TOPICS

- Introduction
- Types of slope movements
- Concepts of Slope Stability Analysis
- Factor of Safety
- **Stability of Infinite Slopes**
- Stability of Finite Slopes with Plane Failure Surface
  - Culmann's Method
- Stability of Finite Slopes with Circular Failure Surface
  - Mass Method
  - Method of Slices

Stability of Infinite Slopes

What is an Infinite slope?

- Slope that extends for a relatively long distance and has consistent subsurface profile can be considered as infinite slope.
- Failure plane parallel to slope surface.
- Depth of the failure surface is small compared to the height of the slope.
- For the analysis, forces acting on a single slice of the sliding mass along the failure surface is considered and end effects is neglected.

Infinite slope – no seepage

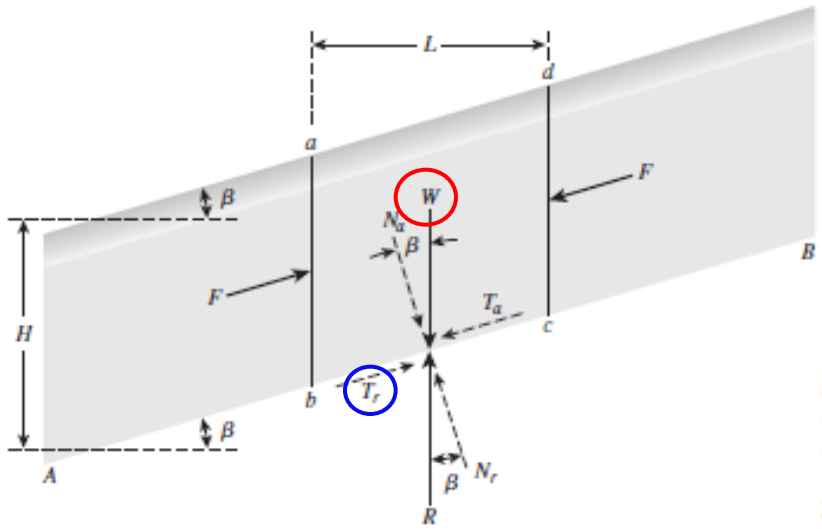
- we will evaluate the factor of safety against a possible slope failure along a plane *AB* located at a depth *H* below the ground surface.
- Let us consider a slope element abcd that has a unit length perpendicular to the plane of the section shown.
- The forces, F, that act on the faces ab and cd are equal and opposite and may be ignored.

➡ The shear stress at the base of the slope element can be given by

$$\tau = \frac{T_a}{\text{Area of base}}$$

Force parallel to the plane AB

$$\tau = \frac{T_a}{\text{Area of base}} = \frac{\gamma LH \sin\beta}{\left(\frac{L}{\cos\beta}\right)} = \gamma H \cos\beta \sin\beta \quad (*)$$



➡ The resistive shear stress is given by

$$\tau_d = c'_d + \sigma' \tan\phi'_d$$

Infinite slope – no seepage

The effective normal stress at the base of the slope element is given by

$$\sigma' = \frac{N_a}{\text{Area of base}} = \frac{\gamma LH \cos\beta}{\left(\frac{L}{\cos\beta}\right)} = \gamma H \cos^2\beta$$

$$\tau_d = c'_d + \gamma H \cos^2\beta \tan\phi'_d \quad (**)$$

Equating R.H.S. of Eqs. (*) and (**) gives

$$\gamma H \sin\beta \cos\beta = c'_d + \gamma H \cos^2\beta \tan\phi'_d$$

$$F_s = F_{c'} = F_{\phi'} \Rightarrow \tan\phi'_d = \frac{\tan\phi'}{F_s} \quad \text{and} \quad c'_d = \frac{c'}{F_s}$$

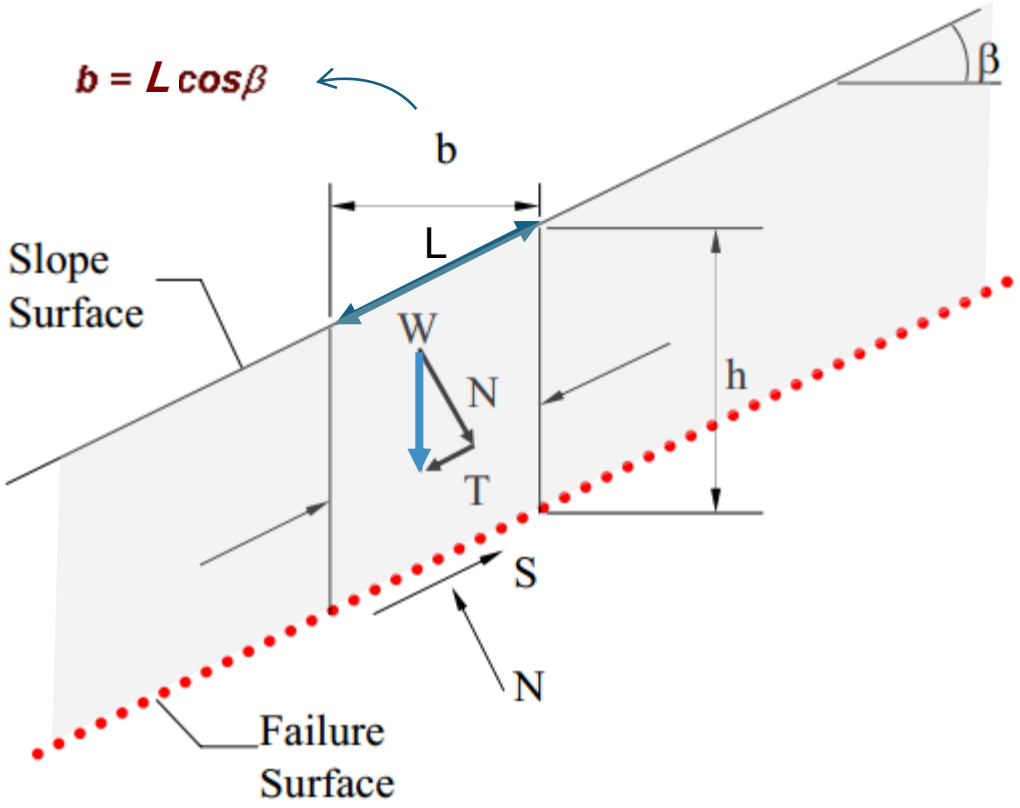
$$F_s = \frac{c'}{\gamma H \cos^2\beta \tan\beta} + \frac{\tan\phi'}{\tan\beta} \quad (***)$$

For Granular Soil (i.e., c = 0)

$$F_s = \frac{\tan\phi'}{\tan\beta}$$

This means that in case of infinite slope in sand, the value of F_s is independent of the height H and the slope is stable as long

Case of Granular soil – Derivation From Simple Statics



Equilibrium of forces on a slice:

$$W = \gamma\, b\, h$$

$$N = W \cos\beta$$

$$T = W \sin\beta$$

$$S = N \tan\phi$$

$$FS = \frac{\text{Resisting Forces}}{\text{Driving Forces}}$$

$$\boxed{FS = \frac{S}{T} = \frac{N \tan\phi}{W \sin\beta} = \frac{\tan\phi}{\tan\beta}}$$

Infinite slope – no seepage

Critical Depth, H_{cr}

The depth of plane along which critical equilibrium occurs is obtained by substituting $F_s = 1$ and $H = H_{cr}$ into Eq. (***)

$$F_s = \frac{c'}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi'}{\tan \beta}$$

$$H_{cr} = \frac{c'}{\gamma} \frac{1}{\cos^2 \beta (\tan \beta - \tan \phi')}$$

EXAMPLE 15.1

Example 15.1

For the infinite slope shown in Figure 15.8 (consider that there is no seepage through the soil), determine:

a. The factor of safety against sliding along the soil–rock interface
b. The height, H, that will give a factor of safety (F_s) of 2 against sliding along the soil–rock interface

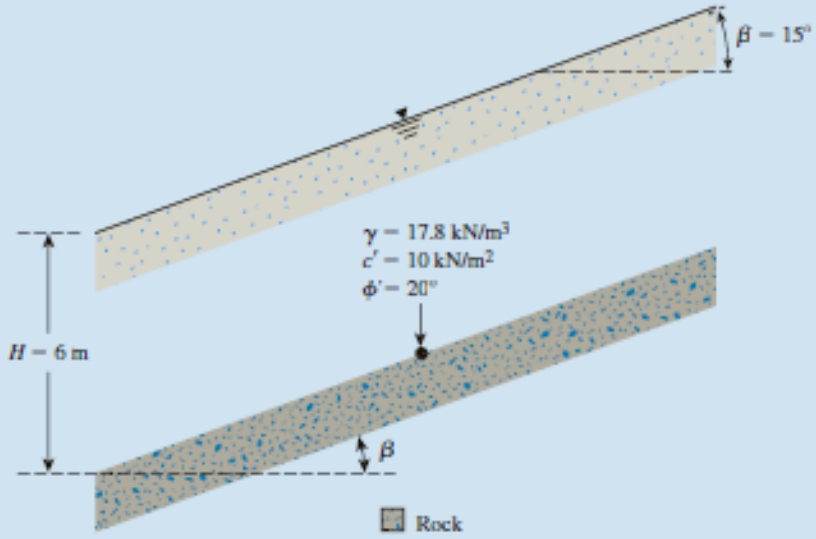


Figure 15.8

EXAMPLE 15.1

Solution

Part a

From Eq. (15.15),

$$F_s = \frac{c'}{\gamma H \cos^2\beta \tan\beta} + \frac{\tan\phi'}{\tan\beta}$$

Given: $c' = 10\ \text{kN/m}^2$, $\gamma = 178\ \text{kN/m}^3$, $\phi' = 20^\circ$, $\beta = 15^\circ$, and $H = 6\ \text{m}$, we have

$$F_s = \frac{10}{(17.8)(6)(\cos^2 15)(\tan 15)} + \frac{\tan 20}{\tan 15} = \mathbf{1.73}$$

Part b

From Eq. (15.15),

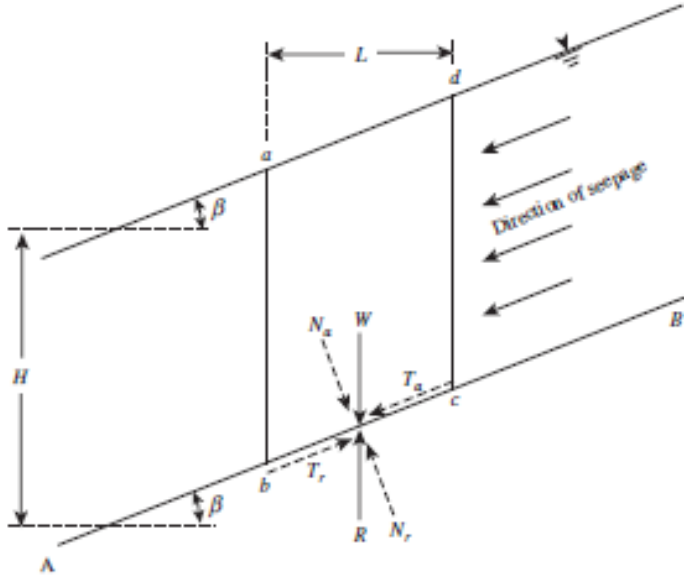
$$F_s = \frac{c'}{\gamma H \cos^2\beta \tan\beta} + \frac{\tan\phi'}{\tan\beta}$$

$$2 = \frac{10}{(17.8)(H)(\cos^2 15)(\tan 15)} + \frac{\tan 20}{\tan 15}$$

$$H = \mathbf{3.5\ m}$$

Infinite slope – with steady state seepage

Seepage is assumed to be parallel to the slope and that the ground water level coincides with the ground surface.



(a)

➡ The shear stress at the base of the slope element can be given

$$\tau = \frac{T_r}{\left(\frac{L}{\cos\beta}\right)} = \gamma_{sat} H \cos\beta \sin\beta \qquad (*)$$

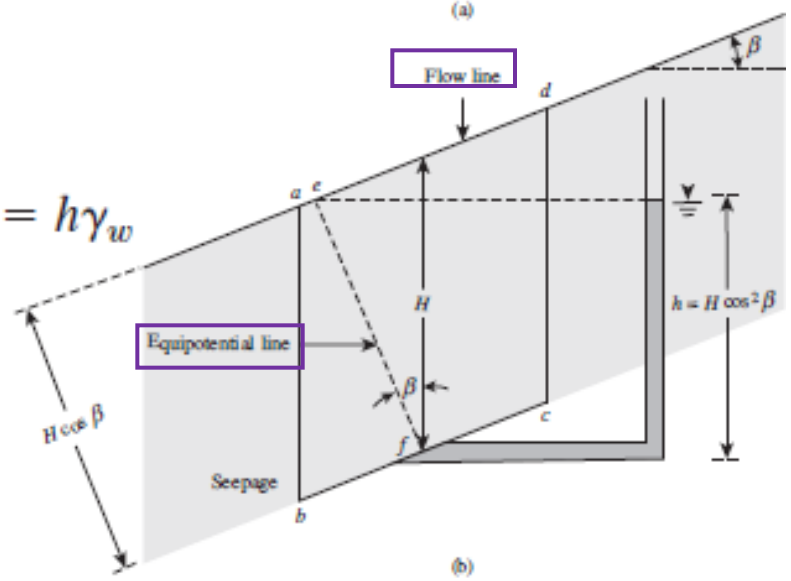
➡ The resistive shear stress developed at the base of the element is given by

$$\tau_d = c'_d + \sigma' \tan\phi_d = c'_d + (\sigma - u)\tan\phi'_d$$

$$u = (\text{Height of water in piezometer placed at } f)(\gamma_w) = h\gamma_w$$

$$u = \gamma_w H \cos^2\beta$$

$$\tau_d = c'_d + (\gamma_{sat} H \cos^2\beta - \gamma_w H \cos^2\beta)\tan\phi'_d$$
$$= c'_d + \gamma' H \cos^2\beta \tan\phi'_d \qquad (**)$$



(b)

Infinite slope – with steady state seepage

Equating the right-hand sides of Eq. (*) and Eq. (**) yields

$$\gamma_{sat} H \cos\beta \sin\beta = c'_d + \gamma' H \cos^2\beta \tan\phi'_d \qquad (***)$$

Recall

$$\tan\phi'_d = \frac{\tan\phi'}{F_s} \quad \text{and} \quad c'_d = \frac{c'}{F_s} \qquad (****)$$

Substituting Eq. (****) Into Eq. (***) and solving for F_s gives

$$F_s = \frac{c'}{\gamma_{sat} H \cos^2\beta \tan\beta} + \frac{\gamma' \tan\phi'}{\gamma_{sat} \tan\beta}$$

$$F_s = \frac{c'}{\gamma H \cos^2\beta \tan\beta} + \frac{\tan\phi'}{\tan\beta}$$

No seepage

Stability of Infinite Slopes

Cohesive Soils

With seepage

$$\tan\phi_d' = \frac{\tan\phi'}{F_s} \qquad c_d' = \frac{c'}{F_s}$$

$$F_s = \frac{c'}{\gamma_{sat} H \cos^2\beta \tan\beta} + \frac{\gamma'}{\gamma_{sat}} \frac{\tan\phi'}{\tan\beta}$$

$$H_{cr} = \frac{c'}{\cos^2\beta(\gamma_{sat}\tan\beta - \gamma'\tan\phi')}$$

No seepage

$$\tan\phi_d' = \frac{\tan\phi'}{F_s} \qquad c_d' = \frac{c'}{F_s}$$

$$F_s = \frac{c'}{\gamma H \cos^2\beta \tan\beta} + \frac{\tan\phi'}{\tan\beta}$$

$$H_{cr} = \frac{c'}{\gamma} \frac{1}{\cos^2\beta(\tan\beta - \tan\phi_d')}$$

Stability of Infinite Slopes

Granular Soils

With seepage

$$c' = 0.0$$

$$F_s = \frac{c'}{\gamma_{sat} H \cos^2 \beta \tan \beta} + \frac{\gamma'}{\gamma_{sat}} \frac{\tan \phi'}{\tan \beta}$$

$$F_s = \frac{\gamma'}{\gamma_{sat}} \frac{\tan \phi'}{\tan \beta}$$

Independant of H

No seepage

$$c' = 0.0$$

$$F_s = \frac{c'}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi'}{\tan \beta}$$

$$F_s = \frac{\tan \phi'}{\tan \beta}$$

Slope is stable as long as $\beta < \phi$

EXAMPLE 15.2

Example 15.2

A cut is to be made in a soil having $\gamma = 16.5\ \text{kN/m}^3$, $c' = 28.75\ \text{kN/m}^2$, and $\phi' = 15°$. The side of the cut slope will make an angle of 45° with the horizontal. What should be the depth of the cut slope that will have a factor of safety (F_s) of 3?

EXAMPLE 15.2

Solution

Given: $\phi' = 15°$; $c' = 28.75$ kN/m^2. If $F_s = 3$, then $F_{c'}$ and $F_{\phi'}$ should both be equal to 3.

$$F_{c'} = \frac{c'}{c'_d}$$

or

$$c'_d = \frac{c'}{F_{c'}} = \frac{c'}{F_s} = \frac{28.75}{3} = 9.58 \text{ kN/m}^2$$

Similarly,

$$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d}$$

$$\tan \phi'_d = \frac{\tan \phi'}{F_{\phi'}} = \frac{\tan \phi'}{F_s} = \frac{\tan 15}{3}$$

or

$$\phi'_d = \tan^{-1}\left[\frac{\tan 15}{3}\right] = 5.1°$$

Substituting the preceding values of c'_d and ϕ'_d in Eq. (15.40),

$$H = \frac{4c'_d}{\gamma}\left[\frac{\sin \beta \cos \phi'_d}{1 - \cos (\beta - \phi'_d)}\right]$$

$$= \frac{4 \times 9.58}{16.5}\left[\frac{\sin 45 \cos 5.1}{1 - \cos(45 - 5.1)}\right]$$

$$= \mathbf{7.03 \text{ m}}$$

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Stability of Finite Slopes with Plane Failure Surface

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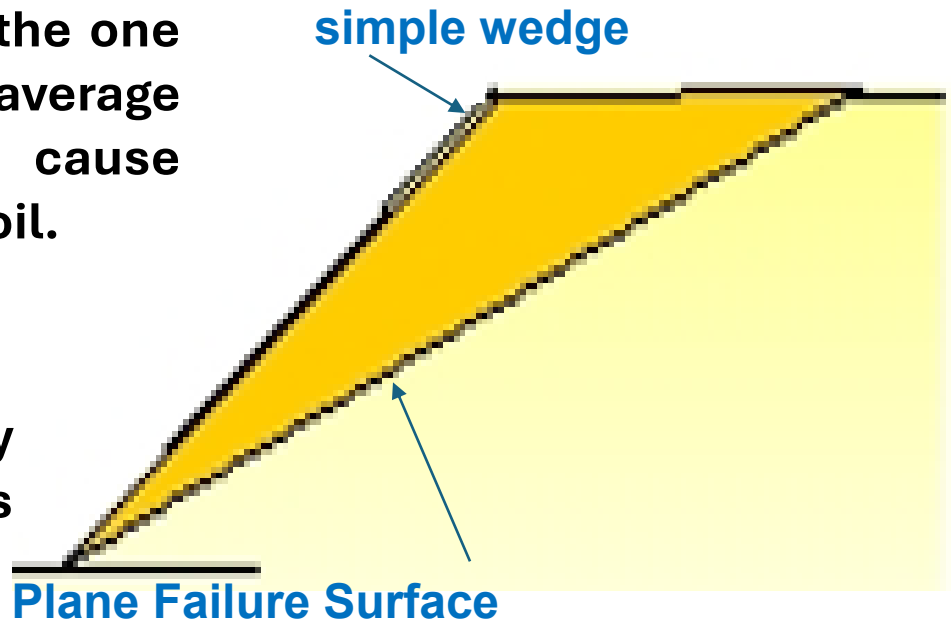
Stability of Finite Slopes with Plane Failure Surface

- For simplicity, when analyzing the stability of a finite slope in a homogeneous soil, we need to make an assumption about the general shape of the surface of potential failure.
- The simplest approach is to approximate the surface of potential failure as a plane.
- However, considerable evidence suggests that slope failures usually occur on curved failure surfaces
- Hence most conventional stability analyses of slopes have been made by assuming that the curve of potential sliding is an arc of a circle.

Culmann's Method

Culmann's Method

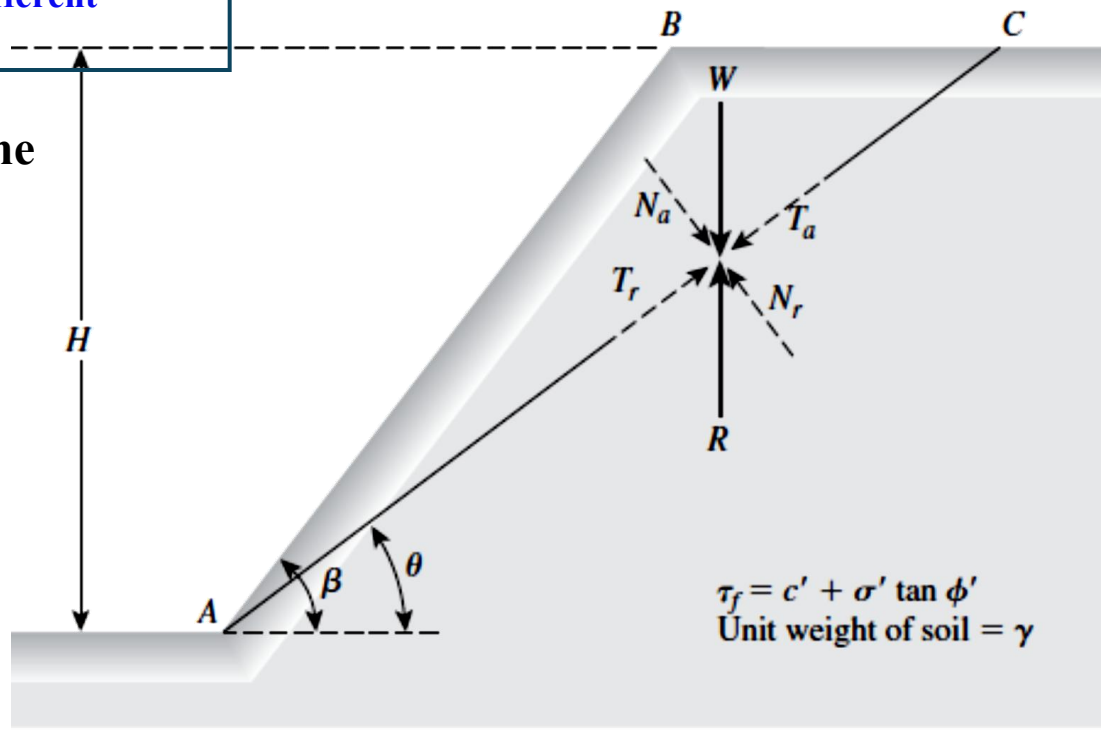
- Culmann's method assumes that the critical surface of failure is a plane surface passing through the toe.
- Culmann's analysis is based on the assumption that the failure of a slope occurs along a plane when the average shearing stress tending to cause the slip is more than the shear strength of the soil.
- Also, the most critical plane is the one that has a minimum ratio of the average shearing stress that tends to cause failure to the shear strength of soil.
- The method gives reasonably accurate results if the slope is vertical or nearly vertical.



Culmann's Method

- A slope of height H and that rises at an angle β is shown below.
- The forces that act on the mass are shown in the figure, where trial failure plane *AC is* inclined at angle θ with the horizontal.

Similar procedures as for infinite slope, only different geometry. Also here we made optimization.



➡ The average shear stress on the plane *AC*

$$\tau = \frac{T_a}{(\overline{AC})(1)} = \frac{T_a}{\left(\frac{H}{\sin\theta}\right)}$$

$T_a = W \sin\theta$

$$W = \frac{1}{2}(H)(\overline{BC})(1)(\gamma) = \frac{1}{2}H(H\cot\theta - H\cot\beta)\gamma$$

$$= \frac{1}{2}\gamma H^2\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]$$

$$\tau = \frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\sin^2\theta \quad (*)$$

Culmann's Method

The average resistive shearing stress (Developed shear strength) developed along the plane *AC* also may be expressed as

$$\tau_d = c'_d + \sigma' \tan \phi'_d$$

$$\sigma' = \frac{N_a}{(\overline{AC})(1)} = \frac{N_a}{\left(\frac{H}{\sin\theta}\right)}$$

N_a = normal component = $W \cos\theta$

$$N_a = \frac{1}{2}\gamma H^2 \left[\frac{\sin(\beta-\theta)}{\sin\beta \sin\theta}\right] \cos\theta$$

$$\sigma' = \frac{1}{2}\gamma H \left[\frac{\sin(\beta-\theta)}{\sin\beta \sin\theta}\right] \cos\theta \sin\theta$$

$$\tau_d = c'_d + \frac{1}{2}\gamma H \left[\frac{\sin(\beta-\theta)}{\sin\beta \sin\theta}\right] \cos\theta \sin\theta \tan\phi'_d \quad (**)$$

Equating the R.H.S of Eqs. (*) and (**) gives

$$c'_d = \frac{1}{2}\gamma H \left[\frac{\sin(\beta-\theta)(\sin\theta - \cos\theta \tan\phi'_d)}{\sin\beta}\right] \quad (***)$$

Culmann's Method

Critical failure plane

- The expression in Eq. (***) is derived for the trial failure plane **AC**.
- To determine the critical failure plane, we must use the principle of **maxima** and **minima** (for F_s=1 and for given values of c', ϕ', g, H, β) to find the critical angle θ:

$$\frac{\partial c'_d}{\partial \theta} = 0 \quad \Rightarrow \quad \theta_{cr} = \frac{\beta + \phi'}{2}$$

- Substitution of the value of $\theta = \theta_{cr}$ into Eq. (***) yields

$$c'_d = \frac{\gamma H}{4}\left[\frac{1 - \cos(\beta - \phi'_d)}{\sin\beta \cos\phi'_d}\right] \qquad (****)$$

Culmann's Method

$$\frac{c'_d}{\gamma H} = m = \frac{1 - \cos(\beta - \phi'_d)}{4 \sin\beta \cos\phi'_d}$$

The maximum height of the slope for which critical equilibrium occurs can be obtained by substituting $c'_d = c'$ and $\phi'_d = \phi'$ (****)

$$H_{cr} = \frac{4c'}{\gamma}\left[\frac{\sin\beta\cos\phi'}{1 - \cos(\beta - \phi')}\right]$$

- For purely cohesive soils c ≠ 0 ϕ = 0.

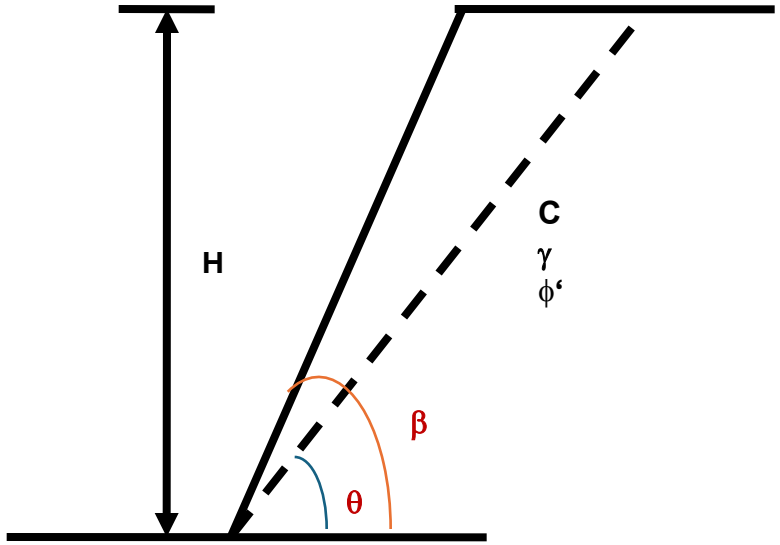
$$\theta_{cr} = \frac{\beta}{2} \longrightarrow H_{cr} = \frac{4c'}{\gamma}\left[\frac{\sin\beta}{1 - \cos\beta}\right]$$

Culmann's Method

- **Steps for Solution**

A. If F_s is given; H is required

1. $F_c = F_\phi = F_s$
2. $c_d' = \dfrac{c'}{F_s}$
3. $\tan\phi_d' = \dfrac{\tan\phi'}{F_s}$
4. $H = \dfrac{4c_d'}{\gamma}\left(\dfrac{\sin\beta\cos\phi_d'}{1-\cos(\beta-\phi_d')}\right)$

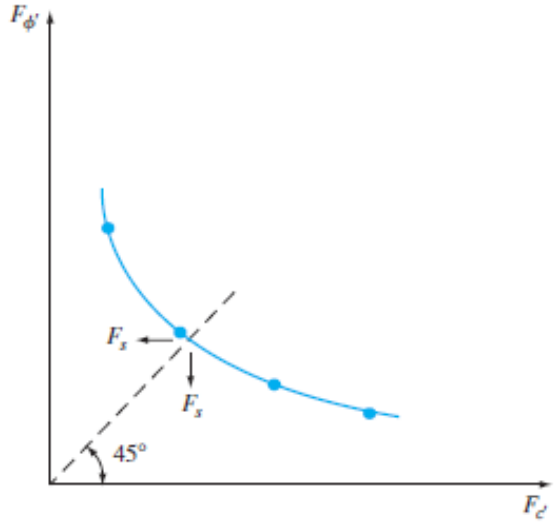


Culmann's Method

- **Steps for Solution**

B. If H is given; F_s is required

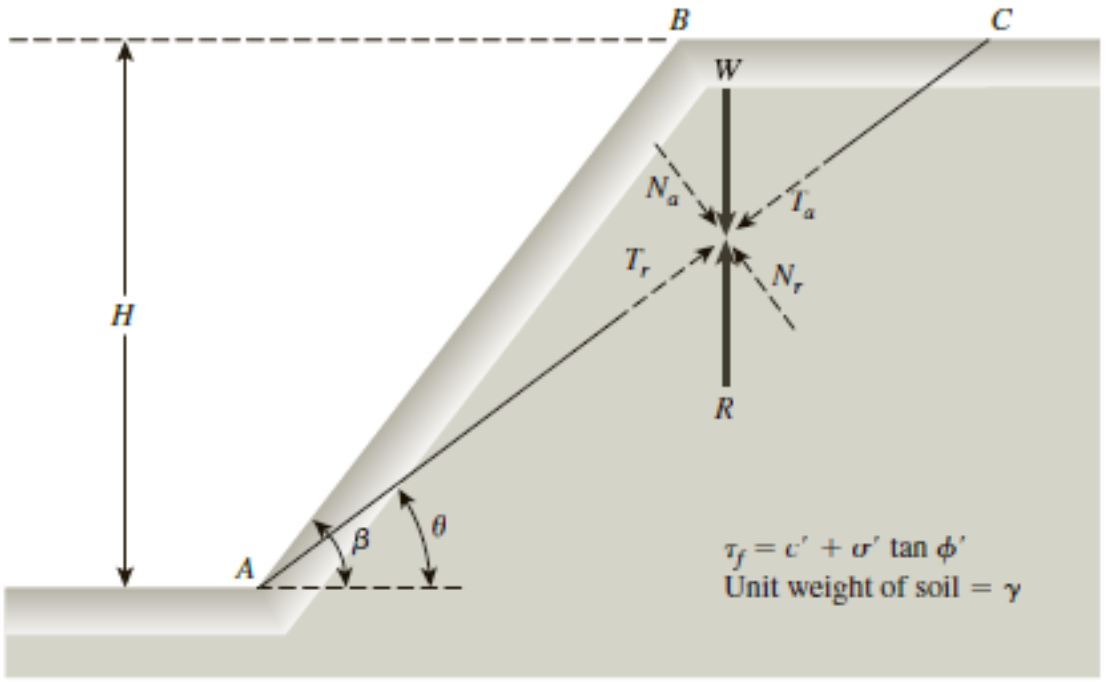
1. $Assume\ F_\phi$
2. $\tan\phi_d' = \dfrac{\tan\phi'}{F_s}$
3. $c_d' = \dfrac{\gamma H}{4}\left(\dfrac{1-\cos(\beta-\phi_d')}{\sin\beta\cos\phi_d'}\right)$
4. $F_c = \dfrac{c'}{c_d'}$
5. $Check\ if\quad F_c = F_\phi \quad \rightarrow F_s = F_c = F_\phi$
6. $If\ \ F_c \neq F_\phi \quad \rightarrow try\ \ another\ \ F_\phi$
7. $Repeat\ steps\ 1 \rightarrow 5$



EXAMPLE 15.3

Example 15.3

Refer to Figure 15.9. For a trial failure surface AC in the slope, given: $H = 5$ m, $\beta = 55°$, $\theta = 35°$, and $\gamma = 17.5$ kN/m^3. The shear strength parameters of the soil are $c' = 25$ kN/m^2 and $\phi' = 26°$. Determine the factor of safety F_s for the trial failure surface.



EXAMPLE 15.3

Solution

From Eq. (15.33), the average shear stress on the plane AC is

$$\tau = \frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)}{\sin\beta\cdot\sin\theta}\right]\sin^2\theta$$

$$= \left(\frac{1}{2}\right)(17.5)(5)\left[\frac{\sin(55-35)}{(\sin 55)(\sin 35)}\right]\sin^2 35 = 10.48\ \text{kN/m}^2$$

From Eq. (15.34), the maximum average shear stress that can be mobilized on the plane AC is

$$\tau_f = c' + \frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\cos\theta\sin\theta\tan\phi'$$

$$= 25 + \left(\frac{1}{2}\right)(17.5)(5)\left[\frac{\sin(55-35)}{(\sin 55)(\sin 35)}\right](\cos 35)(\sin 35)(\tan 26)$$

$$= 32.3\ \text{kN/m}^2$$

So,

$$F_s = \frac{32.3}{10.48} = \mathbf{3.08}$$

EXAMPLE

A cut is to be made in a soil having properties as shown in the figure below.

If the failure surface is assumed to be finite plane, determine the followings:

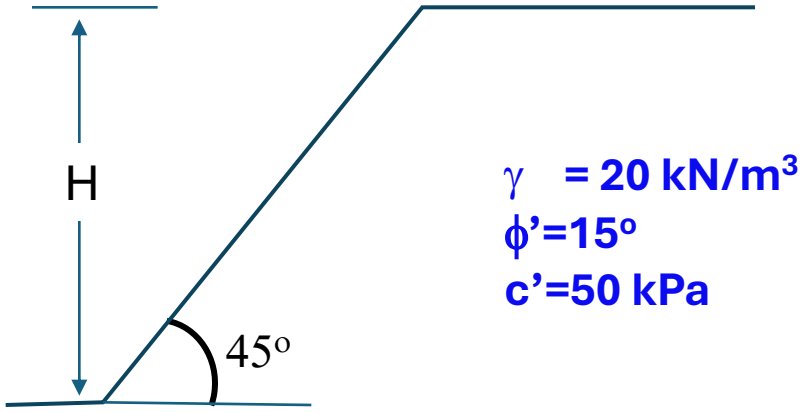
(a) The angle of the critical failure plane.

(b) The critical depth of the cut slope

(c) The safe (design) depth of the cut slope. Assume the factor of safety (F_s=3)?

Given equation:

$$H_{cr} = \frac{4c'}{\gamma}\left[\frac{\sin\beta\cos\phi'}{1-\cos(\beta-\phi')}\right]$$



Key Solution

(a) The angle of the critical failure plane θ can be calculated from:

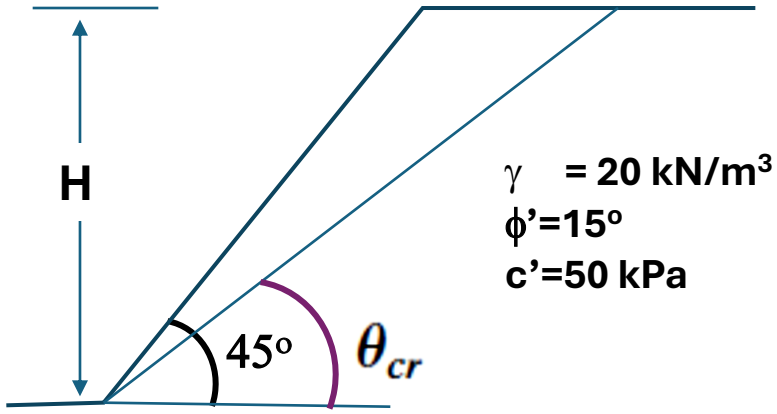
$$\theta_{cr} = \frac{\beta + \phi'}{2}$$

$\beta = 45^o$

$\phi' = 15^o$

(b) The critical depth of the cut slope can be calculated from:

$$H_{cr} = \frac{4c'}{\gamma}\left[\frac{\sin\beta\cos\phi'}{1-\cos(\beta-\phi')}\right]$$



(c) The safe (design) depth of the cut slope.

$$H_d = \frac{4c'_d}{\gamma}\left[\frac{\sin\beta\cdot\cos\phi'_d}{1-\cos(\beta-\phi'_d)}\right]$$

where: c'_d and ϕ'_d can be determined from:

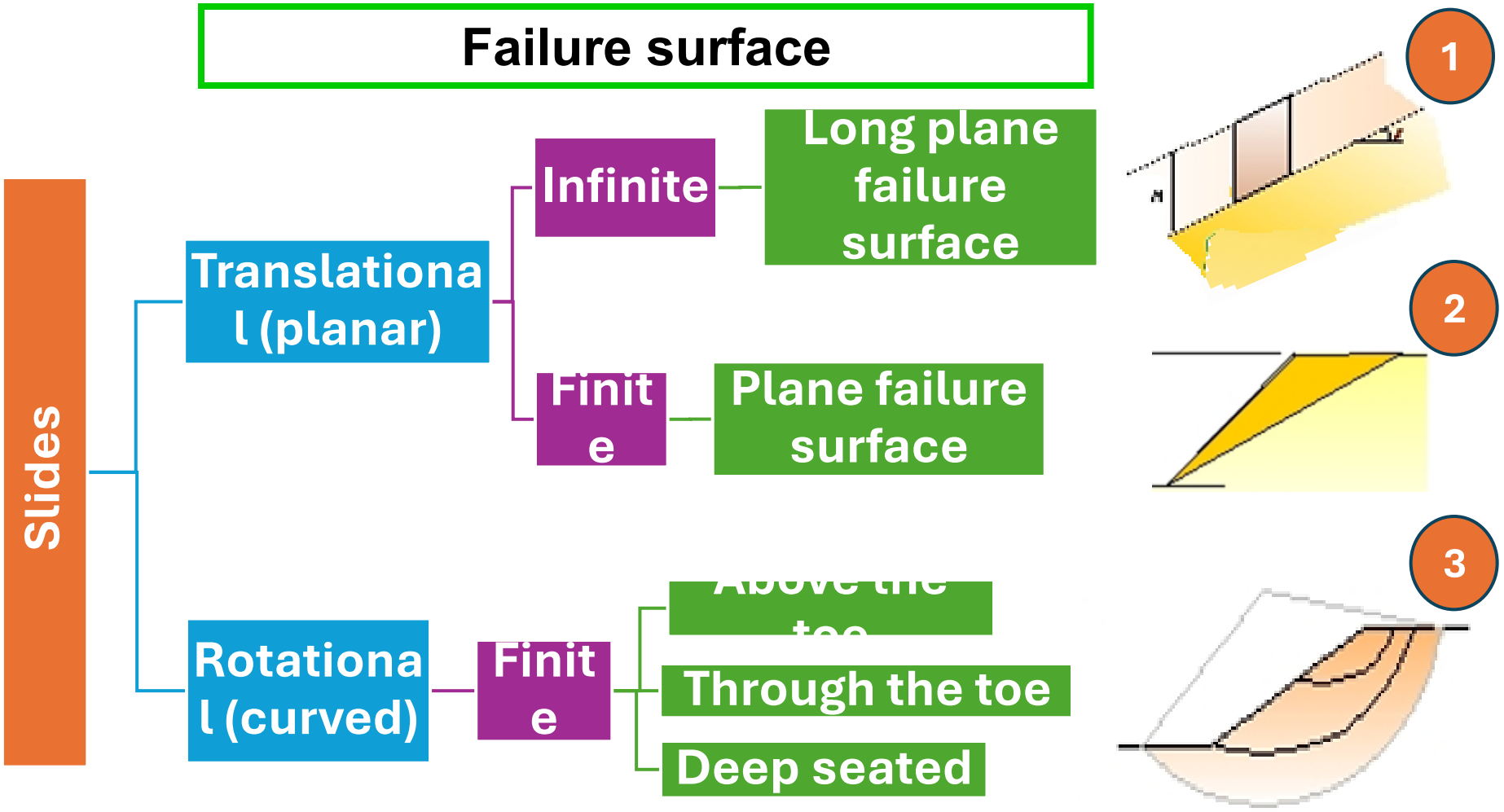
$$c'_d = \frac{c'}{F_{c'}} = \frac{c'}{F_s}$$

$$\tan\phi'_d = \frac{\tan\phi'}{F_{\phi'}} = \frac{\tan\phi'}{F_s}$$

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Types of Failure Surfaces



Finite Slopes with Circular Failure Surface

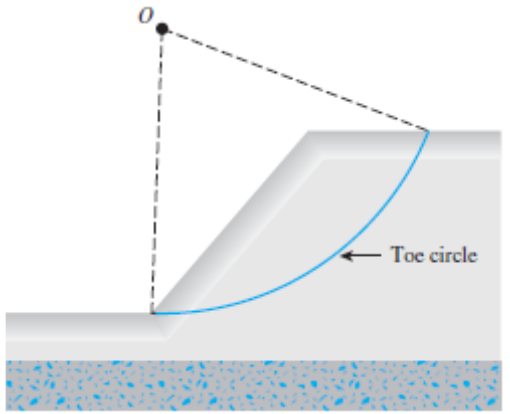
Finite Slopes with Circular Failure Surface

Modes of Failure

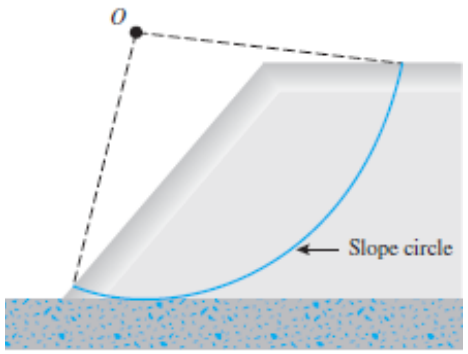
i. Slope failure

- Surface of sliding intersects the slope at or above its toe.

1. The failure circle is referred to as a *toe circle* if it passes through the toe of the slope

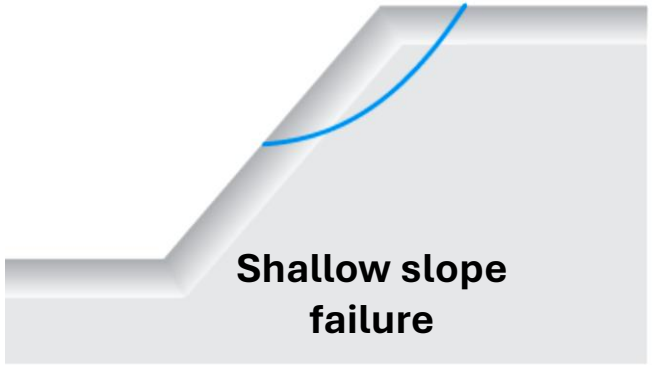


1. The failure circle is referred to as a *slope circle* if it passes above the toe of the slope.



ii. Shallow failure

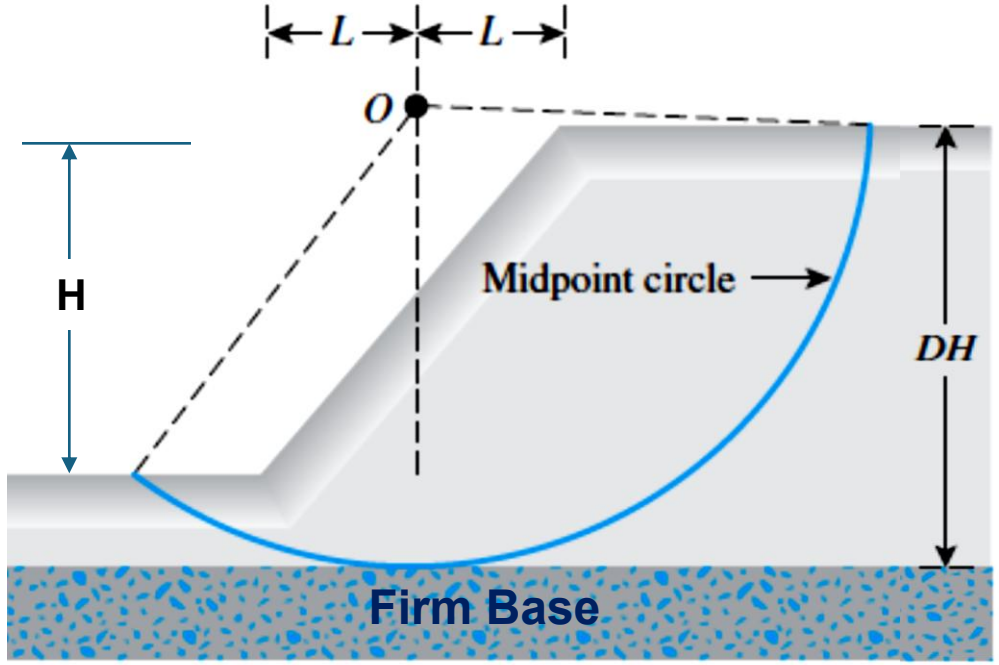
Under certain circumstances, a *shallow slope failure* can occur.



Finite Slopes with Circular Failure Surface

iii. Base failure

- The surface of sliding passes at some distance below the toe of the slope.
- The circle is called the midpoint circle because its center lies on a vertical line drawn through the midpoint of the slope.



- For $\beta > 53^{o}$ always toe
- For $\beta < 53^{o}$ could be toe, slope, or midpoint and that depends on depth function D where:

Depth function:

$$D = \frac{\text{Vertical distance from top of slope to firm base}}{\text{Height of slope}}$$

Finite Slopes with Circular Failure Surface

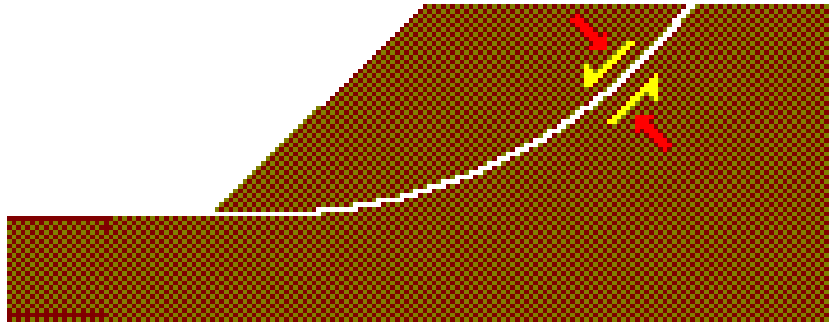
- **Summary**
- **Toe Circle** all circles for soils with $\phi > 3°$ & $\beta > 53°$
- **Slope Circle** always for $D = 0$ & $\beta < 53°$
- **Midpoint Circle** always for $D > 4$ & $\beta < 53°$

Types of Stability Analysis Procedures

Various procedures of stability analysis may, in general, be divided into two major classes:

1. Mass procedure

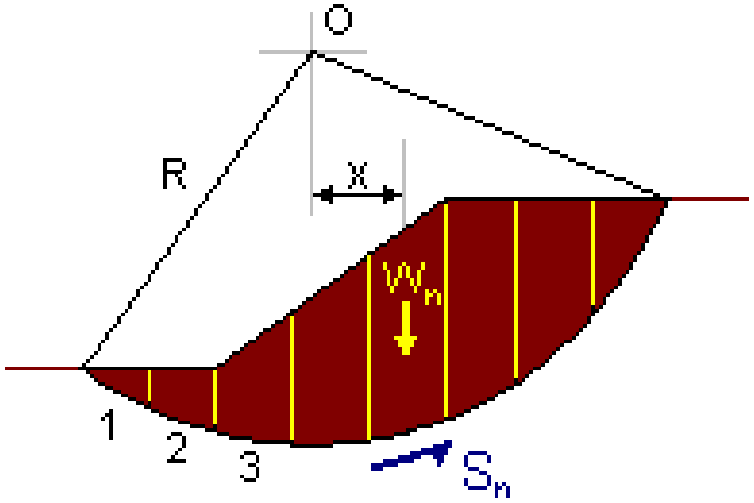
- In this case, the mass of the soil above the surface of sliding is taken as a unit.
- This procedure is useful when the soil that forms the slope is assumed to be homogeneous.



2. Method of slices

- Most natural slopes and many man-made slopes consist of more than on soil with different properties.

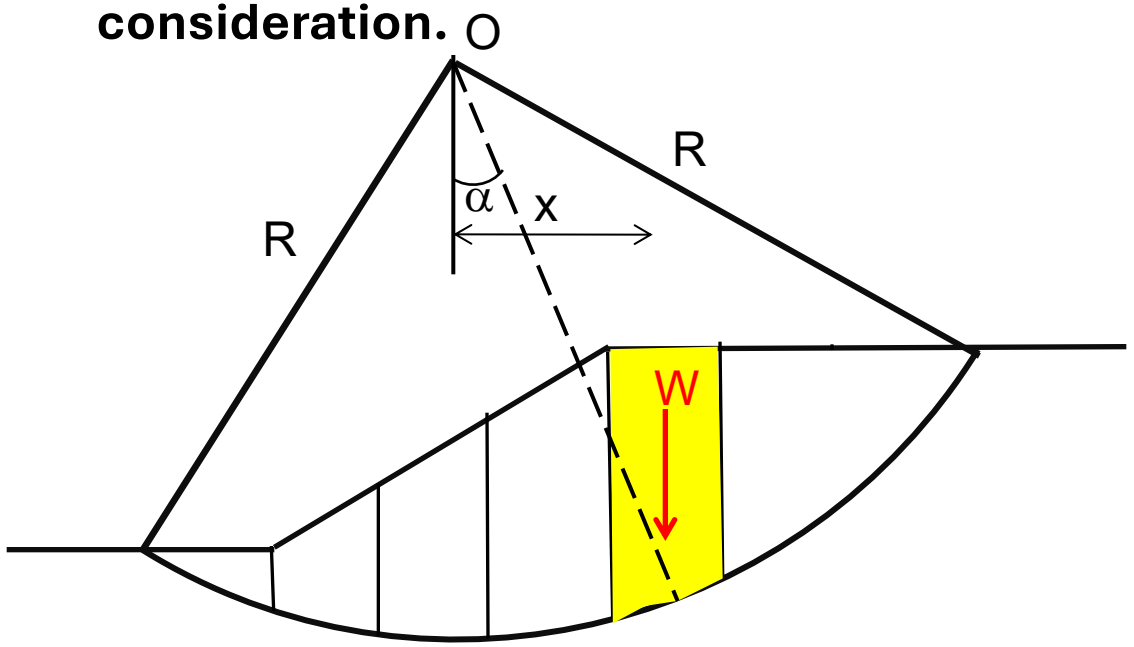
- In this case the use of mass procedure is inappropriate.

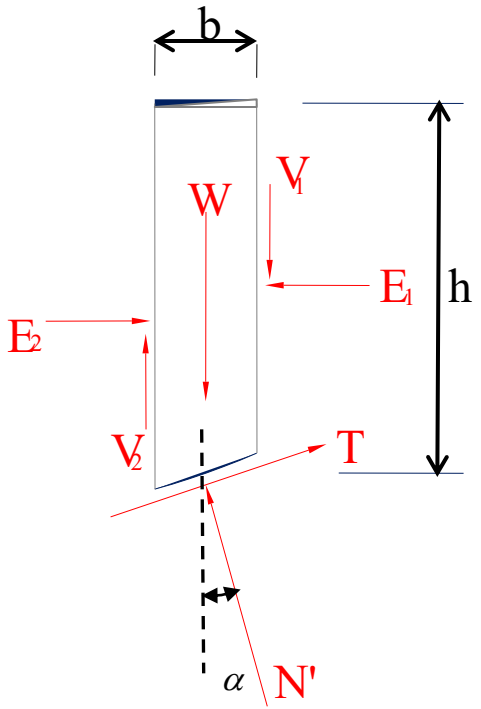


Types of Stability Analysis Procedures

- In the method of slices procedure, the soil above the surface of sliding is divided into a number of vertical parallel slices. The stability of each slice is calculated separately.

- It is a general method that can be used for analyzing irregular slopes in non-homogeneous slopes in which the values of c' and ϕ' are not constant and pore water pressure can be taken into consideration.





TOPICS

- Introduction
- Types of slope movements
- Concepts of Slope Stability Analysis
- Factor of Safety
- Stability of Infinite Slopes
- Stability of Finite Slopes with Plane Failure Surface
  - Culmann's Method
- Stability of Finite Slopes with Circular Failure Surface
  - **Mass Method**
  - Method of Slices

Mass Procedure

Mass Procedure

1.Slopes in purely cohesionless soil with c = 0, $\phi \neq 0$

Failure generally does not take place in the form of a circle. So we will not go into this analysis.

2. Slopes in Homogeneous clay Soil with $c \neq 0$, $\phi = 0$

Determining factor of safety using equilibrium equations (Case I)

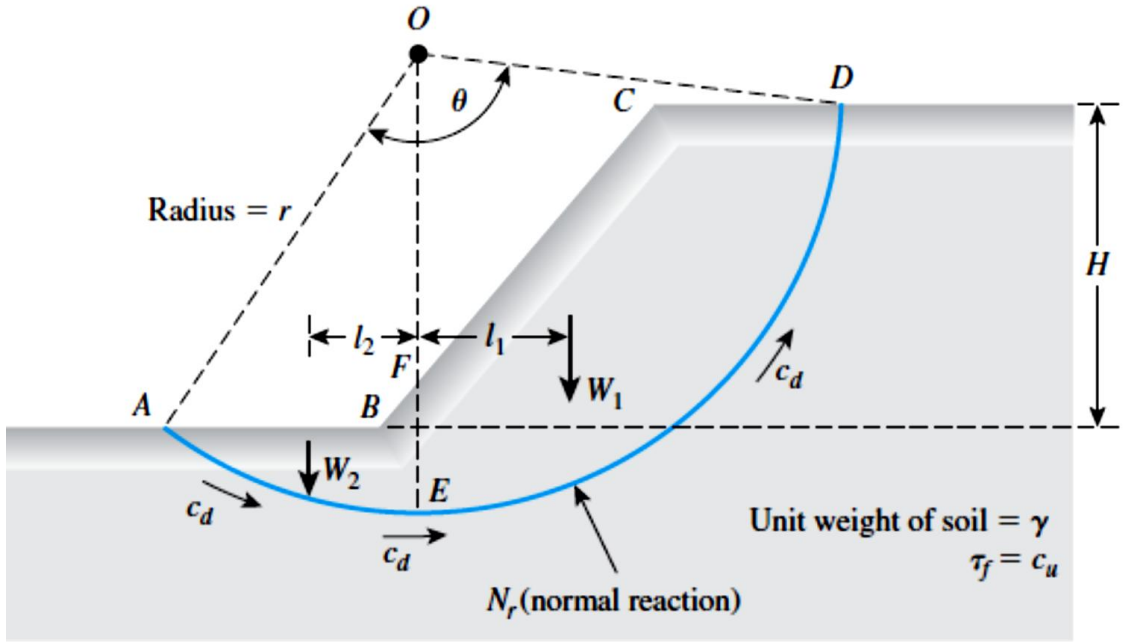
$M_{driving} = M_d = W_1 l_1 - W_2 l_2$

W_1 = (area of FCDEF) γ

W_2 = (area of ABFEA) γ

$M_{resisting} = M_R = c_d\,(AED)\,(1)\,r$

$= c_d\, r^2\theta$



For equilibrium, $M_R = M_d$; thus,

$$c_d = \frac{W_1 l_1 - W_2 l_2}{r^2 \theta}$$

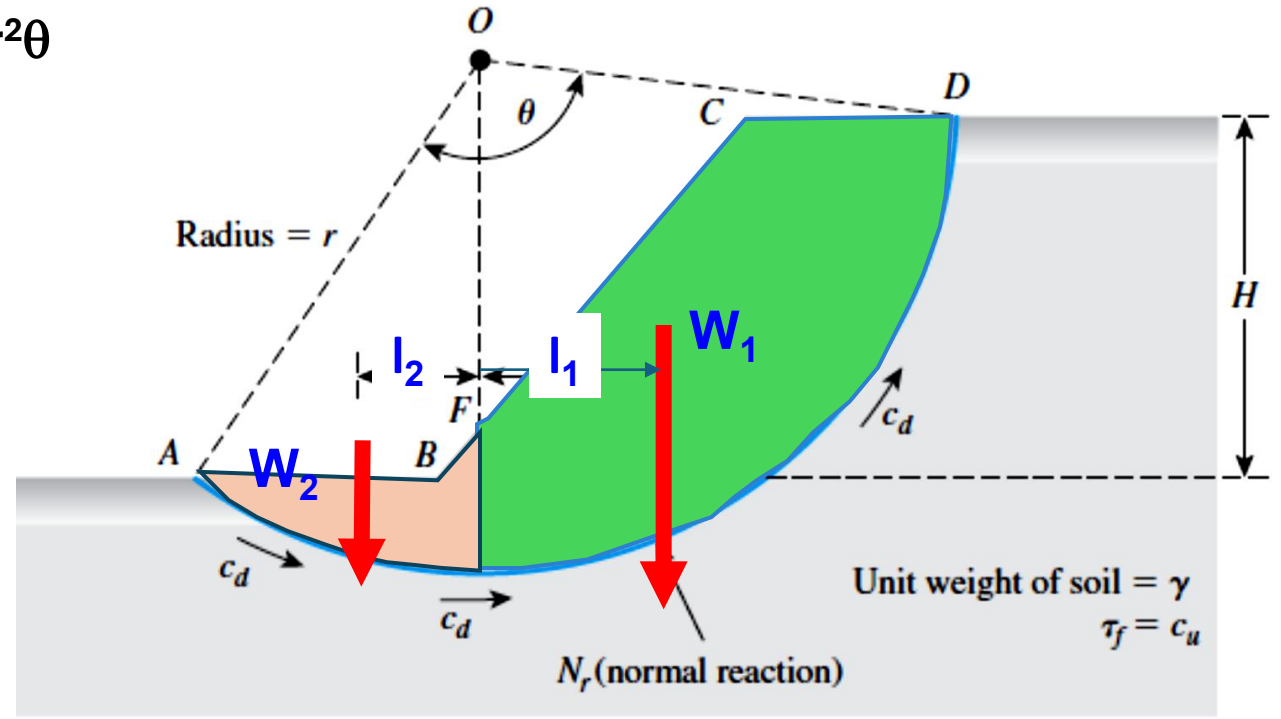
$$F_s = \frac{\tau_f}{c_d} = \frac{c_u}{c_d}$$

$$\boldsymbol{F_s = \frac{r^2 \theta c_u}{w_1 l_1 - w_2 l_2}}$$

Mass Procedure

$M_{driving} = M_d = W_1 l_1 - W_2 l_2$

$M_{resisting} = M_R = c_d r^2 \theta$



$$F.S. = \frac{M_{resisting}}{M_{driving}}$$

$$F_s = \frac{\tau_f}{c_d} = \frac{c_u}{c_d}$$

$$F_s = \frac{r^2 \theta c_u}{w_1 l_1 - w_2 l_2}$$

REMARKS

- The potential curve of sliding, *AED*, was chosen arbitrarily.
- The critical surface is that for which the ratio of C_u to C_d is a minimum. In other words, C_d is maximum.
- To find the critical surface for sliding, one must make a number of trials for different trial circles.
- The minimum value of the factor of safety thus obtained is the factor of safety against sliding for the slope, and the corresponding circle is the critical circle.

Finite Slopes with Circular Failure Surface

- Fellenius (1927) and Taylor (1937) have analytically solved for the ***minimum factor of safety*** and ***critical circles.***

- They expressed the developed cohesion as

$$c_d = \gamma H m$$

Where

m = Stability number

H = height of slope

γ = unit weight of soil

or $m = \dfrac{c_d}{\gamma H}$

  - We then can calculate the min F_s as

$$F_s = \frac{\tau_f}{c_d} = \frac{c_u}{c_d}$$

$$\boldsymbol{F_s = \frac{C_u}{\gamma H m}}$$

- The critical height (i.e., *Fs* 1) of the slope can be evaluated by substituting $H = H_{cr}$ and $c_d = c_u$ (full mobilization of the undrained shear strength) into the preceding equation. Thus,

$$\boldsymbol{H_{cr} = \frac{C_u}{\gamma m}}$$

The end