Ch.2: Motion in One Dimension

Physics 103: Classical Mechanics

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Outline

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1. Motion

2. Instantaneous Velocity and Speed

3. Acceleration

4. One-Dimensional Motion with Constant Acceleration

5. Freely Falling Objects

1.1 Three Types of Motion

- Motion represents a continuous change in the position of an object.

1. Translational Motion: Movement in a straight line.
   - Example: A car moving down a highway.
2. Rotational Motion: Movement around an axis.
   - Example: the Earth's spin on its axis.
3. Oscillatory Motion: Back-and-forth movement.
   - Example: movement of a pendulum.

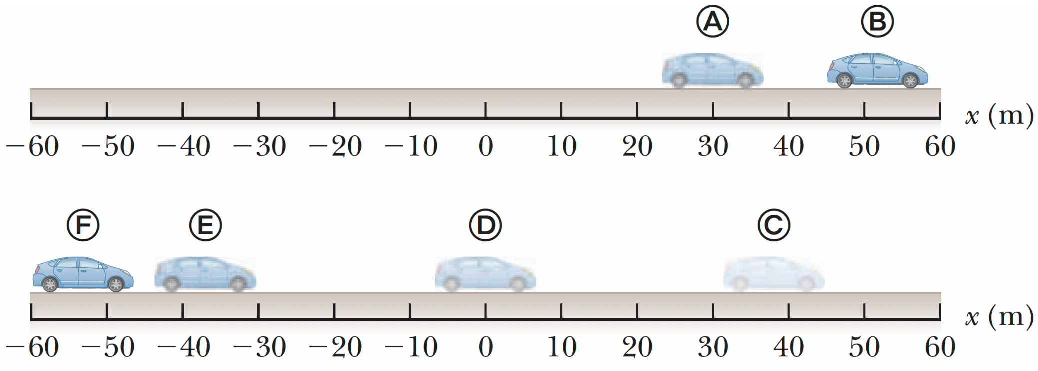
1.2 Translational Motion

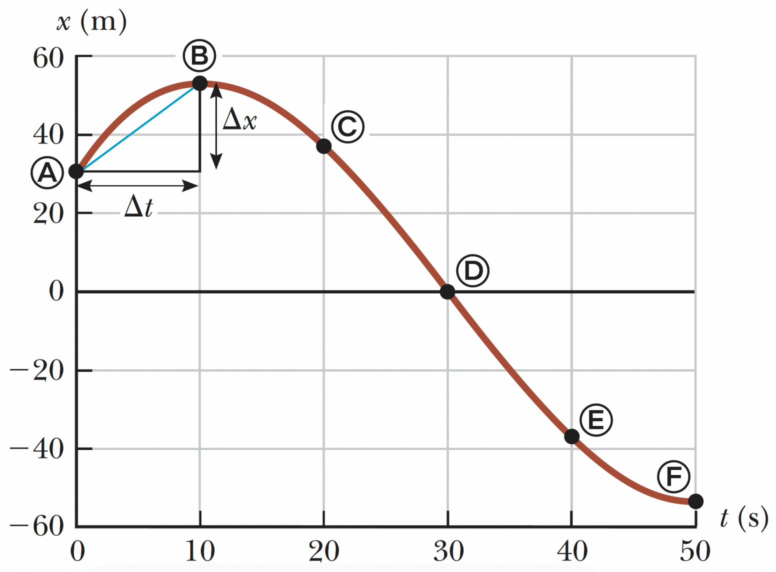
- In this and the next few chapters, we are concerned only with *translational* motion.
- In our study of translational motion, we use what is called the *particle model*, which describes the moving object as a particle (point mass) regardless of its size.
- Typically, motion takes place in a 3-D space, but we can simplify our analysis by considering motion in a 1-D or 2-D.

1.3 Position

- A particle's position is the location of the particle with respect to a chosen reference point that we can consider to be the origin of a coordinate system.
- For a racing car, the origin could be the starting line of the race.
- The choice of the origin is arbitrary (Typically what makes the analysis easier), but once it is chosen, it must be used consistently.
- The motion of a particle is completely known if the particle's position (x) in space is known at all times (t).
- Example: the figure below shows the position of a car moving along a straight road to the right, then to the left:

1.3 Position



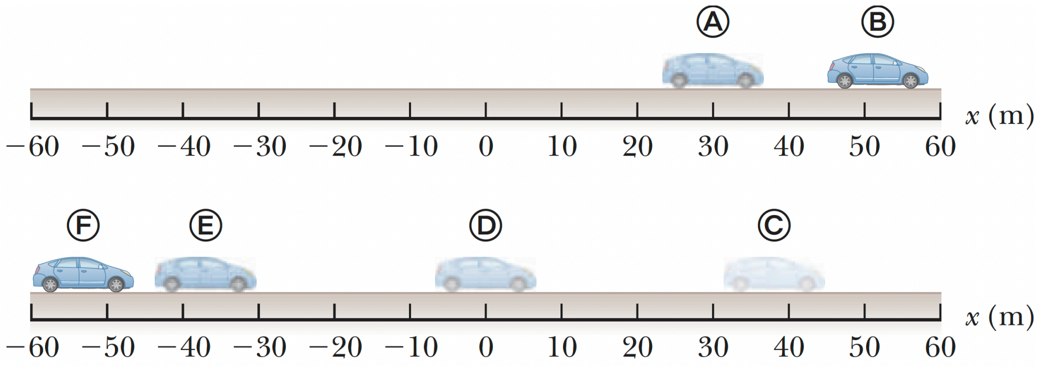


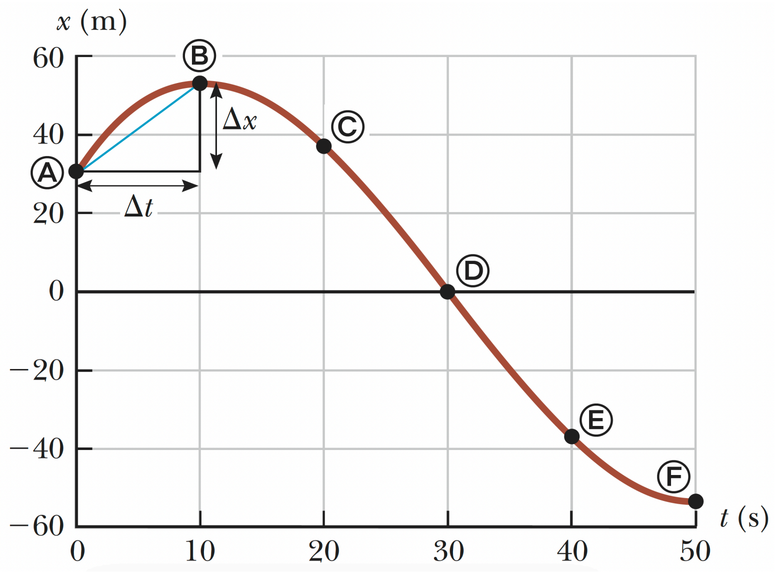
1. Take snapshots of the motion every $\Delta t = 10$ s, and label them (A) to (F).

2. Put position data on the y-axis and time on the x -axis.

3. Fit a smooth curve through the points to show the **position-time** graph.

1.4 Displacement and Distance





Displacement: is the change in position, which can be positive or negative:

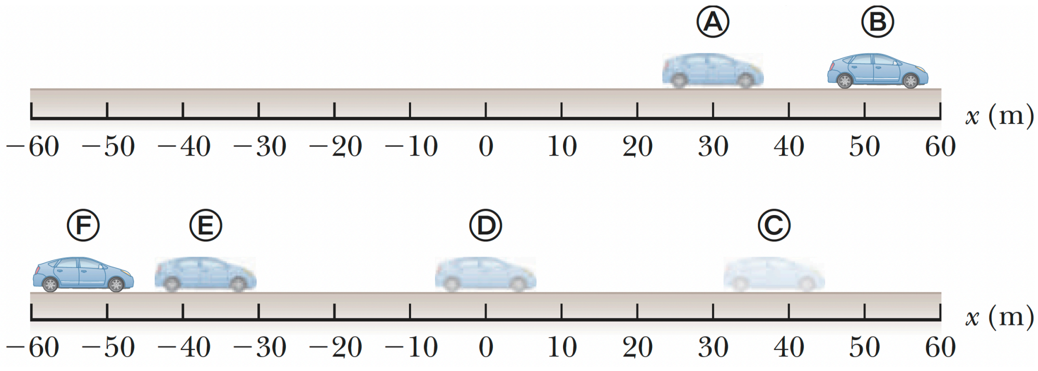
$$\Delta x = x_f - x_i.$$

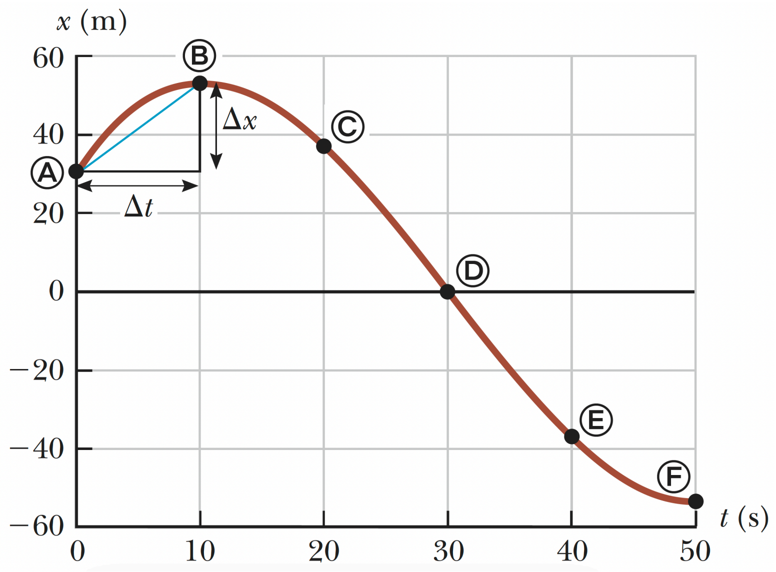
Examples:

$$\Delta x_{\mathrm{AB}} = x_B - x_A = 52 - 30 = 22 \text{ m}$$

$$\Delta x_{\mathrm{AF}} = x_F - x_A = (-53) - 30 = -83 \text{ m}$$

1.4 Displacement and Distance





Distance: is the total path length traveled by an object, regardless of direction:

$$d \geq |\Delta x|.$$

Examples:

$$d_{\mathrm{AB}} = x_B - x_A = 52 - 30 = 22 \text{ m}$$

$$d_{\mathrm{AF}} = d_{\mathrm{AB}} + d_{\mathrm{BF}} = 22 + 52 + 53 = 127 \text{ m}$$

1.5 Average Velocity

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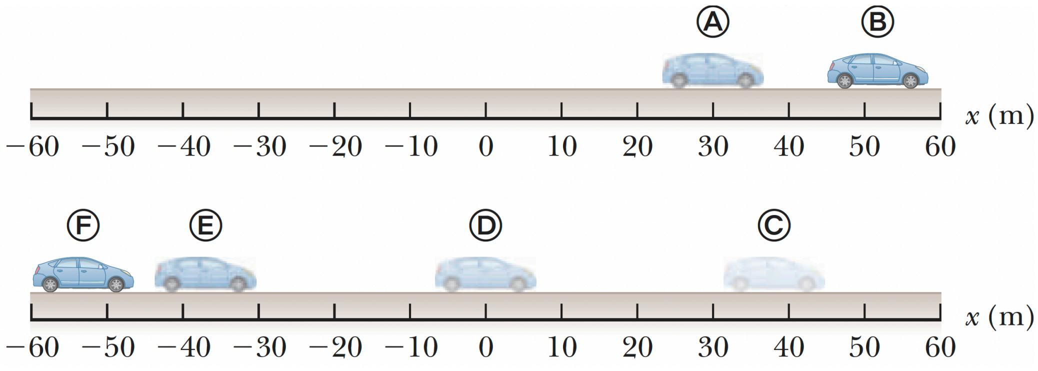
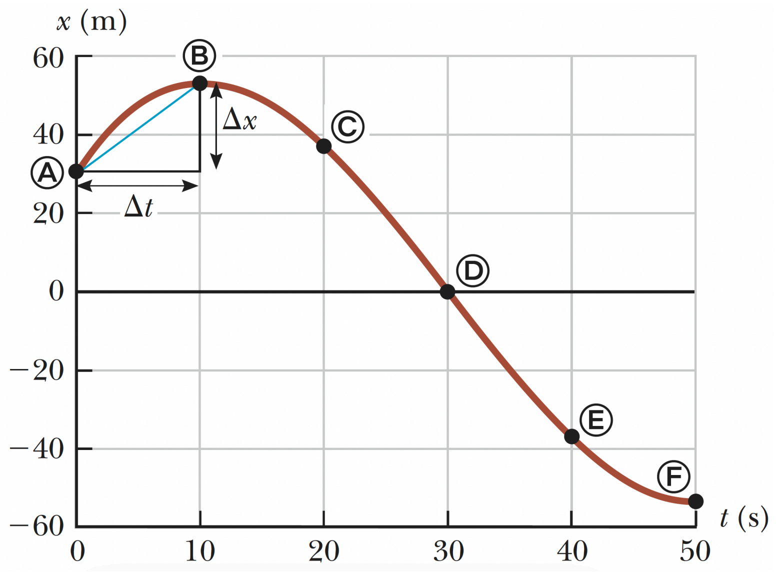




Average Velocity: The *displacement* traveled within a time interval:

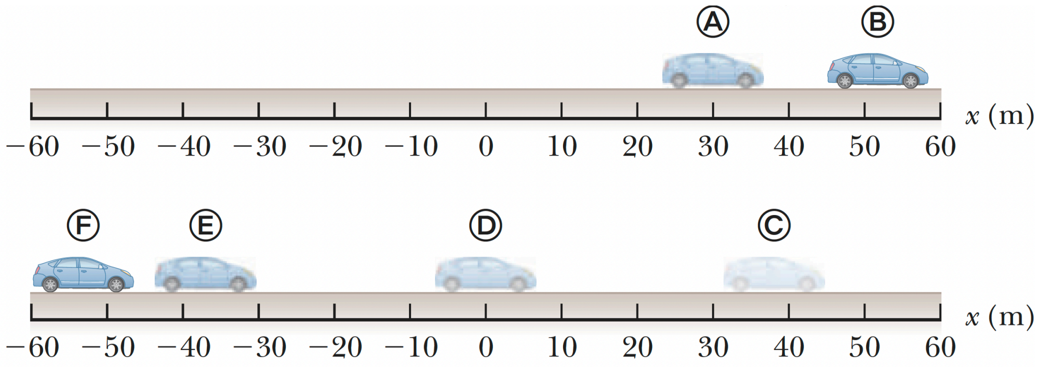
$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \text{Slope of } x(t).$$

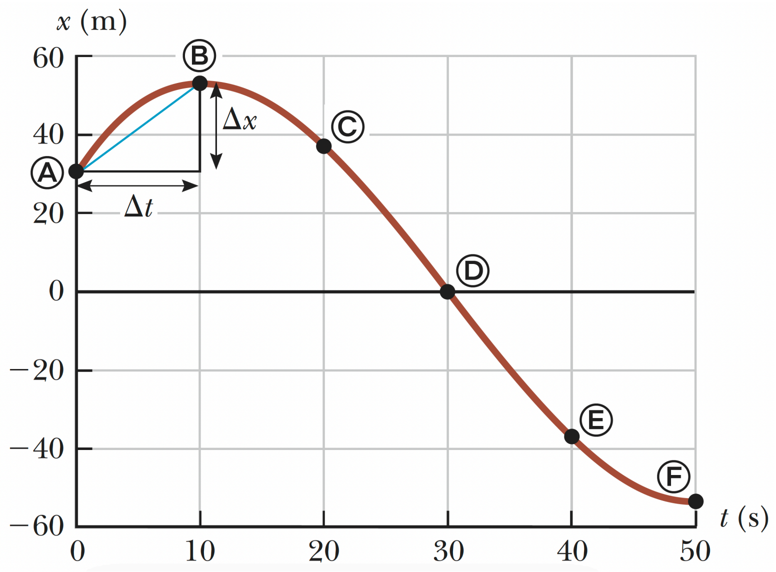
Examples:

$$\bar{v}_{\text{AB}} = \frac{22 \text{ m}}{10 \text{ s}} = 2.2 \text{ m/s}$$

$$\bar{v}_{\text{AF}} = -83 \text{ m} \;/\; 50 \text{ s} = -1.66 \text{ m/s}$$

1.6 Average Speed





Average Speed: The *total distance* traveled within a time interval:

$$\bar{s} = \frac{d}{\Delta t} \geq |\bar{v}|.$$

Examples:

$$\bar{s}_{\text{AB}} = \frac{22\ \text{m}}{10\ \text{s}} = 2.2\ \text{m/s}$$

$$\bar{s}_{\text{AF}} = 127\ \text{m}\ /\ 50\ \text{s} = 2.54\ \text{m/s}$$

1. Motion

2. Instantaneous Velocity and Speed

3. Acceleration

4. One-Dimensional Motion with Constant Acceleration

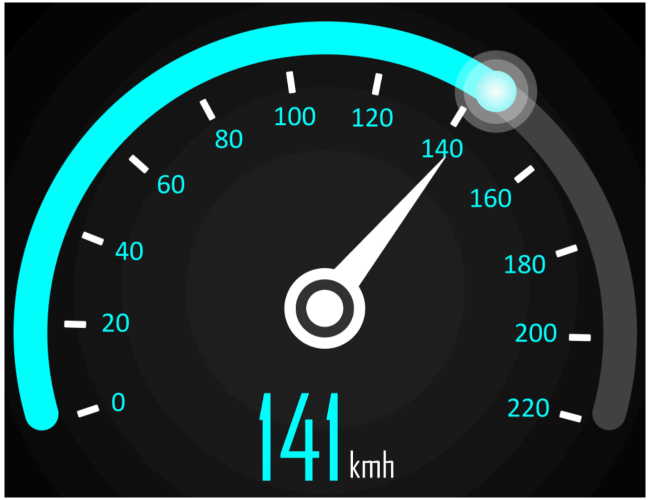
5. Freely Falling Objects

2.1 Definitions

- **Instantaneous Velocity**: The *velocity* of an object at a specific moment in time, or the limit of the average velocity as the time interval approaches zero:

$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}.$$

- **Instantaneous Speed** is the *magnitude* of the instantaneous velocity: $s = |v|$.
- The slope of the position-time graph $x(t)$ at a specific point gives the instantaneous velocity at that point.



2.2 Examples

Example 2.1

Consider the following one-dimensional motions: (A) A ball thrown directly upward rises to a highest point and falls back into the thrower's hand. (B) A race car starts from rest and speeds up to 100 m/s. (C) A spacecraft drifts through space at constant velocity. Are there any points in the motion of these objects at which the instantaneous velocity has the same value as the average velocity over the entire motion? If so, identify the point(s).

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Solution 2.1

(A) The average velocity for the thrown ball is zero because the ball returns to the starting point; thus its displacement is zero. There is one point at which the instantaneous velocity is zero—at the top of the motion.

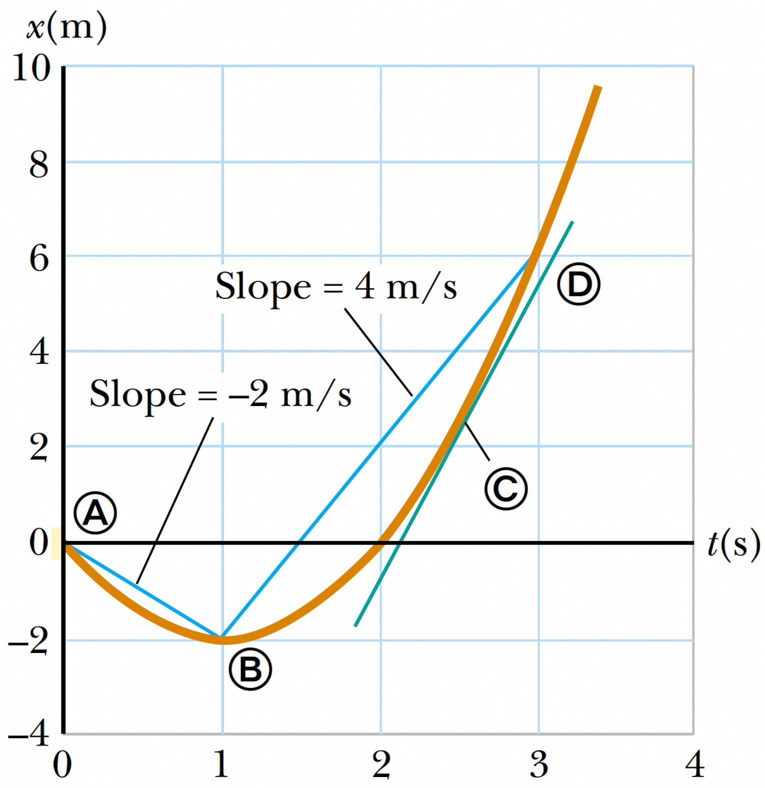
(B) The car's average velocity cannot be evaluated with the information given, but it must be some value between 0 and 100 m/s. Because the car will have every instantaneous velocity between 0 and 100 m/s at some time during the interval, there must be some instant at which the instantaneous velocity is equal to the average velocity.

(C) Because the spacecraft's instantaneous velocity is constant, its instantaneous velocity at any time and its average velocity over any time interval are the same.

2.2 Examples

Example 2.2

A particle moves along the x axis. Its position varies with time according to the expression $x(t) = -4t + 2t^2$, where x is in meters and t is in seconds. The position-time graph for this motion is shown. Note that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment $t = 1$ s, and moves in the positive x direction at times $t > 1$ s.



2.2 Examples

(A) Determine the displacement of the particle in the time intervals:
$(t_A = 0 \text{ s} \longrightarrow t_B = 1 \text{ s})$ and $(t_B = 1 \text{ s} \longrightarrow t_D = 3 \text{ s})$.

Solution 2.2

- For the first interval,

$$\begin{aligned}\Delta x_{A\to B} &= x(1) - x(0) = \left(-4(1) + 2(1)^2\right) - \left(-4(0) + 2(0)^2\right)\\ &= -4 + 2 = -2 \text{ m}.\end{aligned}$$

- For the second interval,

$$\Delta x_{B\to D} = x(3) - x(1) = \left(-4(3) + 2(3)^2\right) - \left(-4(1) + 2(1)^2\right) = +8 \text{ m}.$$

- These displacements can also be read directly from the position-time graph.

2.2 Examples

Example 2.2

(B) Calculate the average velocity during these two time intervals.

Solution 2.2

- The average velocity during the first interval is given by:

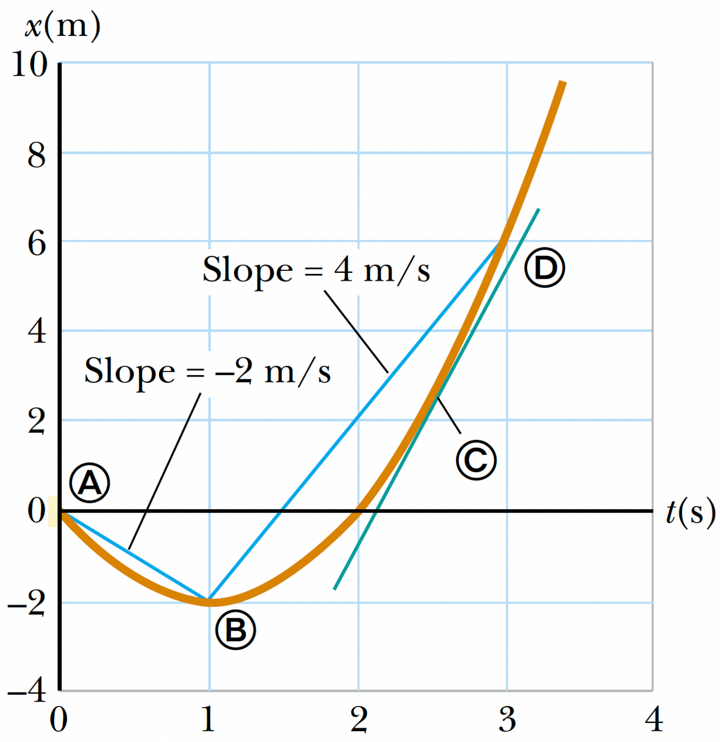
$$\bar{v}_{A\to B} = \frac{\Delta x_{A\to B}}{\Delta t_{A\to B}} = \frac{-2\ \text{m}}{1s - 0s} = -2\ \text{m/s}.$$

- The average velocity during the second interval is given by:

$$\bar{v}_{B\to D} = \frac{\Delta x_{B\to D}}{\Delta t_{B\to D}} = \frac{8\ \text{m}}{3s - 1s} = 4\ \text{m/s}.$$

2.2 Examples

- These values are the same as the slopes of the lines joining these points in the shown figure.



Example 2.2

(C) Find the instantaneous velocity of the particle at $t = 2.5$ s.

Solution 2.2

- The instantaneous velocity is the derivative of the position function $x(t)$ with respect to time:

$$v(t) = \frac{d}{dt}x(t) = \frac{d}{dt}\left(-4t + 2t^2\right) = -4 + 4t.$$

- Evaluating this at t = 2.5 s gives:

$$v(2.5) = -4 + 4(2.5) = -4 + 10 = 6 \text{ m/s}.$$

2.2 Examples

- The instantaneous velocity is the slope of the tangent line to the position-time graph at $t_C = 2.5s$ (see the graph above).

1. Motion

2. Instantaneous Velocity and Speed

3. Acceleration

4. One-Dimensional Motion with Constant Acceleration

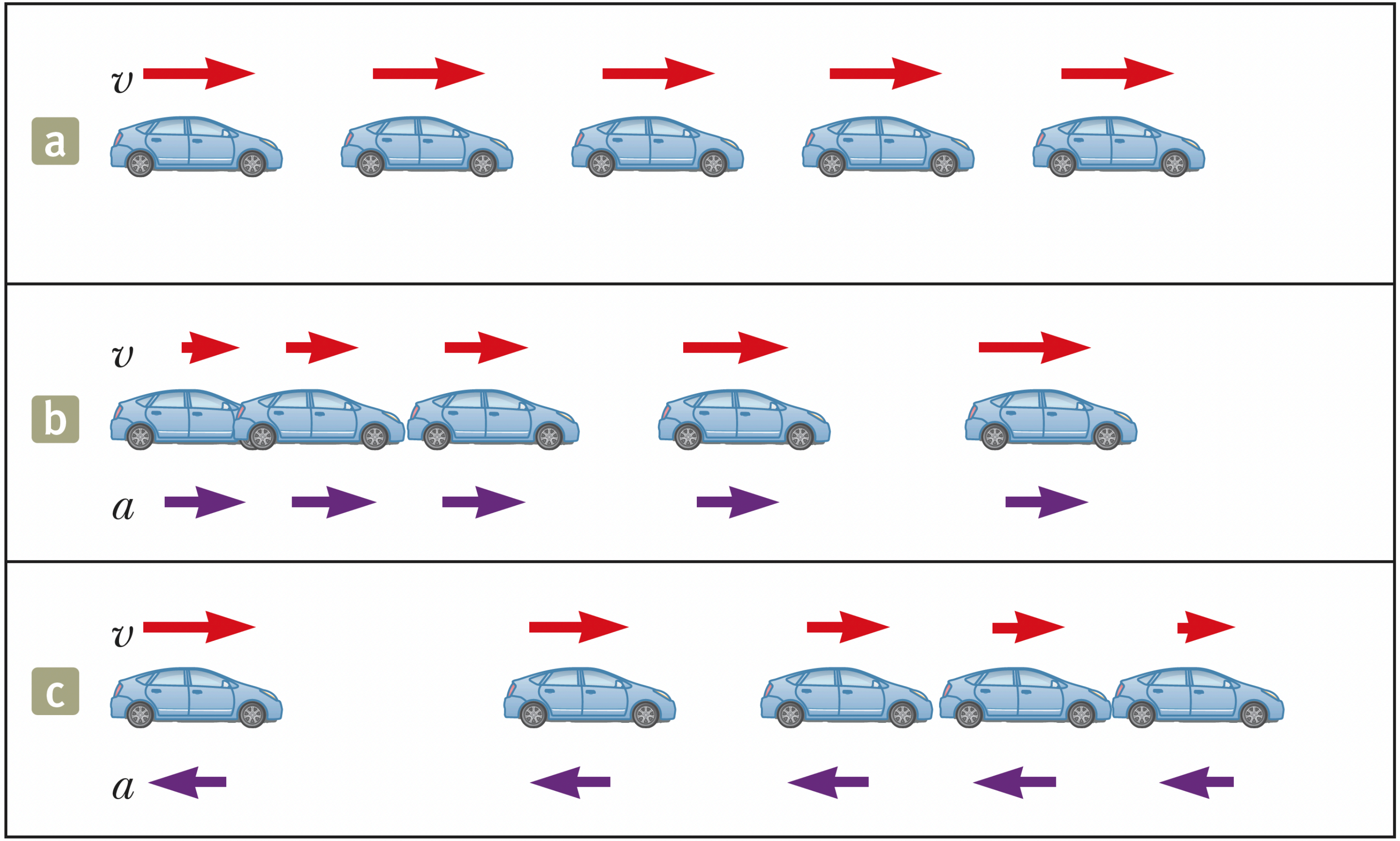
5. Freely Falling Objects

3.1 What is acceleration?

$$v = \text{const}$$
$$\Delta v = 0$$
$$a = 0$$

$$v \propto t$$
$$\Delta v > 0$$
$$a > 0$$

$$v \propto 1/t$$
$$\Delta v < 0$$
$$a < 0$$



3.2 Average and Instantaneous Acceleration

- **Acceleration:** The rate of change of velocity with respect to time.

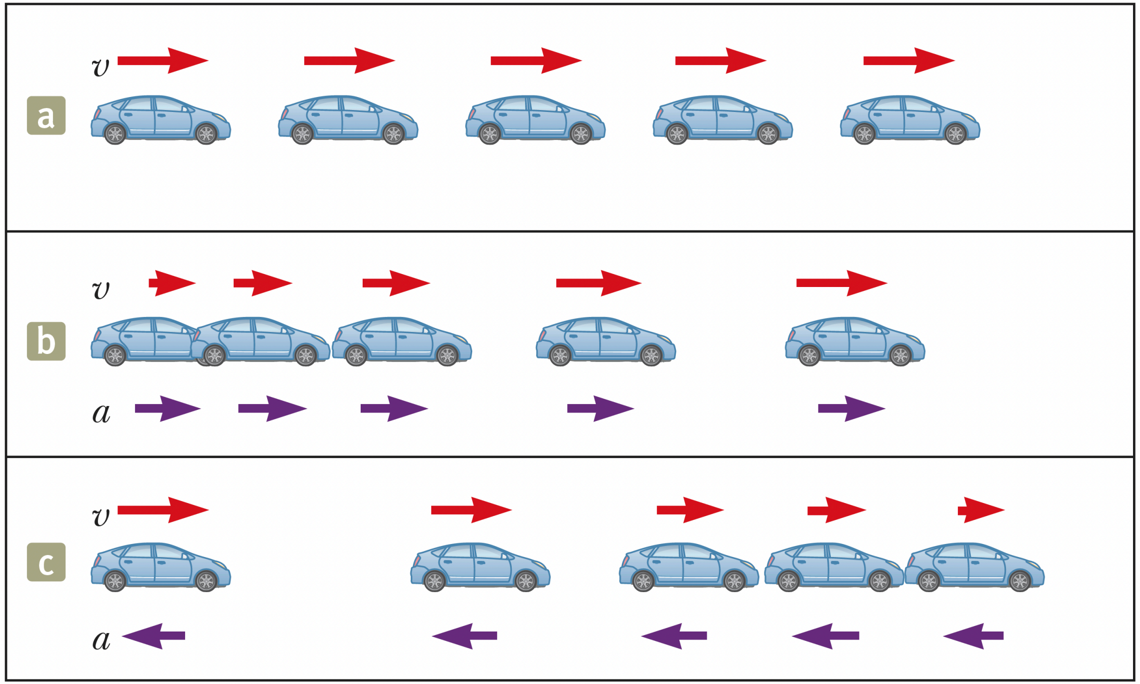
- **Average Acceleration:**

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}.$$

- **Instantaneous Acceleration:**

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}.$$

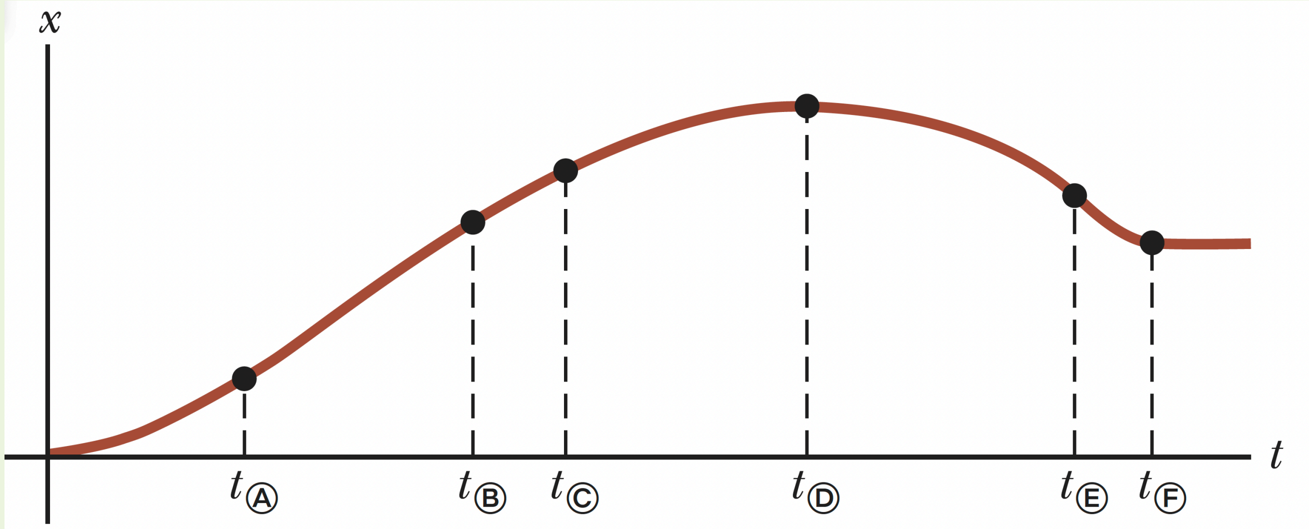
- **Unit:** L/T^2 $(\mathrm{m/s^2})$



3.3 Examples

Example 3.3

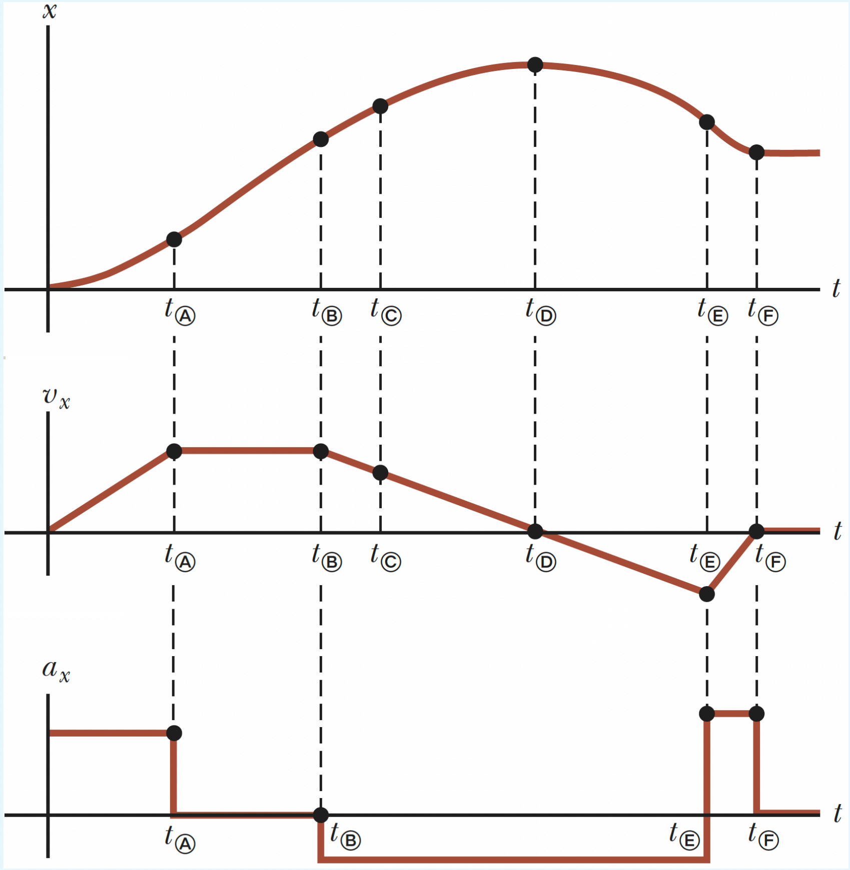
The position of an object moving along the x axis varies with time as shown in the Figure. Graph the velocity versus time and the acceleration versus time for the object.



3.3 Examples

Solution 3.3

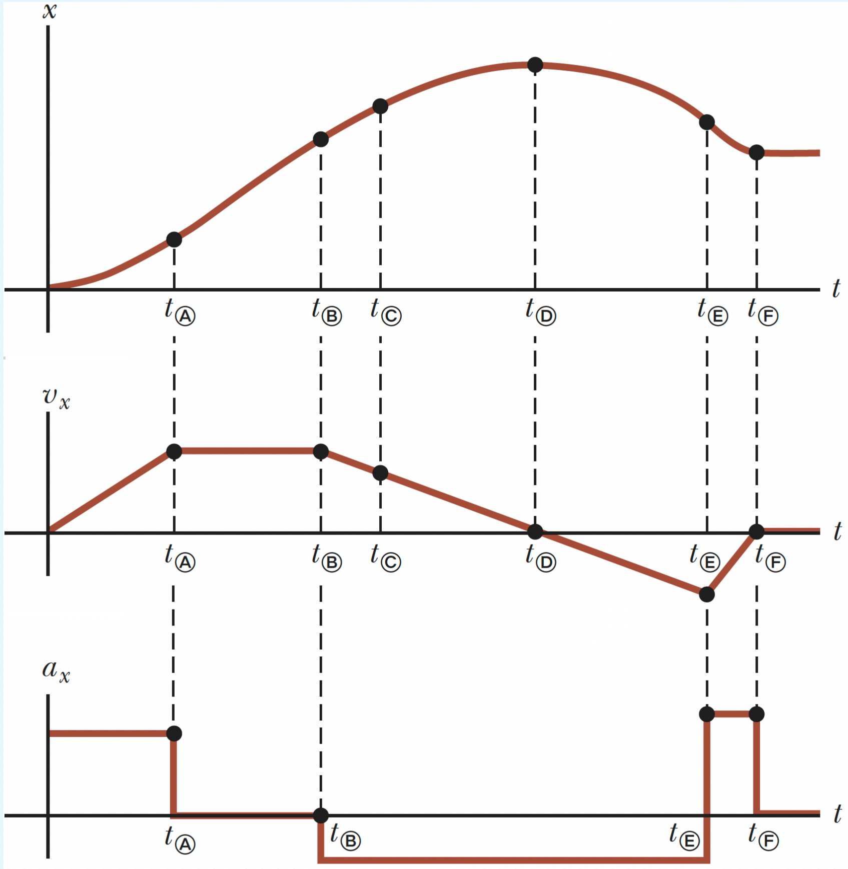
- Between $t = 0$ and $t = t_A$, the slope of the x-t graph increases uniformly, and so the velocity increases linearly.
- Between t_A and t_B, the slope of the x-t graph is constant, and so the velocity remains constant.
- Between t_B and t_E, the slope of the x-t graph decreases, and so the velocity decreases linearly.



3.3 Examples

Solution 3.3

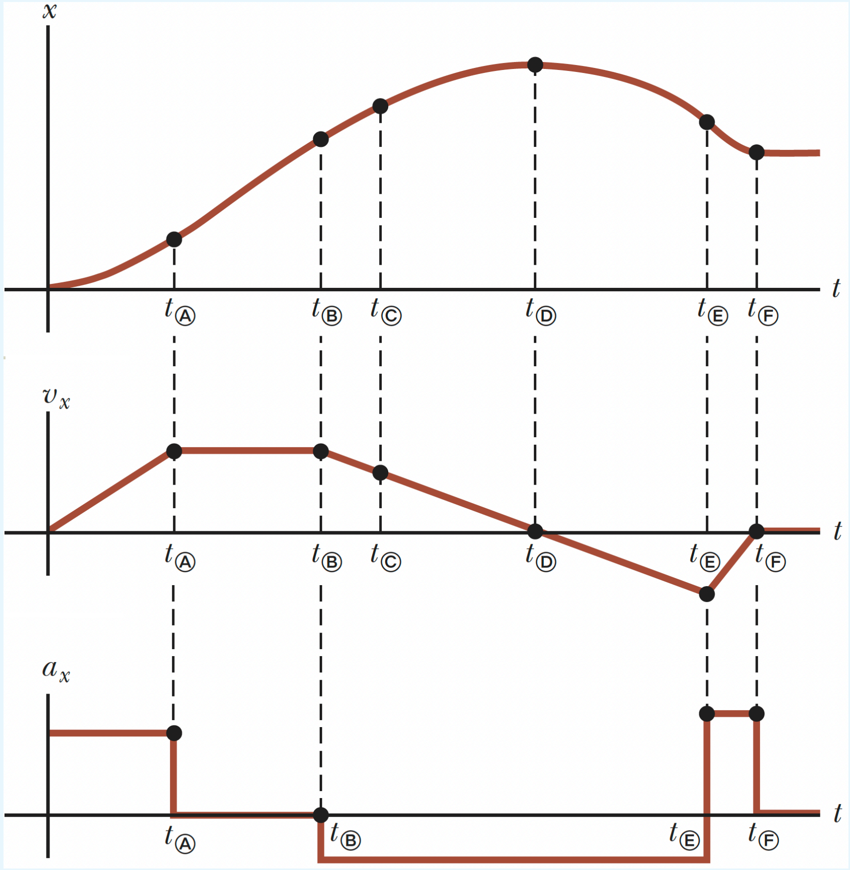
- At t_D, the slope of the x-t graph is zero, so the velocity is zero at that instant.
- Between t_D and t_E, the slope of the x-t graph is negative, and so the velocity decreases linearly.
- At t_F, the slope of the x-t graph is zero again, so the velocity is zero.



3.3 Examples

Solution 3.3

- The acceleration is constant and positive between 0 and t_A, where the slope of the v_x-t graph is positive.
- It is zero between t_A and t_B and for $t > t_F$ because the slope of the v_x-t graph is zero at these times.
- It is negative between t_B and t_E because the slope of the v_x-t graph is negative during this interval.



Example 3.4

The velocity of a particle moving along the x axis varies in time according to the expression $v_x = (40 - 5t^2)$ m/s, where t is in seconds.
(A) Find the average acceleration in the time interval ($t = 0 \longrightarrow 2.0$ s).

Solution 3.4

The average acceleration is given by:

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_x(t=2) - v_x(t=0)}{2-0}.$$

$$= \frac{(40 - 5(2)^2) - (40 - 5(0)^2)}{2-0} = \frac{20-40}{2} = -10 \text{ m/s}^2.$$

3.3 Examples

Example 3.4

(B) Determine the acceleration at $t = 2$ s.

Solution 3.4

- The *instantaneous* acceleration is given by the *derivative* of the velocity function:

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(40 - 5t^2) = -10t.$$

- Evaluating this at t = 2 s gives:

$$a_x(t = 2) = -10(2) = -20 \text{ m/s}^2.$$

1. Motion

2. Instantaneous Velocity and Speed

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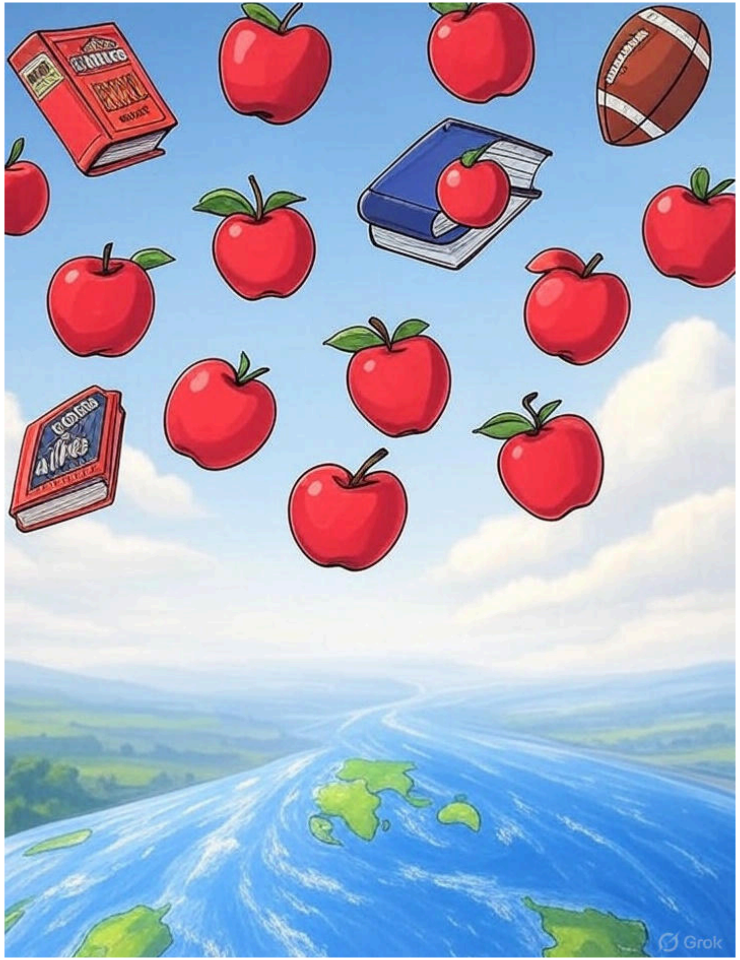
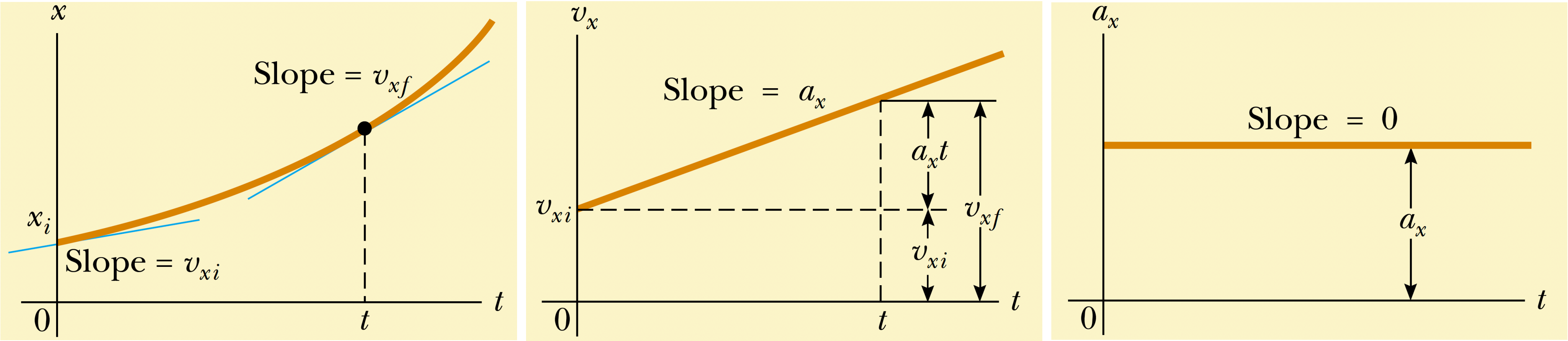
4.1 Why Constant Acceleration?

- In many real-world situations, objects experience constant acceleration.
- It simplifies the analysis of motion and allows us to develop very useful equations of motion.
- Examples of constant acceleration include:
  - Objects in free fall near the Earth's surface (neglecting air resistance).
  - Vehicles accelerating uniformly from rest.



4.2 Deriving the Equations of Motion

- The equations of motion describe the relationship between an object's position, velocity, acceleration, and time.



4.2 Deriving the Equations of Motion

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- From the definition of average acceleration and having *constant* acceleration, we can write:

$$a_x = \bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - 0} = \text{constant}$$

Solving for v_{xf}, we get:

$$\boxed{v_{xf} = v_{xi} + a_x t} \quad (1)$$

- Since the average velocity in a constant acceleration is:

4.2 Deriving the Equations of Motion

$$\bar{v}_x = \frac{v_{xf} + v_{xi}}{2}$$

- Equating this expression with the definition of average velocity, $\bar{v}_x = \frac{\Delta x}{\Delta t}$, gives:

$$\frac{v_{xf} + v_{xi}}{2} = \frac{x_f - x_i}{t_f - t_i}$$

- Solving for x_f and substituting $(t_i = 0; t \equiv t_f)$, gives:

$$\boxed{x_f = x_i + \frac{1}{2}(v_{xf} + v_{xi})t} \qquad (2)$$

4.2 Deriving the Equations of Motion

- Using Eq (1) to eliminate v_{xf} in Eq (2), we get:

$$x_f = x_i + \frac{1}{2}[(v_{xi} + a_x t) + v_{xi}]t$$

- By simplifying the expression, we get:

$$\boxed{x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2} \qquad (3)$$

4.2 Deriving the Equations of Motion

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- It is useful to eliminate time from the equations of motion, to do so, we substitute Eq (1) into Eq (2):

$$x_f = x_i + \frac{1}{2}(v_{xf} + v_{xi})\left[\frac{v_{xf} - v_{xi}}{a_x}\right]$$

- Simplifying the expression, we get:

$$\boxed{v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)} \qquad (4)$$

4.3 Four Steps to Solve Problems

1. Identify the known quantities.
2. Convert to SI units, if needed.
3. Identify the relevant equations.
4. Solve for the asked unknowns.

Equations of Motion at Constant Acceleration	
$v_{xf} = v_{xi} + a_x t$	(1)
$x_f = x_i + \frac{1}{2}(v_{xf} + v_{xi})t$	(2)
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	(3)
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	(4)

Quantity	x_i	x_f	v_{xi}	v_{xf}	a_x	t
Equations	2,3,4	2,3,4	all	1,2,4	1,3,4	1,2,3

4.4 Examples

Example 4.5

(A) Estimate your average acceleration as you drive up the entrance ramp to an interstate highway.

- Assume your final velocity is 30 m/s (about 100 km/h)
- Assume your initial velocity is 10 m/s.
- Assume that it takes 10 seconds to reach the final velocity.

4.4 Examples

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Solution 4.5

Quantity	x_i	x_f	v_{xi}	v_{xf}	$\bar{a}_x$	t
Value	0 m	-	10 m/s	30 m/s	?	10 s

- The average acceleration is given by Eq (1):

$$\bar{a}_x = a_x = \frac{v_{xf} - v_{xi}}{t} = \frac{30 - 10}{10} = 2\ \mathrm{m/s^2}.$$

Example 4.5

(B) How far did you go during the first half of the time interval during which you accelerated?

Solution 4.5

Quantity	x_i	x_f	v_{xi}	v_{xf}	a_x	$t/2$
Value	0 m	?	10 m/s	30 m/s	2 m/s^2	5 s

Eq (3):

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$
$$= 0 + (10)(5) + \frac{1}{2}(2)(5^2) = 50 + 25 = 75 \text{ m.}$$

4.4 Examples

Example 4.6

A jet lands on an aircraft carrier at 63 m/s. (A) What is its acceleration (assumed constant) if it stops in 2 s, due to an arresting cable that brings it to a stop?

Solution 4.6

Quantity	x_i	x_f	v_{xi}	v_{xf}	a_x	t
Value	0 m	-	63 m/s	0 m/s	?	2 s

Using Eq (1):

$$a_x = \frac{v_{xf} - v_{xi}}{t} = \frac{0 - 63}{2.0} = -31.5 \text{ m/s}^2.$$

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Example 4.6

(B) If the plane touches down at position $x_i = 0$, what is the final position of the plane?

Solution 4.6

Quantity	x_i	x_f	v_{xi}	v_{xf}	a_x	t
Value	0 m	?	63 m/s	0 m/s	−31.5 m/s^2	2 s

Using Eq (2), (3) or (4) is possible, and all of them give the same result:

$$\text{Eq (2):} \quad x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t = 0 + \frac{1}{2}(63 + 0)(2.0) = 63 \text{ m.}$$

4.4 Examples

Example 4.6

(C) Suppose the landing space has a length of 75 m. what is the maximum *initial* speed at which the plane can land and still stop within the landing space?

4.4 Examples

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Solution 4.6

Quantity	x_i	x_f	v_{xi}	v_{xf}	a_x	t
Value	0 m	75 m	?	0 m/s	$-31.5\ \text{m/s}^2$	-

- Since t is unknown, we can only use Eq (4) and solve for v_{xi} :

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$\Rightarrow v_{xi} = \pm\sqrt{v_{xf}^2 - 2a_x(x_f - x_i)}$$

$$= \pm\sqrt{0 - 2(-31.5)(75 - 0)}$$

$$= +\ 68.7\ \text{m/s}.$$

- The negative root is not physically meaningful in this context; therefore, we discard it.

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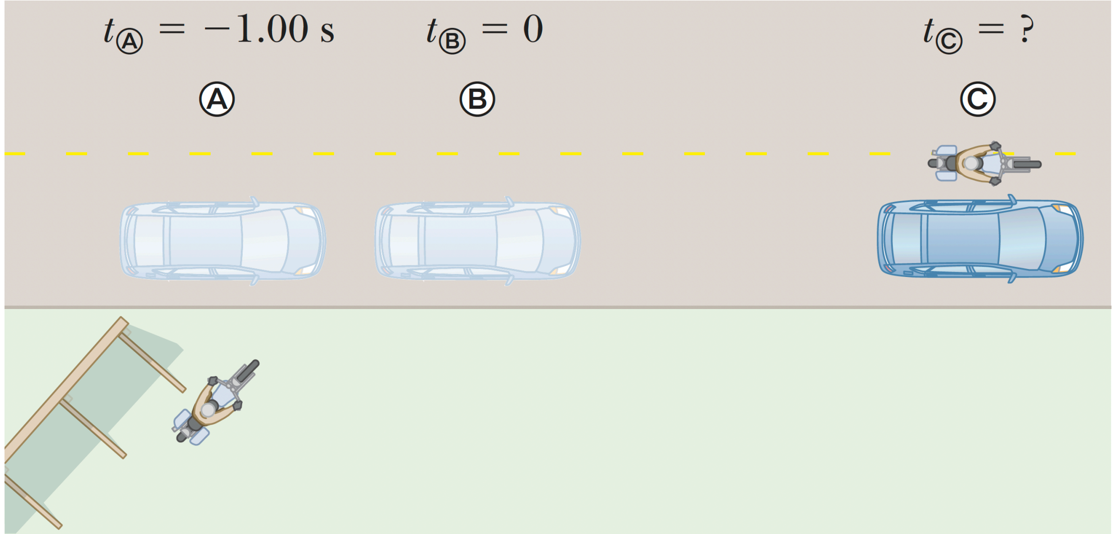
Example 4.7

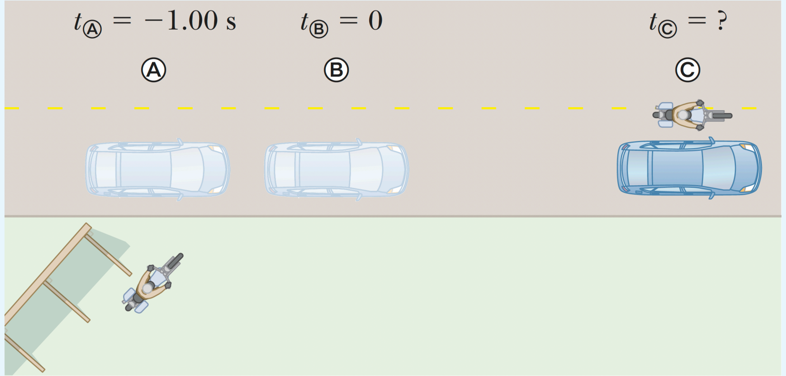
A car traveling at a constant speed of 45 m/s passes a trooper (شرطي متخفي) hidden behind a billboard (لوحة إعلانات). One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch it, accelerating at a constant rate of 3 m/s^2. How long does it take him to overtake the car?



4.4 Examples

Solution 4.7

Car	x_i	x_f	v_{xi}	v_{xf}	a_x	t_i	t_f
Value	x_A	x_C	45 m/s	45 m/s	0 m/s^2	-1 s	t_C
Trooper	x_i	x_f	v_{xi}	v_{xf}	a_x	t_i	t_f
Value	x_A	x_C	0 m/s	-	3 m/s^2	0 s	t_C



4.4 Examples

- Using Eq (3) for both the car and the trooper, we find the position of each object as a function of time:
  - The trooper starts from rest, so his position is given by:

$$x_{\text{trooper}} = x_A + v_{xi}t + \frac{1}{2}at^2 = 0 + (0)t + \frac{1}{2}(3)t^2 = 1.5t^2.$$

  - The car is moving at constant speed, so its position is given by:

$$x_{\text{car}} = x_A + v_{\text{x,car}}\left(t_f - t_i\right) = 0 + 45(t - (-1)) = 45(t + 1).$$

- Finally, we set positions equal to each other to find the time t_C when the trooper overtakes the car:

4.4 Examples

$$x_C = x_{\text{trooper}}(t_C) = x_{\text{car}}(t_C)$$

$$1.5\ t_C^2 = 45 + 45\ t_C$$

$$\Rightarrow 1.5\ t_C^2 - 45\ t_C - 45 = 0$$

- Applying the quadratic formula to find t_C:

$$\Rightarrow t_c = \frac{45 \pm \sqrt{(45)^2 - 4(1.5)(-45)}}{2(1.5)}$$

$$\Rightarrow t_c = +31\ \text{s} \qquad \text{(ignore the negative solution)}$$

1. Motion

2. Instantaneous Velocity and Speed

3. Acceleration

4. One-Dimensional Motion with Constant Acceleration

5. Freely Falling Objects

5.1 What is Free Fall?

- Free fall is the motion of an object under the influence of gravitational force *only*.
- Gravity causes all freely falling objects to accelerate downward at a constant rate, typically denoted as $g \approx 9.81 \text{ m/s}^2$ near the surface of the Earth.
- This means that the object's velocity increases linearly with time, and its position changes quadratically with time, according the equations of motion.

5.1 What is Free Fall?

Quiz

After a ball is thrown upward and is in the air, its speed:
(a) increases (b) decreases (c) increases and then decreases (d) decreases and then increases (e) remains the same?

Answer

The speed of the ball decreases as it rises, reaches a maximum at the top of its trajectory, and then increases as it falls back down. Therefore, the correct answer is (d) decreases and then increases.

5.2 Examples

Example 5.8

A sky diver jumps out of a hovering helicopter. A few seconds later, another sky diver jumps out, and they both fall along the same vertical line. Ignore air resistance, so that both sky divers fall with the same acceleration. Does the difference in their speeds stay the same throughout the fall? Does the vertical distance between them stay the same throughout the fall?

Solution 5.8

- Speed:
  - From Eq(1) and assuming the initial speed is zero, we have:

$$v(t) = v_i + at = 0 + (-g)t = -gt$$

5.2 Examples

- The difference in their speeds is given by:

$$v_2(t_2) - v_1(t_1) = -gt_2 - (-gt_1) = -g(t_2 - t_1).$$

- Since g and $(t_2 - t_1)$ are constant numbers, the difference in their speeds will be constant throughout the fall.

- Distance:
  - From Eq(3) and assuming the initial position and velocity are zero, we have:

$$y(t) = y_i + v_i t + \frac{1}{2}at^2 = 0 + 0 - \frac{1}{2}gt^2 = -\frac{1}{2}gt^2.$$

  - The difference in their positions is given by:

$$y_2(t_2) - y_1(t_1) = -\frac{1}{2}g(t_2^2 - t_1^2) = -\frac{1}{2}g(t_2 - t_1)(t_1 + t_2).$$

5.2 Examples

- Notice that $t_2 - t_1$ is always constant, however, $t_1 + t_2$ is not.
- Therefore, the vertical distance between the two sky divers will increase throughout the fall.

Example 5.9

A ball is thrown straight up with an initial speed of 25 m/s. Estimate its velocity at one second intervals.

Solution 5.9

- Givens:

5.2 Examples

King Saud University

- $v_{yi} = 25$ m/s
- $a_y \approx -10$ m/s^2
- $t = 1, 2, 3, \ldots$ s

- Let's choose the upward direction as positive.
- Therefore, every second, the velocity decreases by 10 m/s.
- The velocity for the first four seconds can be found from Eq (1), as:
  - $t = 1$ s : $v_y = 25 - 10(1) = 15$ m/s
  - $t = 2$ s : $v_y = 25 - 10(2) = 5$ m/s
  - $t = 3$ s : $v_y = 25 - 10(3) = -5$ m/s

5.2 Examples

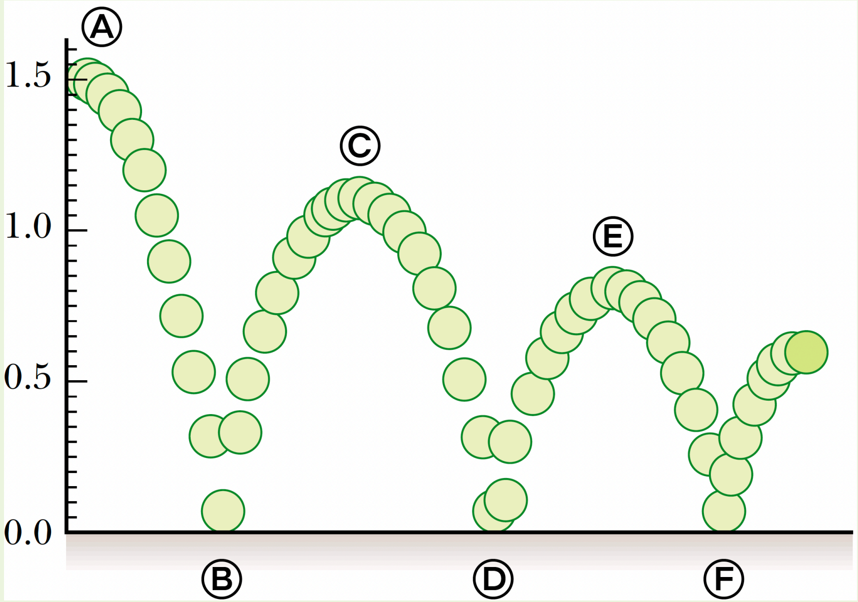
- $t = 4 \text{ s} : v_y = 25 - 10(4) = -15 \text{ m/s}$

- The negative sign indicates that the ball is moving downward at this time.

5.2 Examples

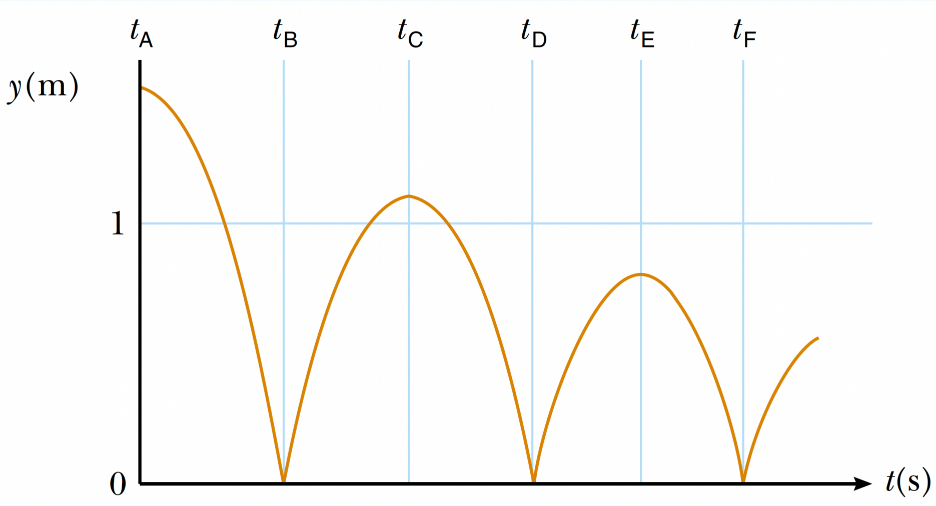
Example 5.10

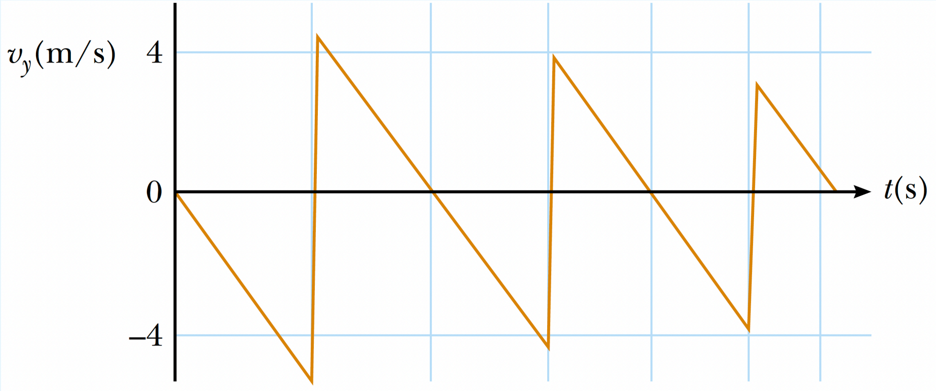
A tennis ball is dropped from shoulder height (about 1.5 m) and bounces three times before it is caught. Sketch graphs of its position, velocity, and acceleration as functions of time, with the +y direction defined as upward.



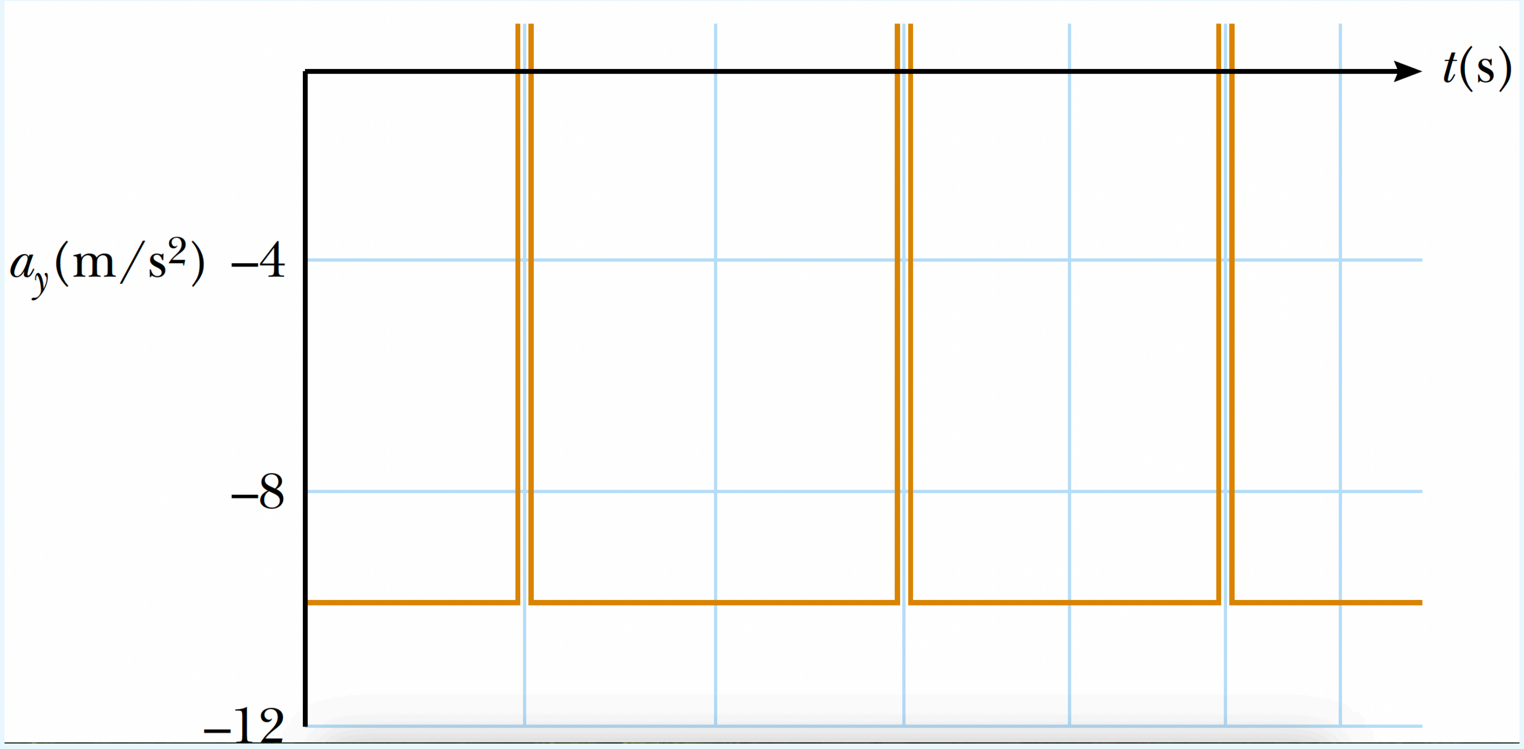
5.2 Examples

Solution 5.10





5.2 Examples



5.2 Examples

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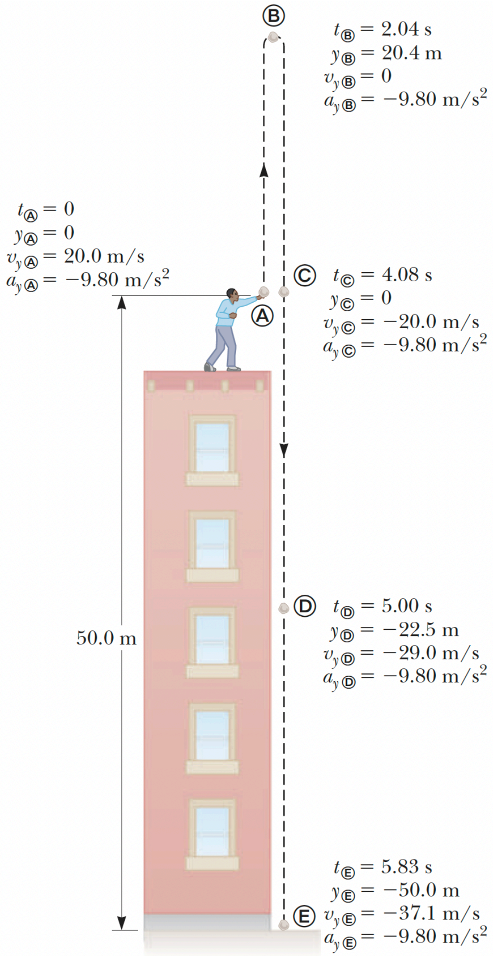
Example 5.11

A stone thrown from the top of a building is given an initial velocity of 20 m/s straight upward. The building is 50 m high, and the stone just misses the edge of the roof on its way down, as shown. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position y_A, determine (A) the time at which the stone reaches its maximum height, (B) the maximum height, (C) the time at which the stone returns to the height from which it was thrown, (D) the velocity of the stone at this instant, and (E) the velocity and position of the stone at $t = 5s$.



5.2 Examples

Solution 5.11

- (A)

Quantity	y_i	y_f	v_{yi}	v_{yf}	a_y	t_i	t_f
Value	$y_A = 0$ m	y_B	20 m/s	0 m/s	$-g$	$t_A = 0$	$t_B =?$

Maximum height means $v_{yf} = v_{yB} = 0$. The time to reach this point is given by Eq (1):

$$v_{yf} = v_{yi} + a_y t \;\Rightarrow\; v_{yB} = v_{yA} + a_y t_B$$

$$\Rightarrow t_B = \frac{v_B - v_{yA}}{a_y} = \frac{0 - 20}{-9.8} = 2.04 \text{ s.}$$

5.2 Examples

- (B) The maximum height can be found from Eq (3) using t_B:

$$y_{\text{max}} = y_B = y_A + v_{yA}\ t_B + \frac{1}{2}a_y t_B^2$$
$$= 0 + 20(2.04) + \frac{1}{2}(-9.8)(2.04)^2 = 20.4 \text{ m} .$$

- (C) Ignoring the air resistance, there is a clear symmetry in the motion. The time taken to reach the maximum height is equal to the time taken to return to the original height. Therefore, the time at which the stone returns to the height from which it was thrown is given by:

$$t_C = 2t_B = 2(2.04) = 4.08 \text{ s.}$$

Alternatively, we can find t_C by determining when the stone reaches the height $y_C = 0$ m. In this case, we can use Eq (3):

5.2 Examples

$$y_C = y_A + v_{yA} t_C + \frac{1}{2} a_y t_C^2$$

$$0 = 0 + 20 t_C + \frac{1}{2}(-9.8) t_C^2$$

$$\Rightarrow t_C(20 - 4.9 t_C) = 0$$

$$\Rightarrow (t_A = 0 \text{ s}) \quad \text{or} \quad (t_C = 4.08 \text{ s}).$$

- (D) Since the motion is symmetrical, the velocity at this point will be equal in magnitude but opposite in direction to the initial velocity:

$$v_C = -v_{yA} = -20 \text{ m/s}.$$

Alternatively, we can find v_C by using the equation:

5.2 Examples

$$v_C = v_{yA} + a_y t_C = 20 - 9.8(4.08) = -20 \text{ m/s}.$$

- (E) To find the velocity and position of the stone at $t = 5s$, we can use Eq (1) and (3):
  - Velocity:

$$v_D = v_{yA} + a_y t_D = 20 - 9.8(5) = -29 \text{ m/s}.$$

  - Position:

$$y_D = y_A + v_{yA} t_D + \frac{1}{2} a_y t_D^2 = 0 + 20(5) + \frac{1}{2}(-9.8)(5)^2 = -22.5 \text{ m}.$$

5.3 Suggested Problems

4, 5, 11, 15, 16, 20, 21, 22, 23, 25, 27, 28, 29, 32, 33, 40, 42, 43, 46, 48, 51, 52