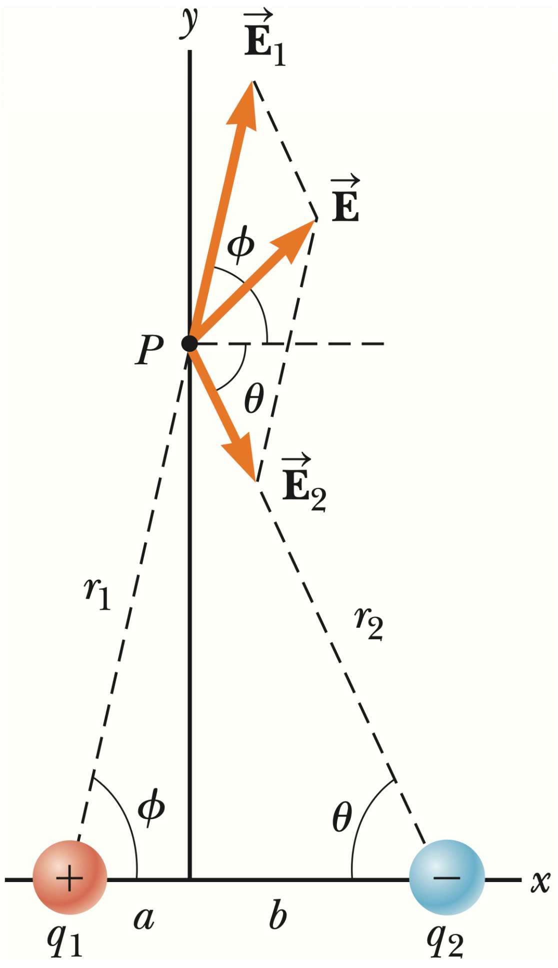
$$\implies \quad q = -\frac{mg}{E} = \frac{(3 \times 10^{-12})(9.8)}{6 \times 10^3} = -4.9 \times 10^{-15}\ \mathrm{C}$$

2.4 Examples:

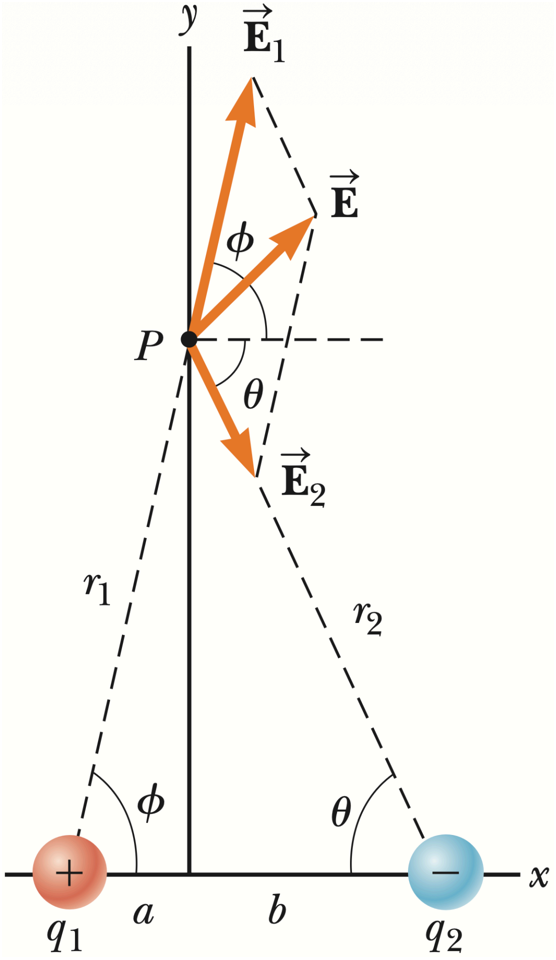
Example 2.6

Charges q_1 and q_2 are located on the x-axis, at distances a and b, respectively, from the origin as shown in the Figure.

(A) Find the components of the net electric field at the point P, which is at position $(0, y)$.



2.4 Examples:



Solution 2.6

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$E_x\hat{\imath} + E_y\hat{\jmath} = \left(E_{1,x}\ \hat{\imath} + E_{1,y}\ \hat{\jmath}\right) + \left(E_{2,x}\ \hat{\imath} + E_{2,y}\ \hat{\jmath}\right)$$

$$= E_1(\cos\varphi\hat{\imath} + \sin\varphi\hat{\jmath}) + E_2(\cos\theta\hat{\imath} - \sin\theta\hat{\jmath})$$

$$\Rightarrow \quad E_x = E_1\cos\varphi + E_2\cos\theta$$

$$\Rightarrow \quad E_y = E_1\sin\varphi - E_2\sin\theta$$

where E_1 and E_2 are found by:

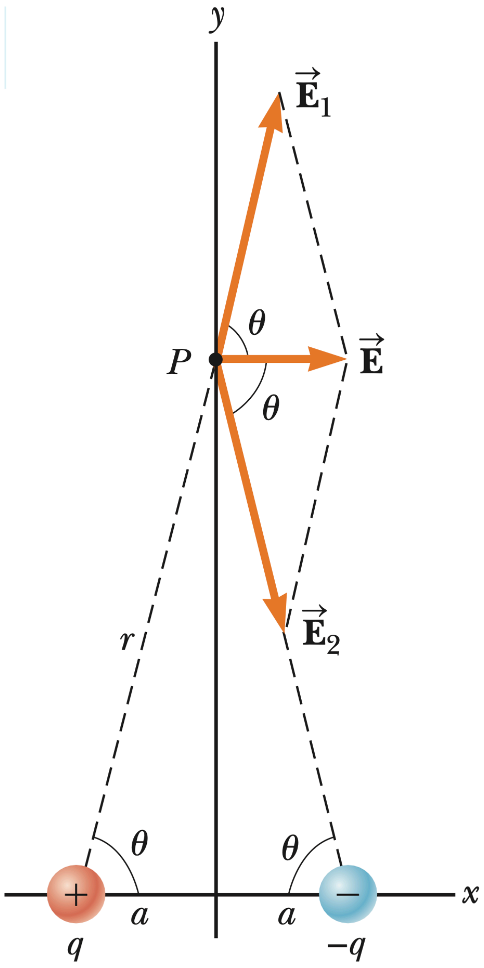
$$E_1 = k_e\frac{|q_1|}{r_1^2} = k_e\frac{|q_1|}{a^2 + y^2}$$

$$E_2 = k_e\frac{|q_2|}{r_2^2} = k_e\frac{|q_2|}{b^2 + y^2}$$

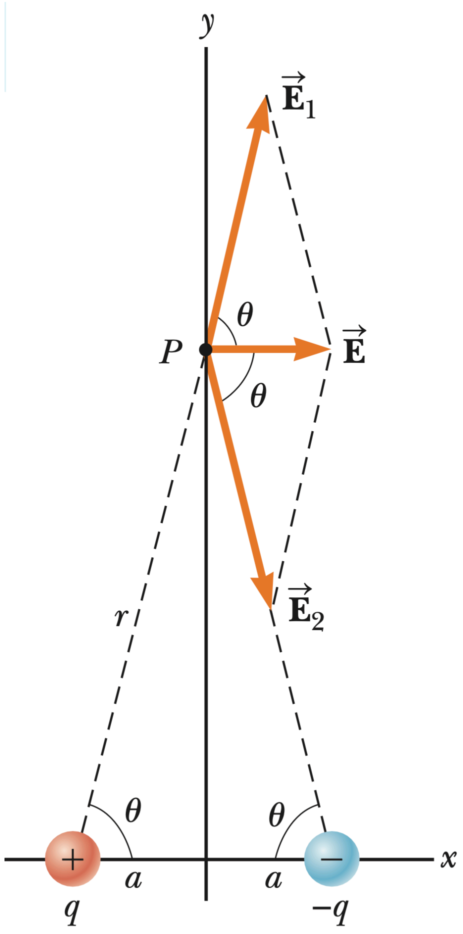
2.4 Examples:

(B) Evaluate the electric field at point P in the special case that

$$|q_1| = |q_2| \qquad \text{and} \qquad a = b$$



2.4 Examples:



At these conditions, we have:

$$\varphi = \theta \quad \text{and} \quad E_1 = E_2$$

Therefore,

$$E_y = E_1 \sin\theta - E_1 \sin\theta$$

$$E_y = 0$$

$$E_x = E_1 \cos\theta + E_1 \cos\theta = 2E_1 \cos\theta$$

$$E_x = 2k_e \frac{q}{a^2 + y^2} \cos\theta$$

$$E_x = 2k_e \frac{q}{a^2 + y^2} \left(\frac{a}{\sqrt{a^2 + y^2}} \right) = \frac{2k_e q a}{(a^2 + y^2)^{3/2}}$$

2.4 Examples:

(c) Find the electric field due to the electric dipole when point P is a distance $y \gg a$ from the origin.

When $y \gg a$, then $a^2 + y^2 \approx y^2$, therefore $\left(a^2 + y^2\right)^{3/2} \approx y^3$. Thus,

$$E_x = \frac{2k_e qa}{\left(a^2 + y^2\right)^{3/2}} \approx \frac{2k_e qa}{y^3}$$

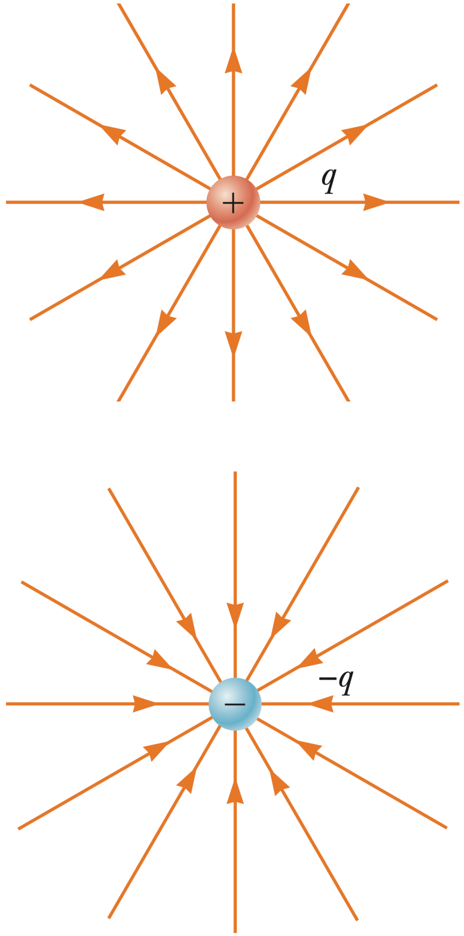
1. Coulomb’s Law

2. Analysis Model: Particle in a Field (Electric)

3. Electric Field Lines

4. Motion of a Charged Particle in a Uniform Electric Field

3.1 Electric Field Lines Concept

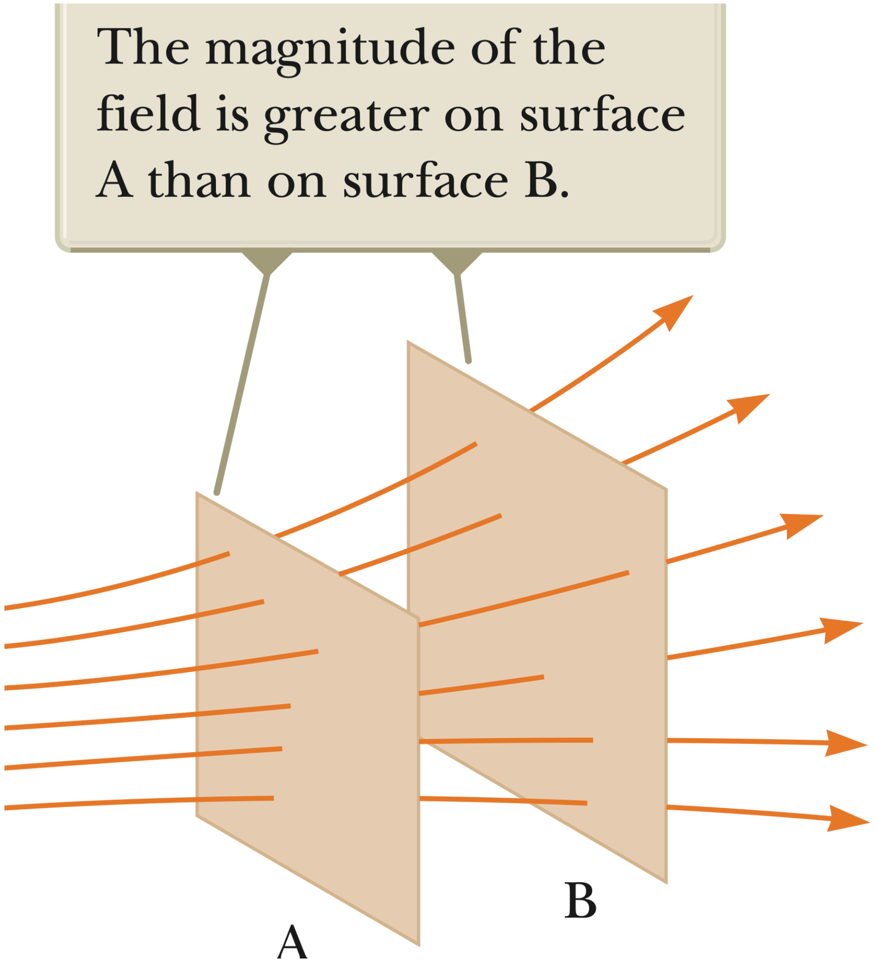


A convenient way of visualizing electric field is to draw lines known as **electric field lines**.

These lines have the following (5) properties:

1. The *direction* of the electric field at any point is tangent to the field line at that point.

3.1 Electric Field Lines Concept



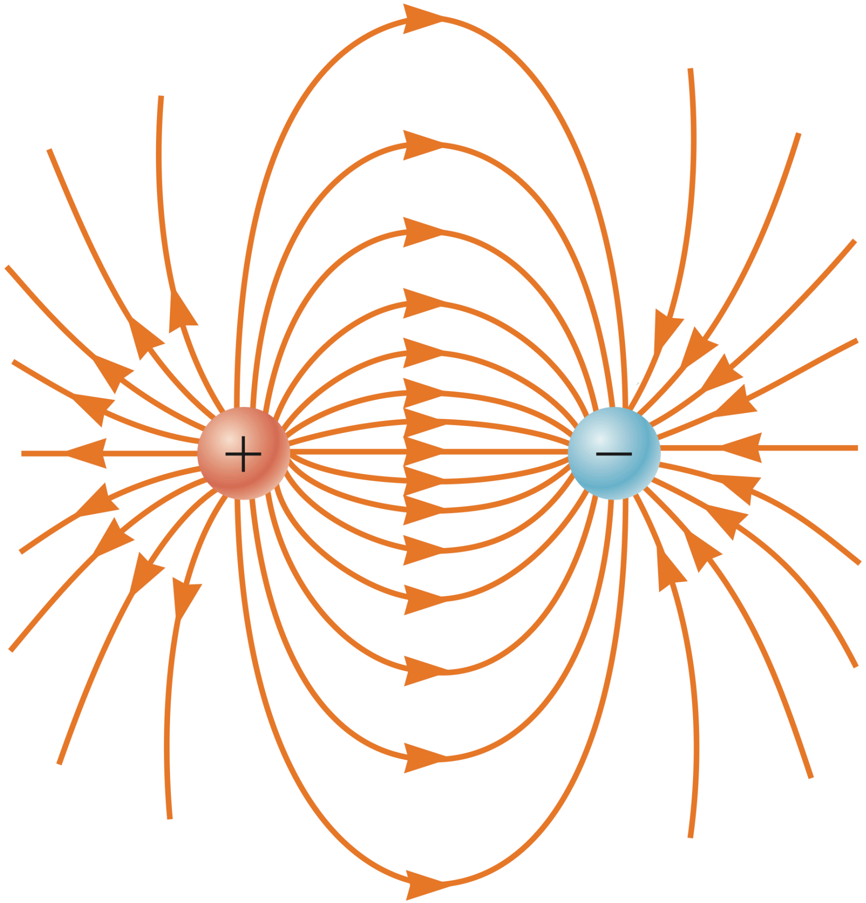
2. The density of the field lines ρ (number of lines N per unit area of a sphere) is proportional to the magnitude of the electric field in that region.

$$\rho = \frac{N}{4\pi r^2} \propto E \propto \frac{|q|}{r^2}$$

- When comparing the number of lines between two charges Q_1 and Q_2,

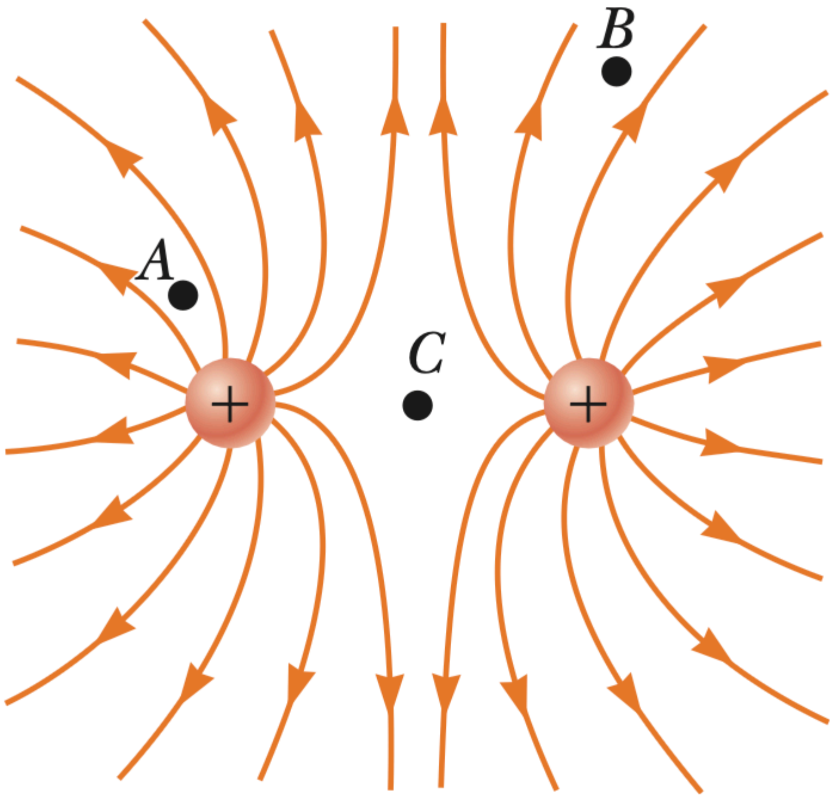
$$\frac{N_2}{N_1} = \frac{|Q_2|}{|Q_1|}$$

3.1 Electric Field Lines Concept



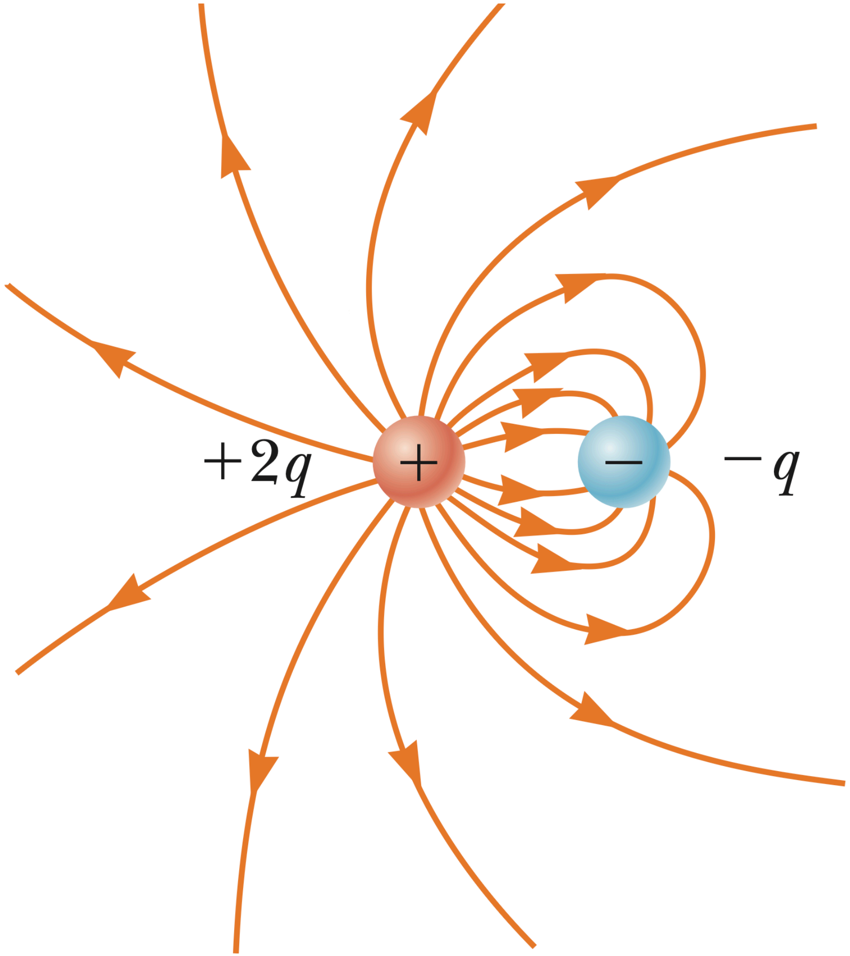
3. Field lines originate on positive charges and terminate on negative charges.
4. The number of field lines leaving one positive charge equals the number of field lines entering one negative charge.
5. Field lines never cross each other.

3.1 Electric Field Lines Concept



- The electric field magnitude at **A** is greater than that at **B**.
- The electric field magnitude at **C** is zero.
- The electric field terminates at infinity.

3.1 Electric Field Lines Concept



- The number of filed lines leaving $+2q$ is two times grater than $-q$,

$$N_{+2q} = 2N_{-q}$$

- Only half of the field lines from $+2q$ terminate on $-q$, while the other half extends to infinity.

1. Coulomb’s Law

2. Analysis Model: Particle in a Field (Electric)

3. Electric Field Lines

4. Motion of a Charged Particle in a Uniform Electric Field

4.1 Newton's Second Law for a Charged Particle in an Electric Field

From Newton's second law, the acceleration of a particle of mass m and charge q in an electric field $\vec{\boldsymbol{E}}$ is given by:

$$\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$$

In the case where the only force acting on the particle is the electric force $\vec{\boldsymbol{F}}_e = q\vec{\boldsymbol{E}}$, then:

$$\boxed{\vec{\boldsymbol{F}}_e = q\vec{\boldsymbol{E}} = m\vec{\boldsymbol{a}}}$$

Therefore, the acceleration of the particle is:

$$\boxed{\vec{\boldsymbol{a}} = \frac{q}{m}\vec{\boldsymbol{E}}}$$

4.2 Recall the Equations of Motion

Equations of Motion at Constant Acceleration	
$\vec{v}_f = \vec{v}_i + \vec{a}t$	(1)
$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$	(2)
$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\left(\vec{r}_f - \vec{r}_i\right)$	(3)

Remember

$$\vec{r} = x\hat{\imath} + y\hat{\jmath}$$

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$$

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$$

Therefore: Equation (1) and (2) becomes:

$$\vec{v}_f = (v_{xi} + a_x t)\hat{\imath} + \left(v_{yi} + a_y t\right)\hat{\jmath}$$

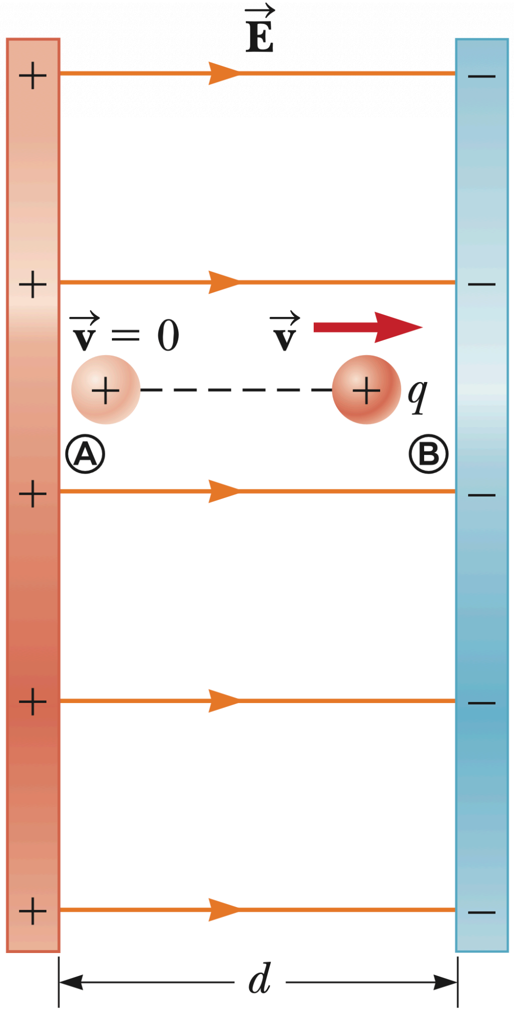
$$\vec{r}_f = \left(x_i + v_{xi}t + \frac{1}{2}a_x t^2\right)\hat{\imath} + \left(y_i + v_{yi}t + \frac{1}{2}a_y t^2\right)\hat{\jmath}$$

4.3 Examples

Example 4.7

A uniform electric field $\vec{E}$ is directed along the x axis between parallel plates of charge separated by a distance d as shown in the Figure. A positive point charge q of mass m is released from *rest* at a point **A** next to the positive plate and accelerates to a point **B** next to the negative plate.

(A) Find the speed of the particle at **B** by modeling it as a particle under constant acceleration.



4.3 Examples

Solution 4.7

Since the motion of the particle is along the x-axis, we can use the scalar form of the equations of motion (motion in one dimension).

$$v_f^2 = v_i^2 + 2a_x(x_f - x_i)$$

- The particle is released from rest at point A, therefore $v_i = 0$.
- The initial position is $x_i = 0$ and the final position is $x_f = d$.
- The acceleration of the particle is given by:

$$a_x = q\frac{E}{m}$$

4.3 Examples

- Therefore, the speed of the particle at point B is:

$$v_f^2 = 0 + 2\left(q\frac{E}{m}\right)(d - 0)$$

$$\implies \quad v_f = \sqrt{\frac{2qEd}{m}}$$

4.3 Examples

(B) Find the speed of the particle at **B** by modeling it as a nonisolated system in terms of energy.

Using the work-energy theorem for a nonisolated system, we have:

$$W = \Delta K$$
$$Fd = K_f - K_i$$
$$(qE)d = \frac{1}{2}mv_f^2 - 0$$
$$\Longrightarrow \quad v_f = \sqrt{\frac{2qEd}{m}}$$

We get the same result as in part (A). Therefore, both methods are valid to analyze the motion of a charged particle in a uniform electric field.

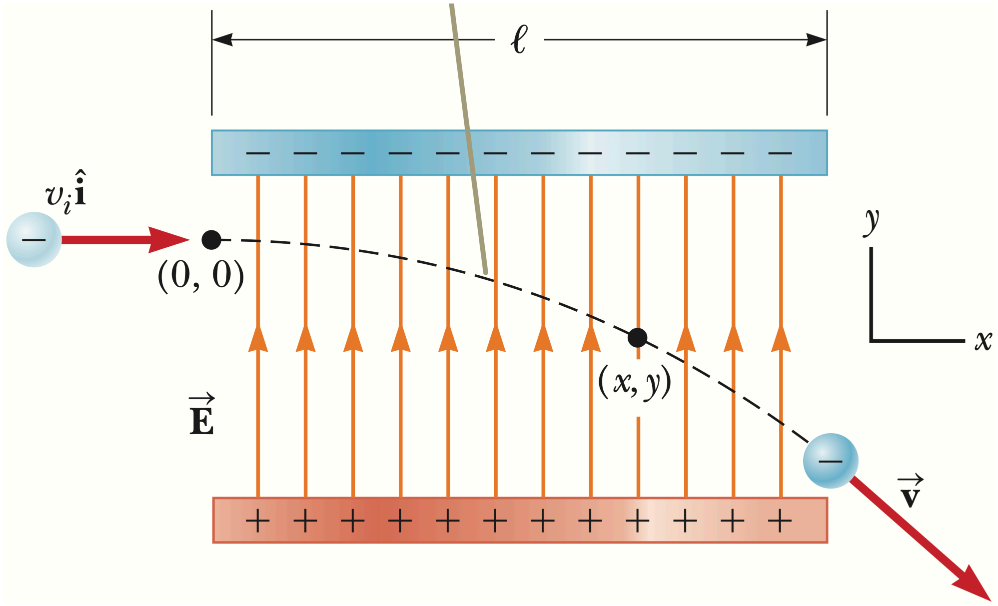
4.3 Examples

King Saud University

Example 4.8

An electron enters the region of a uniform electric field as shown in the Figure, with $v_i = 3 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $l = 0.1$ m.

(A) Find the acceleration of the electron while it is in the electric field.



Solution 4.8

- Since the electric field is along the y-axis, the acceleration of the electron is only along the y-axis, therefore $a_x = 0$.
- The acceleration along the y-axis is given by:

$$a_y = \frac{qE}{m}$$

$$= \frac{(-e)E}{m_e} = \frac{(-1.602 \times 10^{-19}\ \mathrm{C})(200\ \mathrm{N/C})}{9.11 \times 10^{-31}\mathrm{kg}}$$

$$a_y = -3.51 \times 10^{13}\ \mathrm{m/s^2}$$

Finally, the acceleration vector is: $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$

4.3 Examples

(B) Assuming the electron enters the field at time $t = 0$, find the time at which it leaves the field.

$$x_f = x_i + v_i t + \frac{1}{2} a_x t^2$$

since $a_x = 0$, then

$$x_f = x_i + v_i t$$

$$\Longrightarrow \quad t = \frac{x_f - x_i}{v_i} = \frac{l}{v_i} = \frac{0.1}{3 \times 10^6} = 3.33 \times 10^{-8} \text{ s}$$

4.3 Examples

King Saud University

(C) Assuming the vertical position of the electron as it enters the field is $y_i = 0$, what is its vertical position when it leaves the field?

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$= 0 + 0 + \frac{1}{2}\left(-3.51 \times 10^{13}\right)\left(3.33 \times 10^{-8}\right)^2$$
$$= -1.95 \times 10^{-2}\ \text{m}$$

Suggested Problems

9, 12, 15, 20, 21, 24, 25, 30, 35, 38

Book: Serway, R. A., & Jewett, J. W. (2018). Physics for Scientists and Engineers (10th ed.)

Chapter: 22 - Electric Fields