SHEAR STRENGTH OF SOIL

Chapter 10: Sections

10.2

10.3

Chapter 12: All sections except

12.14

12.15

12.18

TOPICS

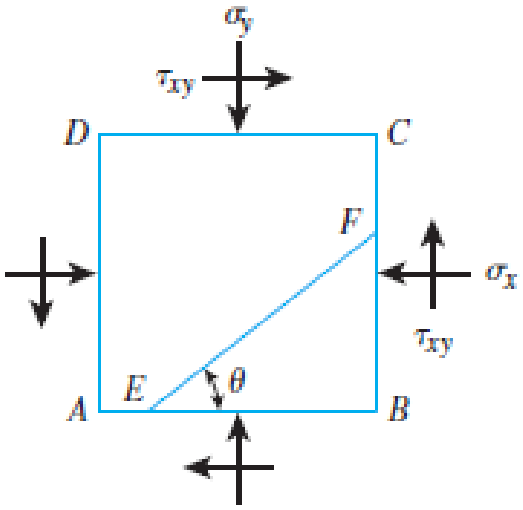
- Normal and Shear Stresses on a Plane
- Components of Shear Strength of Soils
- Mohr-Coulomb Failure Criterion
- Laboratory Shear Strength Testing
  - Direct Shear Test
  - Triaxial Compression Test
  - Unconfined Compression Test
- Field Testing (Vane test)

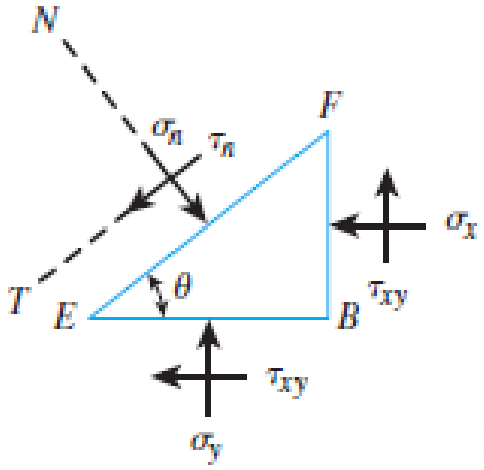
Normal and Shear Stress along a Plane

Chapter 10

- Normal and Shear Stresses along a Plane (Sec. 10.2)
- Pole Method for Finding Stresses along a Plane (Sec. 10.3)

Normal and Shear Stress along a Plane





From geometry

$$\overline{EB} = \overline{EF}\cos\theta$$

$$\overline{FB} = \overline{EF}\sin\theta$$

Sign Convention	Normal Stresses	Shear Stresses
Positive	Compression	Counter clockwise rotation
Negative	Tension	Clockwise rotation

- Note that for convenience our sign convention has **compressive forces and stresses positive** because most normal stresses in geotechnical engineering are compressive.
- These conventions are the ***opposite*** of those normally assumed in **structural mechanics**.

Normal and Shear Stress along a Plane

$$\sum F_N = 0$$

$$\sigma_n * (\overline{EF}) - \sigma_x \sin\theta * (\overline{EF}\sin\theta) - \sigma_y \cos\theta * (\overline{EF}\cos\theta)$$

$$- \tau_{xy} \cos\theta * (\overline{EF}\sin\theta) - \tau_{xy} \sin\theta * (\overline{EF}\cos\theta) = 0$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy} \sin 2\theta$$

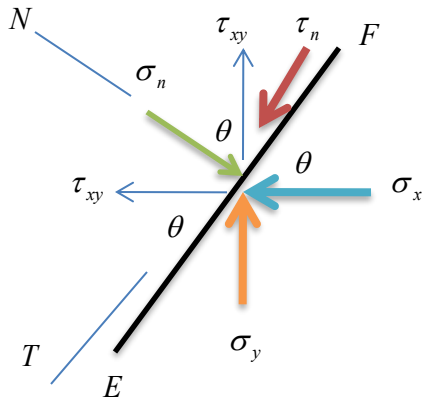
Similarly,

$$\sum F_T = 0$$

$$\tau_n * (\overline{EF}) - \sigma_y \sin\theta * (\overline{EF}\cos\theta) + \sigma_x \cos\theta * (\overline{EF}\sin\theta)$$

$$- \tau_{xy} \sin\theta * (\overline{EF}\sin\theta) + \tau_{xy} \cos\theta * (\overline{EF}\cos\theta) = 0$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy} \cos 2\theta$$



$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy} \cos 2\theta$$

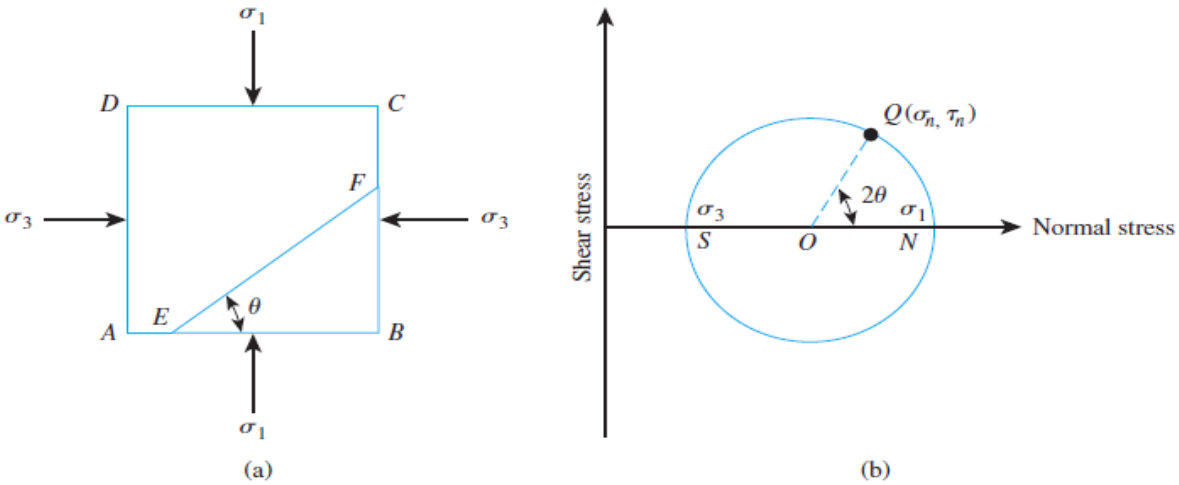
Principal Planes & Principal Stresses

Principal Planes

Planes on which the shear stress is equal to zero

Principal Stresses

Normal stress acting on the principal planes



$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta$$

$$\tau_n = \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta$$

Principal Stresses

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad (1)$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \quad (2)$$

$For\ \tau_n == 0$

$$0 = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$\tan 2\theta_p = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2\tau_{xy}}{\sigma_y - \sigma_x} \quad (3)$$

For any given values of σ_x , σ_y and τ_{xy}

Equation (3) will give two values of θ

which are 90 degrees apart

Two principal planes 90 degrees apart

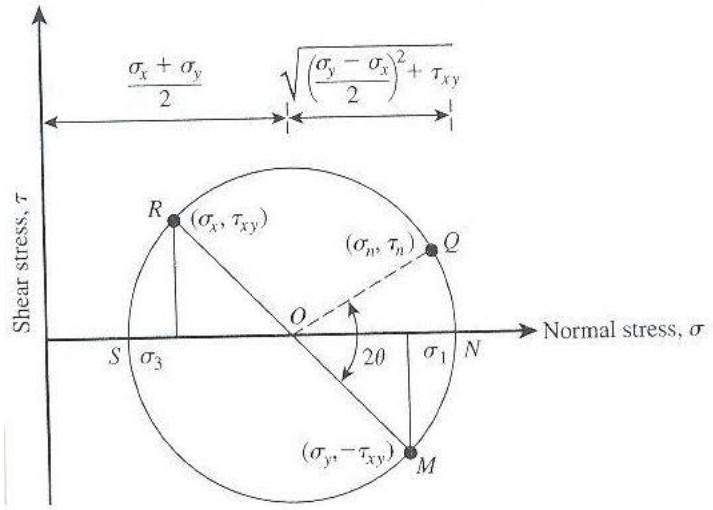
Substitute eq (3) into eq (1)

Major Principal Stress

$$\sigma_n = \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

Minor Principal Stress

$$\sigma_n = \sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$



Example 10.1

Example 10.1

A soil element is shown in Figure 10.4. The magnitudes of stresses are $\sigma_x = 120\ \text{kN/m}^2$, $\tau = 40\ \text{kN/m}^2$, $\sigma_y = 300\ \text{kN/m}^2$, and $\theta = 20^\circ$. Determine

a. Magnitudes of the principal stresses.
b. Normal and shear stresses on plane AB. Use Eqs. (10.3), (10.4), (10.6), and (10.7).

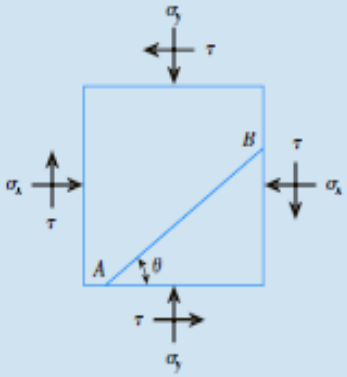


Figure 10.4 Soil element with stresses acting on it

Solution

Part a

From Eqs. (10.6) and (10.7),

$$\left.\begin{matrix}\sigma_3\\ \sigma_1\end{matrix}\right\} = \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2}$$

$$= \frac{300 + 120}{2} \pm \sqrt{\left[\frac{300 - 120}{2}\right]^2 + (-40)^2}$$

$$\sigma_1 = \mathbf{308.5\ kN/m^2}$$

$$\sigma_3 = \mathbf{111.5\ kN/m^2}$$

Part b

From Eq. (10.3),

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau \sin 2\theta$$

$$= \frac{300 + 120}{2} + \frac{300 - 120}{2}\cos(2 \times 20) + (-40)\sin(2 \times 20)$$

$$= \mathbf{253.23\ kN/m^2}$$

From Eq. (10.4),

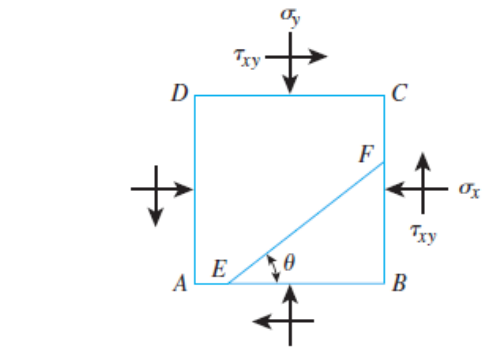
$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau \cos 2\theta$$

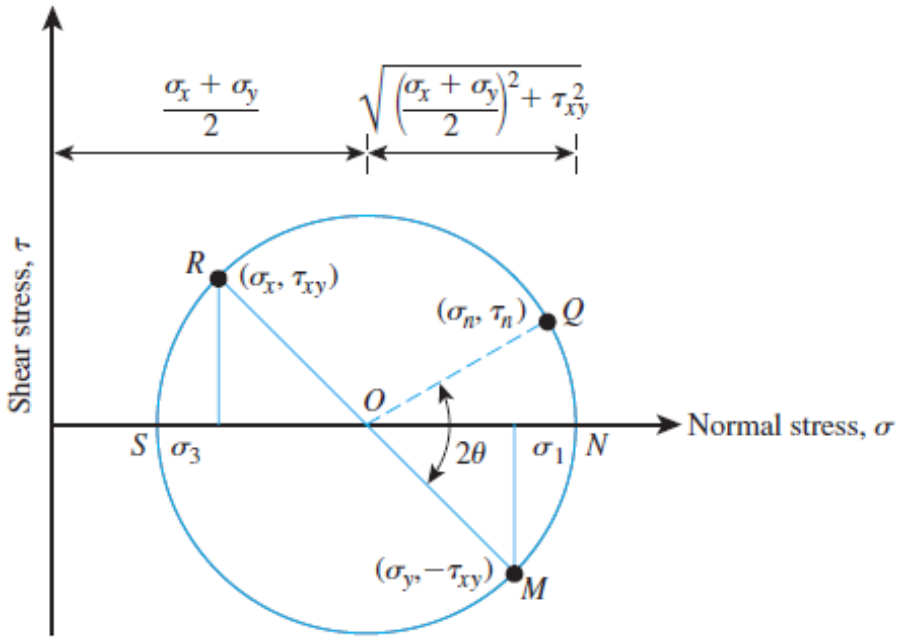
$$= \frac{300 - 120}{2}\sin(2 \times 20) - (-40)\cos(2 \times 20)$$

$$= \mathbf{88.40\ kN/m^2}$$

Construction of Mohr's Circle

1. Plot σ_y, τ_{xy} as point M
2. Plot σ_x, τ_{xy} as point R
3. Connect M and R
4. Draw a circle of diameter of the line RM about the point where the line RM crosses the horizontal axis (denote this as point O)





Sign Convention	Normal Stresses	Shear Stresses
Positive	Compression	Counter clockwise rotation
Negative	Tension	Clockwise rotation

- The points R and M in Figure above represent the stress conditions on plane AD and AB, respectively. O is the point of intersection of the normal stress axis with the line RM.

Pole Method for Finding Stresses on a Plane

There is a unique point on the Mohr's circle called the POLE (origin of planes)

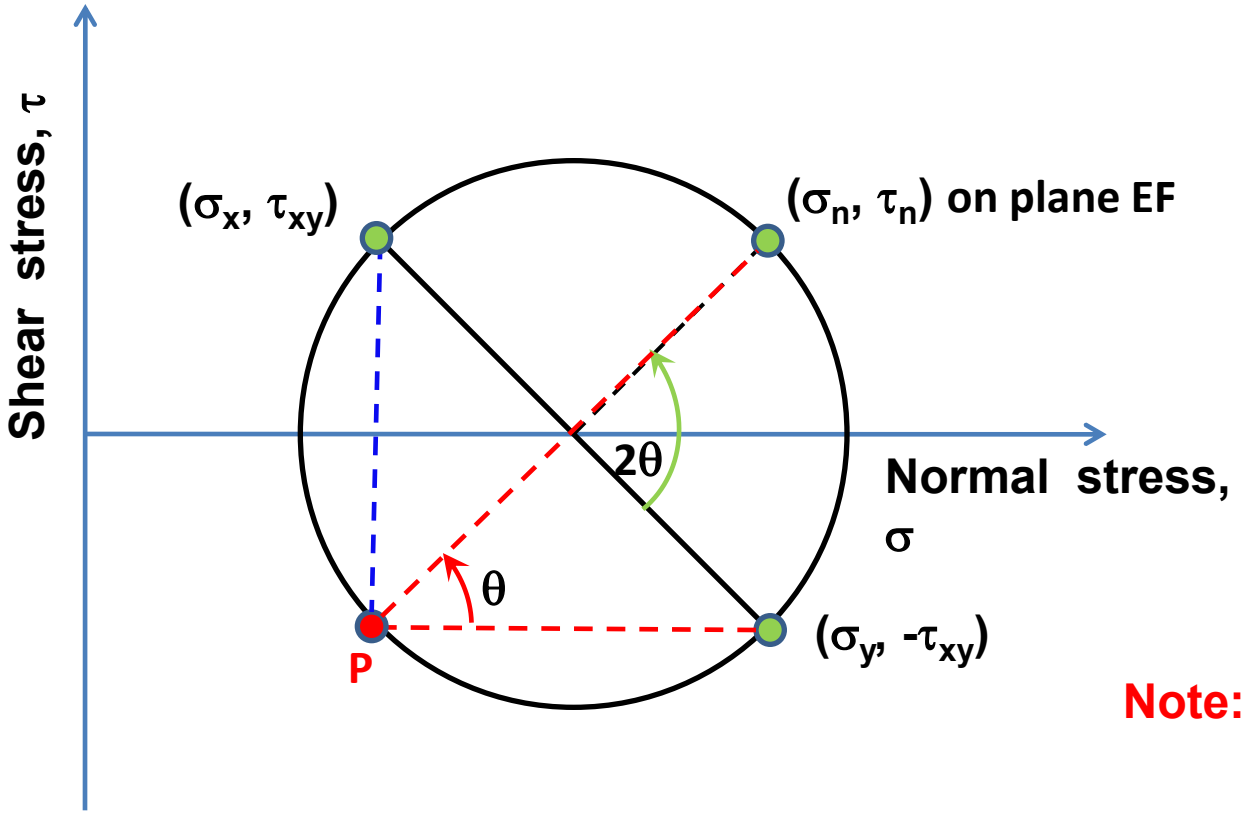
Any straight line drawn through the pole will intersect the Mohr's circle at a point which represents the state of stress on a plane inclined at the same orientation in space as the line.

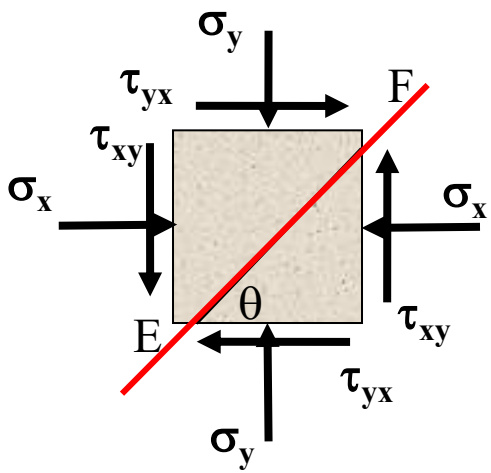
Draw a line parallel to a plane on which you know the stresses, it will intersect the circle in a point (Pole)

Once the pole is known, the stresses on *any plane* can readily be found by simply drawing a line from the pole parallel to that plane; the coordinates of the point of intersection with the Mohr circle determine the stresses on that plane.

Pole Method for Finding Stresses on a Plane

How to determine the location of the Pole?





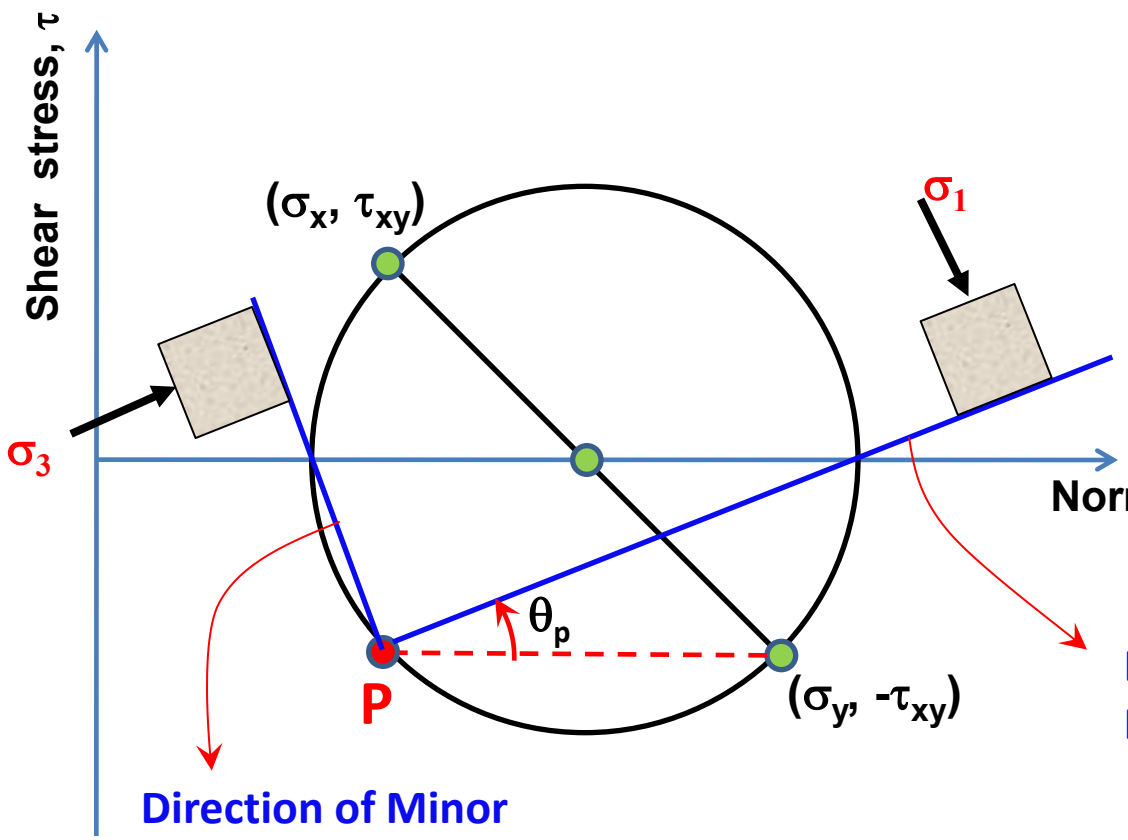
Note: it is assumed that $\sigma_y > \sigma_x$

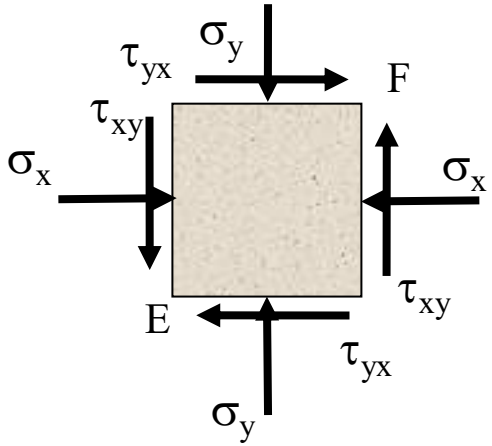
1. From a point of known stress coordinates and plane orientation, draw a line parallel to the plane where the stress is acting on.

2. The line intersecting the Mohr circle is the pole, P.

Normal and Shear Stress along a Plane

Using the Pole to Determine Principal Planes





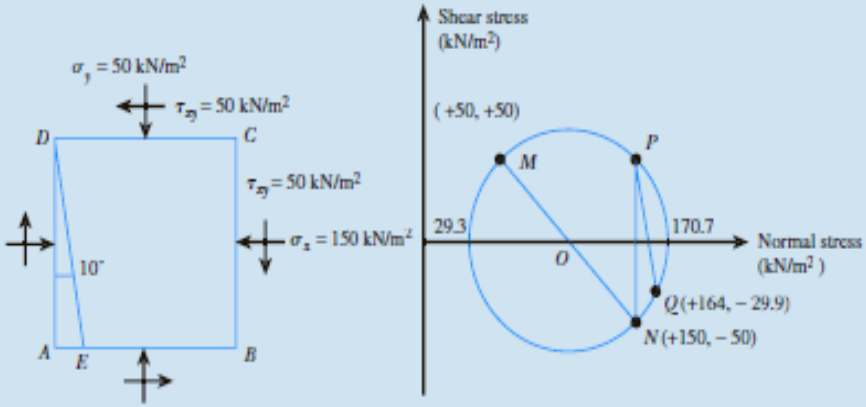
Example 10.2

Example 10.2

For the stressed soil element shown in Figure 10.6a, determine

a. Major principal stress
b. Minor principal stress
c. Normal and shear stresses on the plane DE

Use the pole method.



(a) (b)

Figure 10.6 (a) Stressed soil element; (b) Mohr's circle for the soil element

Solution

Part a

From Eqs. (10.6) and (10.7),

$$\left.\begin{matrix}\sigma_3\\ \sigma_1\end{matrix}\right\} = \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2}$$

$$= \frac{300 + 120}{2} \pm \sqrt{\left[\frac{300 - 120}{2}\right]^2 + (-40)^2}$$

$$\sigma_1 = \mathbf{308.5\ kN/m^2}$$

$$\sigma_3 = \mathbf{111.5\ kN/m^2}$$

Part b

From Eq. (10.3),

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau \sin 2\theta$$

$$= \frac{300 + 120}{2} + \frac{300 - 120}{2}\cos(2 \times 20) + (-40)\sin(2 \times 20)$$

$$= \mathbf{253.23\ kN/m^2}$$

From Eq. (10.4),

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau \cos 2\theta$$

$$= \frac{300 - 120}{2}\sin(2 \times 20) - (-40)\cos(2 \times 20)$$

$$= \mathbf{88.40\ kN/m^2}$$

Example

For the stresses of the element shown across, determine the normal stress and the shear stress on the plane inclined at α = 35° from the horizontal reference plane.

Solution

- **Center of circle** $= \frac{\sigma_1+\sigma_3}{2} = \frac{52+12}{2} = 32$ **kPa**
- **Radius of circle** $= \frac{\sigma_1-\sigma_3}{2} = \frac{52-12}{2} = 20$ **kPa**

- **Plot the Mohr circle to some convenient scale (See the figure across).**
- **Establish the POLE**
- **Draw a line through the POLE inclined at angle α = 35° from the horizontal plane it intersects the Mohr circle at point C.**

σ_α = 39 kPa

τ_α = 18.6 kPa

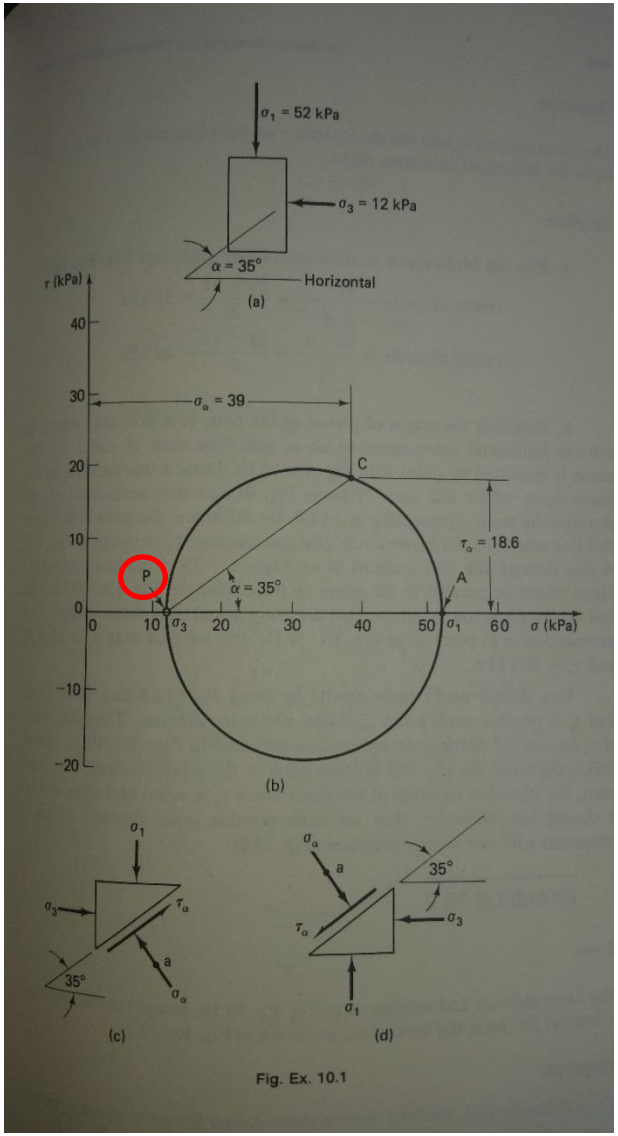


Fig. Ex. 10.1

Example

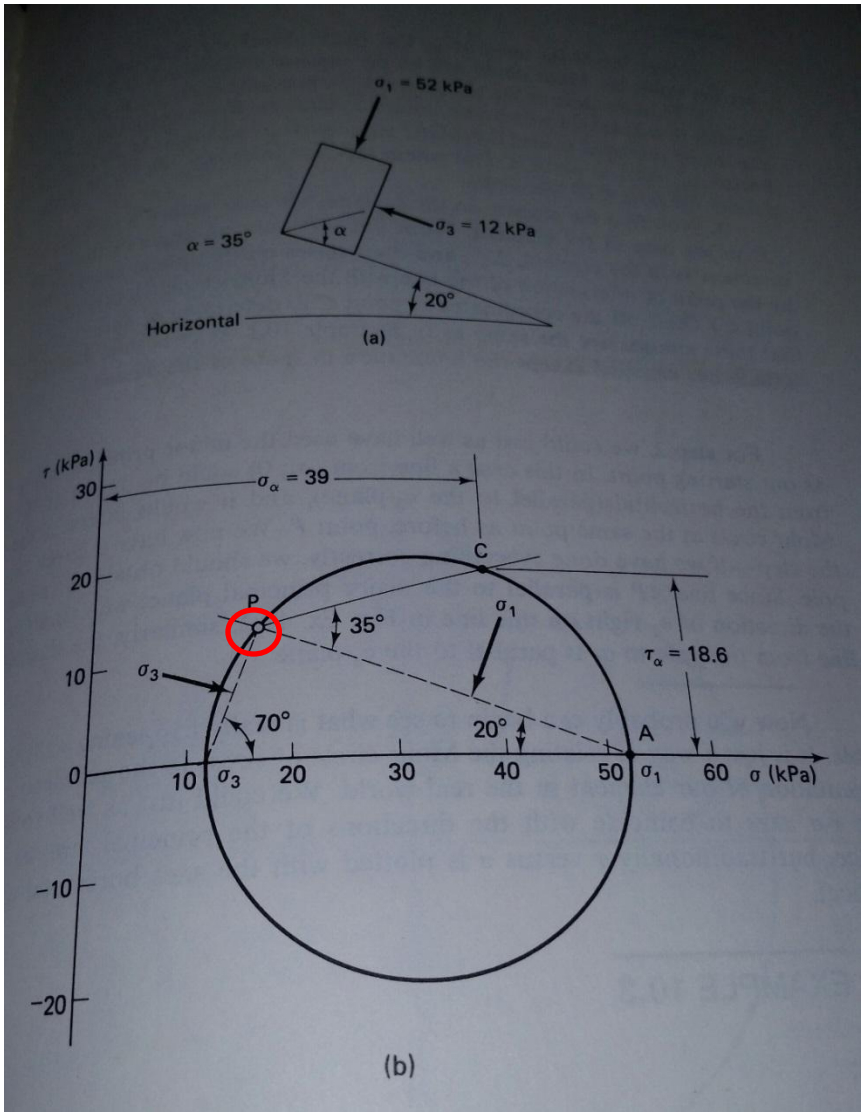
The same element and stresses as in Example 2 except that the element is rotated 20° from the horizontal as shown.

Solution

- **Since the principal stresses are the same, the Mohr circle will be the same as in Example 2.**
- **Establish the POLE.**
- **Draw a line through the POLE inclined at angle $\alpha = 35°$ from the plane of major principal stress. It intersects the Mohr circle at point C.**
- **The coordinates of point C yields**

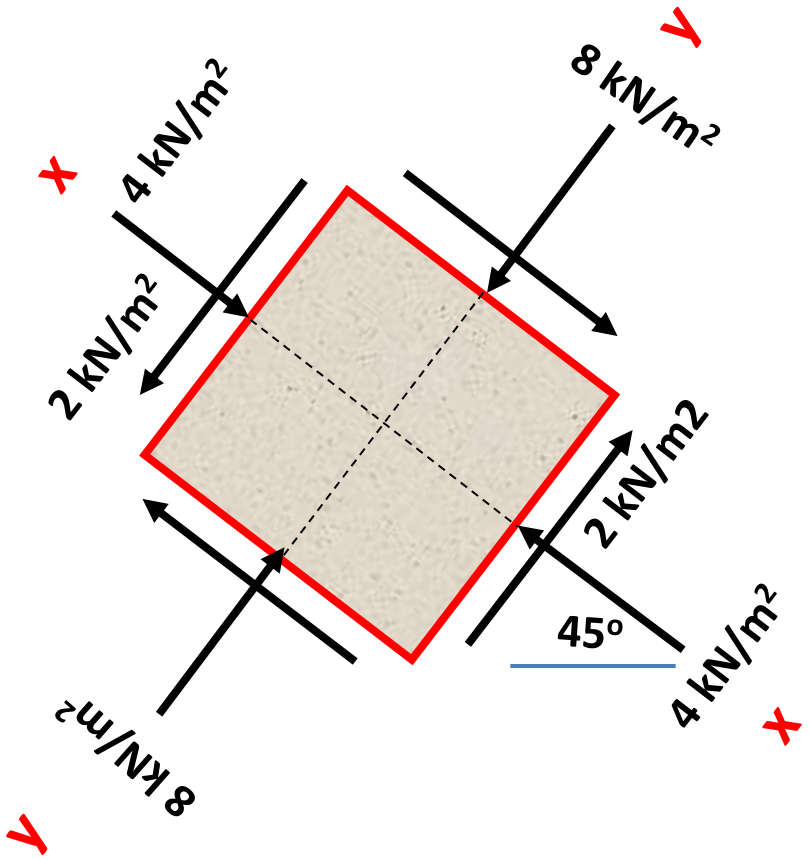
$$\sigma_\alpha = 39 \text{ kPa}$$

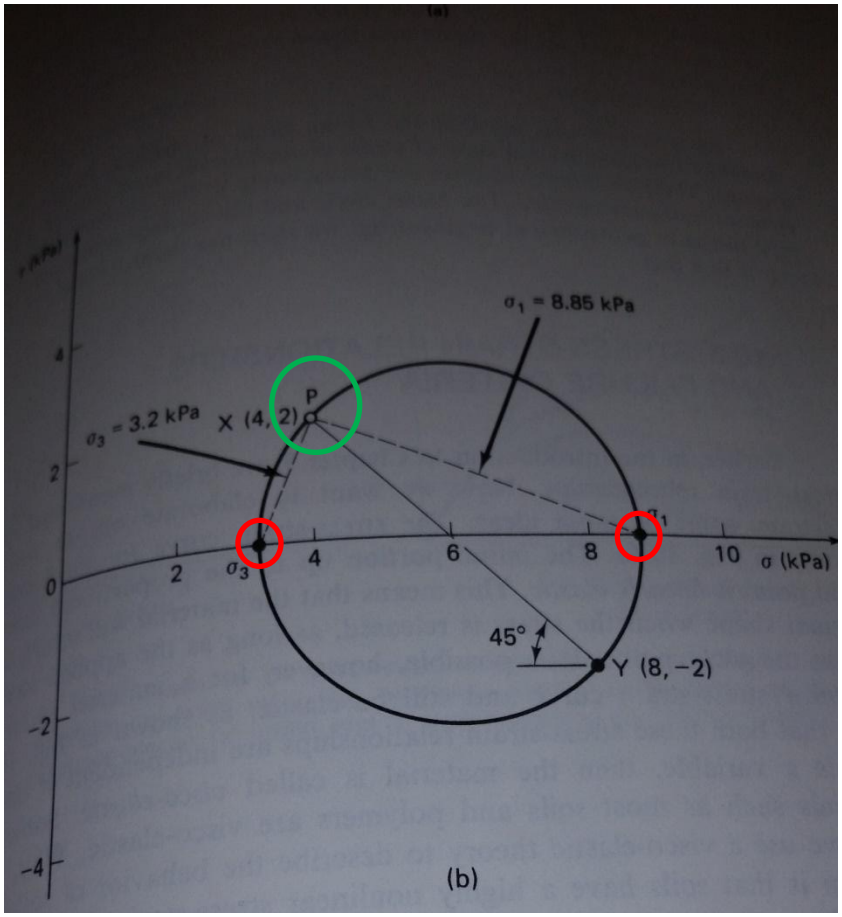
$$\tau_\alpha = 18.6 \text{ kPa}$$



Example

Given the stress shown on the element across. Find the magnitude and direction of the major and minor principal stresses.

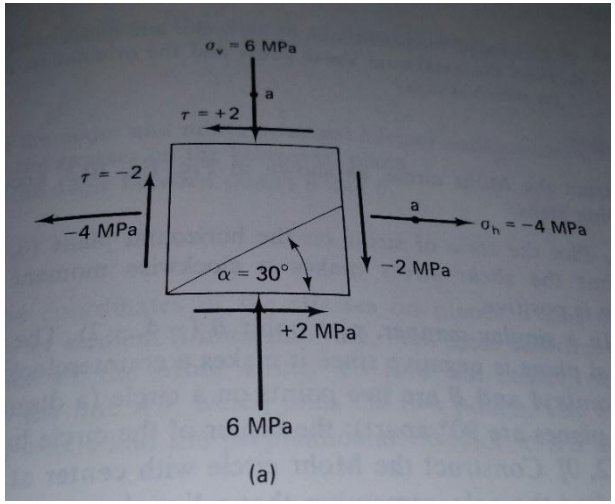


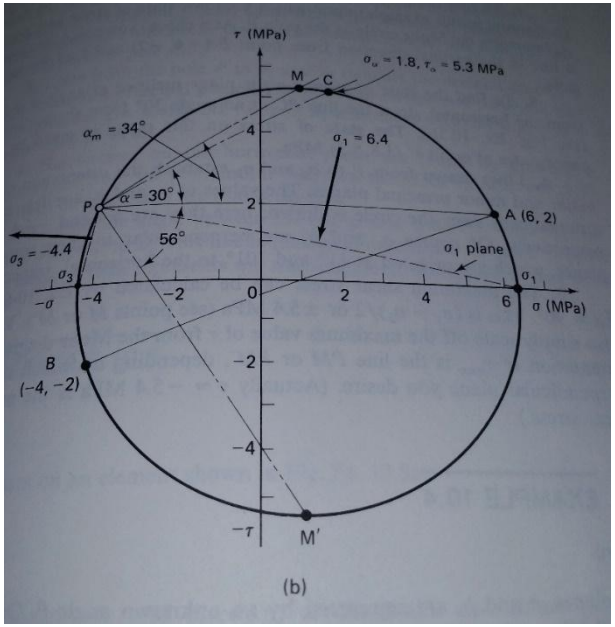


Example

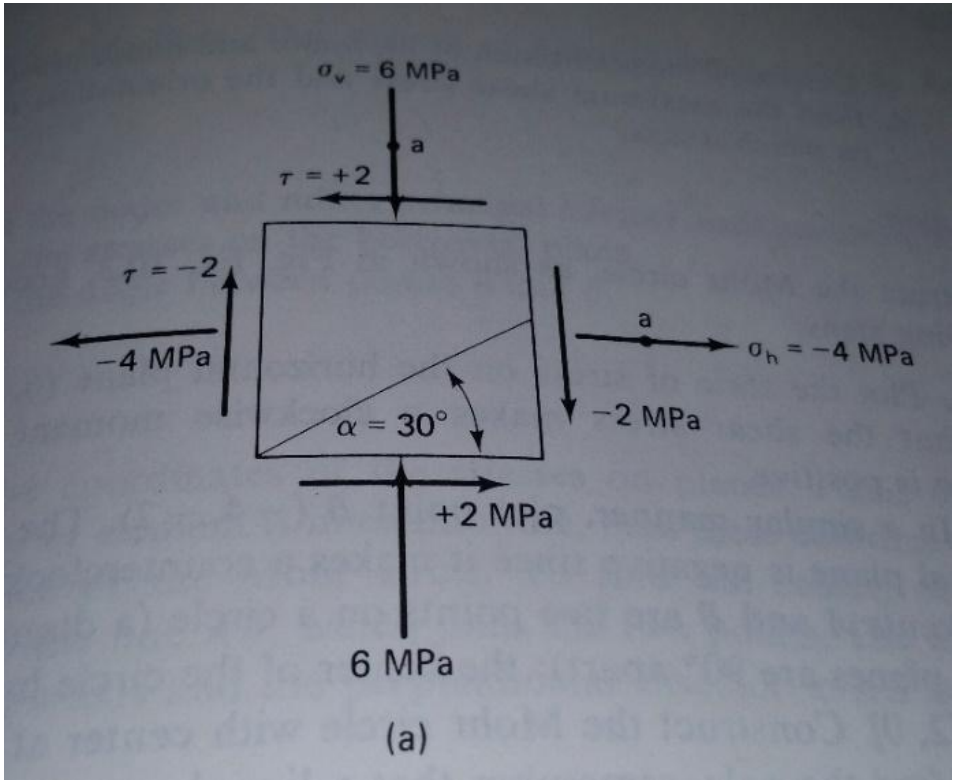
Given the stress shown on the element across. Required:

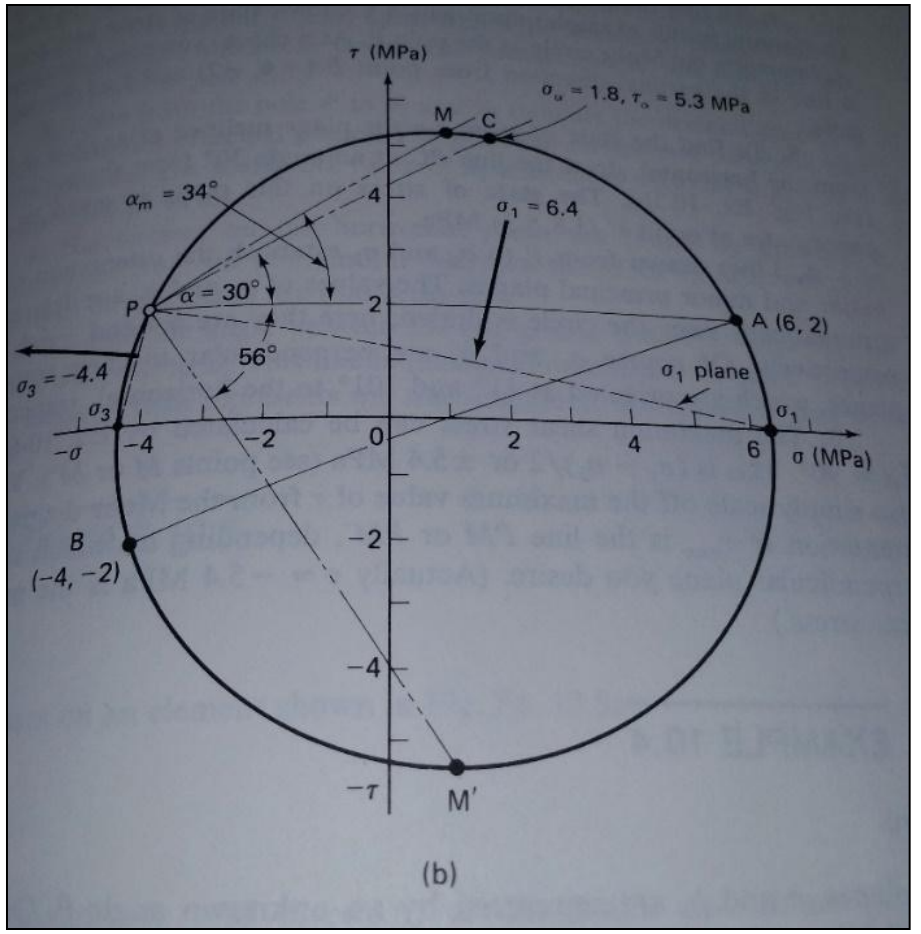
a. Evaluate σ_α and τ_α when $\alpha = 30^o$.
b. Evaluate σ_1 and σ_3.
c. Determine the orientation of the major and minor principal planes.
d. Determine the maximum shear stress and the orientation of the plane on which it acts.





Example


σ_v = 6 MPa
a
τ = +2
τ = −2
−4 MPa
a
σ_h = −4 MPa
−2 MPa
α = 30°
+2 MPa
6 MPa
(a)


τ (MPa)
σ_a = 1.8, τ_a = 5.3 MPa
M
C
α_m = 34°
σ_1 = 6.4
P
α = 30°
A (6, 2)
56°
σ_1 plane
σ_3 = −4.4
σ_3
σ_1
−σ
−4
−2
0
2
4
6
σ (MPa)
B
(−4, −2)
4
2
−2
−4
−τ
M′
(b)

CHAPTER 12

SHEAR STRENGTH OF SOIL

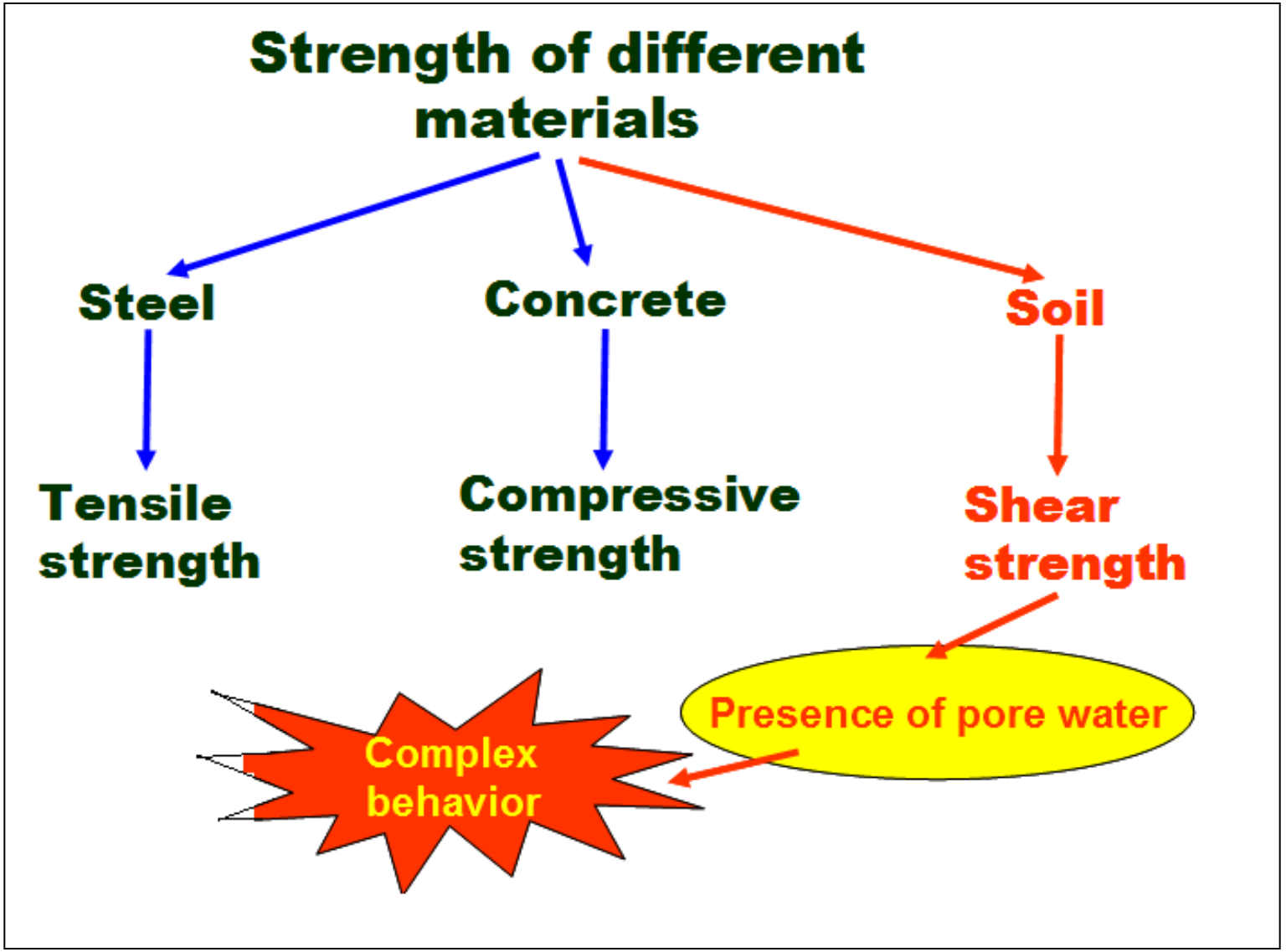
Omitted Sections

12.14
12.15
12.18

SHEAR STRENGTH OF SOIL

- Soil failure usually occurs in the form of "shearing" along internal surface within the soil.
- The *shear strength* of a soil mass is the internal resistance per unit area that the soil mass can offer to resist failure and sliding along any plane inside it.
- The safety of any geotechnical structure is dependent on the strength of the soil.
- Shear strength determination is a very important aspect in geotechnical engineering. Understanding shear strength is the basis to analyze soil stability problems like:
  - Bearing capacity.
  - Lateral pressure on earth retaining structures
  - Slope stability

STRENGTH OF MATERIALS



SHEAR STRENGTH OF SOIL

- Coulomb (1776) observed that there was a stress-dependent component of shear strength and a stress-independent component.
- The stress-dependent component is similar to sliding friction in solids described above. The other component is related to the intrinsic COHESION of the material. Coulomb proposed the following equation for shear strength of soil:

$$\tau_f = C + \sigma_n \tan \emptyset$$

cohesion

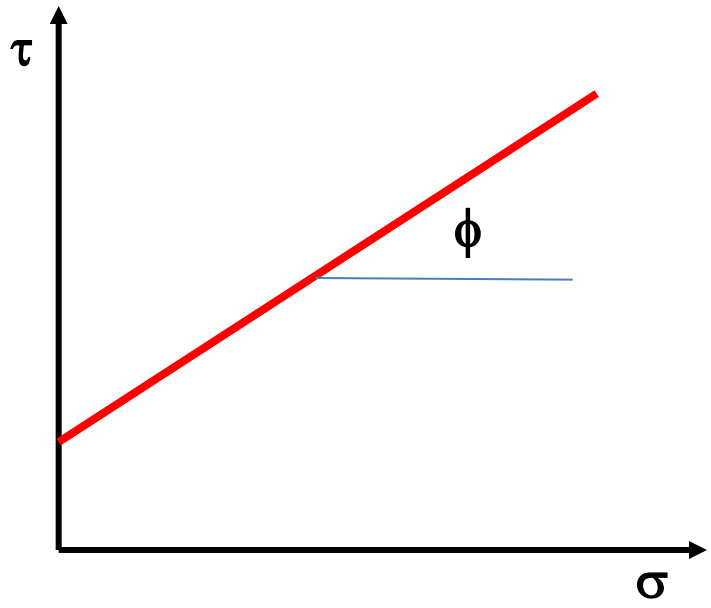
Friction

τ_f = shear strength of soil

σ_n = Applied normal stress

C = Cohesion

ϕ = Angle of internal friction (or angle of shearing resistance)

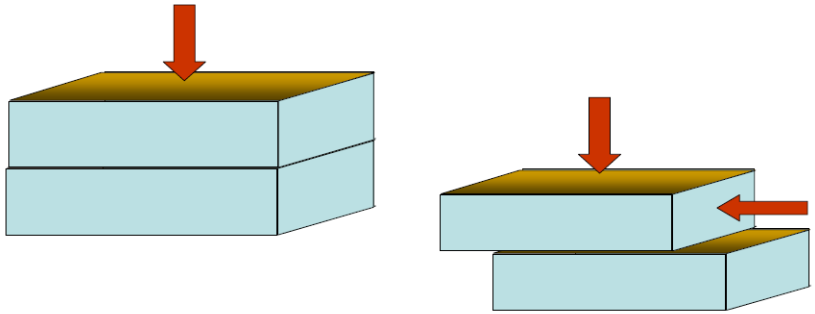


SHEAR STRENGTH OF SOIL

- **Cohesion** (c), is a measure of the forces that cement particles of soils (stress independent).



- **Internal Friction angle (ϕ)**, is a measure of the shear strength of soils due to friction (stress dependent).



- For granular materials, there is no cohesion between particles

$$\tau_f = \sigma_n \tan \emptyset$$

SHEAR STRENGTH OF SOIL

Saturated Soils

$$\tau_f = C' + \sigma'_n \tan \emptyset'$$

But from the principle of effective stress

$$\sigma' = \sigma - u$$

Where u is the pore water pressure (p.w.p.)

Then

$$\tau_f = C' + (\sigma_n - u) \tan \emptyset'$$

- **C , ϕ or C′ , ϕ′ are called strength parameters, and we will discuss various laboratory tests for their determination.**

FAILURE CRITERIA FOR SOILS

- There are many ways of defining failure in real materials, or put another way, there are many failure criteria.
- Various theories are available for different engineering materials. However, no one is general for all materials
- The one generally accepted and used for soil is Mohr Theory of Failure.
- According to Coulomb relation for shear strength

$$\tau_f = C + \sigma_n \tan \emptyset$$

Where σ_n is the normal stress on the failure plane.

From the previous slides we new know how to estimate σ_n but we still need to know the plane of failure.

FAILURE CRITERIA FOR SOILS

- **What is failure Plane?**

•Is it the plane of a maximum shear stress **No**

•Is it the plane of a maximum normal stress

• (major principal stress) **No**

•It is the plane when shear stress reaches some unique function of the normal stress on that plane: $\tau_f = f(\sigma_f)$

•It is the plane , where the failure angle measured

•From the plane of the major principal stress is

$$\theta_f = 45 + \frac{\phi}{2}$$

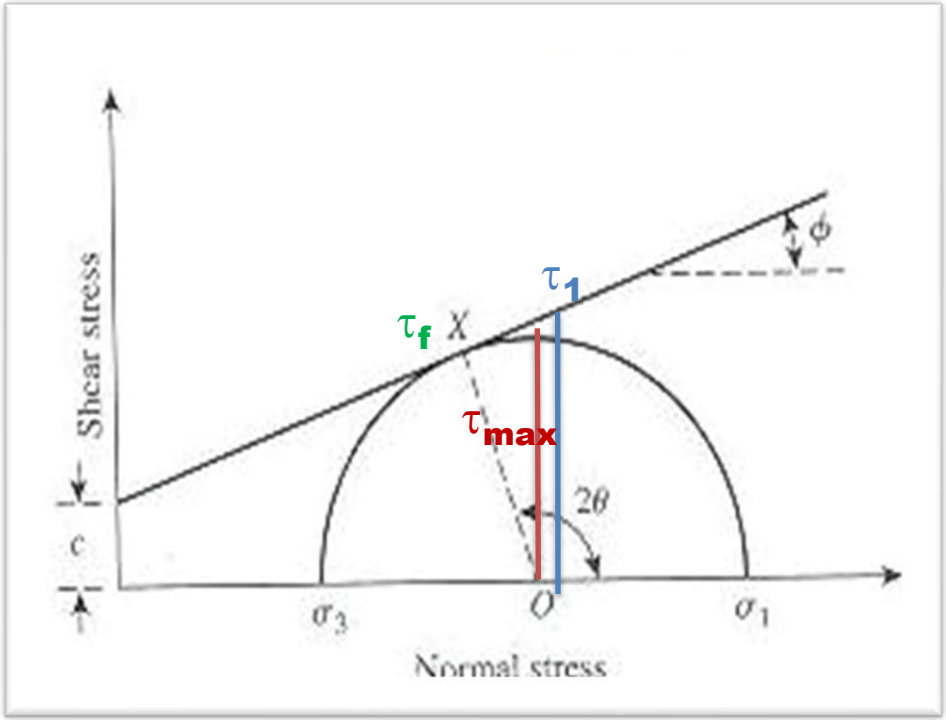
FAILURE CRITERIA FOR SOILS

- **Inclination of failure Plane due to shear**

- **τ_f is not the maximum shear stress ….. Why?**

$$F_s = \frac{\tau_{aviable}}{\tau_{applied}}$$

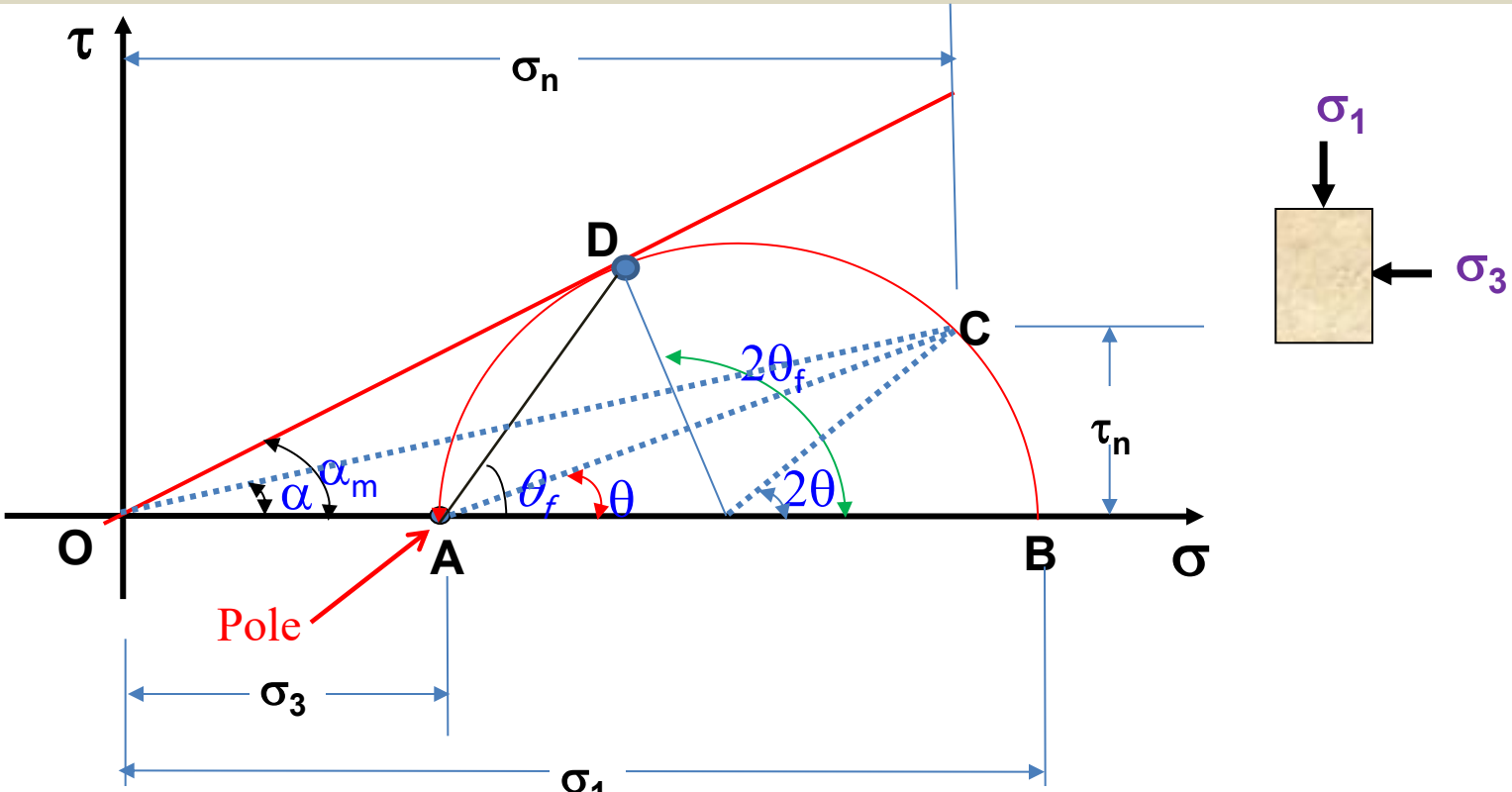
$$F_s = \frac{\tau_1}{\tau_{max}}$$



Mohr Theory of Failure

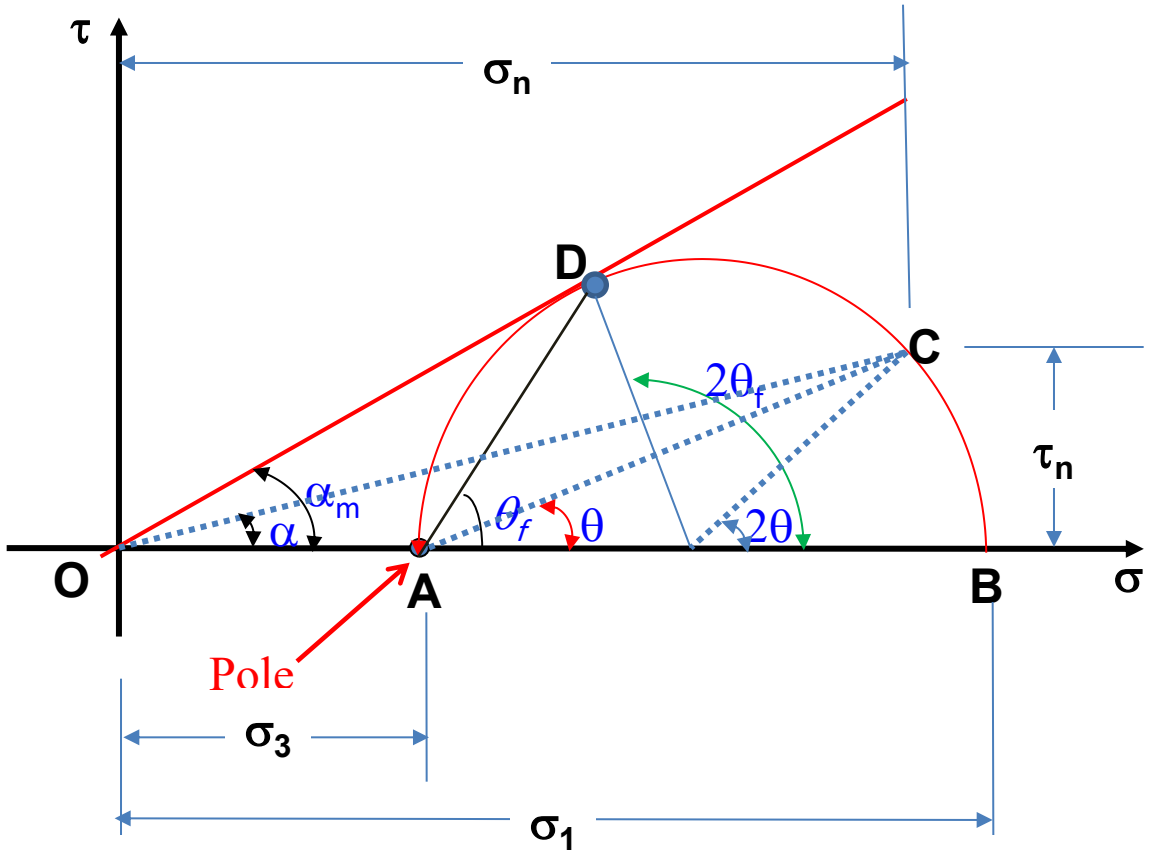
Mohr theory of failure states that:

*A material fails along the plane and at the time at which the angle between the **RESULTANT** of the **NORMAL** and **SHEARING STRESS** and the **NORMAL STRESS** is a maximum; that is , when the combination of NORMAL and SHEARING stresses produces the maximum obliquity angle α_m.*



Mohr Theory of Failure

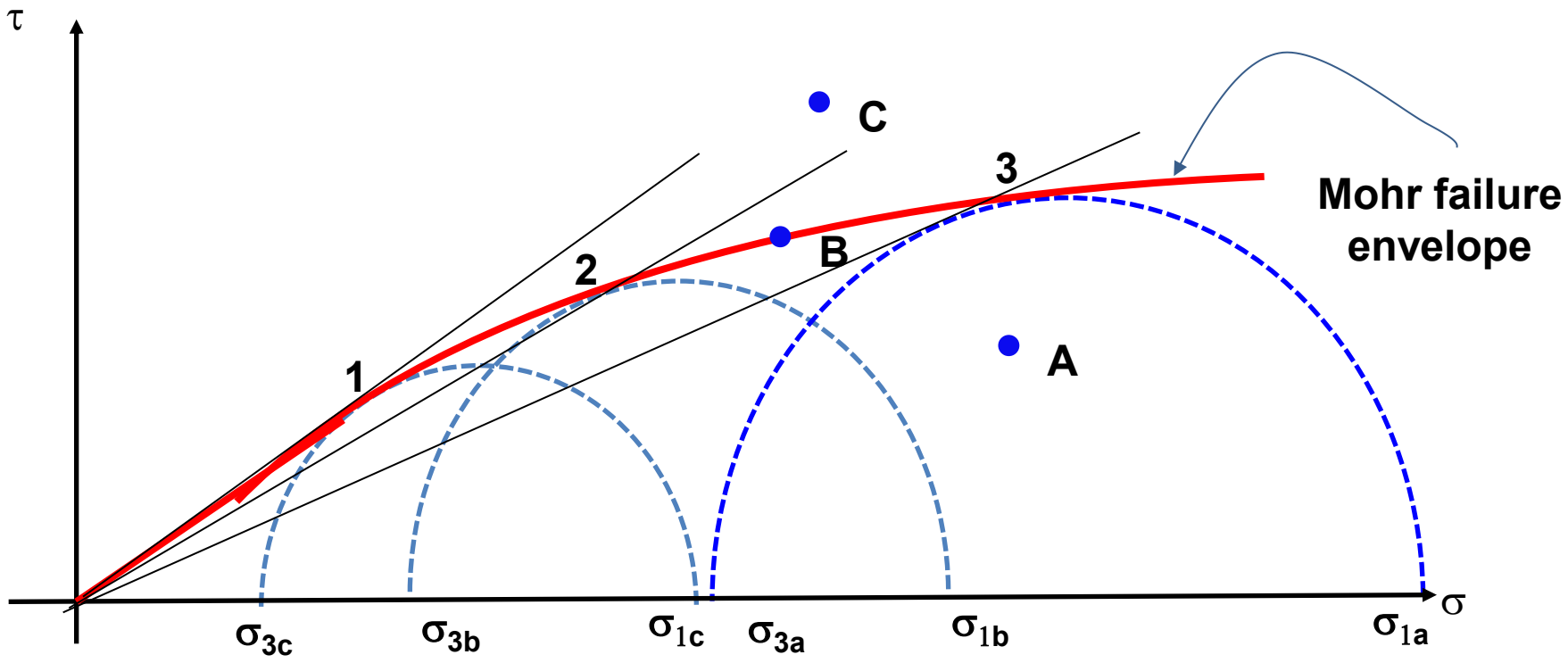
- In the across diagram, it is seen that the optimum stress combination which fulfills Mohr's criterion is that represented by the point D, and the orientation of the failure plane is represented by the line AD which makes an angle θ_f with the maximum principal plane.



- Since, according to the Mohr theory, the tangent line **OD** represents the stress situation at failure, the maximum obliquity angle α_m is equal to the friction angle ϕ, just as indicated in the case of the brick sliding on a horizontal surface.

Mohr Failure Envelope

- By plotting Mohr's circles for different states of stresses and in each case draw a tangent to each circle from the origin we come up with points 1,2,3… etc. If we connect those points we come up with what is called Mohr's failure envelope.



Mohr Failure Envelope

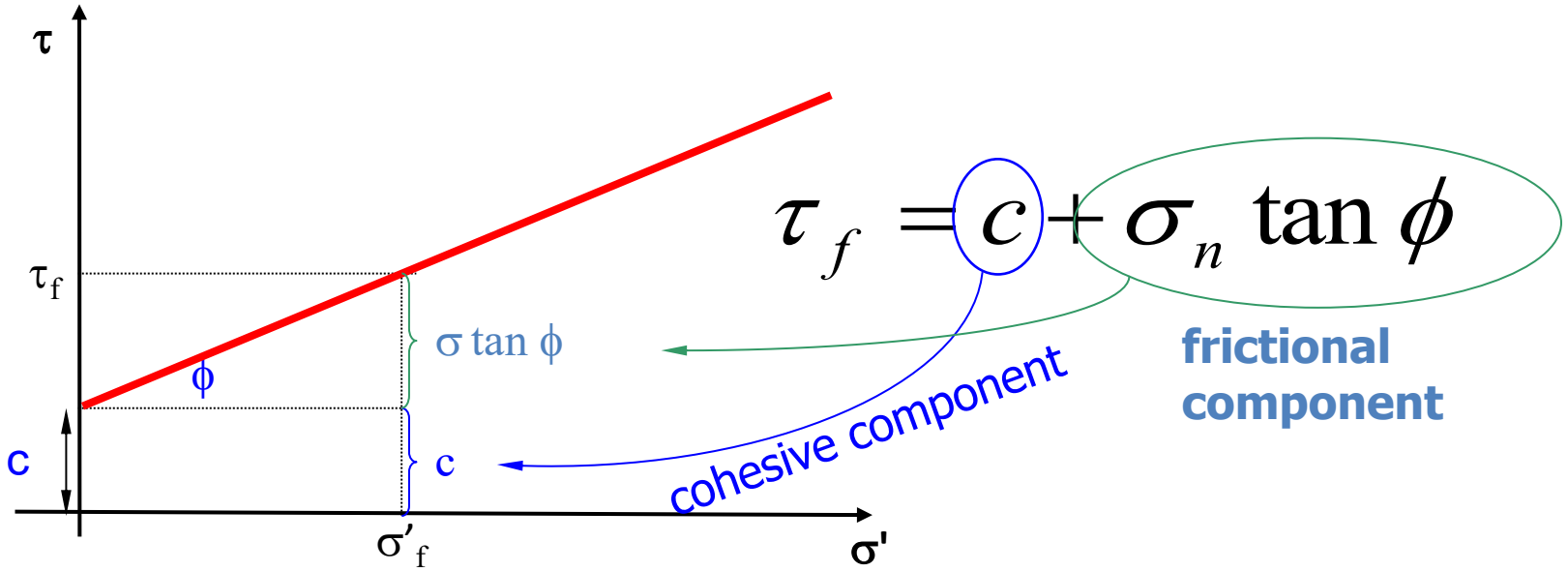
- This envelope separates cases of stresses which cause failure from that which do not.
- For instance if the normal stress and shear stress on a plane in a soil mass are such that they plot as point A , shear failure will not occur along that plane.
- For point B failure takes place, and point C cannot exist, since it plots above failure envelope and shear failure in a soil would have occurred already.
- Therefore, failure occurs only when the combination of shear and normal stress is such that the Mohr circle is TANGENT to the Mohr failure envelope.
- Then once the point of the tangency is determined the angle of failure plane and the stresses $(\sigma_n, \tau)_f$ can be determined using the POLE method.

Mohr-Coulomb Failure Criterion

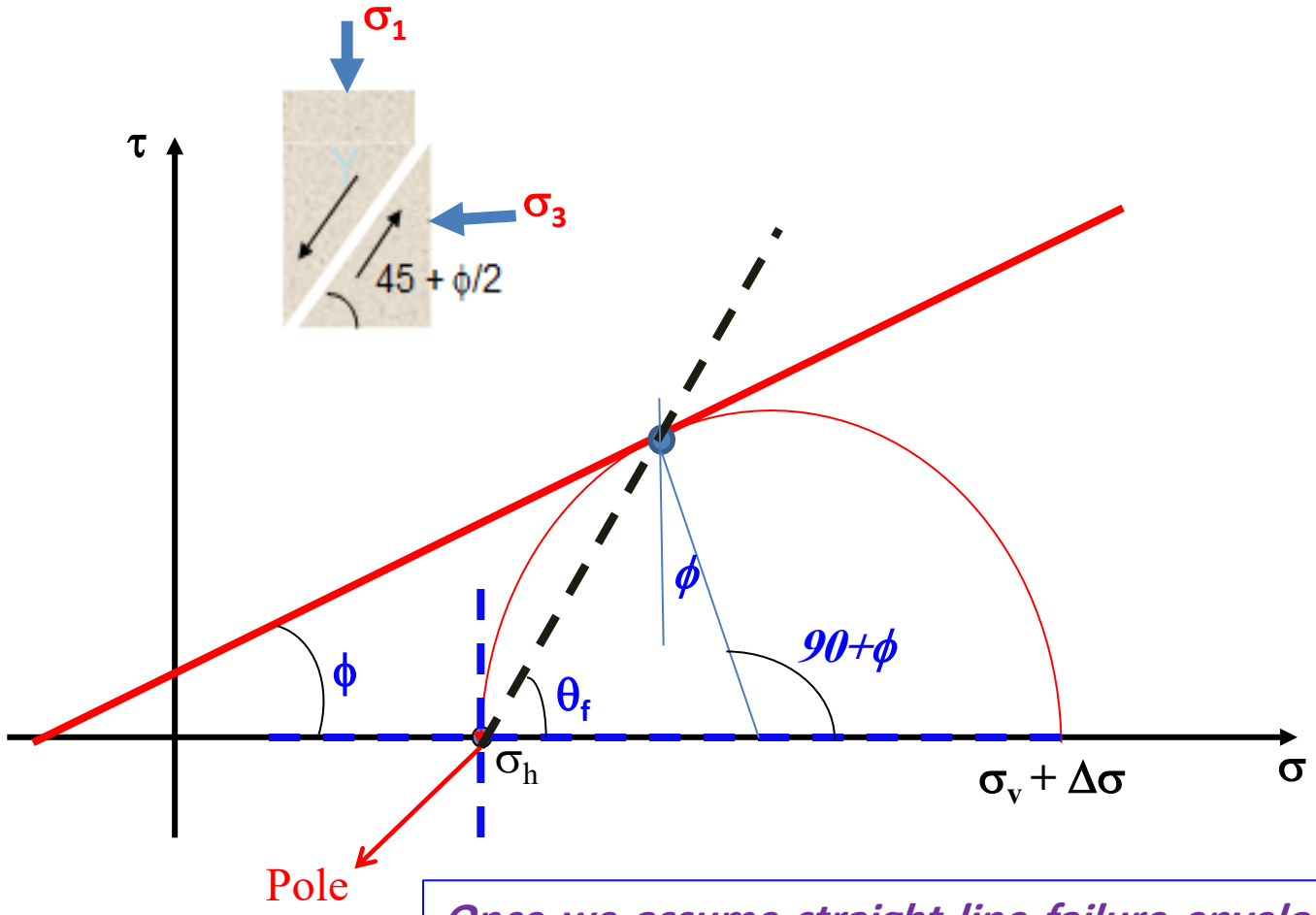
- The disadvantages of **Mohr failure envelope** is that it is a **curved** line, and needs a lot of tests to construct and difficult to use.
- It was then approximated to be a straight line, and the equation for the line was written in terms of the Coulomb strength parameters C and ϕ , as

$$\tau_f = C + \sigma_n \tan\emptyset$$

- This gave the birth to **MOHR-COULOMB FAILURE CRITERION**, which is by far the most popular criterion applied to soils.



Orientation of Failure Plane



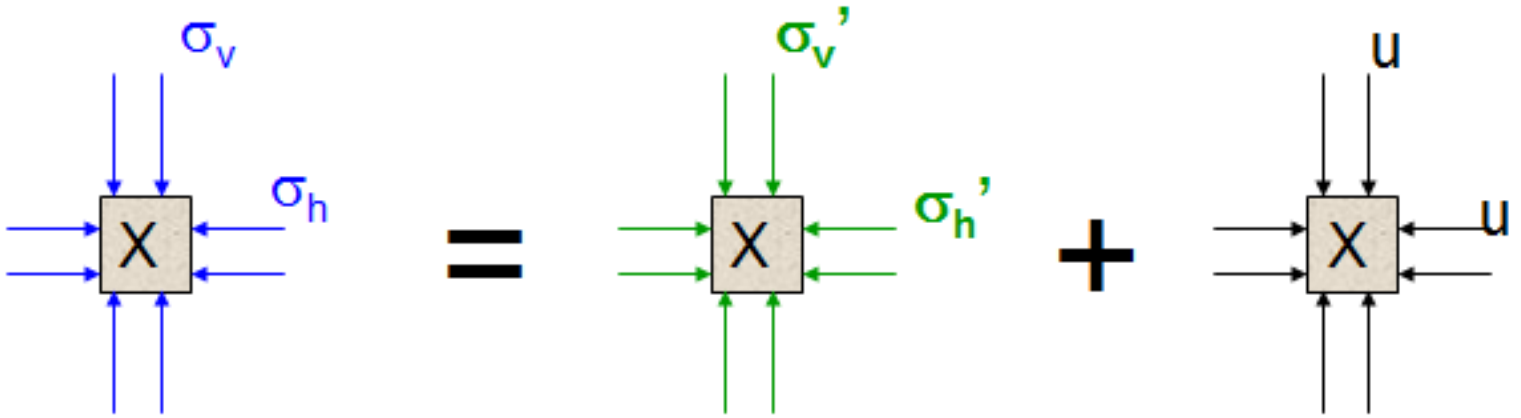
$2\theta_f = 90 + \phi$

$\theta_f = 45 + \phi/2$

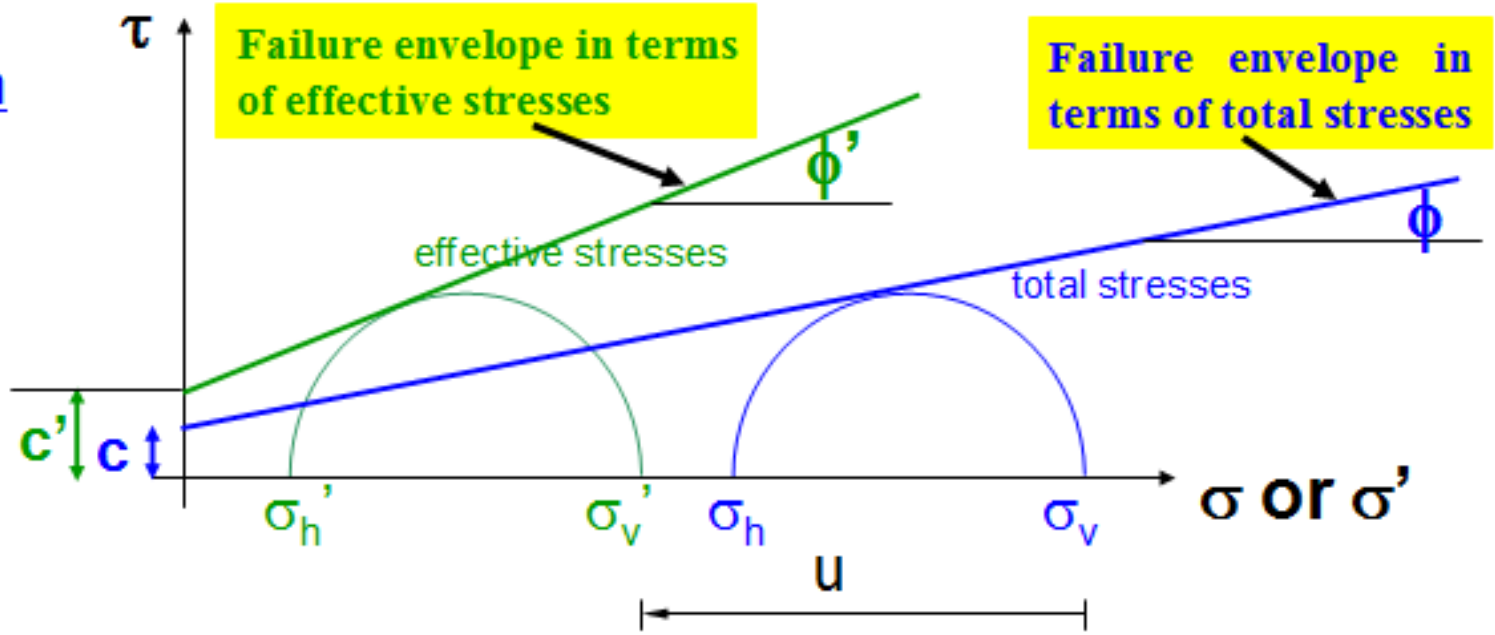
This could be proved analytically (See Das)

Once we assume straight line failure envelope, $\theta_f = 45 + \phi/2$ always and independent of the values of σ_1 and σ_3 (i.e. confinement).

Mohr circles & failure envelope in terms of total and effective stress



If X is on failure

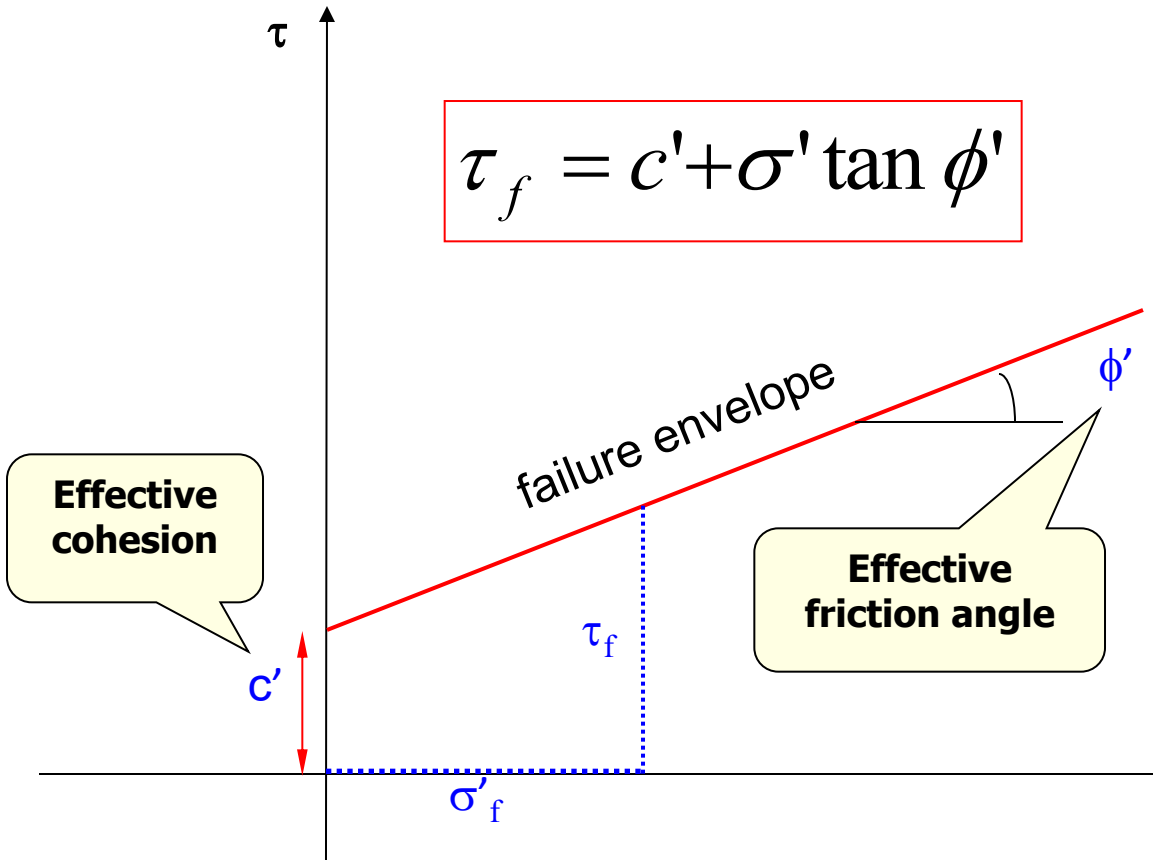


Mohr-Coulomb Failure Criterion in terms of effective stress

$$\tau_f = c' + \sigma' \tan \phi'$$

$$\sigma' = \sigma - u$$

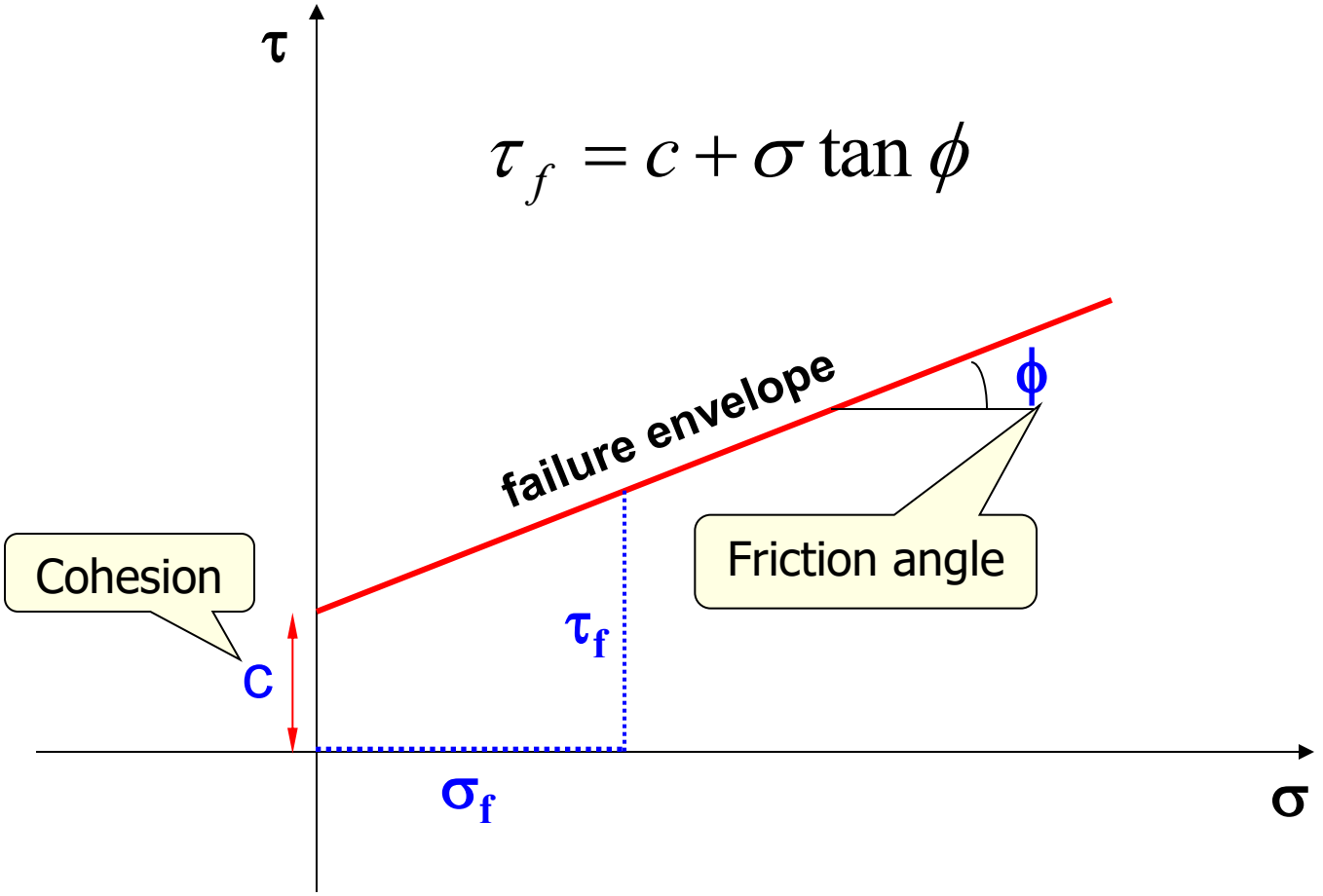
u = pore water pressure



C′ and ϕ' are called the effective strength parameters.

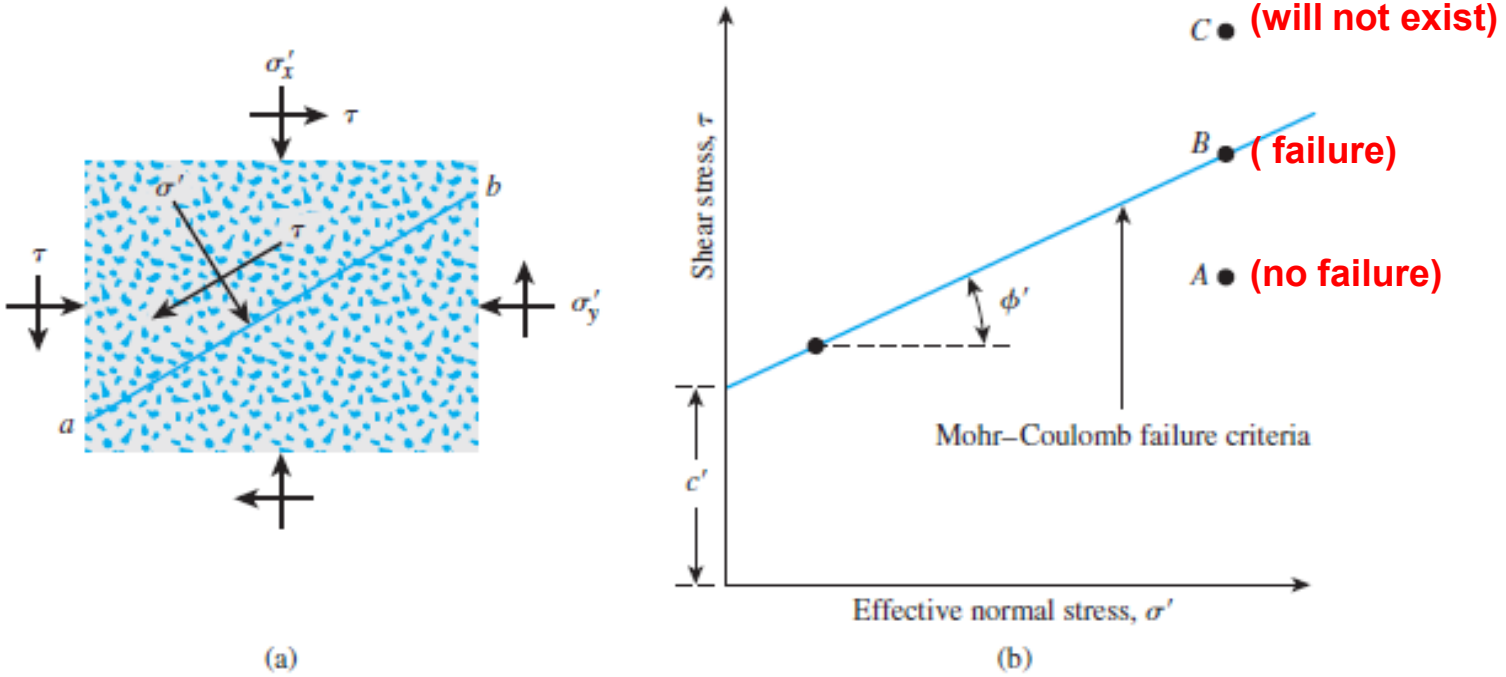
In this case, soil behavior is controlled by effective stresses, and the effective strength parameters are the fundamental strength parameters.

Mohr-Coulomb Failure Criterion in terms of total stress



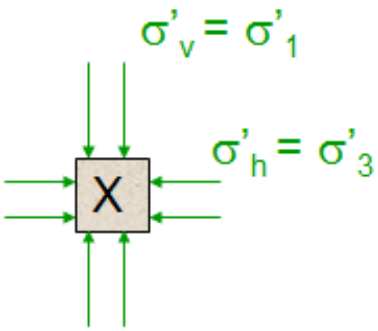
τ_f is the maximum shear stress the soil can take without failure, under normal stress of σ.

What does the Failure Envelope Signify?

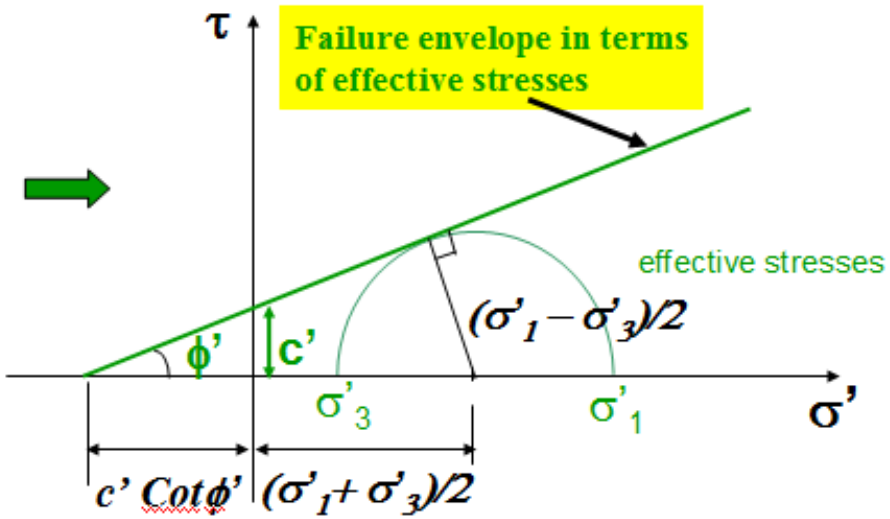


- If the magnitudes of σ and τ on plane *ab* are such that they plot as point *A* shear failure will not occur along the plane.
- If the effective normal stress and the shear stress on plane *ab* plot as point B (which falls on the failure envelope), shear failure will occur along that plane.
- A state of stress on a plane represented by point C cannot exist, because it plots above the failure envelope, and shear failure in a soil would have occurred already.

Relationship between principle stresses and shear strength parameters



X is on failure



Therefore,

$$\left[c'Cot\phi' + \left(\frac{\sigma_1' + \sigma_3'}{2}\right)\right]Sin\phi' = \left(\frac{\sigma_1' - \sigma_3'}{2}\right)$$

$$\sigma_1' = \sigma_3' Tan^2\left(45 + \frac{\phi'}{2}\right) + 2c'Tan\left(45 + \frac{\phi'}{2}\right)$$

REMARKS

- **C and ϕ are measures of shear strength and are called shear strength parameters.**
- **The parameters C, ϕ are in general not soil constants. They depend on the initial state of the soil (e.g. density, water content), and type of loading (drained or undrained).**
- **In case of using effective stress, C' and ϕ' are called effective shear strength parameters.**
- **The value of C' for sand and inorganic silt is 0. For normally consolidated clays, C' can be approximated at 0. Overconsolidated clays have values of C' that are greater than 0.**