Mass Procedure

Mass Procedure

1.Slopes in purely cohesionless soil with c = 0, $\phi \neq 0$

Failure generally does not take place in the form of a circle. So we will not go into this analysis.

2. Slopes in Homogeneous clay Soil with $c \neq 0$, $\phi = 0$

Determining factor of safety using equilibrium equations (Case I)

$M_{driving} = M_d = W_1 l_1 - W_2 l_2$

W_1 = (area of FCDEF) γ

W_2 = (area of ABFEA) γ

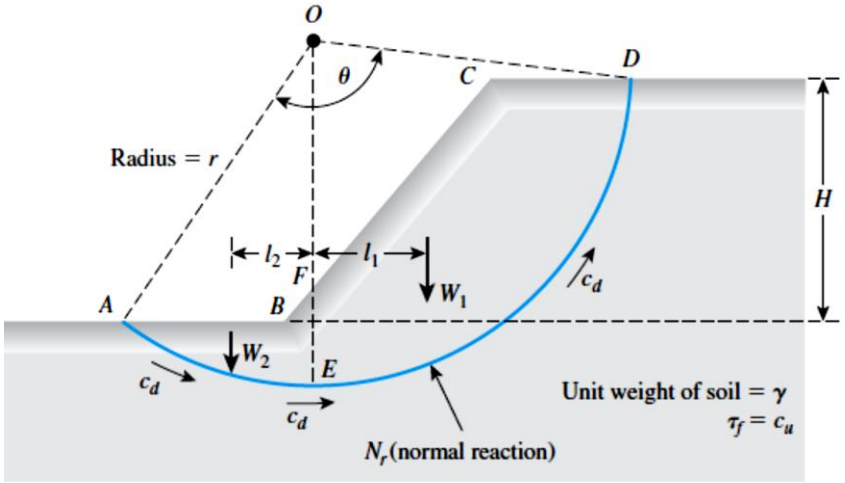
$M_{resisting} = M_R = c_d$ (AED) (1) r

For equilibrium, $M_R = M_d$; thus,

$$c_d = \frac{W_1 l_1 - W_2 l_2}{r^2 \theta}$$

$$F_s = \frac{\tau_f}{c_d} = \frac{c_u}{c_d}$$

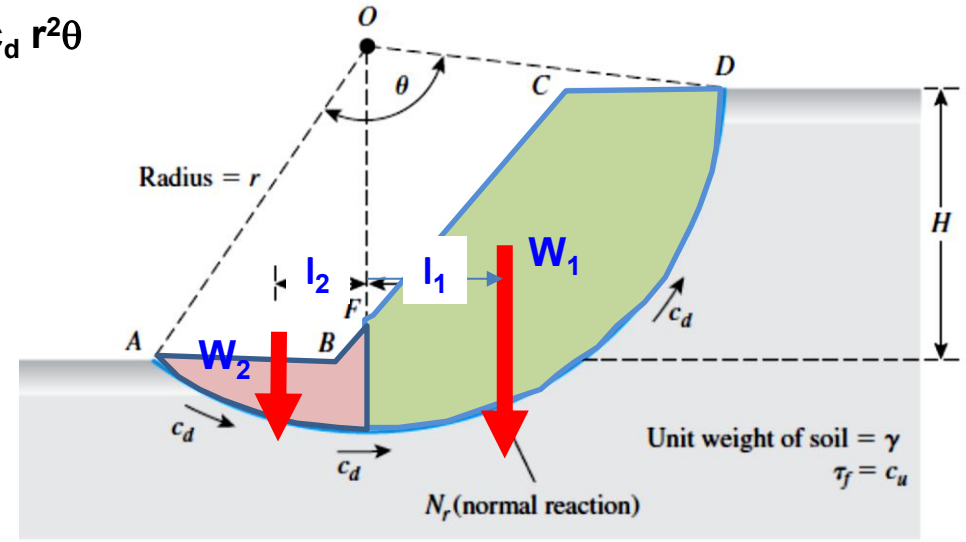
$$F_s = \frac{r^2 \theta c_u}{w_1 l_1 - w_2 l_2}$$



Mass Procedure

$M_{driving} = M_d = W_1 l_1 - W_2 l_2$

$M_{resisting} = M_R = c_d r^2 \theta$



$$F.S. = \frac{M_{resisting}}{M_{driving}}$$

$$F_s = \frac{\tau_f}{c_d} = \frac{c_u}{c_d}$$

$$\boldsymbol{F_s = \frac{r^2 \theta c_u}{w_1 l_1 - w_2 l_2}}$$

REMARKS

- The potential curve of sliding, *AED,* was chosen arbitrarily.
- The critical surface is that for which the ratio of C_u to C_d is a minimum. In other words, C_d is maximum.
- To find the critical surface for sliding, one must make a number of trials for different trial circles.
- The minimum value of the factor of safety thus obtained is the factor of safety against sliding for the slope, and the corresponding circle is the critical circle.

Finite Slopes with Circular Failure Surface

- Fellenius (1927) and Taylor (1937) have analytically solved for the ***minimum factor of safety*** and ***critical circles.***
- They expressed the developed cohesion as
  - We then can calculate the min F_s as

$$F_s = \frac{\tau_f}{c_d} = \frac{c_u}{c_d}$$

$$\boldsymbol{F_s = \frac{C_u}{\gamma H m}}$$

$c_d = \gamma H m$

Where

m = Stability number

H = height of slope

γ = unit weight of soil

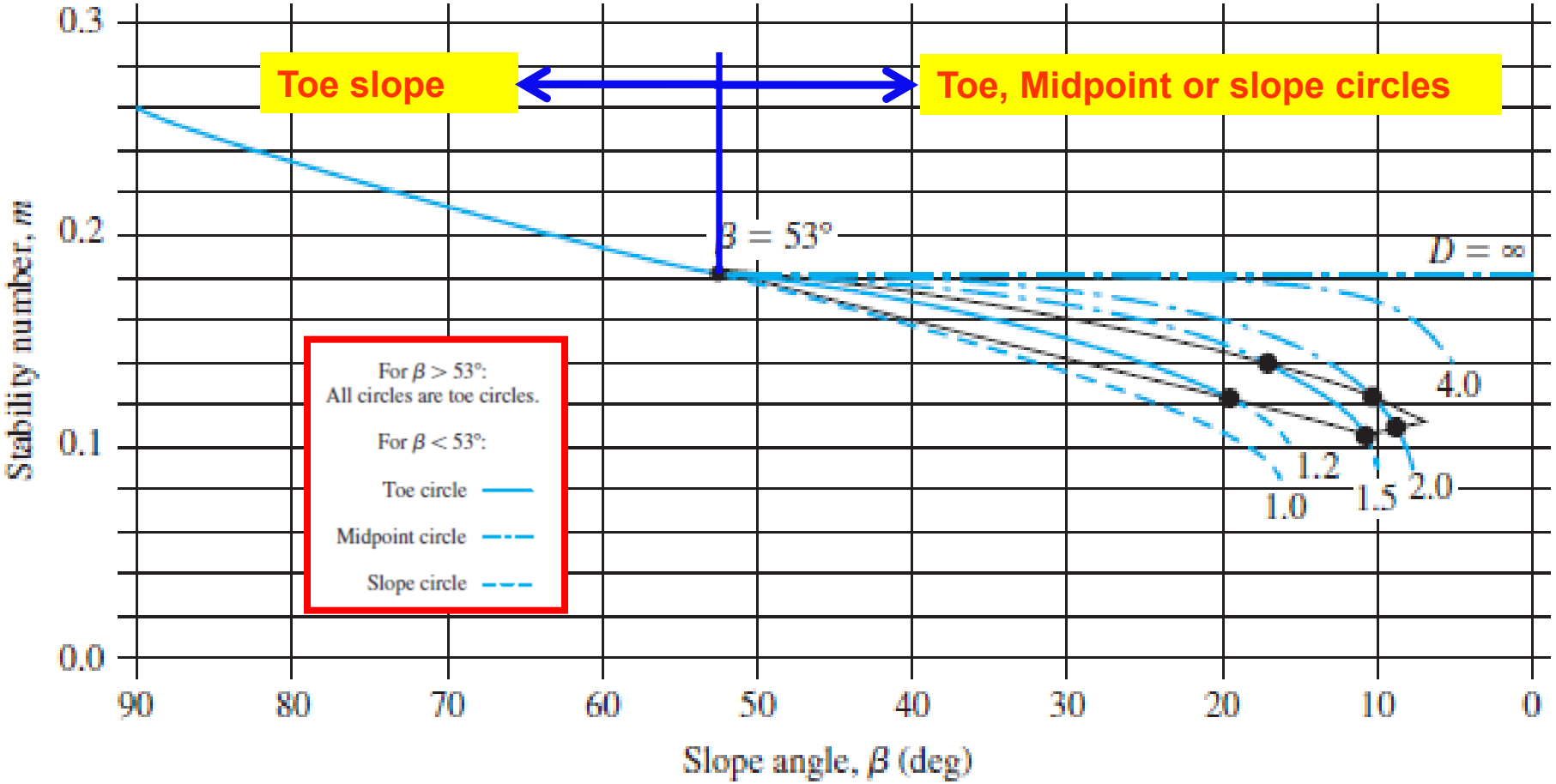
or $m = \frac{c_d}{\gamma H}$

- The critical height (i.e., Fs 1) of the slope can be evaluated by substituting $H = H_{cr}$ and $c_d = c_u$ (full mobilization of the undrained shear strength) into the preceding equation. Thus,

$$\boldsymbol{H_{cr} = \frac{C_u}{\gamma m}}$$

Finite Slopes with Circular Failure Surface

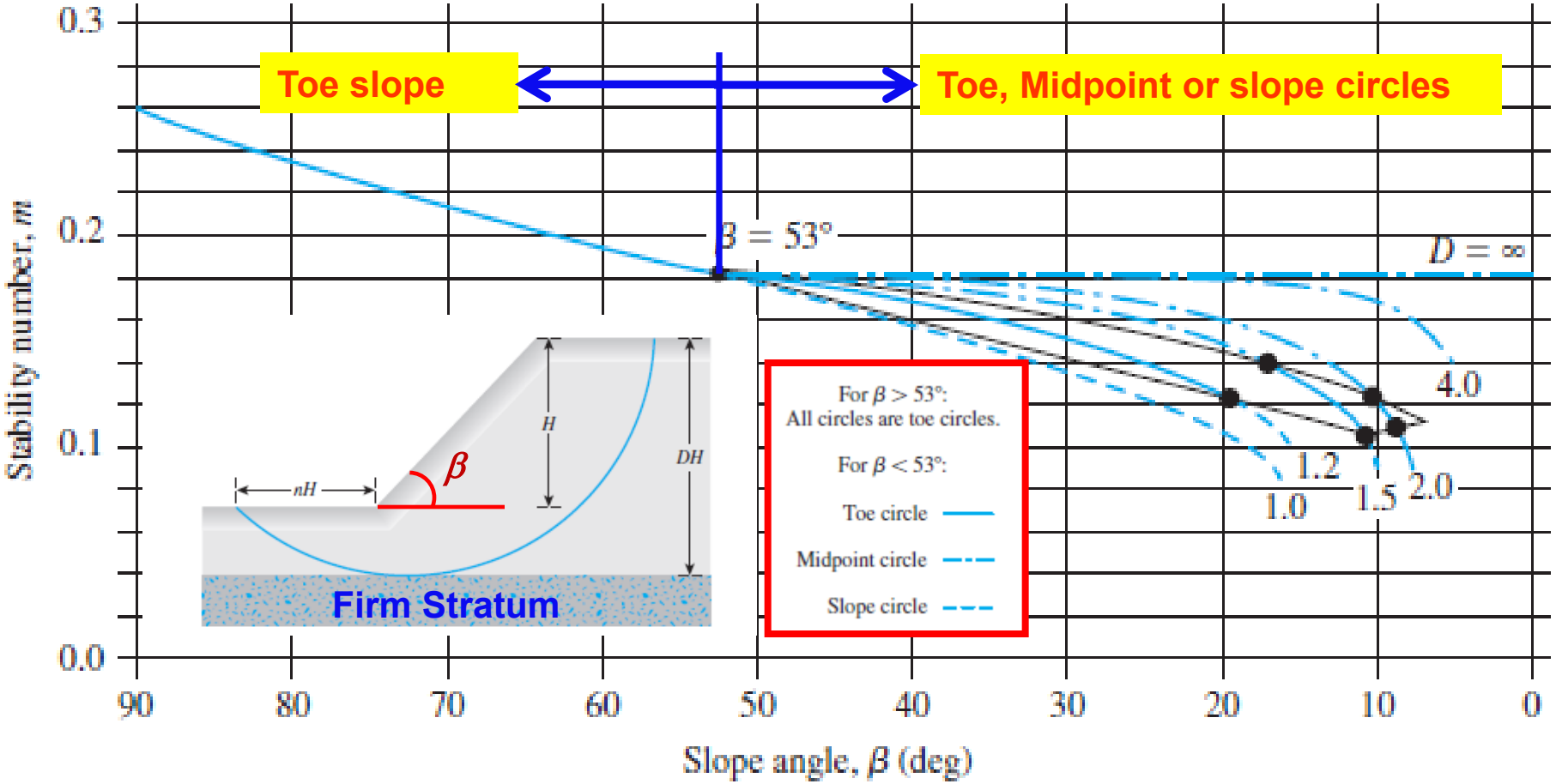
- The results of analytical solution to obtain critical circles was represented graphically as the variation of stability number, m, with slope angle β.



m is obtained from this chart depending on angle β

Finite Slopes with Circular Failure Surface

- The results of analytical solution to obtain critical circles was represented graphically as the variation of stability number, m, with slope angle β.



m is obtained from this chart depending on angle β

Finite Slopes with Circular Failure Surface

For $\beta > 53°$:
All circles are toe circles.

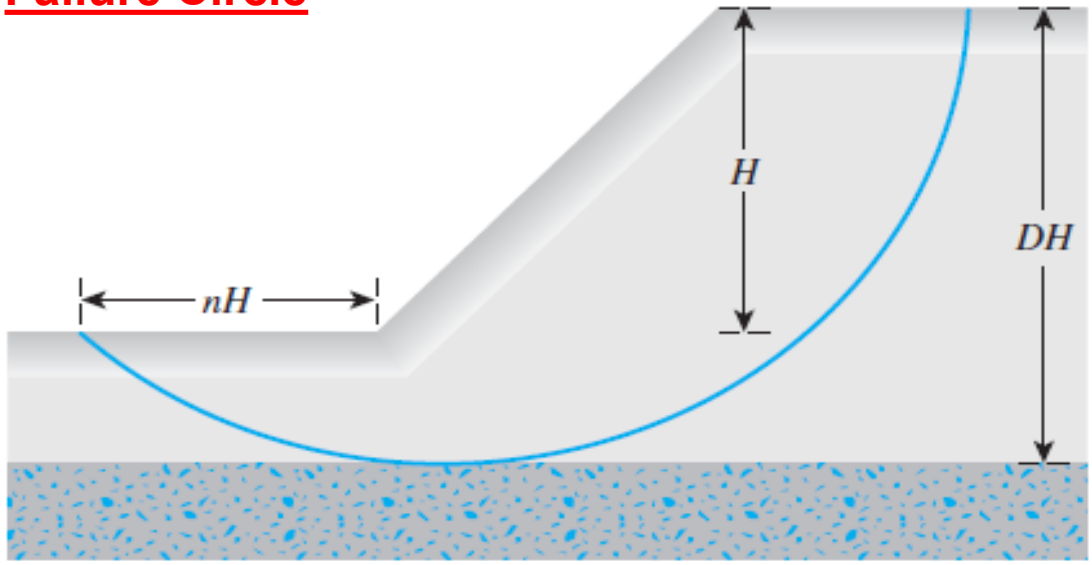
For $\beta < 53°$:

Toe circle ——

Midpoint circle —·—

Slope circle – – –

Failure Circle



- **For a slope angle $\beta > 53°$, the critical circle is always a toe circle. The location of the center of the critical toe circle may be found with the aid of Figure 15.13.**

- **For $\beta < 53°$, the critical circle may be a toe, slope, or midpoint circle, depending on the location of the firm base under the slope. This is called the depth function, which is defined as**

$$D = \frac{\text{Vertical distance from top of slope to firm base}}{\text{Height of slope}}$$

Location of the center of the critical toe circle

For $\beta > 53°$:
All circles are toe circles.

The location of the center of the critical toe circle may be found with the aid of Figure 15.13

(radius)

$$r = \frac{H_{cr}}{2 \sin \alpha \quad \sin \frac{\theta}{2}}$$

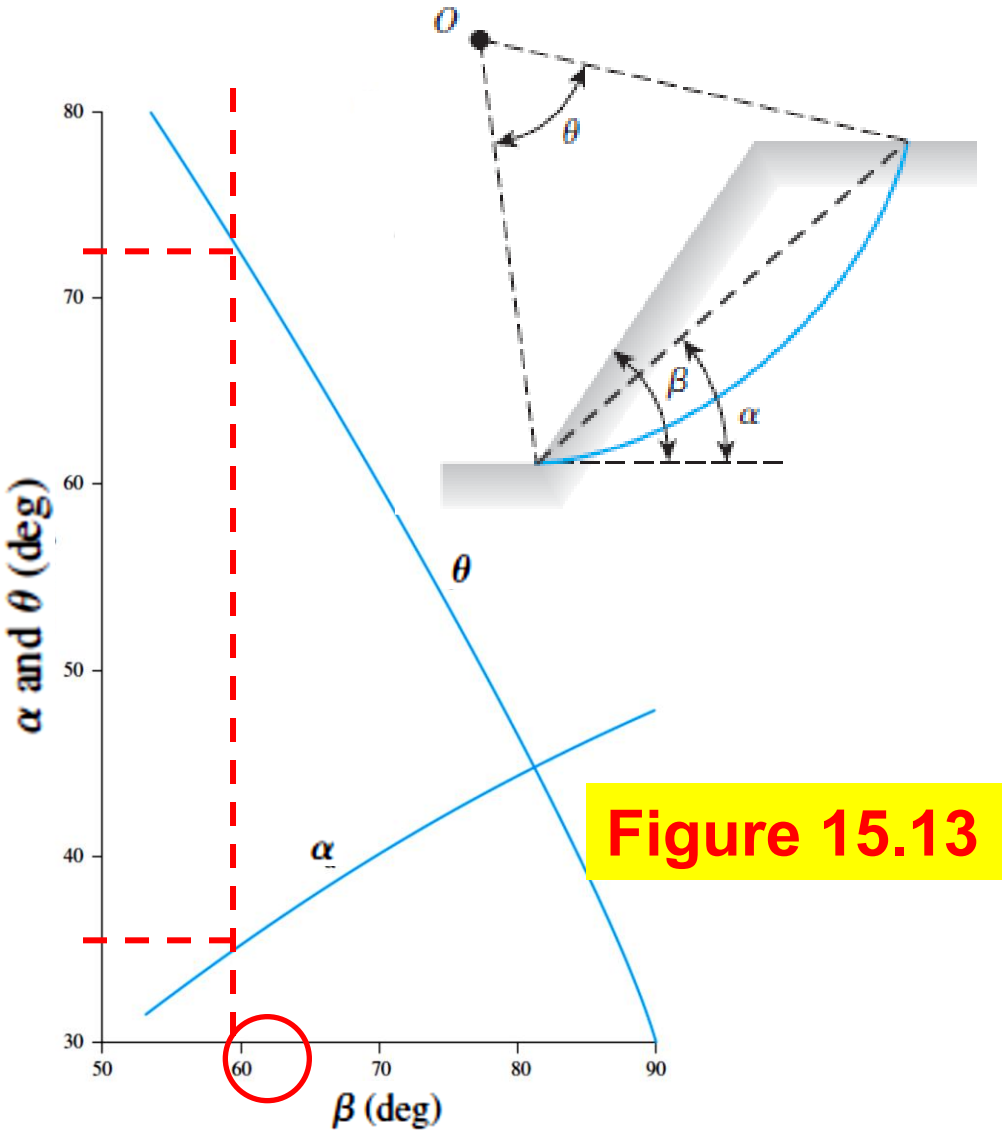


Figure 15.13

Finite Slopes with Circular Failure Surface

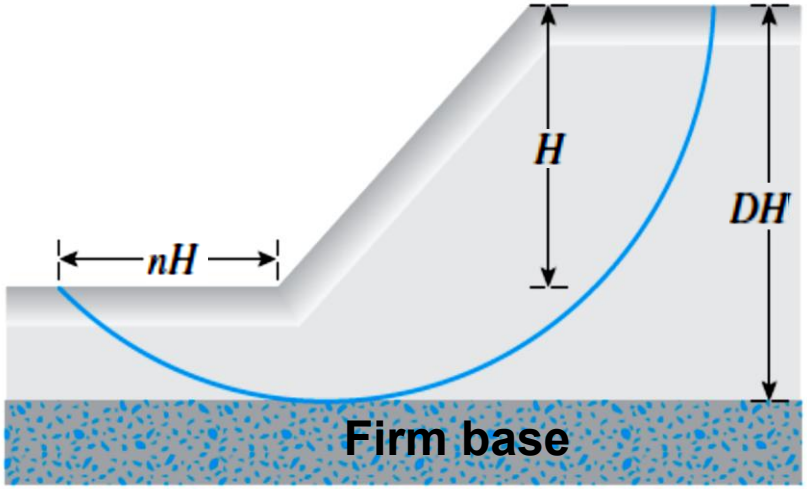
2. For $\beta < 53°$, the critical circle may be a toe, slope, or midpoint circle, depending on the location of the firm base under the slope. This is called the *depth function*, which is defined as

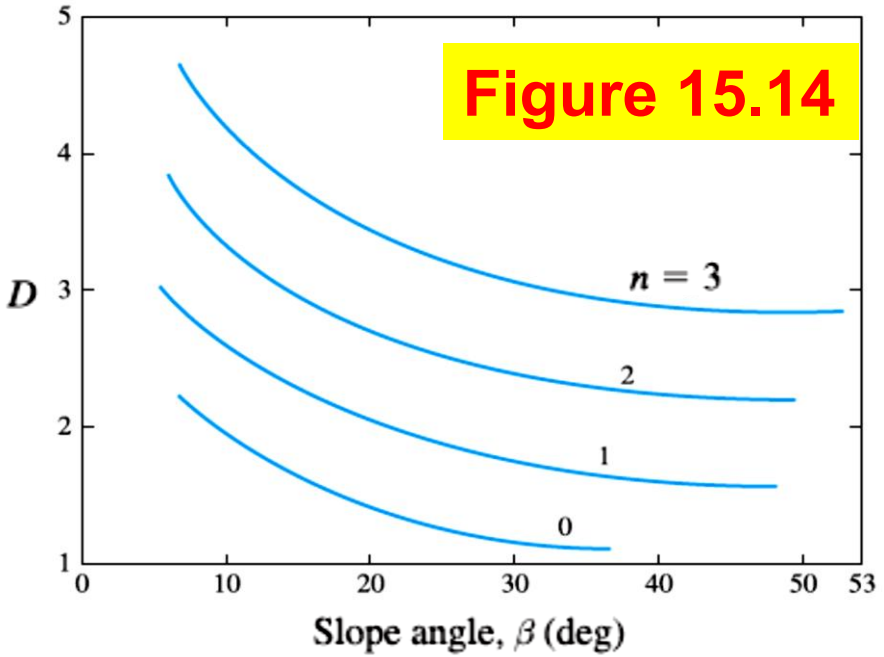
$$D = \frac{\text{Vertical distance from top of slope to firm base}}{\text{Height of slope}} \qquad (15.49)$$

For $\beta < 53°$:

midpoint circle

When the critical circle is a midpoint circle (i.e., the failure surface is tangent to the firm base), its position can be determined with the aid of Figure 15.14.





Critical toe circles for slopes with $\beta < 53°$

For $\beta < 53°$:

toe circles

Table 15.1 Location of the Center of Critical Toe Circles ($\beta < 53°$)

n'	β (deg)	α_1 (deg)	α_2 (deg)
1.0	45	28	37
1.5	33.68	26	35
2.0	26.57	25	35
3.0	18.43	25	35
5.0	11.32	25	37

Note: for notations of n', β, α_1, and α_2, see Figure 15.16

The location of these circles can be determined with the use of Figure 15.15 and Table 15.1.

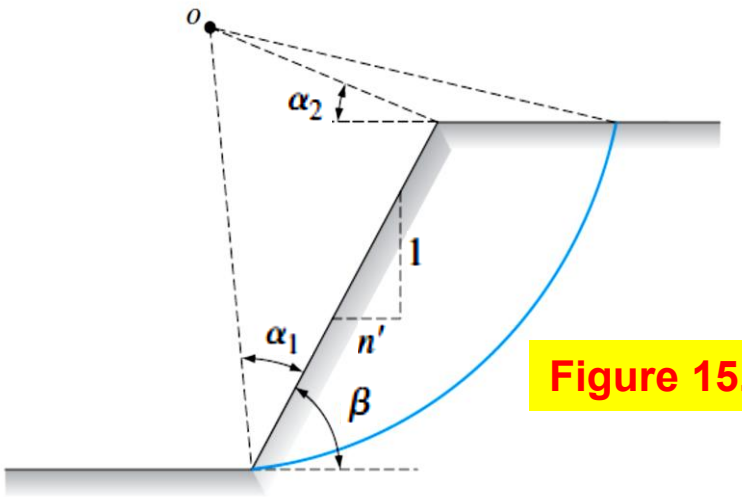
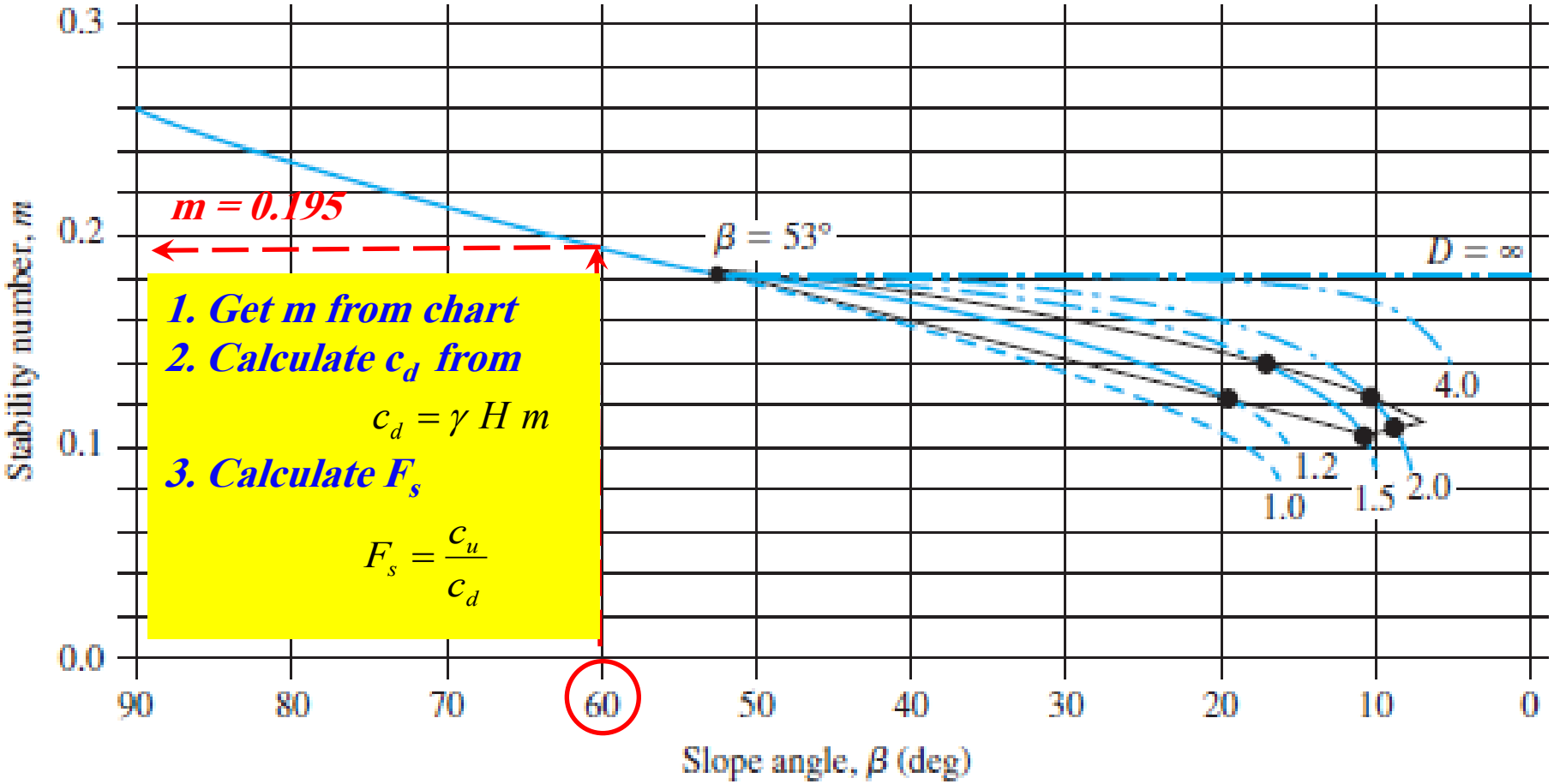


Figure 15.15

Figure 15.16 Location of the center of critical toe circles for $\beta < 53°$

Note that these critical toe circle are not necessarily the most critical circles that exist.

How to use the stability chart? Given: $\beta = 60^{o}$, H, γ, c_u Required: min F_s

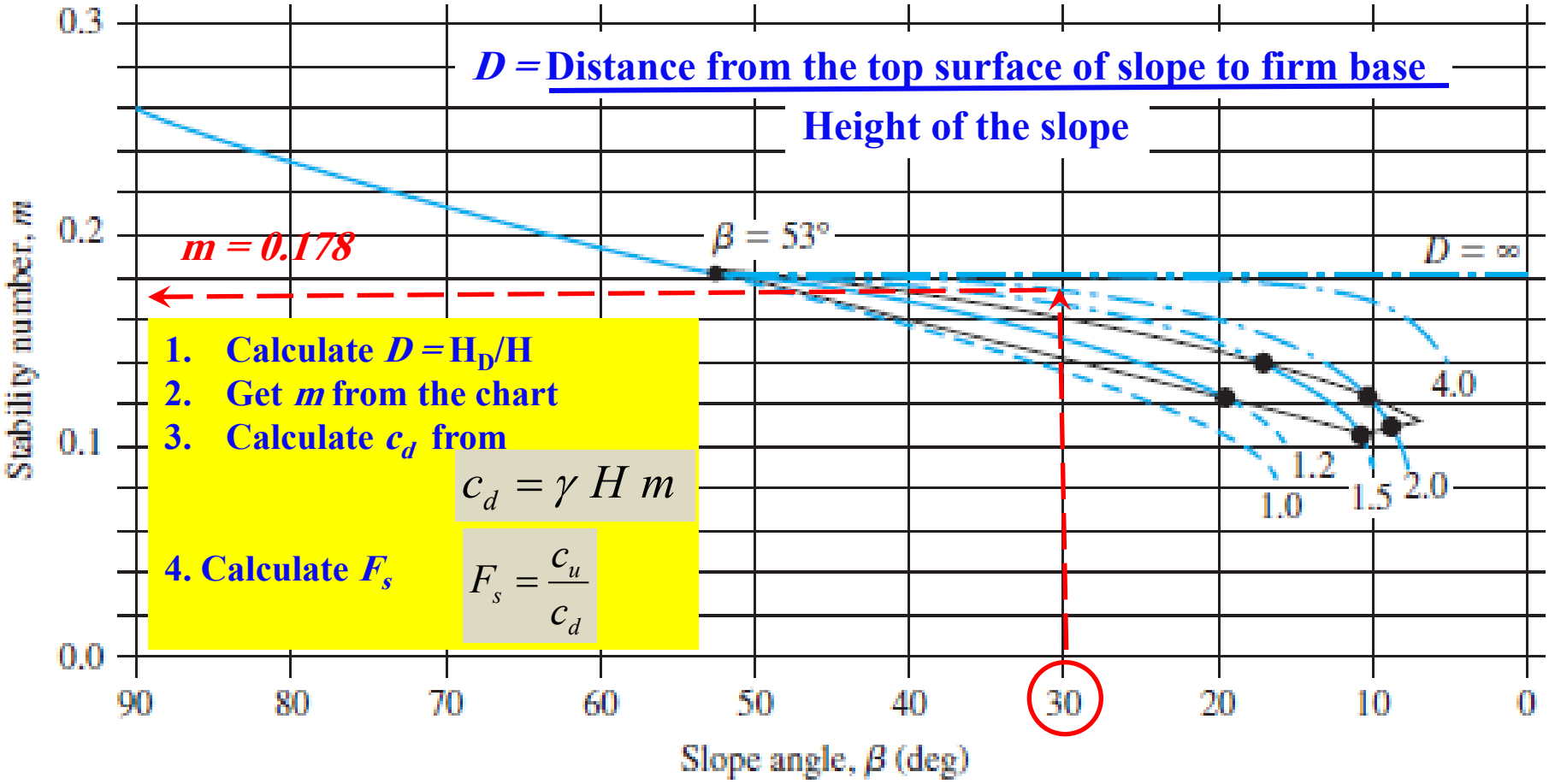


should be careful in using Figure 15.13 and note that it is valid for slopes of saturated clay and is applicable to only undrained conditions ($\phi = 0$).

Terzaghi used the term $\gamma H/c_d$, the reciprocal of m and called it the *stability factor.*

How to use the previous chart?

Given: $\beta = 30^\circ$, H, γ, c_u, H_D (depth to hard stratum) **Required: min.** F_s



EXAMPLE 15.4

Example 15.4

A cut slope in saturated clay (Figure 15.16) makes an angle 56° with the horizontal.

a. Determine the maximum depth up to which the cut could be made. Assume that the critical surface for sliding is circularly cylindrical. What will be the nature of the critical circle (i.e., toe, slope, or midpoint)?
b. How deep should the cut be made if a factor of safety of 2 against sliding is required?

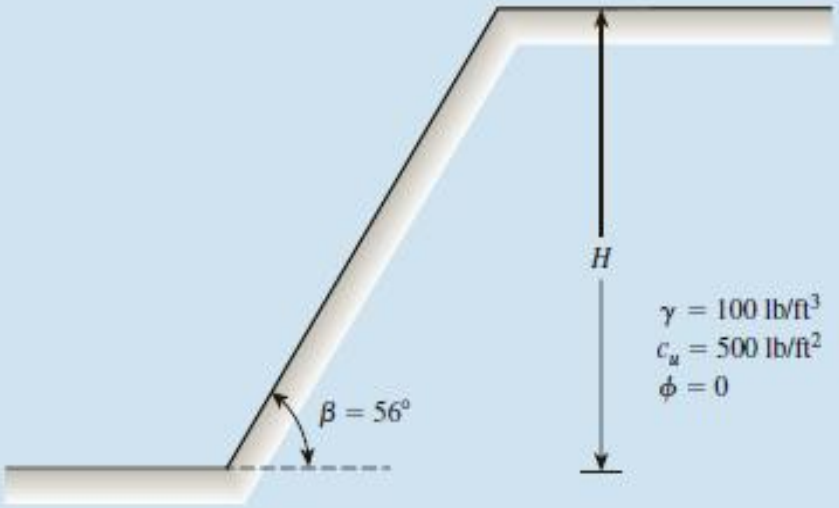


Figure 15.16

EXAMPLE 15.4

Solution

Part a

Since the slope angle $\beta = 56° > 53°$, the critical circle is a **toe circle**. From Figure 15.12 for $\beta = 56°, m = 0.185$. Using Eq. (15.48), we have

$$H_{cr} = \frac{c_u}{\gamma m} = \frac{500}{(100)(0.185)} = \mathbf{27.03\ ft}$$

Part b

The developed cohesion is

$$c_d = \frac{c_u}{F_s} = \frac{500}{2} = 250\ \text{lb/ft}^2$$

From Figure 15.12, for $\beta = 56°, m = 0.185$. Thus, we have

$$H = \frac{c_d}{\gamma m} = \frac{250}{(100)(0.185)} = \mathbf{13.51\ ft}$$

EXAMPLE 15.5

Example 15.5

A cut slope is to be made in a soft saturated clay with its sides rising at an angle of 60° to the horizontal (Figure 15.17).
Given: c_u – 40 kN/m² and γ – 175 kN/m³.

a. Determine the maximum depth up to which the excavation can be carried out.
b. Find the radius, r, of the critical circle when the factor of safety is equal to 1 (Part a).
c. Find the distance $\overline{BC}$.

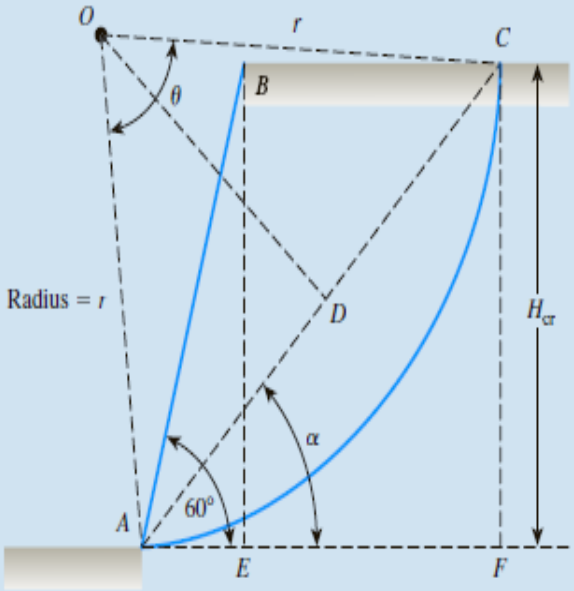


Figure 15.17

EXAMPLE 15.5

Solution

Part a

Since the slope angle $\beta - 60° > 53°$, the critical circle is a toe circle. From Figure 15.12, for $\beta = 60°$, the stability number = 0.195.

$$H_{cr} = \frac{c_u}{\gamma m} = \frac{40}{17.5 \times 0.195} = \mathbf{11.72\ m}$$

Part b

From Figure 15.17,

$$r = \frac{\overline{DC}}{\sin\frac{\theta}{2}}$$

But

$$\overline{DC} = \frac{\overline{AC}}{2} = \frac{\left(\frac{H_{cr}}{\sin\alpha}\right)}{2}$$

so,

$$r = \frac{H_{cr}}{2\ \sin\alpha\ \sin\frac{\theta}{2}}$$

From Figure 15.13, for $\beta = 60°$, $\alpha = 35°$ and $\theta = 72.5°$. Substituting these values into the equation for r, we get

$$r = \frac{H_{cr}}{2\ \sin\alpha\ \sin\frac{\theta}{2}}$$

$$= \frac{11.72}{2(\sin 35)(\sin 36.25)} = \mathbf{17.28\ m}$$

EXAMPLE 15.5

Part c

$$\overline{BC} = \overline{EF} = \overline{AF} - \overline{AE}$$
$$= H_{cr}(\cot \alpha - \cot 60^\circ)$$
$$= 11.72(\cot 35 - \cot 60) = \mathbf{9.97\ m}$$

EXAMPLE 15.6

Example 15.6

A cut slope was excavated in a saturated clay. The slope made an angle of 40° with the horizontal. Slope failure occurred when the cut reached a depth of 7 m. Previous soil explorations showed that a rock layer was located at a depth of 10.5 m below the ground surface. Assuming an undrained condition and $\gamma_{sat} = 18\ kN/m^3$, find the following.

a. Determine the undrained cohesion of the clay (use Figure 15.12).
b. What was the nature of the critical circle?
c. With reference to the toe of the slope, at what distance did the surface of sliding intersect the bottom of the excavation?

EXAMPLE 15.6

Solution

Part a

Referring to Figure 15.12,

$$D = \frac{10.5}{7} = 1.5$$

$$\gamma_{sat} = 18\ \text{kN/m}^3$$

$$H_{cr} = \frac{c_u}{\gamma m}$$

From Figure 15.12, for $\beta = 40^\circ$ and $D = 1.5$, $m = 0.175$. So,

$$c_u = (H_{cr})(\gamma)(m) = (7)(18)(0.175) = \mathbf{22.05\ kN/m^2}$$

Part b

Midpoint circle.

Part c

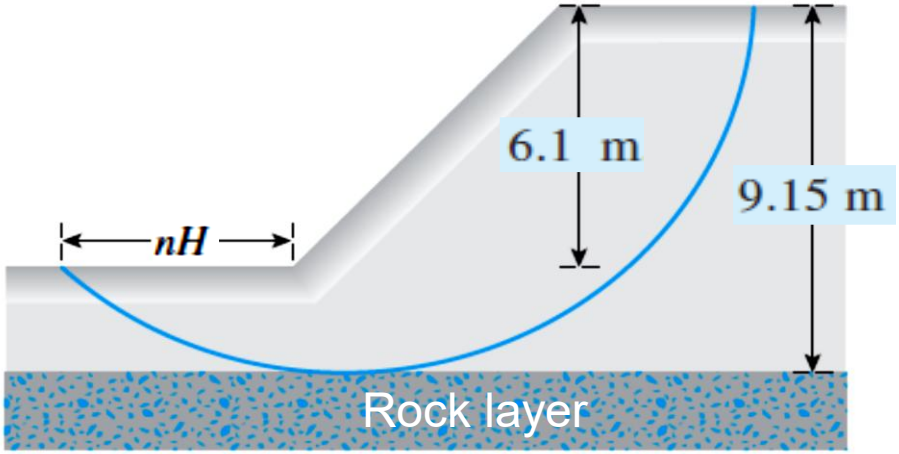
Again, from Figure 15.14, for $D = 1.5$, $\beta = 40^\circ$; $n = 0.9$. So,

$$\text{Distance} = (n)(H_{cr}) = (0.9)(7) = \mathbf{6.3\ m}$$

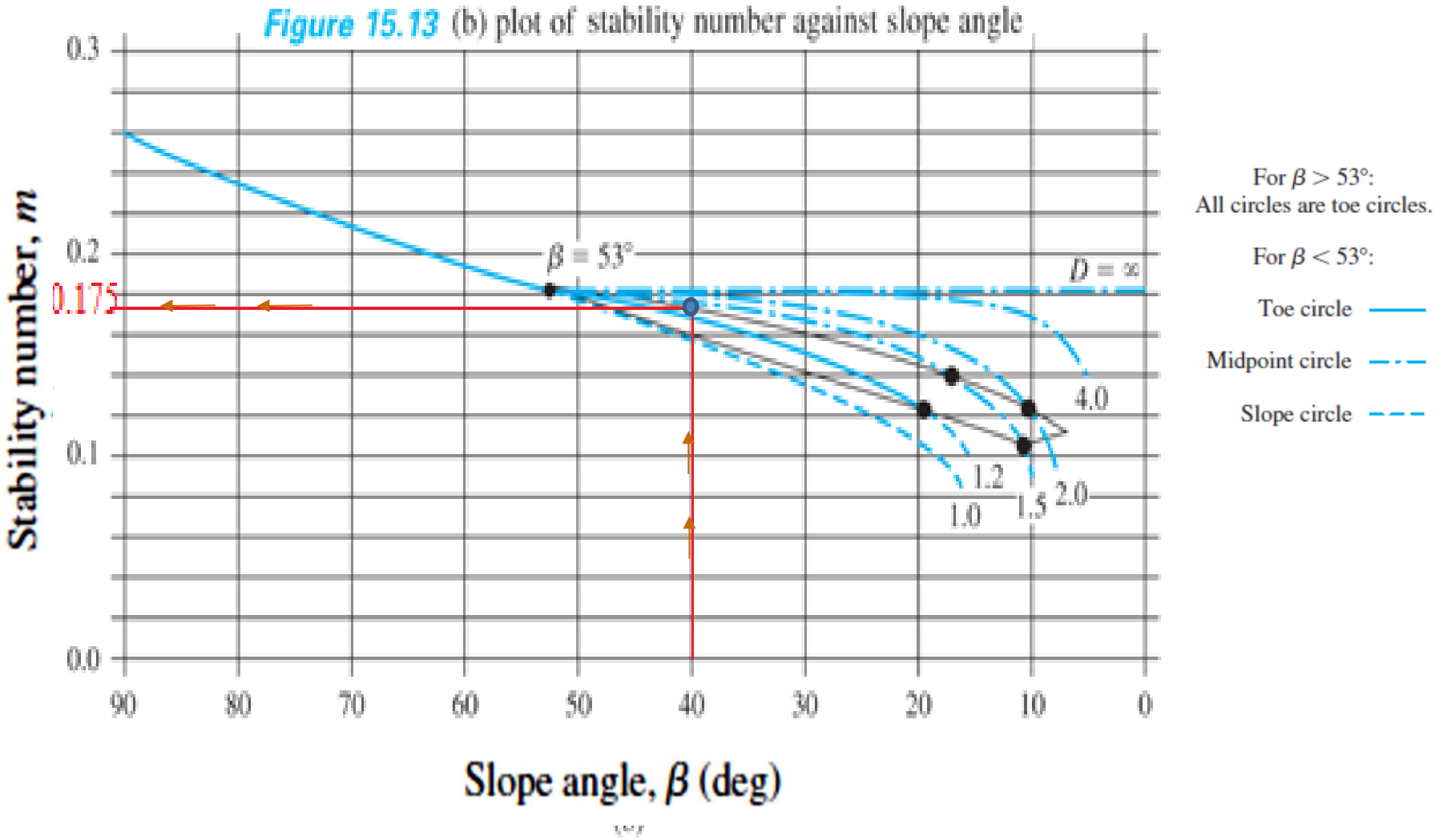
EXAMPLE

A cut slope was excavated in a saturated clay. The slope made an angle of 40° with the horizontal. Slope failure occurred when the cut reached a depth of 6.1 m. Previous soil explorations showed that a rock layer was located at a depth of 9.15 m below the ground surface. Assuming an undrained condition and $\gamma_{sat} = 17.29\ kN/m^3$, find the following.

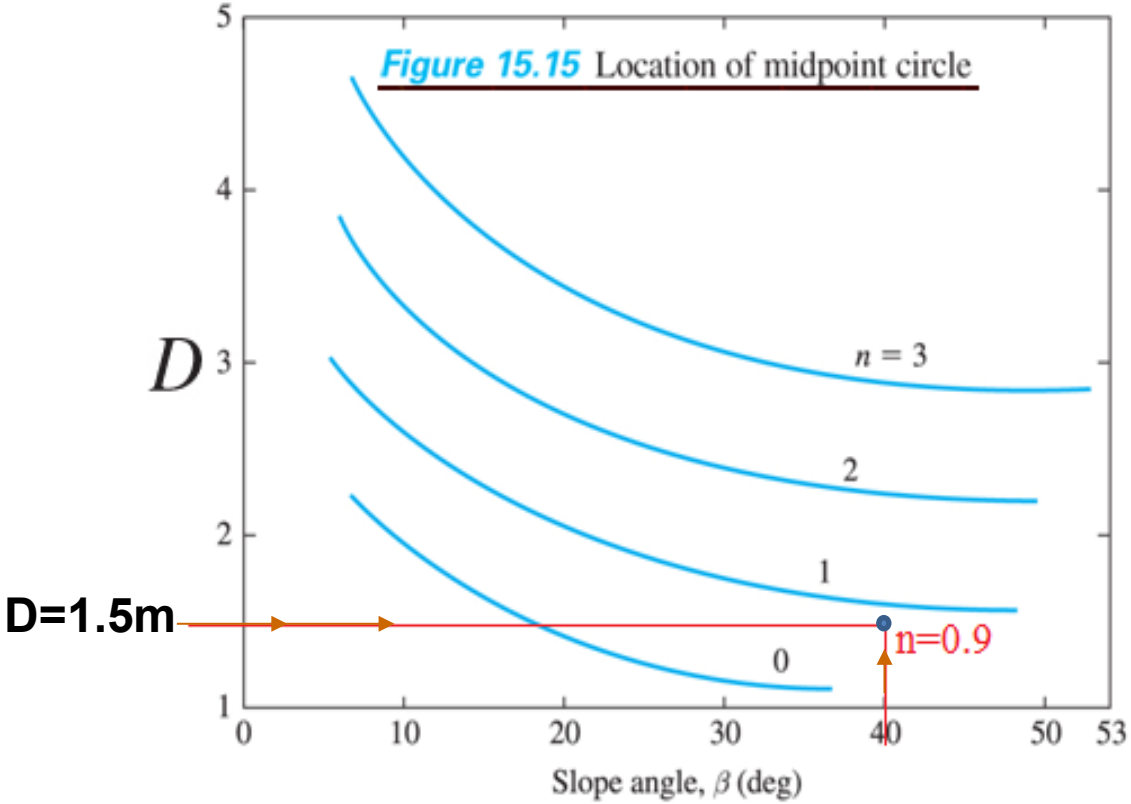
a. Determine the undrained cohesion of the clay (Figure 15.13).
b. What was the nature of the critical circle?
c. With reference to the toe of the slope, at what distance did the surface of sliding intersect the bottom of the excavation?



SOLUTION


Figure 15.13 (b) plot of stability number against slope angle
Stability number, m
0.3
0.2
0.1
0.0
0.175
β = 53°
D = ∞
4.0
2.0
1.5
1.2
1.0
90
80
70
60
50
40
30
20
10
0
Slope angle, β (deg)
For β > 53°:
All circles are toe circles.
For β < 53°:
Toe circle
Midpoint circle
Slope circle

SOLUTION


Figure 15.15 Location of midpoint circle
D
5
4
3
2
1
n = 3
2
1
0
D=1.5m
n=0.9
0
10
20
30
40
50
53
Slope angle, β (deg)

SOLUTION

$$F_s = \frac{C_u}{\gamma H m}$$

Solution

Part a

Referring to Figure 15.13,

$$D = \frac{9.15}{6.1} = 1.5$$

$$\gamma_{sat} = 17.29 \text{ kN/m}^3$$

$$H_{cr} = \frac{c_u}{\gamma m}$$

From Figure 15.13, for $\beta = 40°$ and $D = 1.5$, $m = 0.175$. So,

$$c_u = (H_{cr})(\gamma)(m) = (6.1)(17.29)(0.175) = \mathbf{18.46 \text{ kN/m}^2}$$

Part b

Midpoint circle.

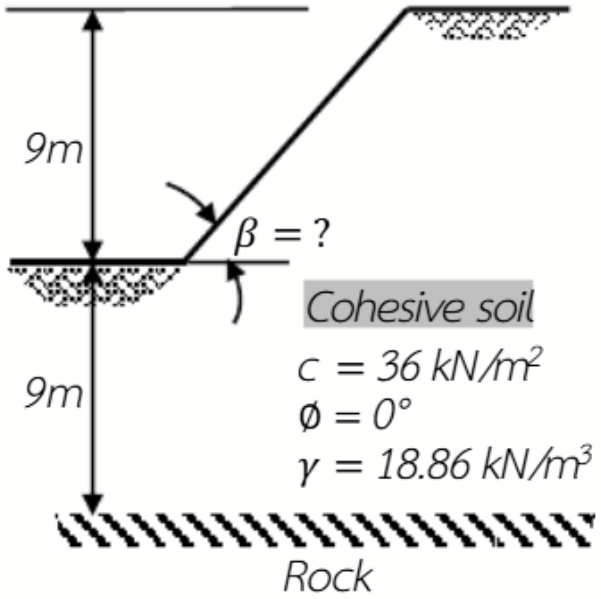
Part c

Again, from Figure 15.15, for $D = 1.5$, $\beta = 40°$; $n = 0.9$. So,

$$\text{Distance} = (n)(H_{cr}) = (0.9)(6.1) = \mathbf{5.49 \text{ m}}$$

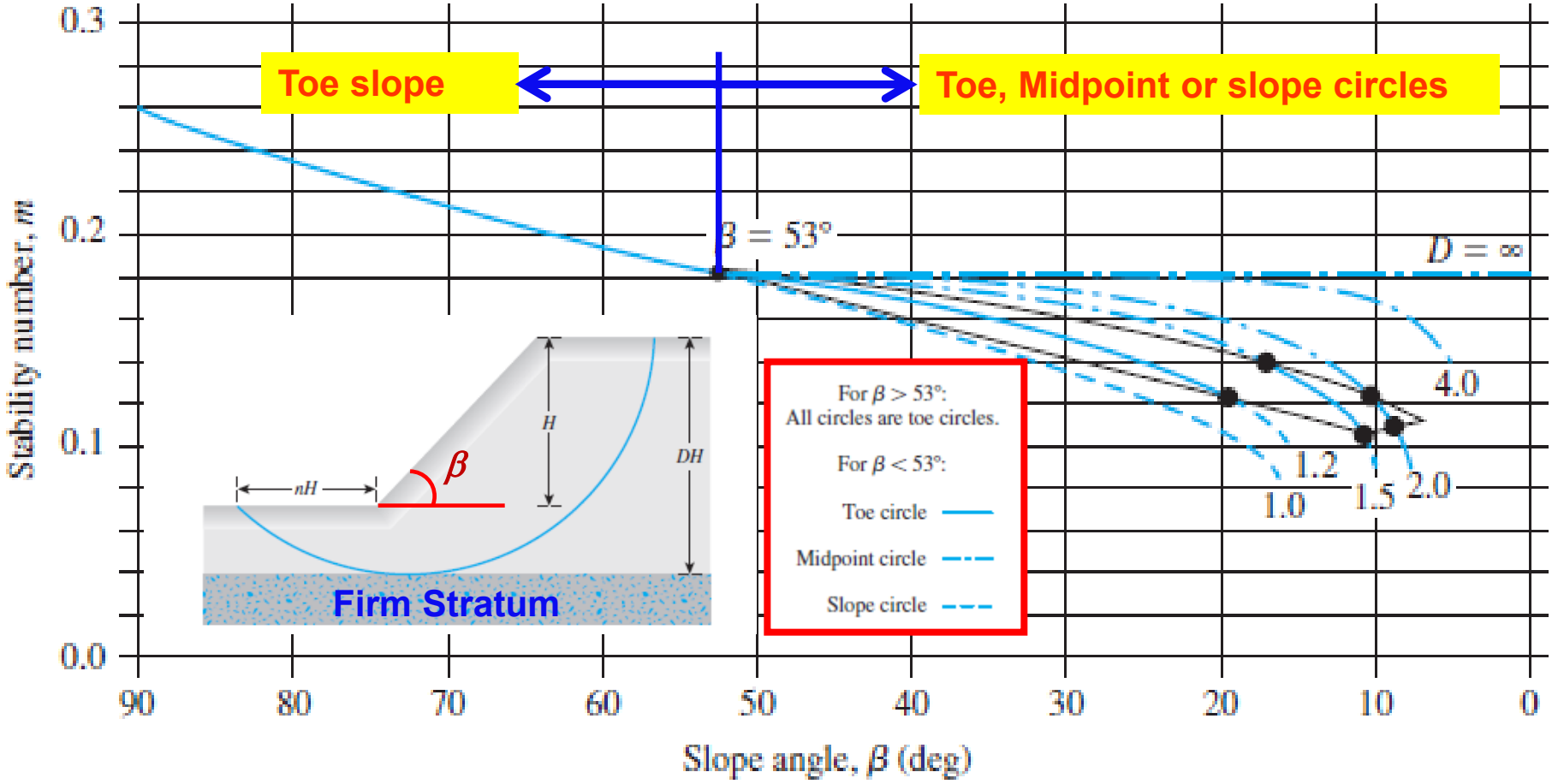
EXAMPLE

A cut 9m deep is to be made in a deposit of a highly cohesive soil that is 18m thick and underlaid by rock. The properties of the soil to be cut are, c = 36 kN/m ^{2}, ∅= 0° and γ = 18.86 kN/m^3. If the factor of safety against slope failure is equal to 1.25, estimate the slope angle (β) at which the cut should be made.



Slopes in Homogeneous clay Soil with c ≠ 0 , ϕ = 0

- The results of analytical solution to obtain critical circles was represented graphically as the variation of stability number, m , with slope angle β.



m is obtained from this chart depending on angle β

REMARKS

- Since we know the magnitude and direction of W and the direction of C_d and F we can draw the force polygon to get the magnitude of C_d.
- We can then calculate c'_d from $c'_d = \frac{C_d}{\overline{AC}}$
- Determination of the magnitude of c'_d described previously is based on a trial surface of sliding.
- Several trails must be made to obtain the most critical sliding surface, minimum factor of safety or along which the developed cohesion is a maximum
- The maximum cohesion developed along the critical surface as

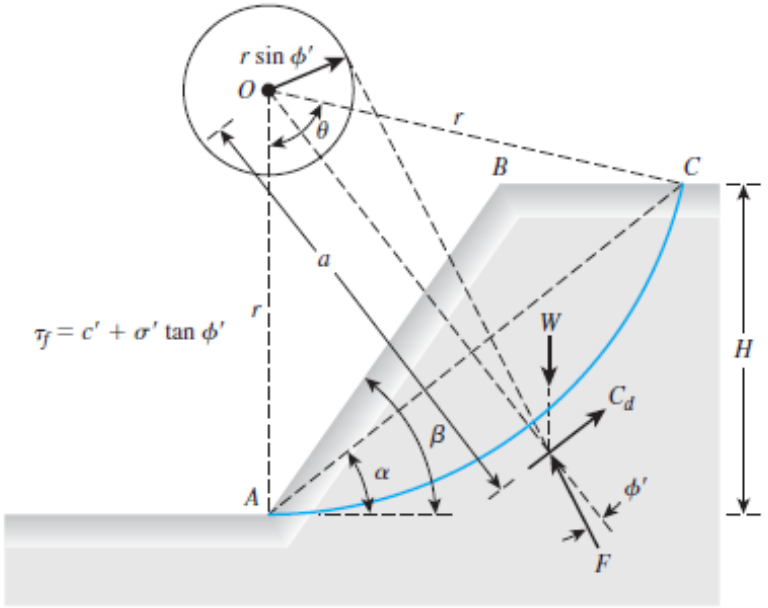
$$c'_d = \gamma \, \mathrm{H}\left[f\left(\alpha, \beta, \theta, \phi'\right)\right] \Rightarrow \frac{c'_d}{\gamma \, \mathbf{H}} = f\left(\alpha, \beta, \theta, \phi'\right) = m = stability\ number$$

- The results of analytical solution to obtain minimum F_s was represented graphically as the variation of stability number, m, with slope angle β for various values of ϕ' (Fig. 15.21).
- Solution to obtain the minimum F_s using this graph is performed by trial-and-error until $F_s = F_{c'} = F_{\phi'}$

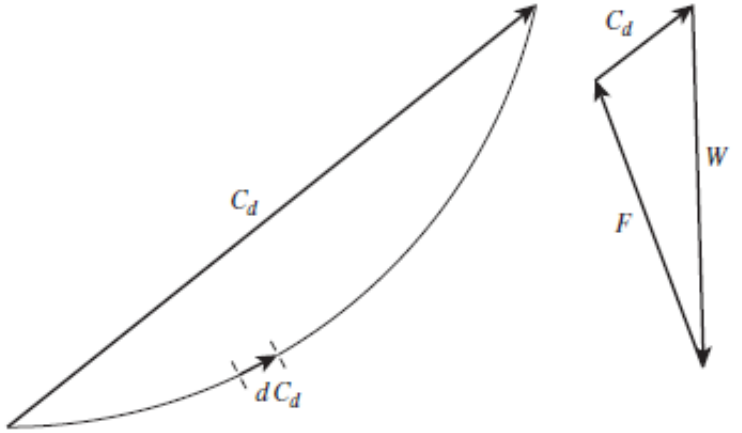
Slopes in Homogeneous C' – ϕ' Soils

Slopes in Homogeneous C' – ϕ' Soils

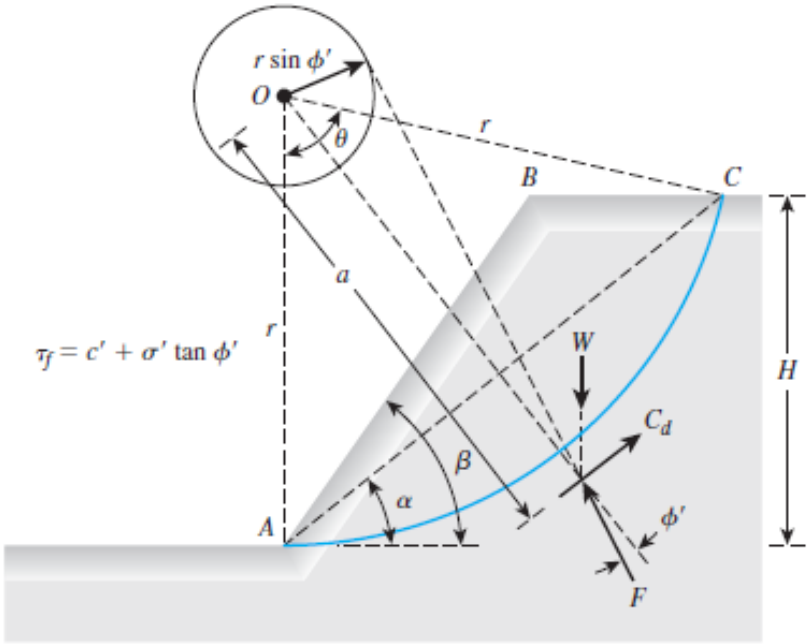
- Here the situation is more complicated than for purely cohesive soils.
- The Friction Circle method (or the ϕ-Circle Method) is very useful for homogenous slopes. The method is generally used when both cohesive and frictional components are to be used.
- $\widehat{AC}$ is a trial circular arc that passes through the toe of the slope, and O is the center of the circle.



- The pore water pressure is assumed to be zero
- F—the resultant of the normal and frictional forces along the surface of sliding. For equilibrium, the line of action of F will pass through the point of intersection of the line of action of W and C_d.

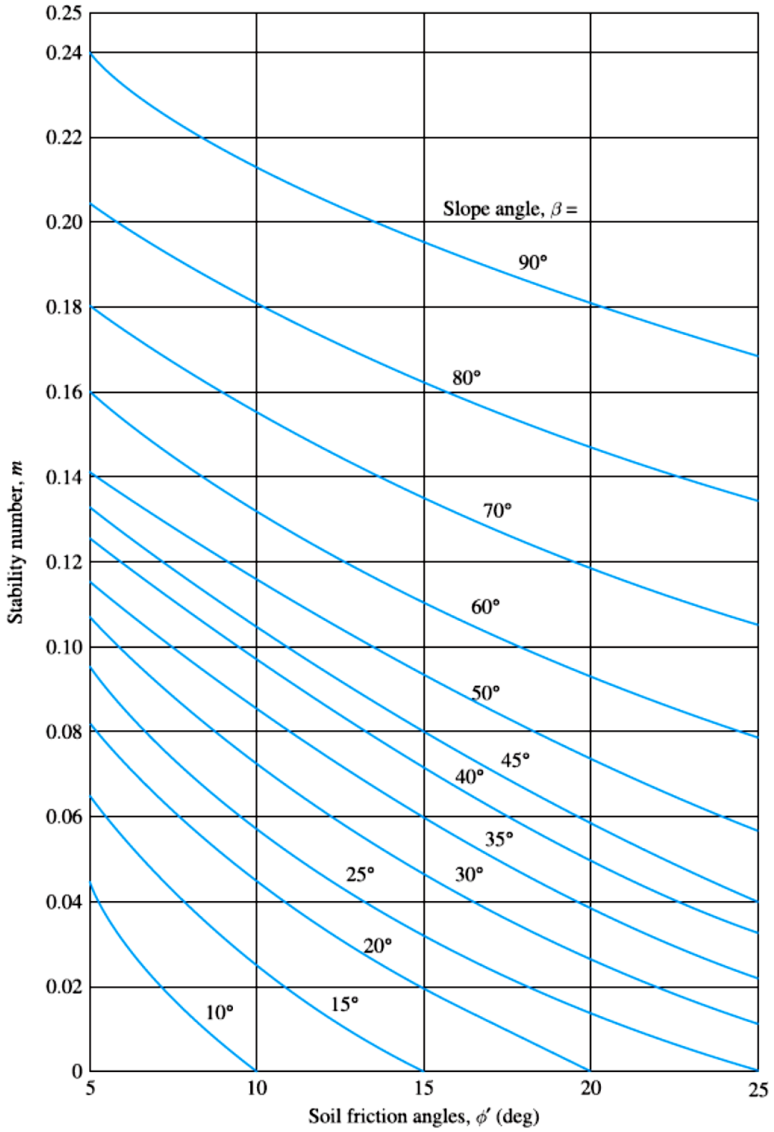


Friction Circle method



$$c'_d = \gamma \, \mathrm{H}\left[f\left(\alpha, \beta, \theta, \phi'\right)\right]$$

$$c_d = \gamma \, H \, m$$



(Fig. 15.21)

Procedures of graphical solution

Given: $H, \beta, \gamma, c', \phi'$ Required: F_s

1. Assume ϕ_d *(Generally start with* ϕ'_d *i.e. full friction is mobilized)*

2. Calculate $$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d}$$

3. With ϕ_d and β Use Chart to get m

4. Calculate $$c_d = \gamma H m$$

5. Calculate $$F_c = \frac{c'}{c_d}$$

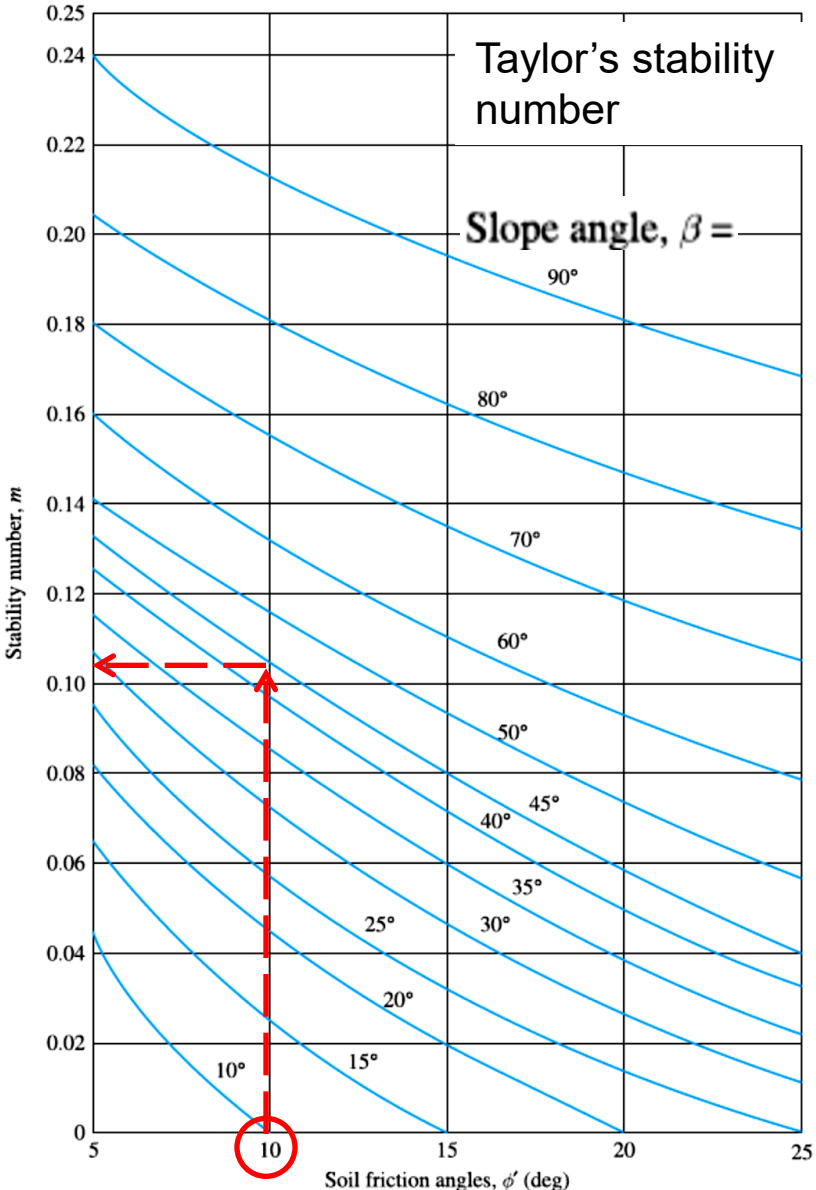
6. If $F_{c'} = F_{\phi'}$, The overall factor of safety

$$F_s = F_{c'} = F_{\phi'}$$

7. If $F_{c'} \neq F_{\phi'}$, reassume ϕ_d and repeat steps 2 through 5 until $F_{c'} = F_{\phi'}$

Or

Plot the calculated points on F_c versus F_ϕ coordinates and draw a curve through the points. [see next slide]. Then Draw a line through the origin that represents $F_s = F_c = F_\phi$



Procedures of graphical solution

Given: $H, \beta, \gamma, c', \phi'$ **Required:** F_s

Calculations have shown that for $\phi > \sim 3°$, the critical circles are all *toe circles*.

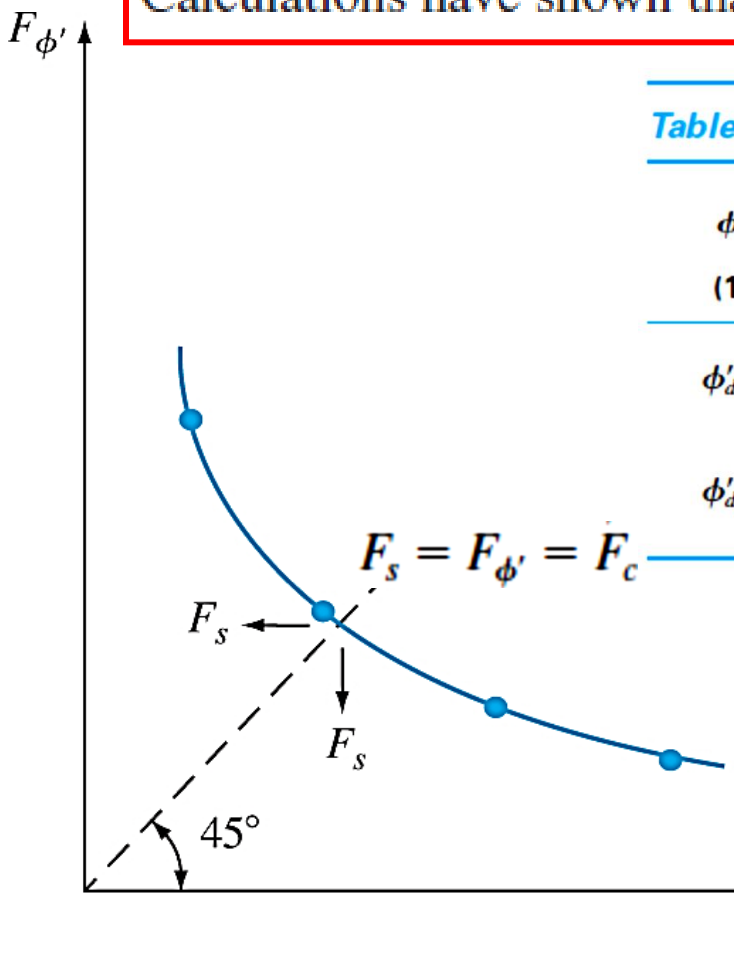


Figure 15.24 Plot of $F_{\phi'}$ versus $F_{c'}$ to determine F_s

Table 15.2 Determination of F_s by Frication Circle Method

ϕ'_d	$F_{\phi'} = \dfrac{\tan\phi'}{\tan\phi'_d}$	m	c'_d	$F_{c'}$
(1)	(2)	(3)	(4)	(5)
$\phi'_{d(1)}$	$\dfrac{\tan\phi'}{\tan\phi'_{d(1)}}$	m_1	$m_1\gamma H = c'_{d(1)}$	$\dfrac{c'}{c'_{d(1)}} = F_{c'(1)}$
$\phi'_{d(2)}$	$\dfrac{\tan\phi'}{\tan\phi'_{d(2)}}$	m_2	$m_2\gamma H = c'_{d(2)}$	$\dfrac{c'}{c'_{d(2)}} = F_{c'(2)}$

Note: Similar to Culmann procedure for planar mechanism but here C_d is found based on m. In Culmann's method C_d is found from analytical equation.

Calculation of Critical Height

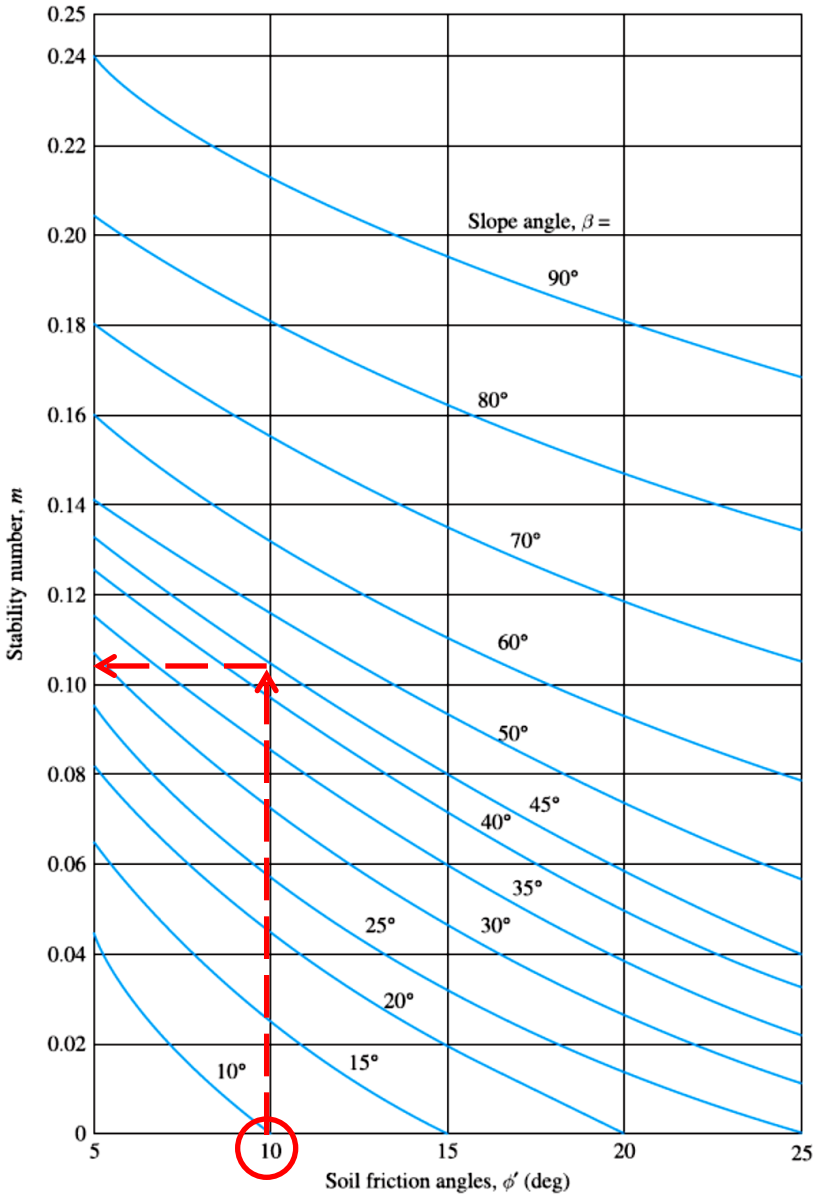
Given: β, γ, C', ϕ' **Required:** H_{cr}

H_{cr} means that $F_{c'} = F_{\phi'} = F_s = 1.0$

1. For the given β and ϕ', use Chart to get m.

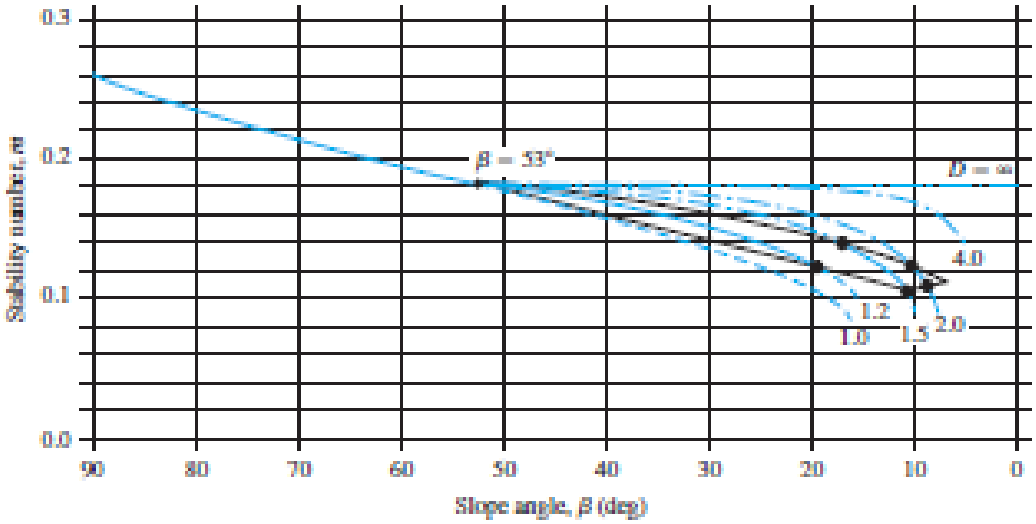
2. Calculate

$$H_{cr} = \frac{c'}{\gamma\, m}$$

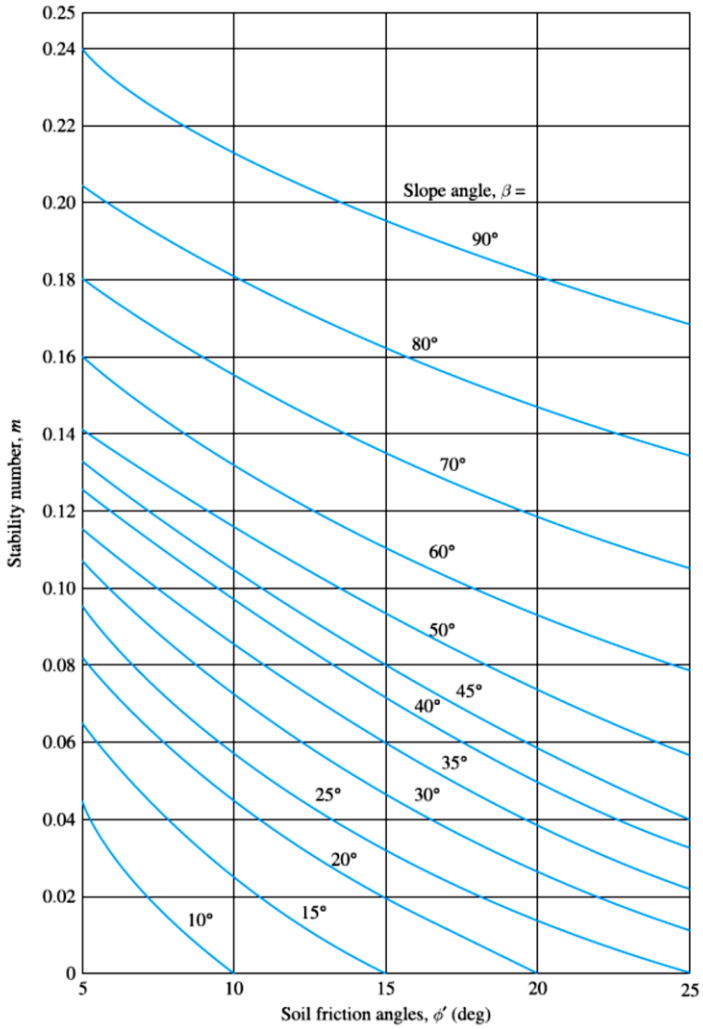


SUMMARY

Mass Procedure – Rotational mechanism need only the use of Taylor's chart.



$\phi = 0$



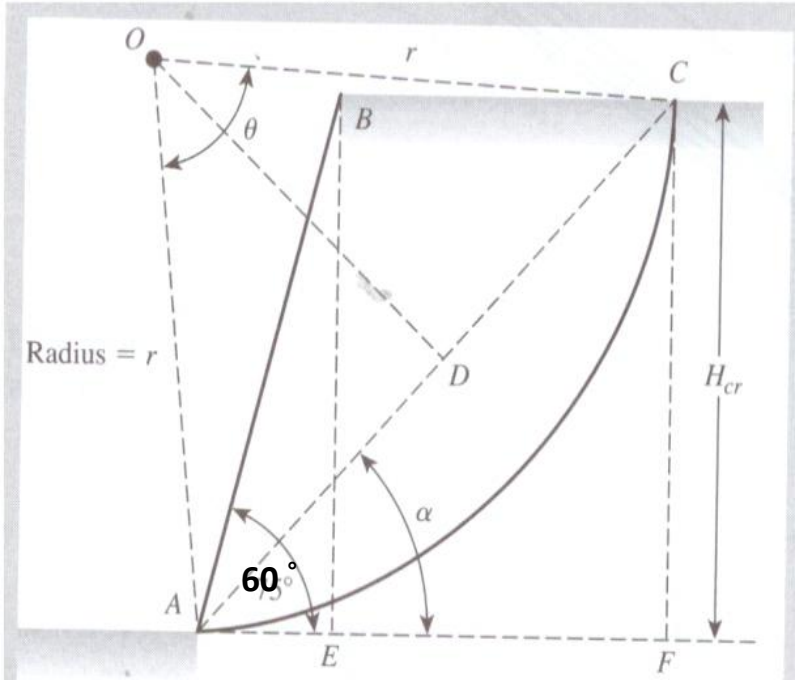
$C - \phi$

EXAMPLE

- **Example**
- **Given: c_u = 40 kN/m² & γ = 17.5 kN/m³**
- **Required:**

1. **Max. Depth**
2. **Radius r when F_s=1**
3. **Distance BC**

- **$\beta = 60^\circ > 53^\circ$ from Fig.15.13 m= 0.195**



$$H_{cr} = \frac{c_u}{\gamma \, \mathrm{m}} = \frac{40}{17.5*0.195} = 11.72\, m$$

$$r = \frac{\overline{DC}}{\sin\frac{\theta}{2}} \qquad \overline{DC} = \frac{\overline{AC}}{2} = \frac{\left(\frac{H_{cr}}{\sin\alpha}\right)}{2}$$

$$From \ \text{Fig. 15.14} \ \text{for} \ \beta = 60^{\circ} \alpha = 35^{\circ} and \ \theta = 72.5^{\circ}$$

$$r = \frac{H_{cr}}{2\sin\alpha \sin\frac{\theta}{2}} = \frac{11.72}{2(\sin 35)(\sin 36.25)} = 17.28 \,\mathrm{m}$$

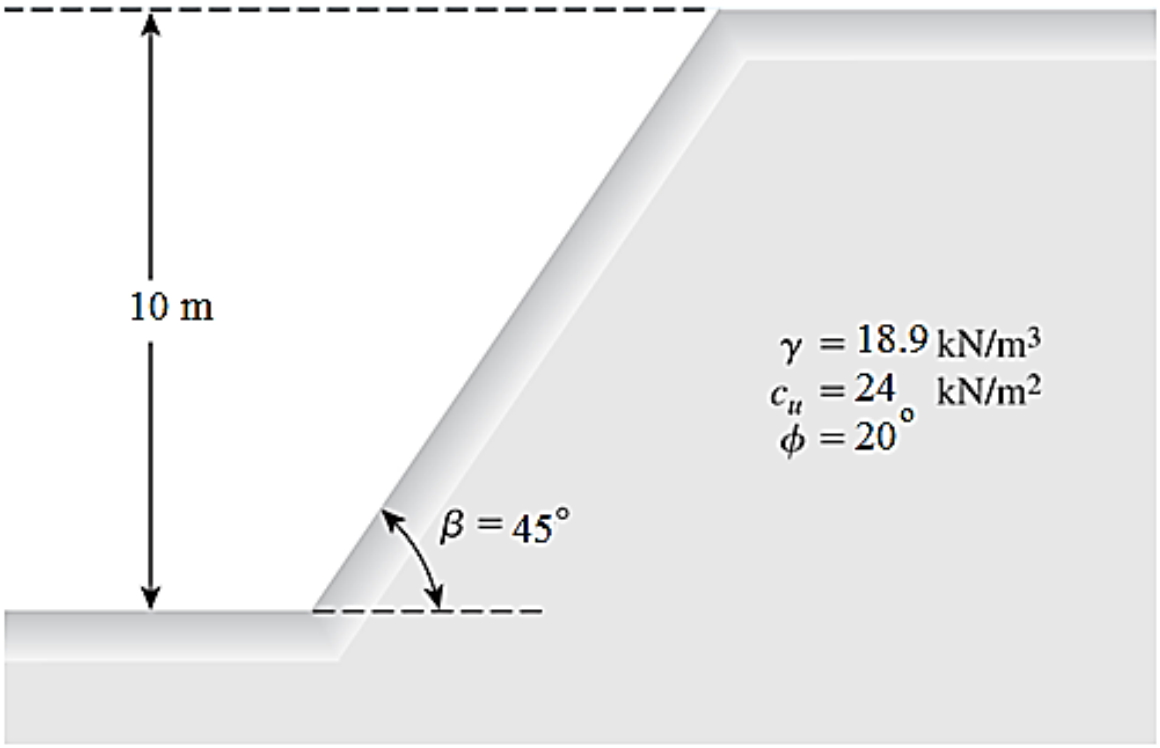
$$\overline{\mathrm{BC}} = \overline{\mathrm{EF}} = \overline{\mathrm{AF}} - \overline{\mathrm{AE}} = H_{cr}\left(\cot\alpha - \cot 60\right) = 9.97 \,\mathrm{m}$$

EXAMPLE 15.8

Example 15.8

A slope with $\beta = 45°$ is to be constructed with a soil that has $\phi' = 20°$ and $c' = 24$ kN/m^2. The unit weight of the compacted soil will be 18.9 kN/m^3.

a. Find the critical height of the slope.
b. If the height of the slope is 10 m, determine the factor of safety with respect to strength.



EXAMPLE 15.8

Solution

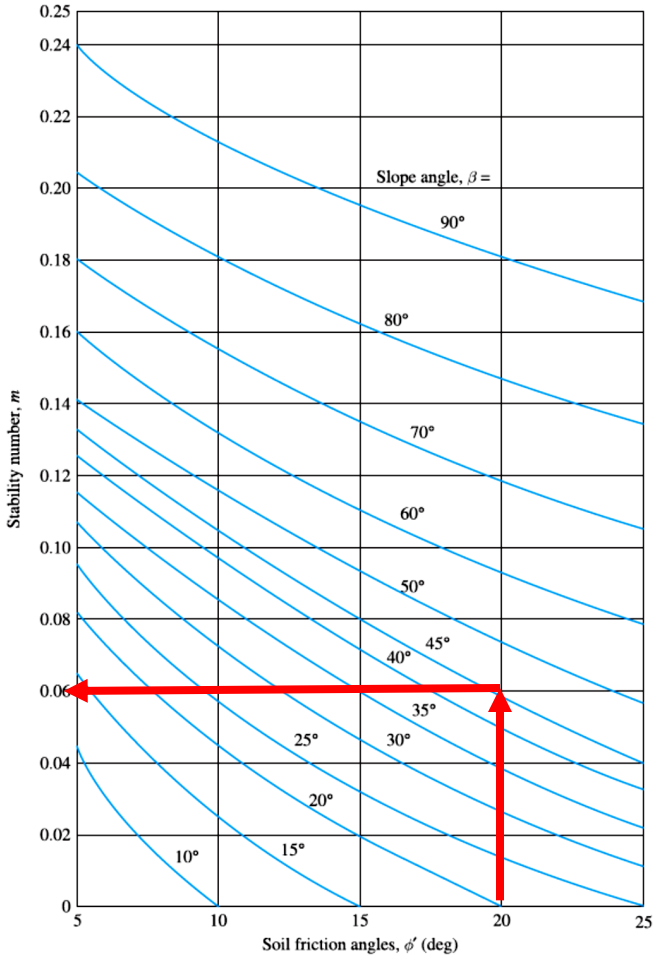
Part a

We have

$$m = \frac{c'}{\gamma H_{cr}}$$

From Figure 15.21, for $\beta = 45°$ and $\phi' = 20°$, $m = 0.06$. So

$$H_{cr} = \frac{c'}{\gamma m} = \frac{24}{(18.9)(0.06)} = \mathbf{21.1\ m}$$



EXAMPLE 15.8

Part b

If we assume that full friction is mobilized, then, referring to Figure 15.21 (for $\beta = 45^\circ$ and $\phi'_d = \phi' = 20^\circ$), we have

$$m = 0.06 = \frac{c'_d}{\gamma H}$$

or

$$c'_d = (0.06)(18.9)(10) = 11.34 \text{ kN/m}^2$$

Thus,

$$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d} = \frac{\tan 20}{\tan 20} = 1$$

and

$$F_{c'} = \frac{c'}{c'_d} = \frac{24}{11.34} = 2.12$$

Since $F_{c'} \neq F_{\phi'}$, this is not the factor of safety with respect to strength.

Now we can make another trial. Let the developed angle of friction, ϕ'_d, be equal to 15°. For $\beta = 45^\circ$ and the friction angle (ϕ'_d) equal to 15°, we find from Figure 15.21.

$$m = 0.083 = \frac{c'_d}{\gamma H}$$

or

$$c'_d = (0.083)(18.9)(10) = 15.69 \text{ kN/m}^2$$

EXAMPLE 15.8

For this trial,

$$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d} = \frac{\tan 20}{\tan 15} = 1.36$$

and

$$F_{c'} = \frac{c'}{c'_d} = \frac{24}{15.69} = 1.53$$

Similar calculations of $F_{\phi'}$ and $F_{c'}$ for various assumed values of ϕ'_d are given in the following table.

ϕ'_d	$\tan \phi'_d$	$F_{\phi'}$	m	c'_d (kN/m^2)	$F_{c'}$
20	0.364	1.0	0.06	11.34	2.12
15	0.268	1.36	0.083	15.69	1.53
10	0.176	2.07	0.105	19.85	1.21
5	0.0875	4.16	0.136	25.70	0.93

The values of $F_{\phi'}$ are plotted against their corresponding values of $F_{c'}$ in Figure 15.26, from which we find

$$F_{c'} = F_{\phi'} = F_s = \mathbf{1.42}$$

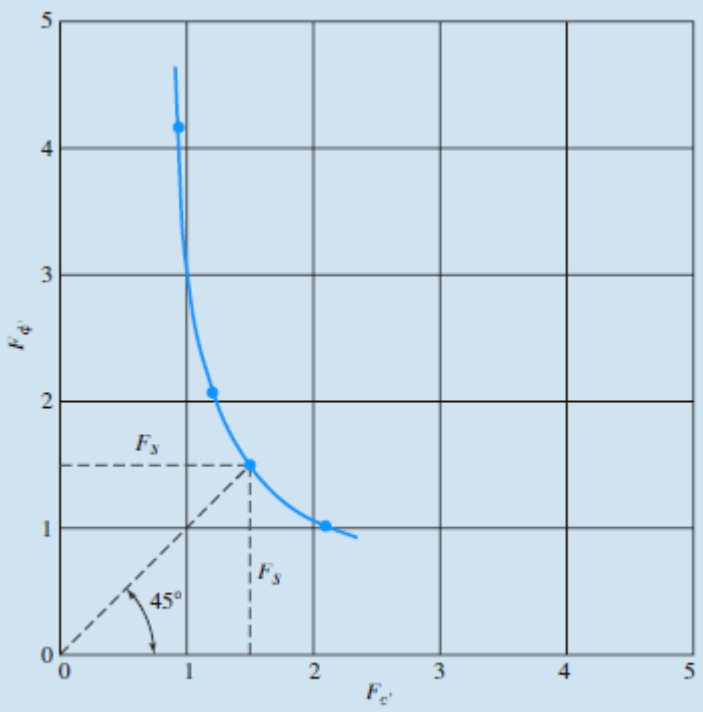
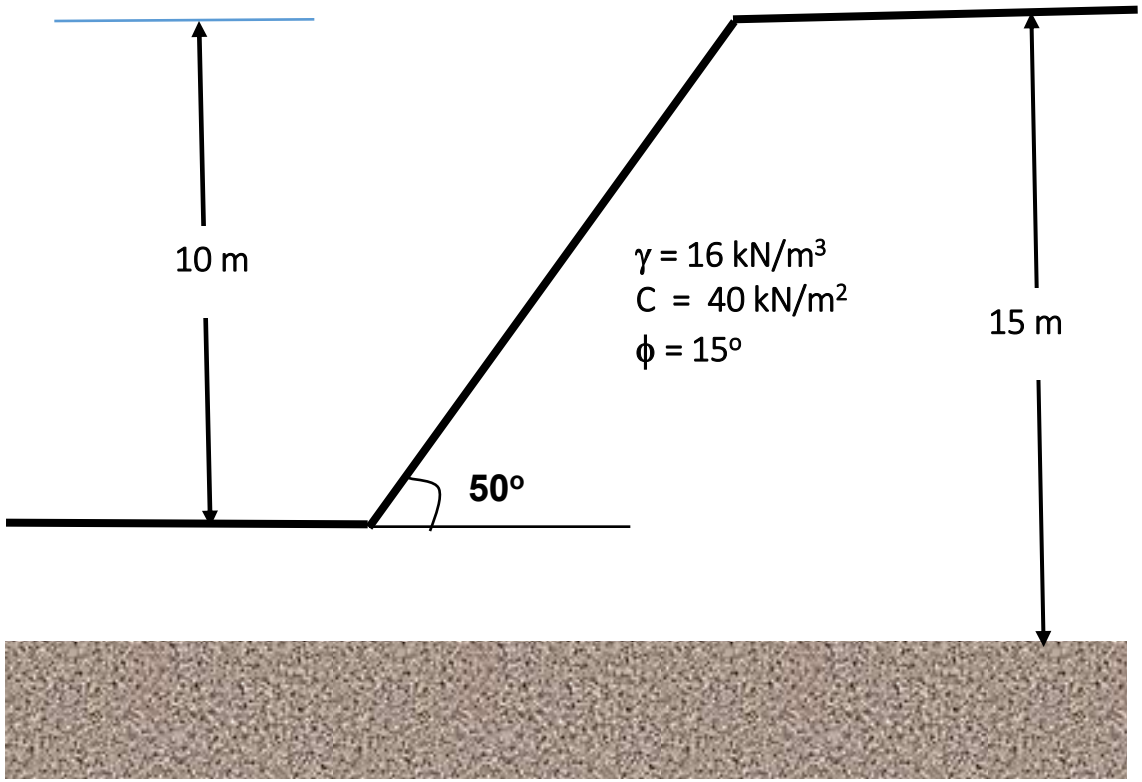


Figure 15.26

Analysis of Michalowski (2002) X
Example 15.9 X

Midterm Exam

- Using Taylor's stability chart determine the factor of safety for the slope shown in Fig.1.
- For the same slope height, what slope angle must be used if a factor of safety of 1.5 is required?



SOLUTION

a)

ϕ_d	$F_\phi = \tan\phi/\tan\phi_d$	m	$C_d = \gamma H m$	$F_c = C/C_d$
15	1	0.092	14.7	2.70
10	1.52	0.116	18.6	2.20
7.5	2.0	0.125	20	2.0

$F_s = 2.0$

b)

ϕ_d

$C_d = 40/1.5 = 26.7$ kpa

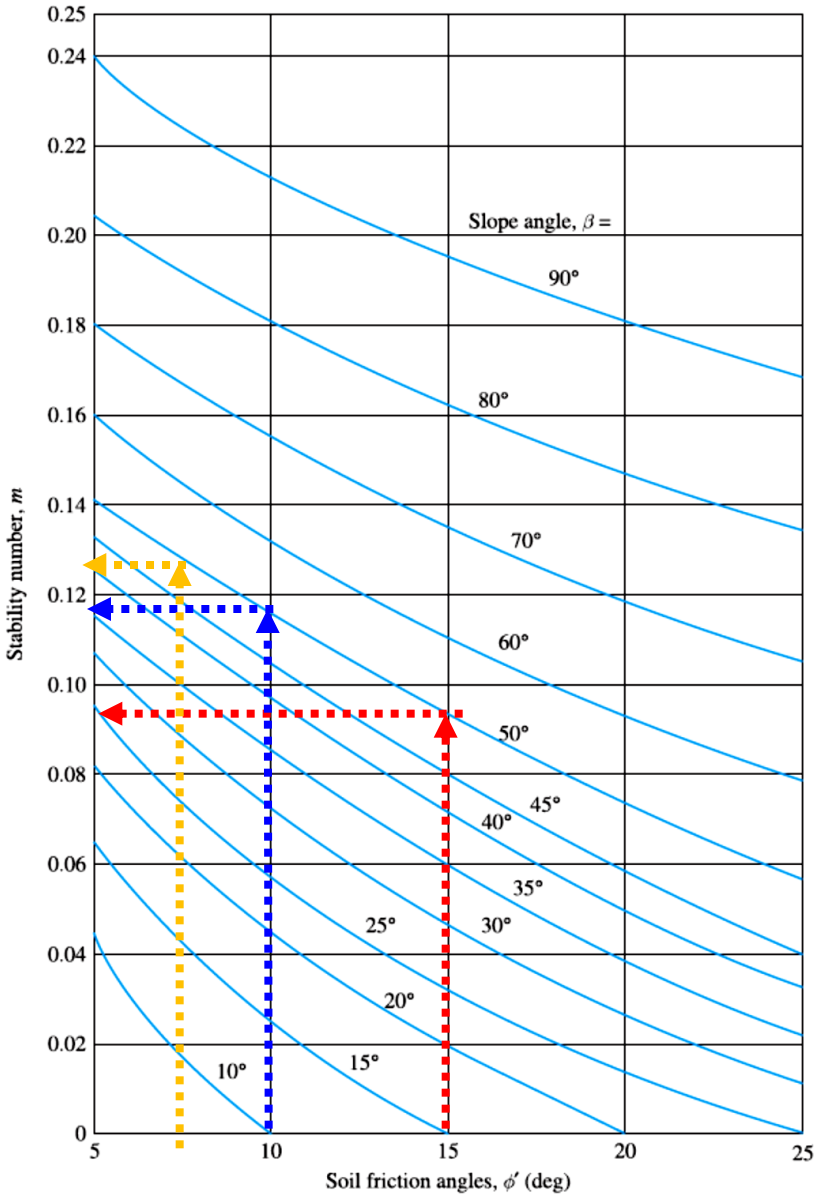
$$c_d = \gamma H m$$

$26.7 = 16 \times 10 \times m$

$m = \underline{0.167}$

$\tan\phi_d = \tan\phi/1.5$ ⟹ $\phi_d = 10.1^o$

At m = 0.167 and $\phi_d = 10.1^o$ from chart $\beta = 75^o$



The end