Integral Calculus

Prof. Mohamad Alghamdi

Department of Mathematics

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Main Contents

1. Antiderivatives.
2. Indefinite Integrals.
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   - Properties of Indefinite Integrals.
3. Integration By Substitution.

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Theorem

If functions F and G are antiderivatives of f on an interval I, there exists a constant c such that $G(x) = F(x) + c$ OR $G(x) - F(x) = c$ for every $x \in I$.

In above example, we have $G(x) - F(x) = x^2 + 2 - x^2 = 2$

Note: The function $F(x) = x^2 + c$ is the general form of the antiderivatives (the family) of the function $f(x) = 2x$.

Section 2: Indefinite Integrals

Definition

Let f be a continuous function on an interval I. The indefinite integral of f is the general antiderivative of f on I:

$$\int f(x)\,dx = F(x) + c.$$

The function f is called the integrand, the symbol $\int$ is the integral sign, x is called the variable of the integration and c is the constant of the integration.

Note: From the example $F^{'}(x) = \frac{d}{dx}(x^2 + 3x) = 2x + 3 = f(x)$, we say that the function $F(x)$ is an antiderivative of $f(x)$. So,

$$\int (2x+3)\,dx = \underbrace{x^2 + 3x}_{F(x)} + c.$$

Properties of Indefinite Integrals

Theorem

Assume f and g have antiderivatives on an interval I, then

1. $\dfrac{d}{dx}\displaystyle\int f(x)\,dx = f(x).$
2. $\displaystyle\int \frac{d}{dx}(F(x))\,dx = F(x) + c.$
3. $\displaystyle\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx.$
4. $\displaystyle\int kf(x)\,dx = k\int f(x)\,dx$, *where k is a constant.*

Integration as an Inverse Process of Differentiation

Rule 1: Power of x.

$$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n \;\Rightarrow\; \int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad \text{for} \quad n \neq -1 \; .$$

Note

Note that **Rule 1** cannot be applied for $n = -1$.

For this value, the formula gives

$$\int x^{-1} \, dx = \frac{x^0}{0} = \infty \; .$$

Special case: For $n = 0$, we have $\int 1 \, dx = x + c$.

From this, $\int 2 \, dx = 2x + c$ and $\int 3 \, dx = 3x + c$ etc.

Example

Evaluate the integral.

1. $\int x \, dx$
2. $\int x^3 \, dx$

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Solution:

1. $\int x \, dx = \frac{x^2}{2} + c$
2. $\int x^3 \, dx = \frac{x^4}{4} + c$

Example

Evaluate the integral.

1. $\int 4\,x\,dx$
2. $\int 7\,x^3\,dx$
3. $\int \frac{1}{x^2}\,dx$
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$$\int kf(x)\ dx = k \int f(x)\ dx$$

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3. $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-1}}{-1} + c = -\,\frac{1}{x} + c$ $\qquad$ *Remember:* $x^{-n} = \frac{1}{x^n}$

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   Remember: $x^{-n} = \frac{1}{x^n}$

4. $\int \frac{1}{\sqrt{x}}\,dx = \int \frac{1}{x^{\frac{1}{2}}}\,dx = \int x^{-\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

   $-\frac{1}{2} + \frac{1}{1} = \frac{-1+2}{2} = \frac{1}{2}$

   Remember:

   (1) $\sqrt{x} = x^{\frac{1}{2}}$ and $\sqrt[m]{x^n} = x^{\frac{n}{m}}$

   (2) $\frac{a}{b} \pm \frac{c}{d} = \frac{a\times d \pm c\times b}{b\times d}$

Example

Evaluate the integral.

1. $\int (x + 1)\ dx$
2. $\int (4x^3 + 2x^2 + 1)\ dx$
3. $\int \left(x^2 - \frac{1}{x^3}\right)\ dx$

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$$\int \left(f(x) \pm g(x)\right)\ dx = \int f(x)\ dx \pm \int g(x)\ dx.$$

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2. $\int (4x^3 + 2x^2 + 1)\ dx = \int 4x^3\ dx + \int 2x^2\ dx + \int 1\ dx = \frac{4x^4}{4} + \frac{2}{3}x^3 + x + c = x^4 + \frac{2}{3}x^3 + x + c\ .$

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2. $\int (4x^3 + 2x^2 + 1)\ dx = \int 4x^3\ dx + \int 2x^2\ dx + \int 1\ dx = \frac{4x^4}{4} + \frac{2}{3}x^3 + x + c = x^4 + \frac{2}{3}x^3 + x + c\ .$
3. $\int \left(x^2 - \frac{1}{x^3}\right)\ dx = \int x^2\ dx - \int x^{-3}\ dx = \frac{x^3}{3} - \frac{x^{-2}}{-2} + c = \frac{x^3}{3} + \frac{1}{2x^2} + c\ .$

Rule 2: Trigonometric Functions.

- $\frac{d}{dx}(\sin\ x) = \cos\ x \Rightarrow \int \cos\ dx = \sin\ x + c$
- $\frac{d}{dx}(\cos\ x) = -\sin\ x \Rightarrow \int -\sin\ x\, dx = \cos\ x + c$ OR $\int \sin\ x\, dx = -\cos\ x + c$
- $\frac{d}{dx}(\tan\ x) = \sec^2\ x \Rightarrow \int \sec^2\ x\, dx = \tan\ x + c$
- $\frac{d}{dx}(\cot\ x) = -\csc\ x \Rightarrow \int -\csc^2\ x\, dx = \cot\ x + c$ OR $\int \csc^2\ x\, dx = -\cot\ x + c$
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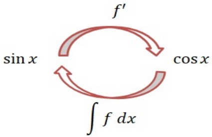
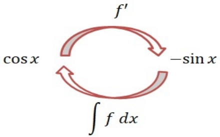
Section 2: Indefinite Integrals





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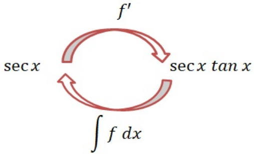

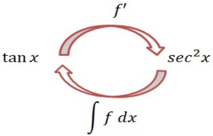

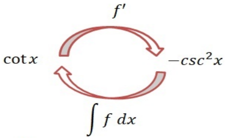

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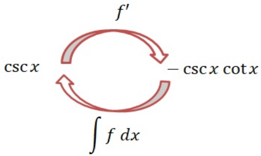

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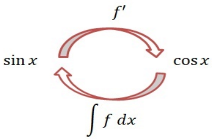
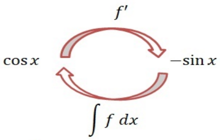
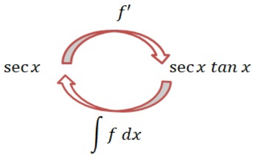
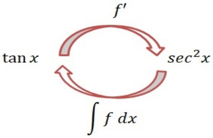
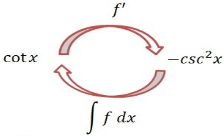




$$\int \sin x \;\; dx = -\cos x + c$$







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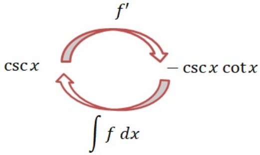

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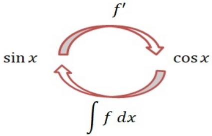
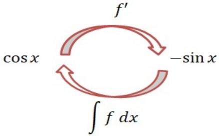
Example

Evaluate the integral $\int (\cos x + \sec x \;\; \tan x) \; dx$





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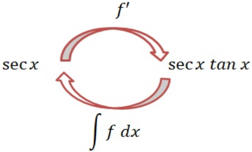

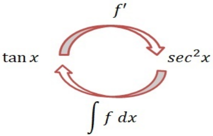

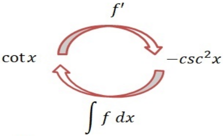

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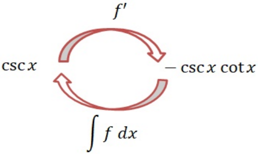

$$\int csc\, x \cot x \;\; dx = -\csc x + c$$

Example

Evaluate the integral $\int (\cos x + \sec x \;\; \tan x) \; dx$

Solution:

$$\int (\cos x + \sec x \;\; \tan x) \; dx = \int \cos x \; dx + \int \sec x \;\; \tan x \; dx = \sin x + \sec x + c$$

Example

Evaluate the integral.

1. $\int \left(\frac{1}{\cos^2 x} - \sin x\right)\, dx$
2. $\int \sec x\,(\sec x + \tan x)\, dx$

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   $\sec x = \frac{1}{\cos x} \Rightarrow \sec^2 x = \frac{1}{\cos^2 x}$

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2. $\int \sec x \, (\sec x + \tan x) \, dx = \int \sec^2 x \, dx + \int \sec x \, \tan x \, dx = \tan x + \sec x + c$.

Example

Evaluate the integral.

1. $\int (4x+3)\ dx$
2. $\int (2\sin\ x + 3\cos\ x)\ dx$
3. $\int (\sqrt{x} + \sec^2\ x)\ dx$
4. $\int \frac{d}{dx}(\sin\ x)\ dx$
5. $\frac{d}{dx}\int \sqrt{x+1}\ dx$

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1. $\int (4x+3)\ dx = 4\int x\ dx + \int 3\ dx = \frac{4x^2}{2} + 3x + c = 2x^2 + 3x + c.$

   $\int (f(x) \pm g(x))\ dx = \int f(x)\ dx \pm \int g(x)\ dx.$ and $\int kf(x)\ dx = k\int f(x)\ dx$

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5. $\frac{d}{dx}\int \sqrt{x+1}\ dx = \sqrt{x+1}.$

   $\frac{d}{dx}\int f(x)\ dx = f(x).$

Integration By Substitution

Remember Rule 1:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad \text{for} \quad n \neq -1 \qquad \qquad Example: \int x^3 \, dx = \frac{x^4}{4} + c$$

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Can we use **Rule 1** to evaluate $\int 2x \, (x^2 + 1)^3 \, dx$?

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Theorem

Let g be a differentiable function on an interval I where the derivative is continuous. Let f be continuous on an interval J that contains the range of the function g. If $\int f(x) \, dx = F(x) + c$*, then*

$$\int f(g(x)) \, g'(x) \, dx = F(g(x)) + c, \quad \forall x \in I.$$

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Evaluate the integral $\int 2x \, (x^2+1)^3 \, dx$.

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$$\int f(g(x)) \, g'(x) \, dx = F(g(x)) + c, \quad \forall x \in I.$$

Example

Evaluate the integral $\int 2x \, (x^2+1)^3 \, dx$.

Solution: Let $f(x) = x^3$ and $g(x) = x^2 + 1$, then $(f \circ g)(x) = f(g(x)) = (x^2+1)^3$.

$$g(x) = x^2 + 1 \Rightarrow g'(x) = 2x$$

From the theorem, we have

$$\int \underbrace{2x}_{g'(x)} \underbrace{(x^2+1)^3}_{f(g(x))} \, dx = \frac{(x^2+1)^4}{4} + c \, .$$

Integration By Substitution

We can end with the same solution by using the five steps of the substitution method given below.

Steps of the integration by substitution:

Step 1: Choose a new variable u.
Step 2: Determine the value of du.
Step 3: Make the substitution i.e., eliminate all occurrences of x in the integral by making the entire integral in terms of u.
Step 4: Evaluate the new integral.
Step 5: Return the evaluation to the initial variable x.

Exercise: Evaluate the integral $\int u^3 \, du$

$$\int u^3 \, du = \frac{u^4}{4} + c$$

Example

Evaluate the integral $\int 2x \, (x^2+1)^3 \, dx$.

Solution: Let

$$u = x^2 + 1 \Rightarrow du = 2x \, dx \Rightarrow \frac{du}{2x} = dx$$

. By substituting that into the original integral, we have

$$\int 2x \, u^3 \, \frac{du}{2x} = \int u^3 \, du = \frac{u^4}{4} + c = \underbrace{\frac{(x^2+1)^4}{4}}_{\text{Returning the evaluation to } x} + c$$

Integration By Substitution

Exercise: Evaluate the integral $\int \sqrt{x}\, dx$

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$$\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{x^{\frac{3}{2}}}{3/2} + c = \frac{2x^{\frac{3}{2}}}{3} + c$$

Example

Evaluate the integral $\int x^2\sqrt{2x^3 - 5}\,dx$

Exercise: Evaluate the integral $\int \sqrt{x}\, dx$

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Example

Evaluate the integral $\int x^2 \sqrt{2x^3 - 5}\, dx$

Solution: $\int x^2 \sqrt{2x^3 - 5}\, dx = \int x^2 (2x^3 - 5)^{\frac{1}{2}}\ dx$

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Example

Evaluate the integral $\int x^2\sqrt{2x^3 - 5}\ dx$

Solution: $\int x^2\sqrt{2x^3 - 5}\ dx = \int x^2(2x^3 - 5)^{\frac{1}{2}}\ dx$

Let $f(x) = x^{\frac{1}{2}}$ and $g(x) = 2x^3 - 5$, then $(f \circ g)(x) = f\big(g(x)\big) = (2x^3 - 5)^{\frac{1}{2}}$.

$$g(x) = 2x^3 - 5 \Rightarrow g'(x) = 6x^2$$

From the theorem $\int f(g(x))g'(x)\ dx = F(g(x)) + c$, we have

$$\frac{1}{6}\int \underbrace{6x^2}_{g'(x)}\underbrace{(2x^3 - 5)^{\frac{1}{2}}}_{f(g(x))}\ dx = \frac{1}{6}\ \frac{(2x^3 - 5)^{\frac{3}{2}}}{3/2} + c = \frac{1}{6}\ \frac{2}{3}\ (2x^3 - 5)^{\frac{3}{2}} + c = \frac{(2x^3 - 5)^{\frac{3}{2}}}{9} + c\ .$$

Example

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Evaluate the integral $\int x^2\sqrt{2x^3-5}\,dx$

Solution:

$$\int x^2\sqrt{2x^3-5}\,dx = \int x^2(2x^3-5)^{\frac{1}{2}}\,dx$$

Let

$$u = 2x^3 - 5 \Rightarrow du = 6x^2\,dx \Rightarrow \frac{du}{6x^2} = dx$$

By substitution, we have

$$\int x^2\, u^{\frac{1}{2}}\,\frac{du}{6x^2} = \frac{1}{6}\int u^{\frac{1}{2}}\,du = \frac{1}{6}\frac{u^{\frac{3}{2}}}{3/2} + c = \frac{1}{6}\frac{2}{3}u^{\frac{3}{2}} + c = \frac{u^{\frac{3}{2}}}{9} + c = \underbrace{\frac{(2x^3-5)^{\frac{3}{2}}}{9}}_{\text{Returning the evaluation to } x} + c$$

Integration By Substitution

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Evaluate the integral $\int \frac{\sec^2 \ \sqrt{x}}{\sqrt{x}} \ dx$

Integration By Substitution

Exercise: Evaluate the integral $\int \sec^2 x \, dx$

$\int \sec^2 x \, dx = \tan x + c$

Example

Evaluate the integral $\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} \, dx$

Solution: Let $f(x) = \sec^2 x$ and $g(x) = \sqrt{x}$, then $(f \circ g)(x) = f(g(x)) = \sec^2 \sqrt{x}$.

$$g(x) = \sqrt{x} \Rightarrow g^{'}(x) = \frac{1}{2\sqrt{x}}$$

From the theorem $\int f(g(x))g^{'}(x) \, dx = F(g(x)) + c$, we have

$$\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} \, dx = 2 \int \sec^2 \sqrt{x} \, . \, \frac{1}{2\sqrt{x}} \, dx = 2\tan \sqrt{x} + c.$$

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Evaluate the integral $\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} \, dx$

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From the theorem $\int f(g(x))g'(x) \, dx = F(g(x)) + c$, we have

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Example

Evaluate the integral $\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} \, dx.$

Solution: Let $u = \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}} \, dx \Rightarrow 2\sqrt{x} \, du = dx$. By substitution, we obtain

$$\int \frac{\sec^2 u}{\sqrt{x}} \, 2\sqrt{x} \, du = 2 \int \sec^2 u \, du = 2 \tan u + c = 2 \tan \sqrt{x} + c$$

Example

Evaluate the integral

1. $\int \sqrt{2x-5}\; dx$
2. $\int \cos\,(3x+4)\; dx$

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$$\int u^{\frac{1}{2}}\ \frac{du}{2}$$

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Evaluate the integral

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   Let $u = 2x-5 \Rightarrow du = 2\; dx \Rightarrow dx = \frac{du}{2}$. By substitution, we have

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OR

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Integration By Substitution

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Evaluate the integral

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Integration By Substitution

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Evaluate the integral

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2. $\int \cos\ (3x+4)\ dx = \frac{1}{3}\ \int 3\ \cos\ (3x+4)\ dx$

Example

Evaluate the integral

1. $\int \sqrt{2x-5}\, dx$
2. $\int \cos(3x+4)\, dx$

Solution:

1. $\int \sqrt{2x-5}\, dx = \int (2x-5)^{\frac{1}{2}}\, dx$

   Let $u = 2x-5 \Rightarrow du = 2\, dx \Rightarrow dx = \frac{du}{2}$. By substitution, we have

   $$\int u^{\frac{1}{2}} \frac{du}{2} = \frac{1}{2} \int u^{\frac{1}{2}}\, du = \frac{1}{2} \frac{u^{\frac{3}{2}}}{3/2} + c = \frac{2}{2} \frac{u^{\frac{3}{2}}}{3} + c = \frac{(2x-5)^{\frac{3}{2}}}{3} + c$$

   OR

   $$\int \sqrt{2x-5}\, dx = \int (2x-5)^{\frac{1}{2}}\, dx = \frac{1}{2} \int 2\,(2x-5)^{\frac{1}{2}}\, dx = \frac{1}{2} \frac{(2x-5)^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{(2x-5)^{\frac{3}{2}}}{3} + c$$

2. $\int \cos(3x+4)\, dx = \frac{1}{3} \int 3 \cos(3x+4)\, dx = \frac{1}{3} \sin(3x+4) + c$

Example

Evaluate the integral

1. $\int 5x(x^2+3)^7 \; dx$
2. $\int \sec^2(4x) \; dx$

Example

Evaluate the integral

1. $\int 5x(x^2+3)^7 \, dx$
2. $\int \sec^2(4x) \, dx$

Solution:

1. $5 \int x(x^2+3)^7 \, dx$

Example

Evaluate the integral

1. $\int 5x(x^2+3)^7 \; dx$
2. $\int \sec^2(4x) \; dx$

Solution:

1. $5\int x(x^2+3)^7 \; dx = \frac{5}{2}\int 2x(x^2+3)^7 \; dx$

Example

Evaluate the integral

1. $\int 5x(x^2+3)^7\ dx$
2. $\int \sec^2(4x)\ dx$

Solution:

1. $5\int x(x^2+3)^7\ dx = \frac{5}{2}\int 2x(x^2+3)^7\ dx = \frac{5}{2}\frac{(x^2+3)^8}{8} + c = 5\frac{(x^2+3)^8}{16} + c$

Integration By Substitution

Example

Evaluate the integral

1. $\int 5x(x^2+3)^7\ dx$
2. $\int \sec^2(4x)\ dx$

Solution:

1. $5\int x(x^2+3)^7\ dx = \frac{5}{2}\int 2x(x^2+3)^7\ dx = \frac{5}{2}\frac{(x^2+3)^8}{8} + c = 5\frac{(x^2+3)^8}{16} + c$
2. $\frac{1}{4}\int 4\sec^2(4x)\ dx$

Example

Evaluate the integral

1. $\int 5x(x^2+3)^7 \ dx$
2. $\int \sec^2(4x) \ dx$

Solution:

1. $5\int x(x^2+3)^7 \ dx = \frac{5}{2}\int 2x(x^2+3)^7 \ dx = \frac{5}{2}\frac{(x^2+3)^8}{8} + c = 5\frac{(x^2+3)^8}{16} + c$
2. $\frac{1}{4}\int 4\sec^2(4x) \ dx = \frac{1}{4}\tan(4x) + c$

Example

Evaluate the integral

1. $\int 5x(x^2+3)^7 \; dx$
2. $\int \sec^2(4x) \; dx$

Solution:

1. $5\int x(x^2+3)^7 \; dx = \frac{5}{2}\int 2x(x^2+3)^7 \; dx = \frac{5}{2}\frac{(x^2+3)^8}{8} + c = 5\frac{(x^2+3)^8}{16} + c$
2. $\frac{1}{4}\int 4\sec^2(4x) \; dx = \frac{1}{4}\tan(4x) + c$