SHEAR STRENGTH OF SOIL

CHAPTER 12

FAILURE CRITERIA FOR SOILS

- There are many ways of defining failure in real materials, or put another way, there are many failure criteria.
- Various theories are available for different engineering materials. However, no one is general for all materials
- The one generally accepted and used for soil is Mohr Theory of Failure.
- According to Coulomb relation for shear strength

$$\boxed{\tau_f = C + \sigma_n \tan \emptyset}$$

Where σ_n is the normal stress on the failure plane.

From the previous slides we new know how to estimate σ_n but we still need to know the plane of failure.

FAILURE CRITERIA FOR SOILS

- **What is failure Plane?**
- **Is it the plane of a maximum shear stress** **No**
- **Is it the plane of a maximum normal stress**
- **(major principal stress)** **No**
- **It is the plane when shear stress reaches some unique function of the normal stress on that plane:** $\tau_f = f(\sigma_f)$

- **It is the plane , where the failure angle measured**
- **From the plane of the major principal stress is**

$$\theta_f = 45 + \frac{\phi}{2}$$

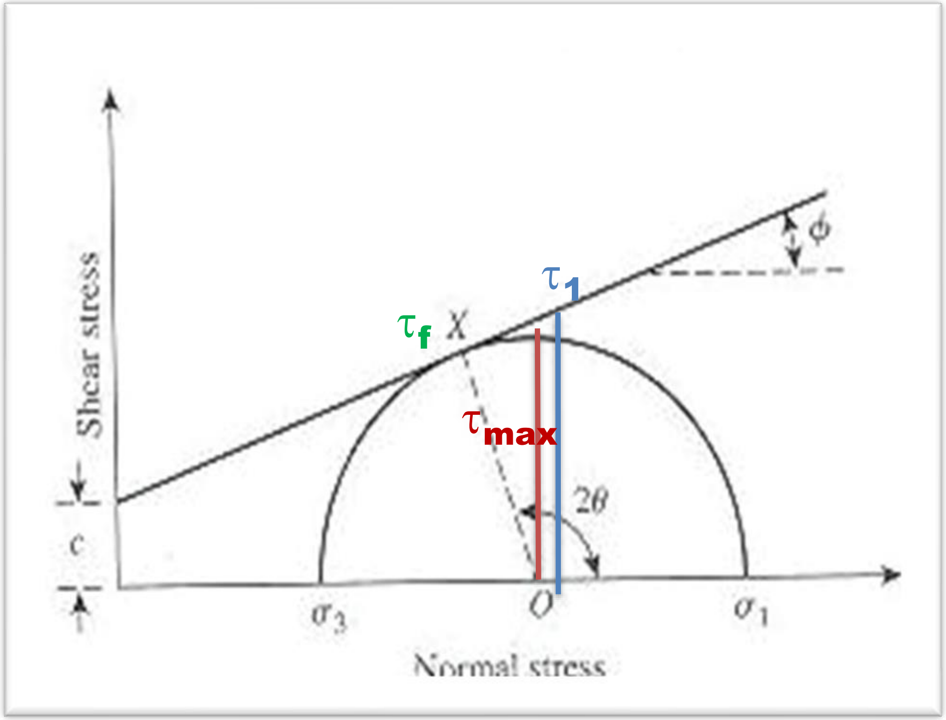
FAILURE CRITERIA FOR SOILS

- **Inclination of failure Plane due to shear**

- τ_f **is not the maximum shear stress ….. Why?**

$$F_s = \frac{\tau aviable}{\tau applied}$$

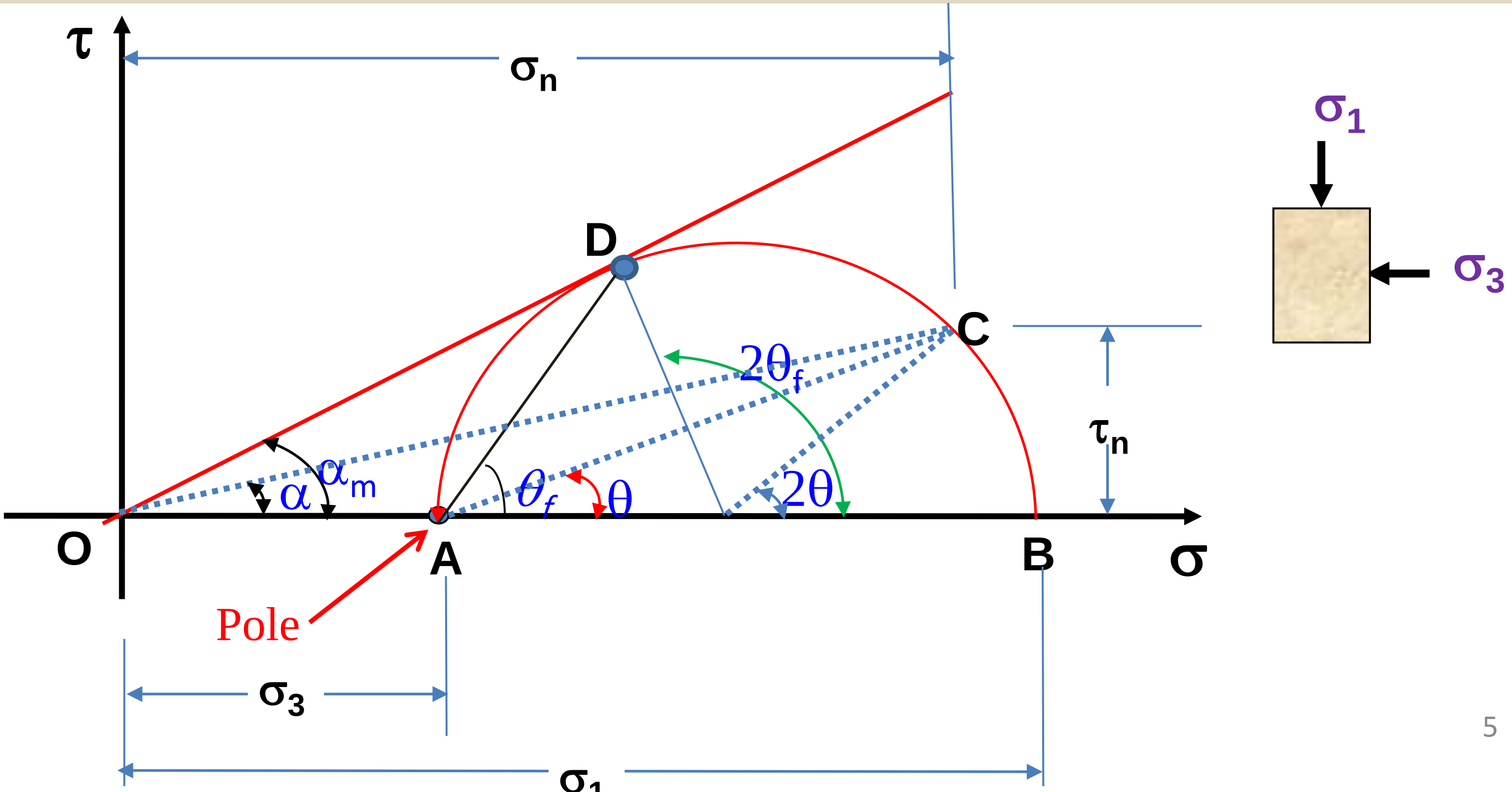
$$F_s = \frac{\tau 1}{\tau \max}$$



Mohr Theory of Failure

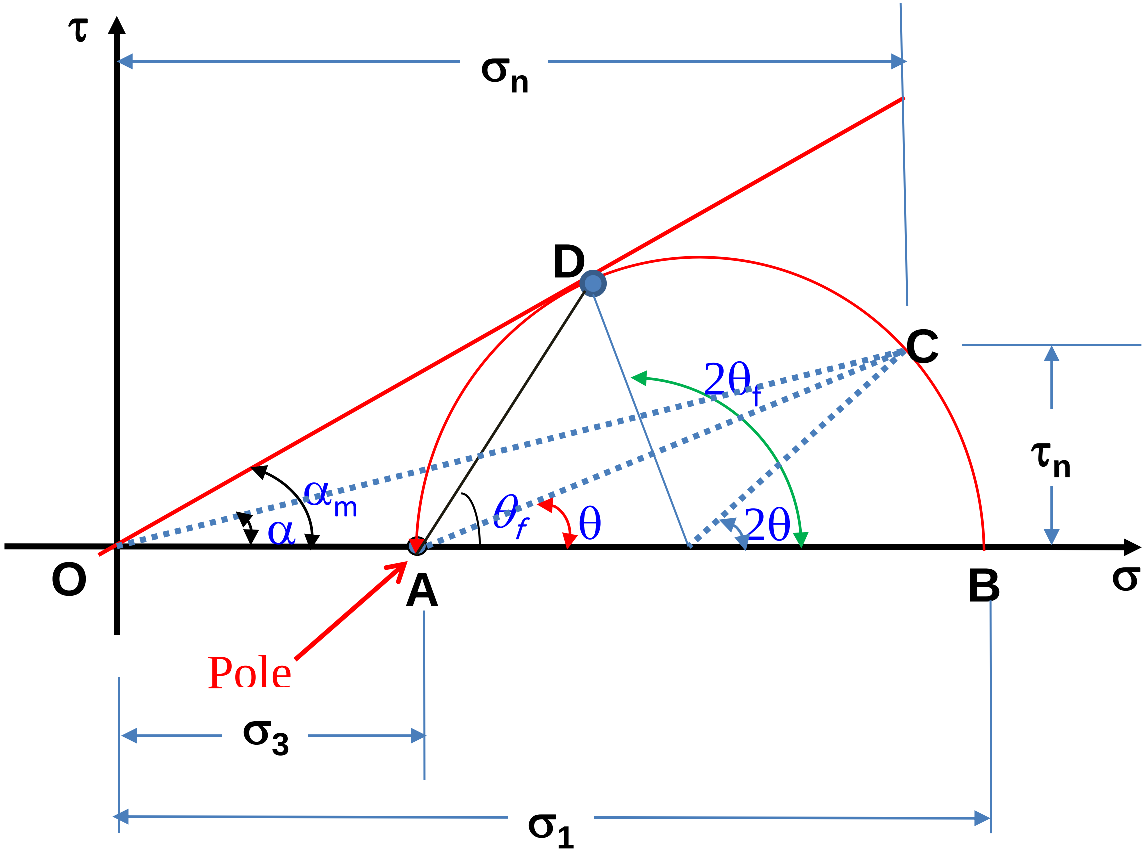
Mohr theory of failure states that:

> *A material fails along the plane and at the time at which the angle between the RESULTANT of the NORMAL and SHEARING STRESS and the NORMAL STRESS is a maximum; that is , when the combination of NORMAL and SHEARING stresses produces the maximum obliquity angle α_m.*



Mohr Theory of Failure

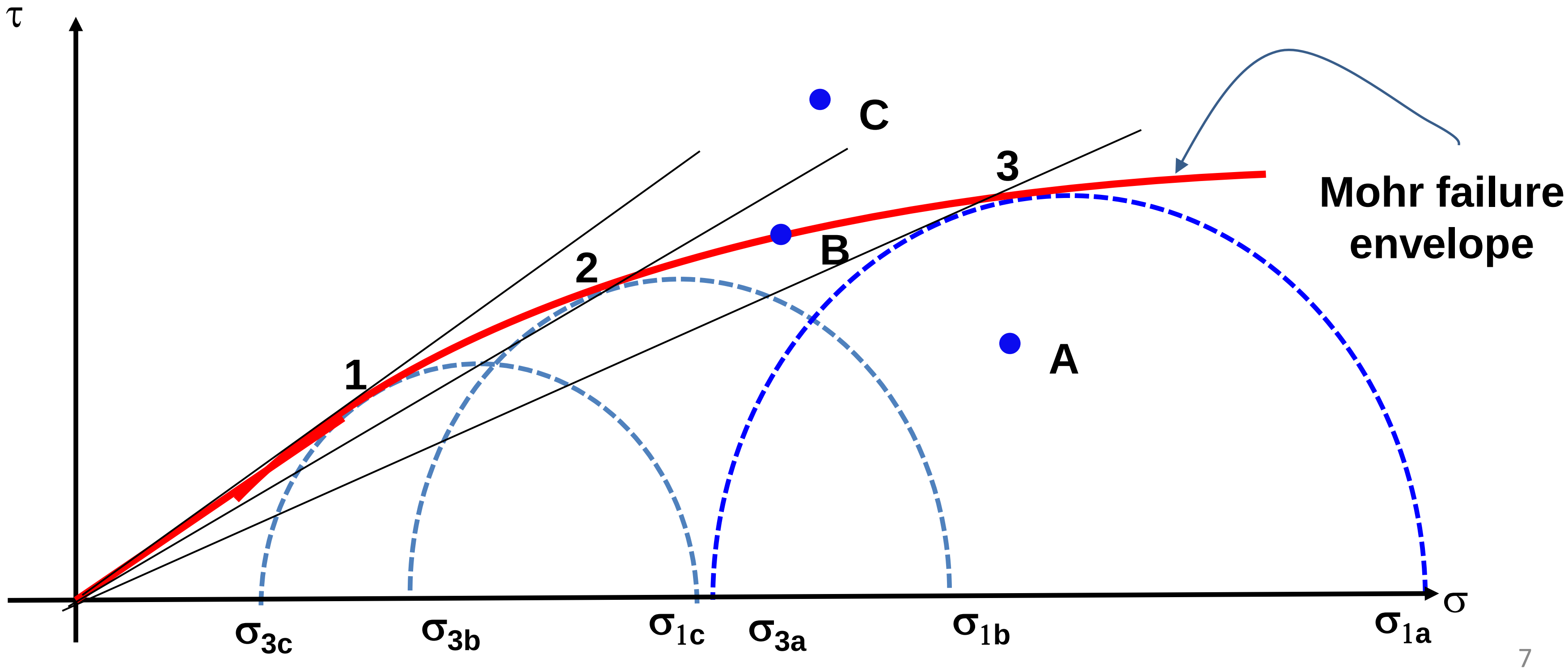
- In the across diagram, it is seen that the optimum stress combination which fulfills Mohr's criterion is that represented by the point D, and the orientation of the failure plane is represented by the line AD which makes an angle θ_f with the maximum principal plane.



- Since, according to the Mohr theory, the tangent line **OD** represents the stress situation at failure, the maximum obliquity angle α_m is equal to the friction angle ϕ, just as indicated in the case of the brick sliding on a horizontal surface.

Mohr Failure Envelope

- By plotting Mohr's circles for different states of stresses and in each case draw a tangent to each circle from the origin we come up with points 1,2,3… etc. If we connect those points we come up with what is called Mohr's failure envelope.



Mohr Failure Envelope

- This envelope separates cases of stresses which cause failure from that which do not.

- For instance if the normal stress and shear stress on a plane in a soil mass are such that they plot as point **A** , shear failure **will not occur** along that plane.

- For point **B** failure takes place, and point **C** cannot exist, since it plots above failure envelope and shear failure in a soil would have occurred already.

- Therefore, failure occurs only when the combination of shear and normal stress is such that the Mohr circle is **TANGENT** to the Mohr failure envelope.

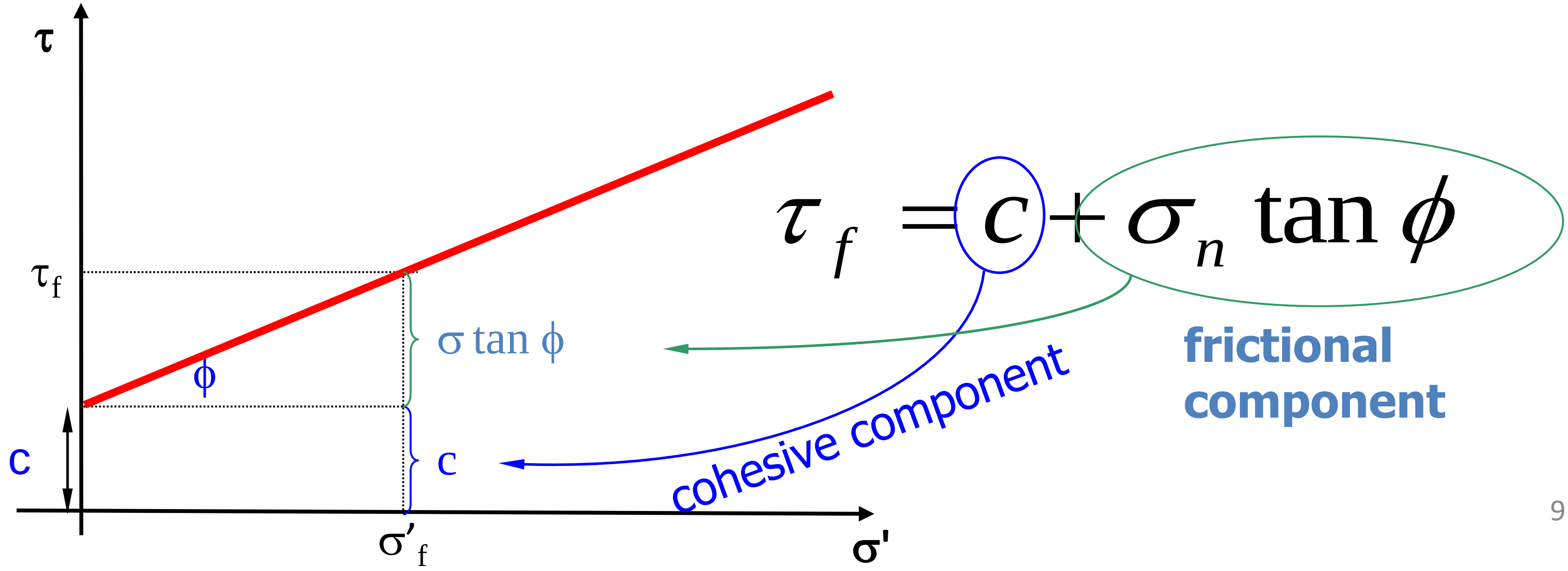
- Then once the point of the tangency is determined the angle of failure plane and the stresses $(\sigma_n, \tau)_f$ can be determined using the **POLE** method.

Mohr-Coulomb Failure Criterion

- The disadvantages of Mohr failure envelope is that it is a curved line, and needs a lot of tests to construct and difficult to use.
- It was then approximated to be a straight line, and the equation for the line was written in terms of the Coulomb strength parameters C and ϕ , as

$$\tau_f = C + \sigma_n \tan\emptyset$$

- This gave the birth to **MOHR-COULOMB FAILURE CRITERION**, which is by far the most popular criterion applied to soils.

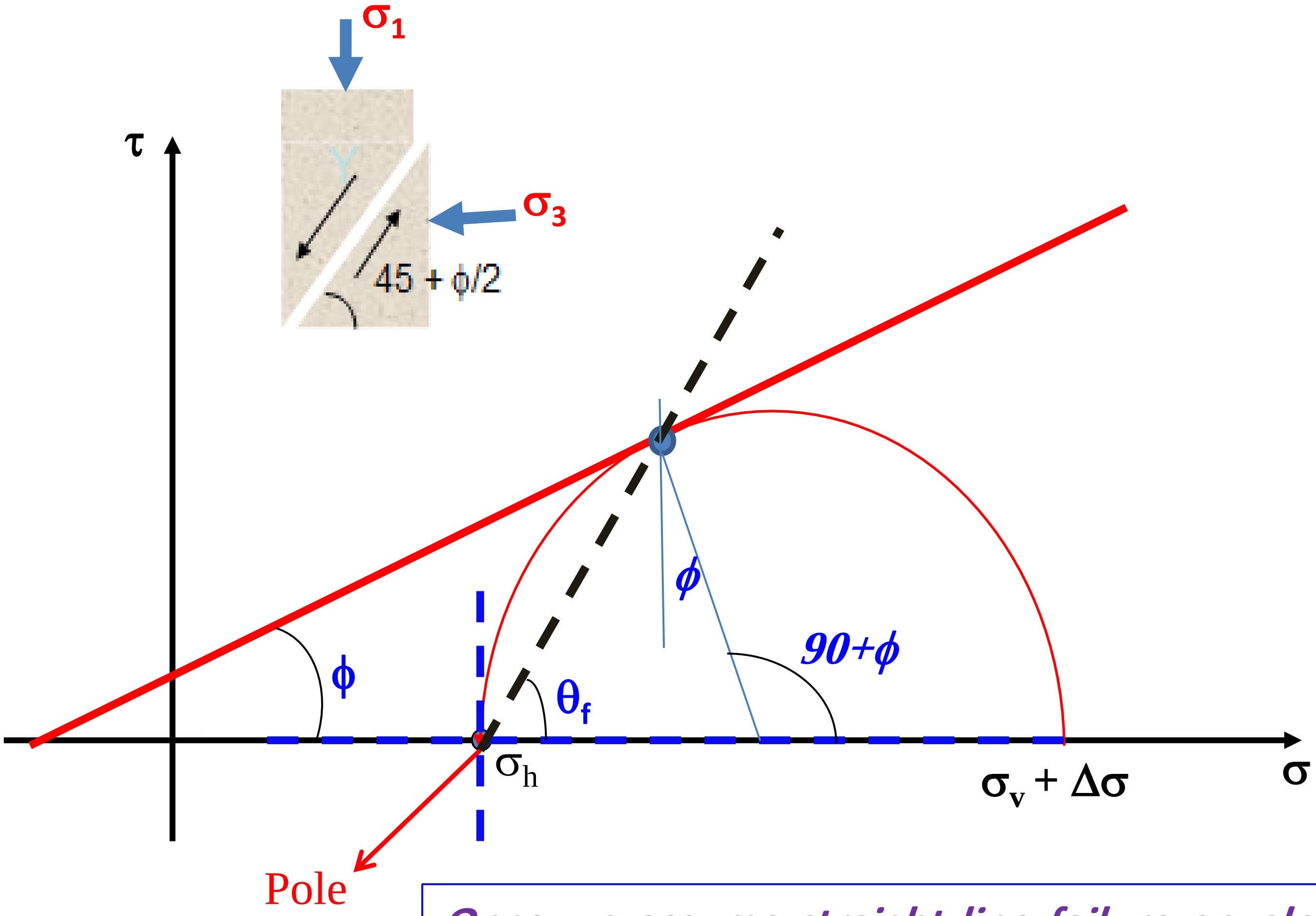


$$\tau_f = \boxed{c} + \sigma_n \tan\phi$$

cohesive component

frictional component

Orientation of Failure Plane



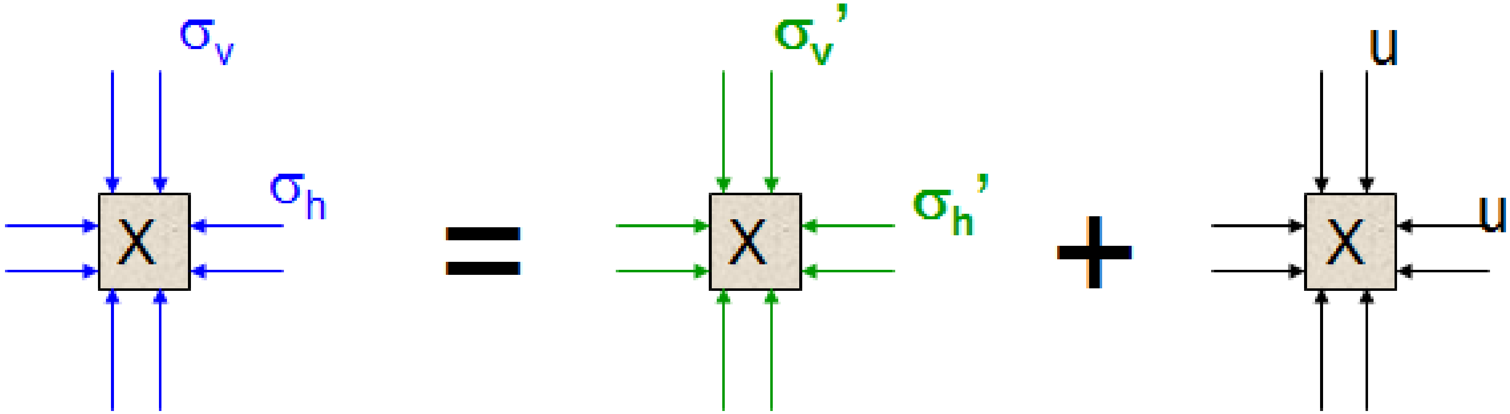
$$2\theta_f = 90 + \phi$$

$$\theta_f = 45 + \phi/2$$

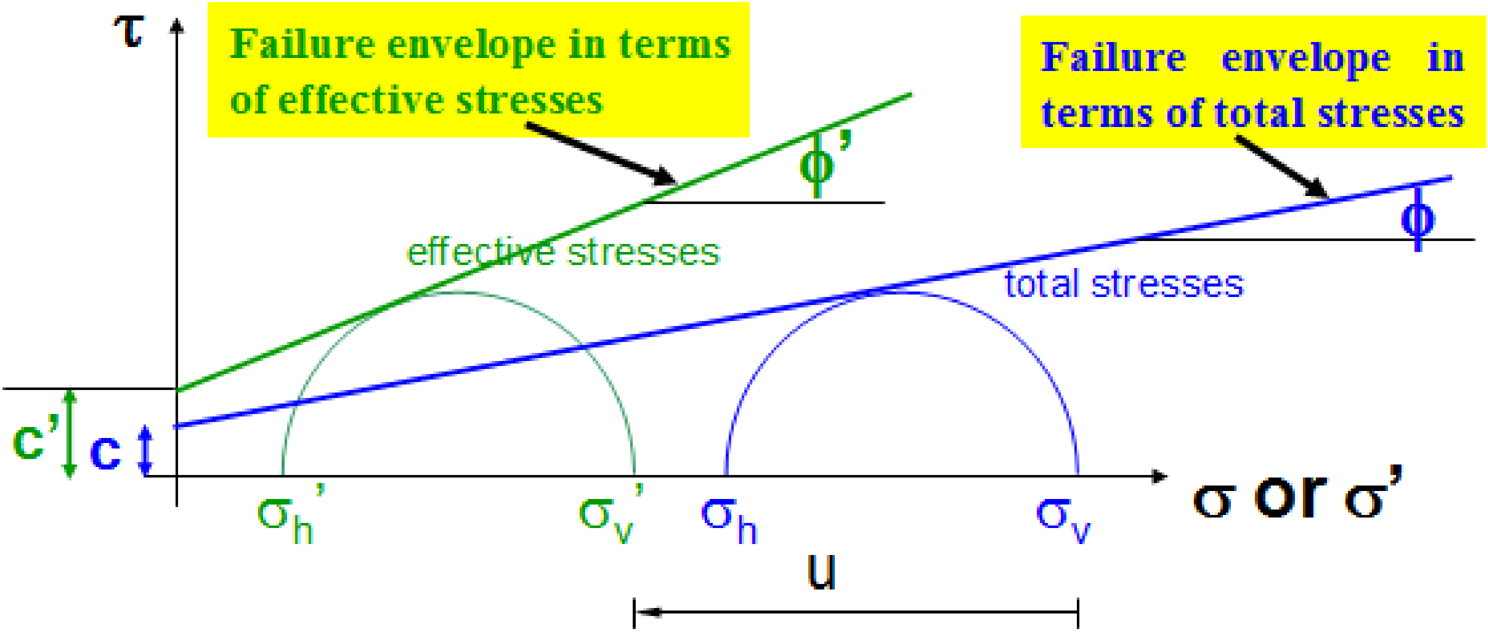
This could be proved analytically (See Das)

Once we assume straight line failure envelope, $\theta_f = 45 + \phi/2$ always and independent of the values of σ_1 and σ_3 (i.e. confinement).

Mohr circles & failure envelope in terms of total and effective stress



[If X is on failure]



Failure envelope in terms of effective stresses

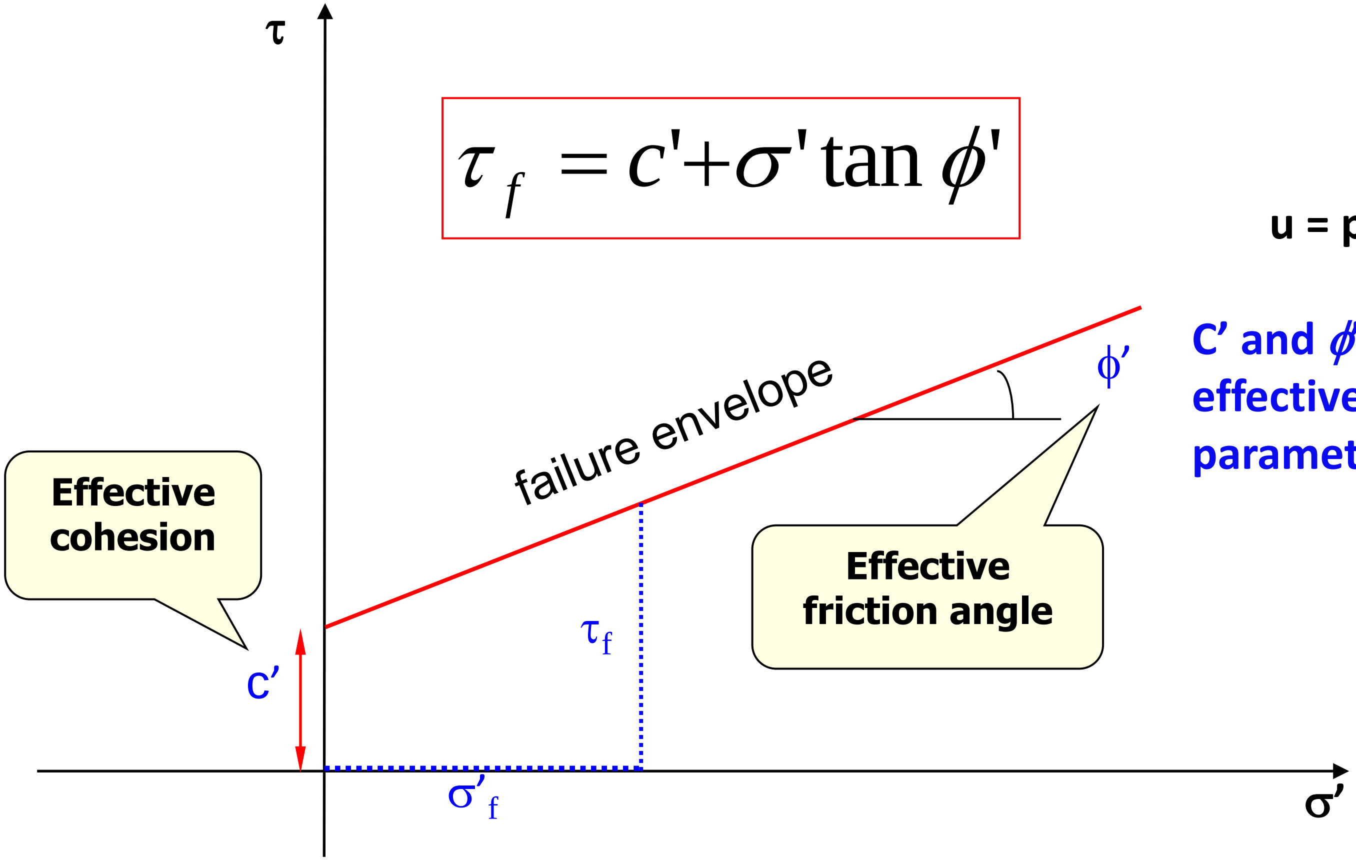
Failure envelope in terms of total stresses

Mohr-Coulomb Failure Criterion in terms of effective stress

$$\boxed{\tau_f = c' + \sigma' \tan \phi'}$$

$$\boxed{\sigma' = \sigma - u}$$

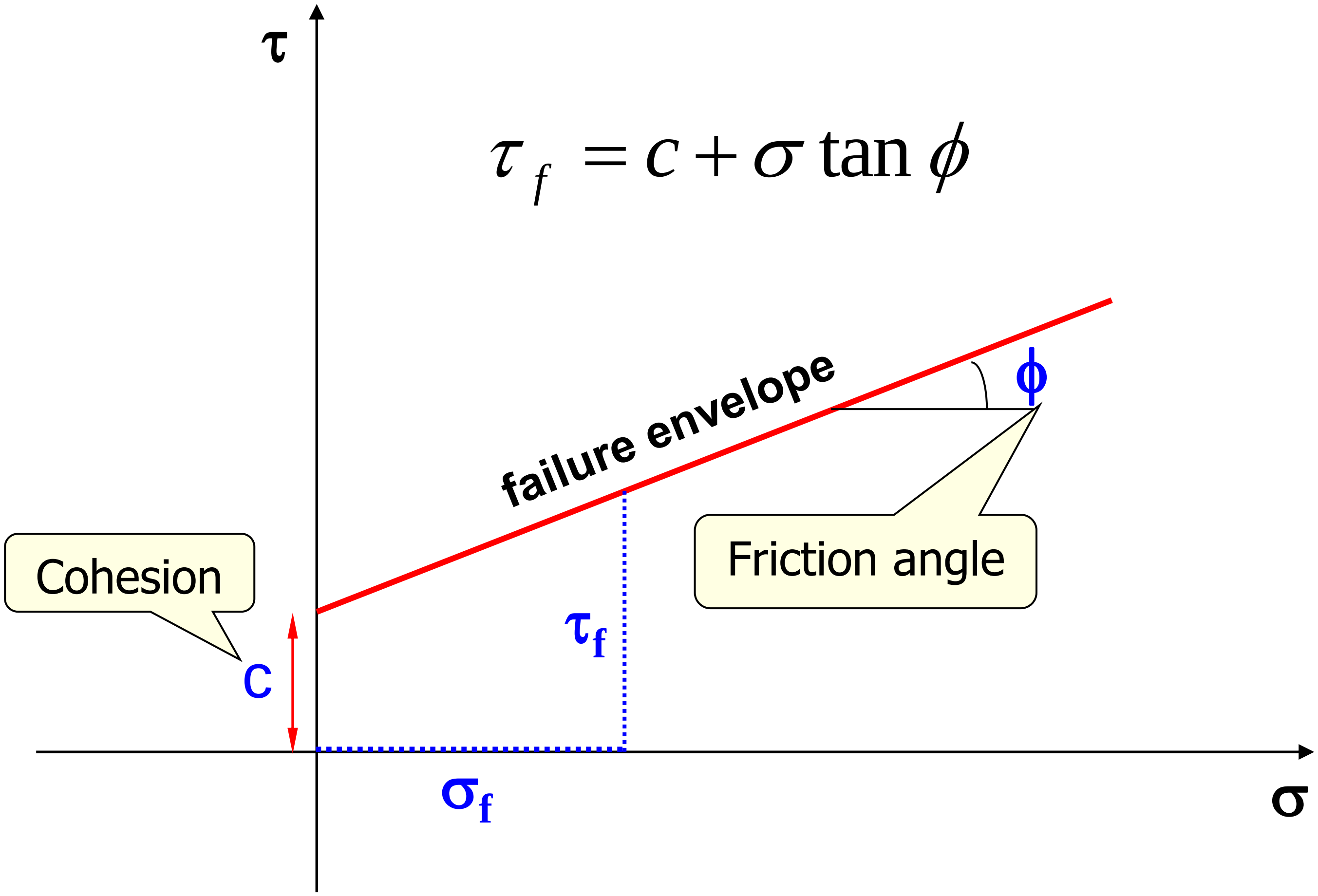
u = pore water pressure



C’ and ϕ' are called the effective strength parameters.

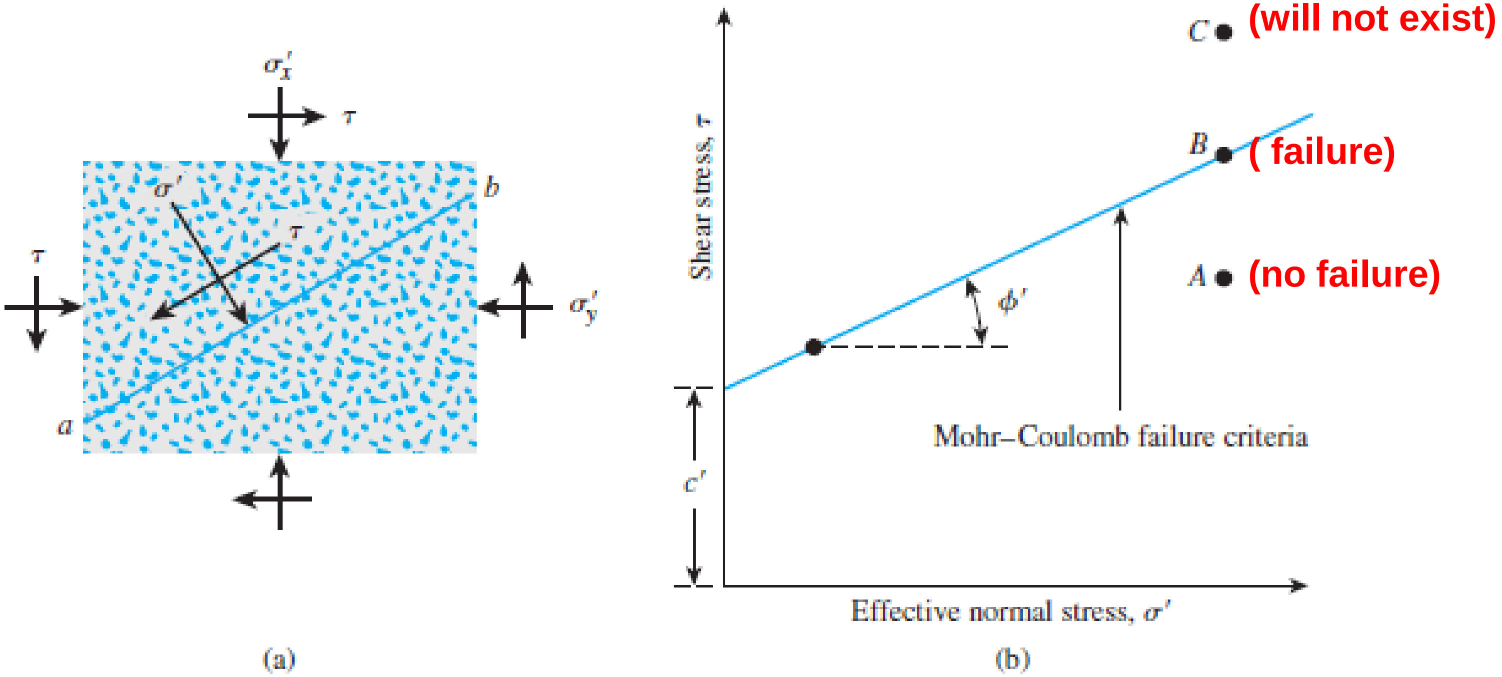
In this case, soil behavior is controlled by effective stresses, and the effective strength parameters are the fundamental strength parameters.

Mohr-Coulomb Failure Criterion in terms of total stress



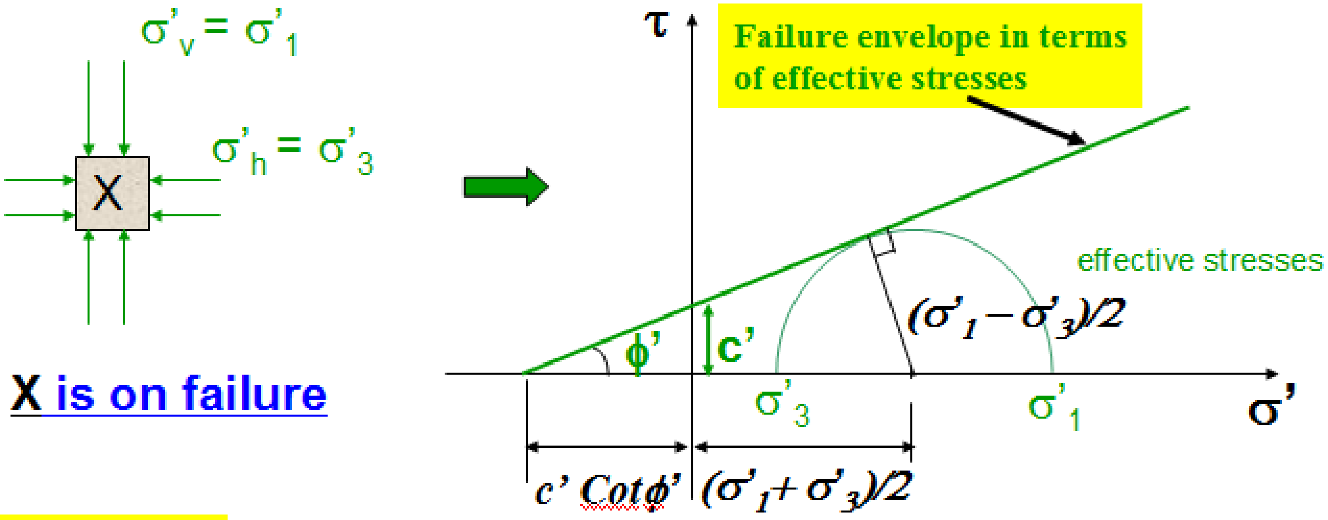
τ_f is the maximum shear stress the soil can take without failure, under normal stress of σ.

What does the Failure Envelope Signify?



- ❑ If the magnitudes of σ and τ on plane ***ab*** are such that they plot as point ***A*** shear failure will not occur along the plane.
- ❑ If the effective normal stress and the shear stress on plane ***ab*** plot as point **B** (which falls on the failure envelope), shear failure will occur along that plane.
- ❑ A state of stress on a plane represented by point **C** cannot exist, because it plots above the failure envelope, and shear failure in a soil would have occurred already.

Relationship between principle stresses and shear strength parameters



Failure envelope in terms of effective stresses

X is on failure

Therefore,

$$\left[c'Cot\phi' + \left(\frac{\sigma_1^{'} + \sigma_3^{'}}{2}\right)\right]Sin\phi' = \left(\frac{\sigma_1^{'} - \sigma_3^{'}}{2}\right)$$

$$\sigma_1^{'} = \sigma_3^{'}Tan^2\left(45 + \frac{\phi'}{2}\right) + 2c'Tan\left(45 + \frac{\phi'}{2}\right)$$

REMARKS

- C and ϕ are measures of shear strength and are called shear strength parameters.
- The parameters C, ϕ are in general not soil constants. They depend on the initial state of the soil (e.g. density, water content), and type of loading (drained or undrained).
- In case of using effective stress, C’ and ϕ’ are called effective shear strength parameters.
- The value of C′ for sand and inorganic silt is 0. For normally consolidated clays, C′ can be approximated at 0. Overconsolidated clays have values of C′ that are greater than 0.

Laboratory Shear Strength Testing

- The purpose of laboratory testing is to determine the shear strength parameters of soil (C, ϕ or C', ϕ') through the determination of failure envelope.
- The shear strength parameters for a particular soil can be determined by means of laboratory tests on specimens taken from representative samples of the in-situ soil.
- There are several laboratory methods available to determine the shear strength parameters (i.e. C, ϕ, C', ϕ'). Some of the tests are rather complicated .
- For further details you should consult manuals and books on laboratory testing, especially those by the ASTM.

Determination of shear strength parameters of soils

Determination of shear strength parameters of soils (c, ϕ or c′, ϕ')

Laboratory Tests

Most common laboratory tests to determine the shear strength parameters are,

1. Direct shear test
2. Triaxial shear test

Other laboratory tests include:

- Direct simple shear test
- Torsional or ring shear test
- Hollow cylinder test
- Plane strain triaxial test
- Laboratory vane shear test
- Laboratory fall cone test.

Field Tests

1. Standard penetration test Pressuremeter
2. Vane shear test
3. Pocket penetrometer
4. Static cone penetrometer

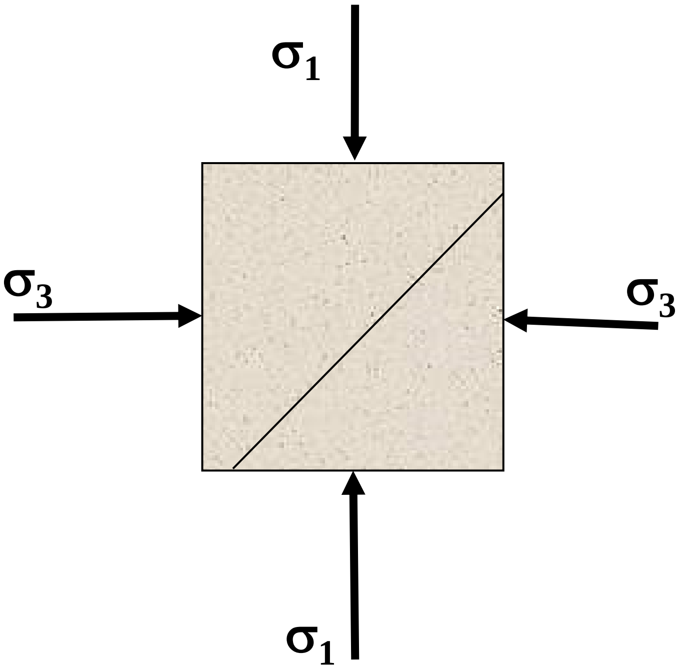
Field test equipment and test methods are described in most textbooks on foundation engineering.

Laboratory Shear Strength Testing

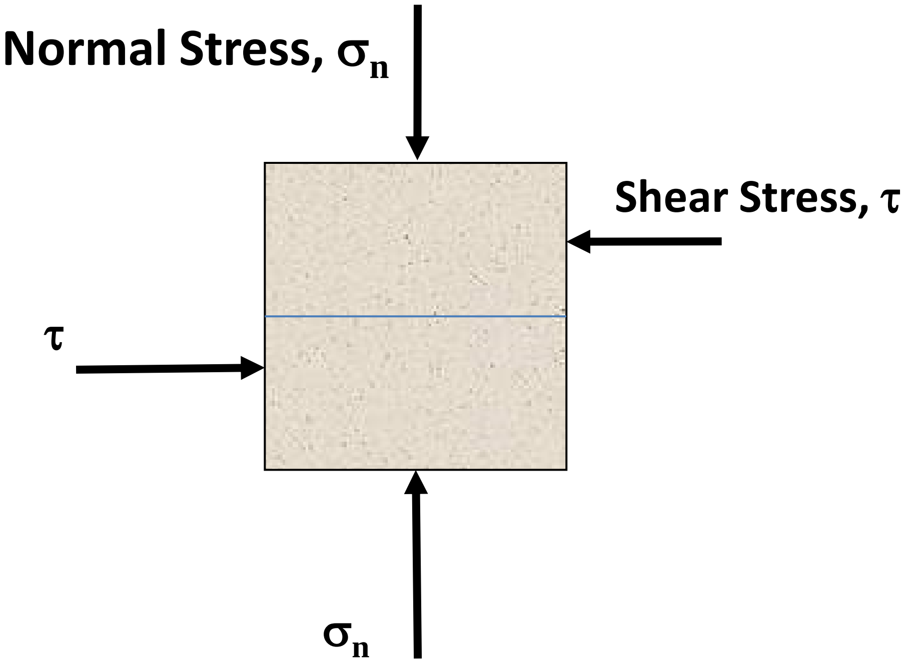
- In laboratory, a soil sample can fail in two ways:

Way 1: Increase normal stress (σ_1) to failure with confining stress (σ_3) constant

Way 2: Normal stress is applied and held constant then shear stress is applied to failure

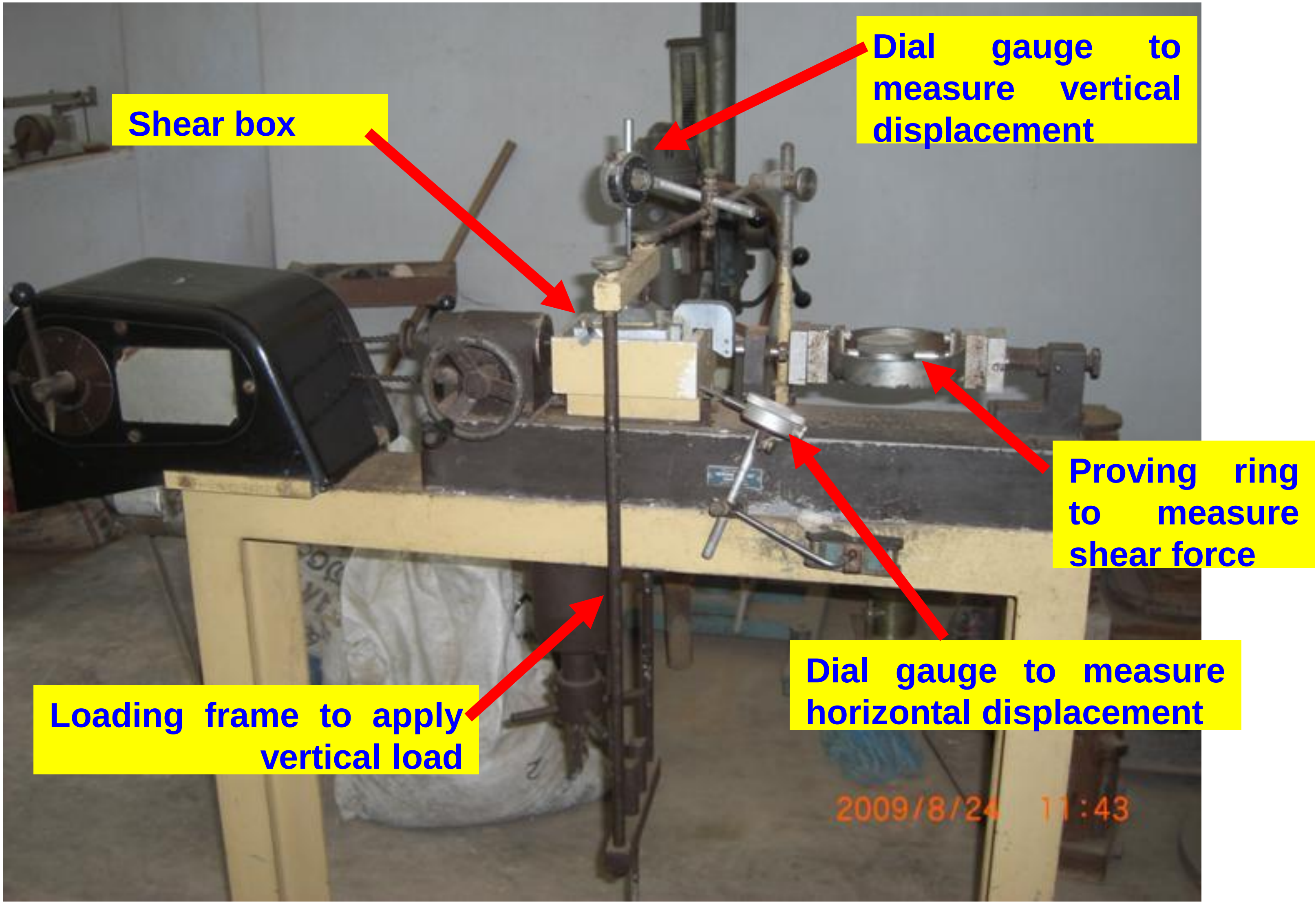


Triaxial Shear Test

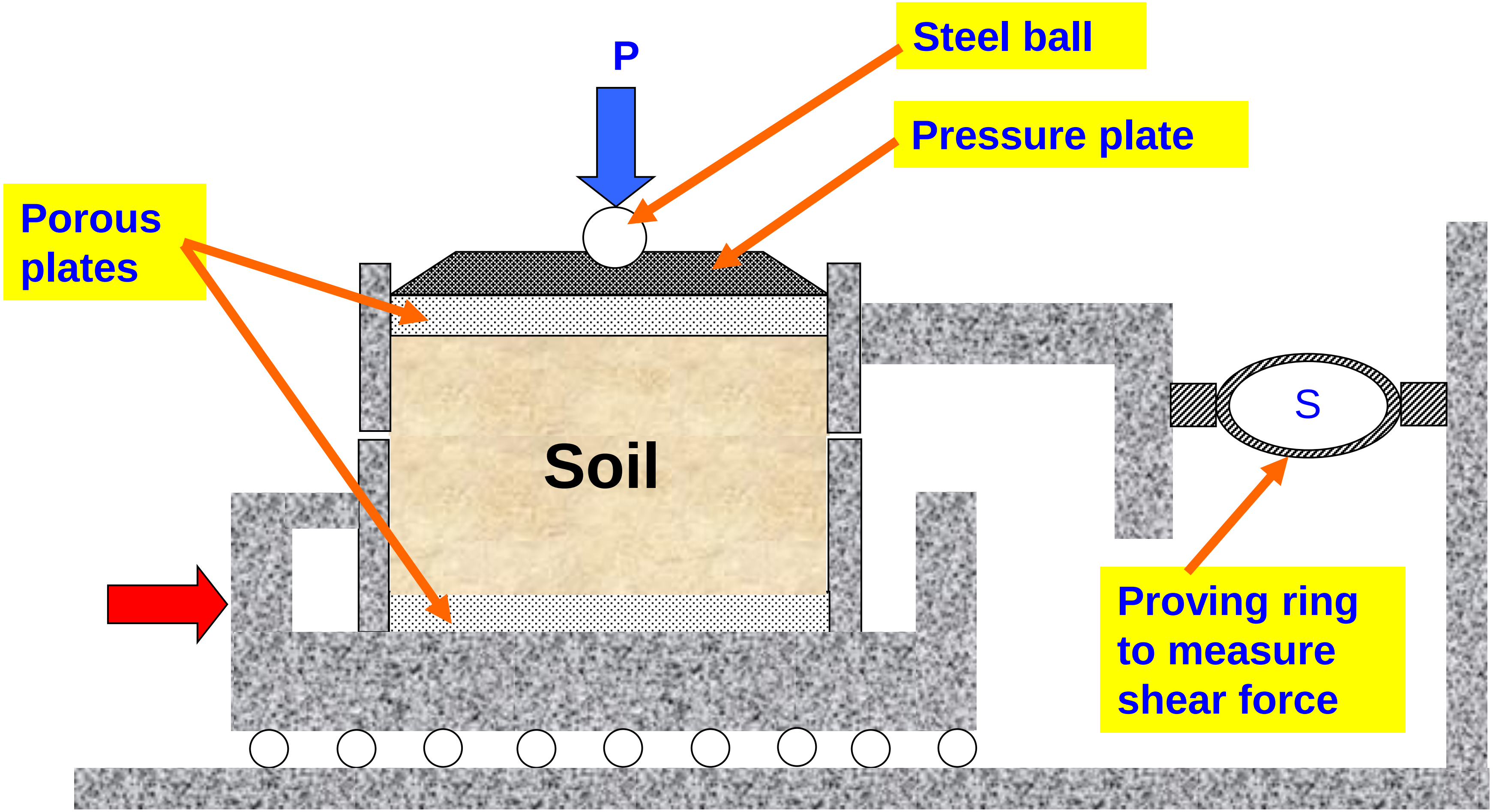


Direct Shear Test

Direct Shear Test



Direct Shear Test



Test Procedure

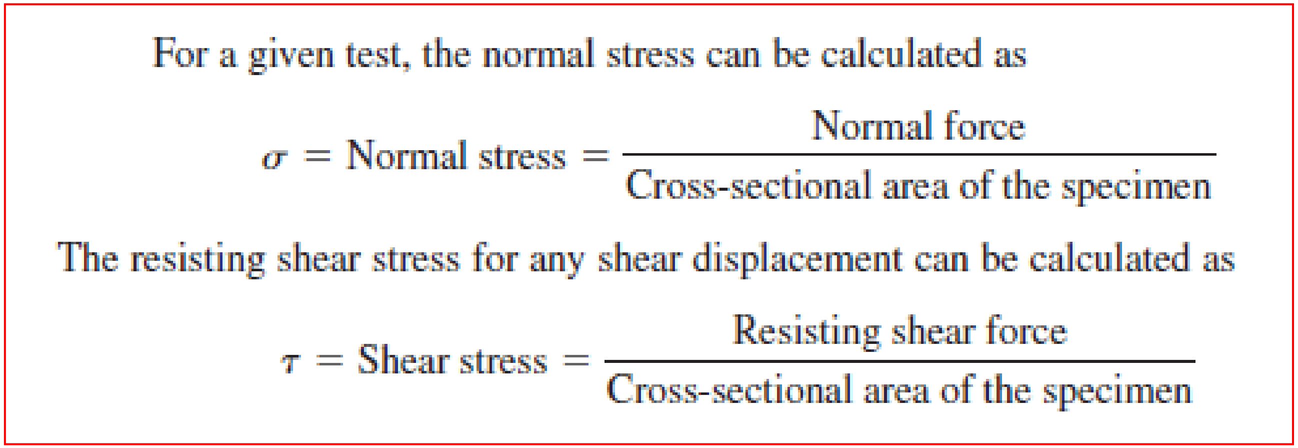
- A constant vertical force (normal stress) is applied through a metal platen.
- Shear force is applied by moving one half of the box relative to the other and increased to cause failure in the soil sample.
- The tests are repeated on similar specimens at various normal stresses
- The normal stresses and the corresponding values of τ_f obtained from a number of tests are plotted on a graph from which the shear strength parameters are determined.

TEST RESULTS

Normal Load : __________ kg

Area of Sample: __________ cm^2

Horizontal Dial Reading (mm)	Vertical Dial Reading (mm)	Horizontal Shear Force (N)	Shear Stress (kPa)



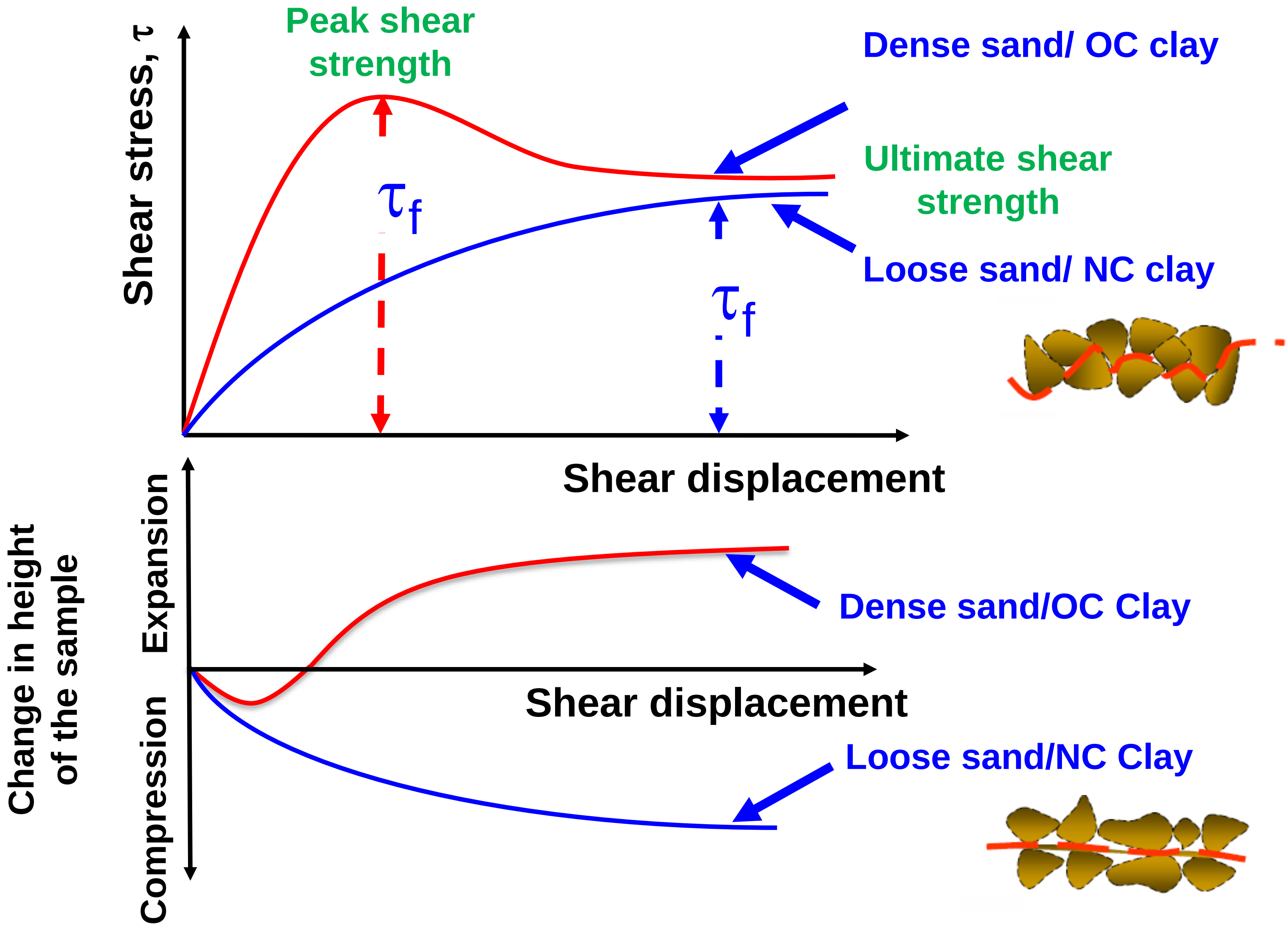
For a given test, the normal stress can be calculated as

$$\sigma = \text{Normal stress} = \frac{\text{Normal force}}{\text{Cross-sectional area of the specimen}}$$

The resisting shear stress for any shear displacement can be calculated as

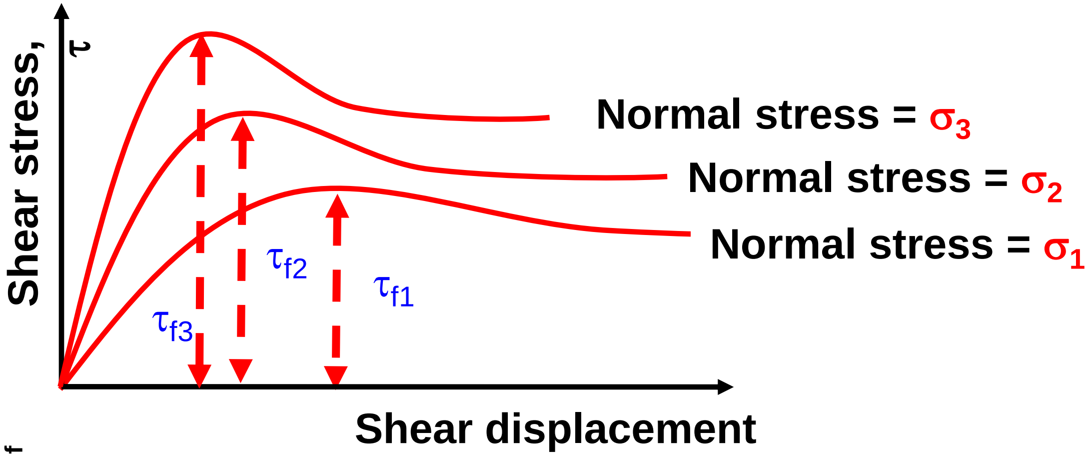
$$\tau = \text{Shear stress} = \frac{\text{Resisting shear force}}{\text{Cross-sectional area of the specimen}}$$

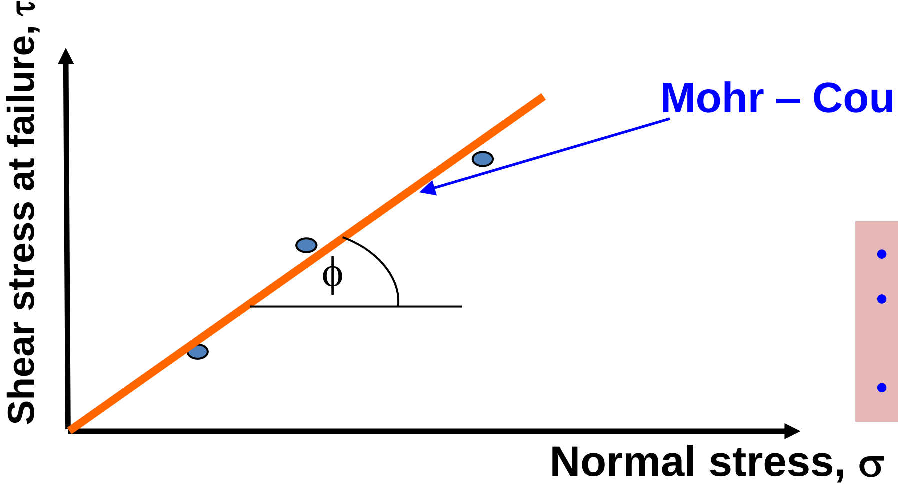
Stress-Strain Relationship



Determining strength parameters C and ϕ

An example of testing three samples of a sand at the same relative density just before shearing.



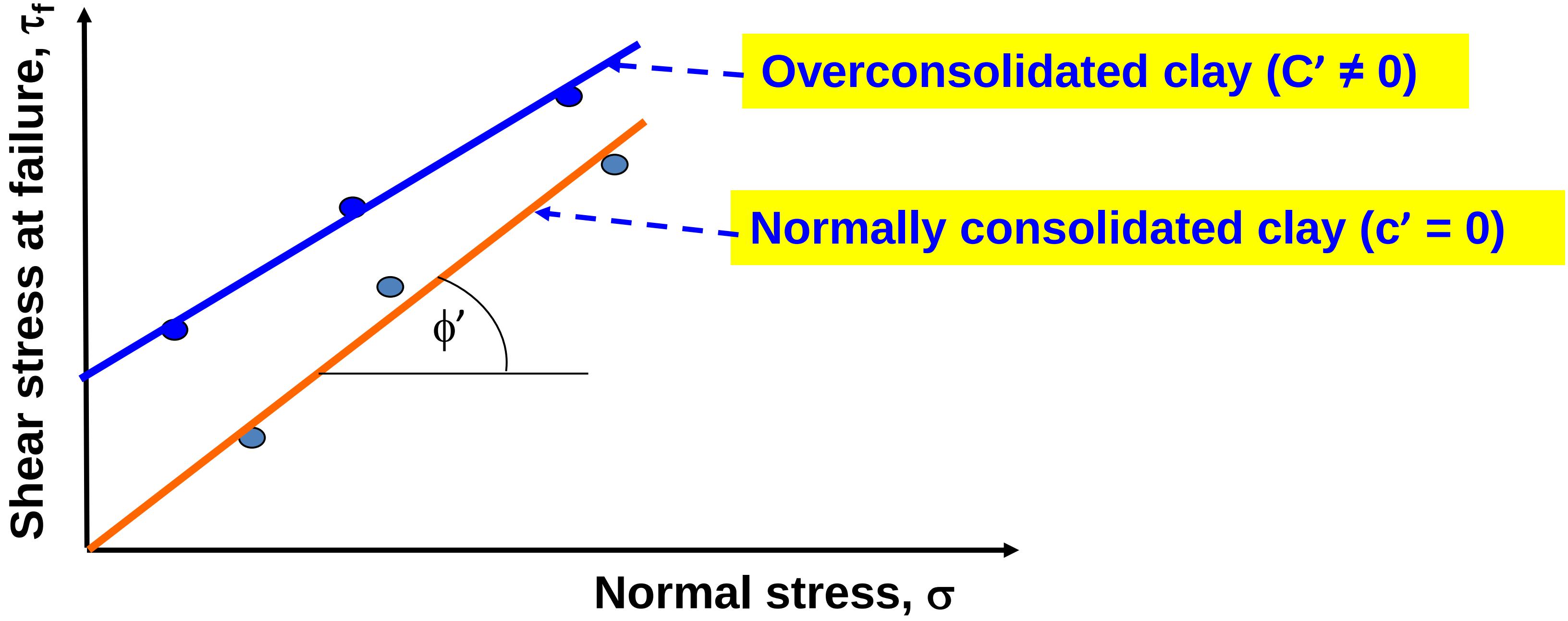


- Sand is cohesionless, then C = 0
- Direct shear tests are drained and pore water pressures are dissipated, then u = 0
- Therefore, $\phi = \phi'$

Determining strength parameters C and ϕ

In case of clay, horizontal displacement should be applied at a very slow rate to allow dissipation of pore water pressure (therefore, one test would take several days to finish)

Failure envelopes for clay from drained direct shear tests



Overconsolidated clay ($C' \neq 0$)

Normally consolidated clay ($c' = 0$)

Notes on Direct Shear Test

- **This test is probably the oldest strength test because Coulomb used a type of shear box test more than two centuries ago to determine the necessary parameters for his strength equation.**
- **The test is quick and inexpansive and common in practice.**
- **Used to determine the shear strength of both cohesive as well as non-cohesive soils.**
- **The test equipment consists of a metal box in which the soil specimen is placed. The box is split horizontally into two halves.**
- **The shear test can be either stress controlled or strain controlled.**
- **Tests on sands and gravels are usually performed dry. Water does not significantly affect the (drained) strength.**

Notes on Direct Shear Test

- Usually only relatively slow drained tests are performed in shear box apparatus. For clays rate of shearing must be chosen to prevent excess pore pressures building up. For sands and gravels tests can be performed quickly.
- If there are no excess pore pressures and as the pore pressure is approximately zero the total and effective stresses will be identical.
- The failure stresses thus define an effective stress failure envelope from which the effective (drained) strength parameters C’, ϕ’ can be determined.
- Normally consolidated clays (OCR = 1) and loose sands do not show separate peak and ultimate failure loci, and for soils in these states C’ = 0.
- Overconsolidated clays and dense sands have peak strengths with C’ > 0.

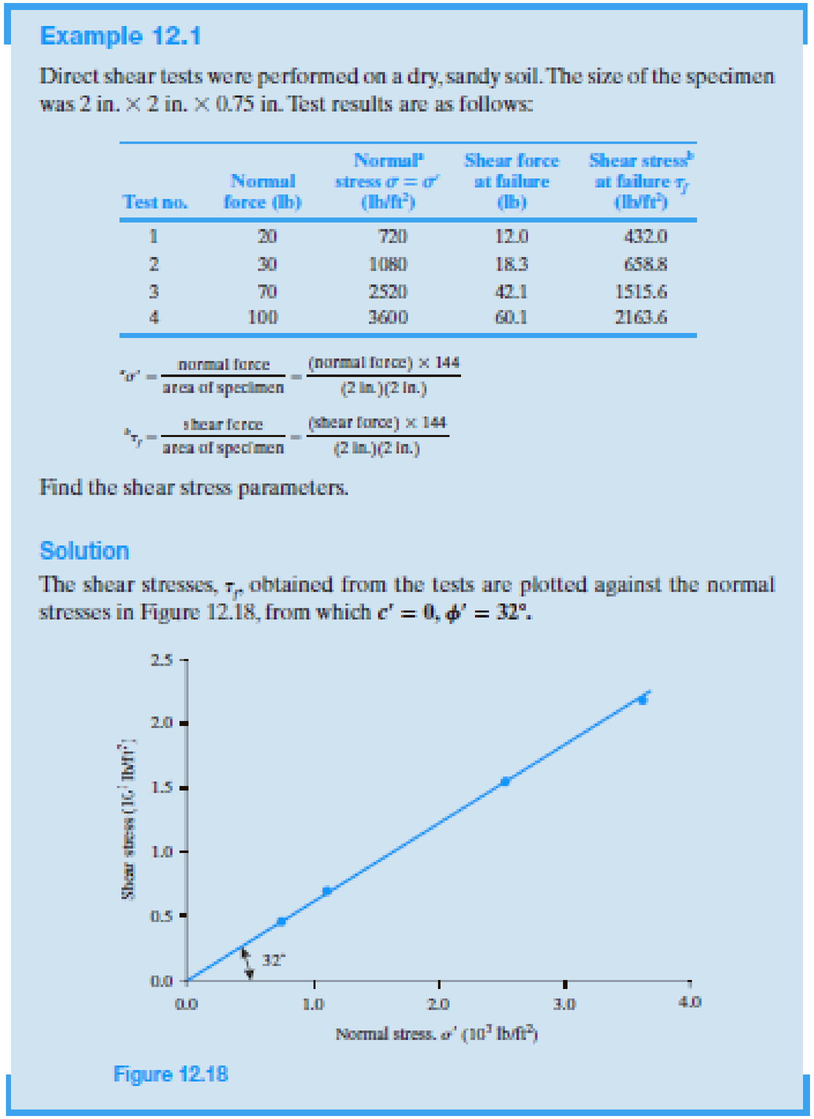
Advantages of direct shear test

- Inexpensive, fast, and simple, especially for granular materials.
- Easiness of sample preparation in case of sand.
- Due to the smaller thickness of the sample, rapid drainage can be achieved
- Large deformations can be achieved by reversing shear direction. This is useful for determining the residual strength of a soil.
- Samples may be sheared along predetermined planes. This is useful when the shear strengths along fissures or an interface is required.

Disadvantages of direct shear test

- Failure occurs along a predetermined failure plane which may not be the weakest plane.
- Non-uniform of shear stresses along failure surface in the specimen. There are rather stress concentrations at the sample boundaries, which lead to highly nonuniform stress conditions within the test specimen itself.
- There is no means of estimating pore pressures, so effective stresses cannot be determined and only the total normal stress can be determined.
- It is very difficult if not impossible to control drainage, especially for fine-grained soils. Consequently, the test is not suitable for other than completely drained conditions.

Example 12.1



Example 12.1

Direct shear tests were performed on a dry, sandy soil. The size of the specimen was 2 in. × 2 in. × 0.75 in. Test results are as follows:

Test no.	Normal force (lb)	Normal[a] stress $\sigma = \sigma'$ (lb/ft^2)	Shear force at failure (lb)	Shear stress[b] at failure τ_f (lb/ft^2)
1	20	720	12.0	432.0
2	30	1080	18.3	658.8
3	70	2520	42.1	1515.6
4	100	3600	60.1	2163.6

[a] $\sigma' = \dfrac{\text{normal force}}{\text{area of specimen}} = \dfrac{(\text{normal force}) \times 144}{(2\text{ in.})(2\text{ in.})}$

[b] $\tau_f = \dfrac{\text{shear force}}{\text{area of specimen}} = \dfrac{(\text{shear force}) \times 144}{(2\text{ in.})(2\text{ in.})}$

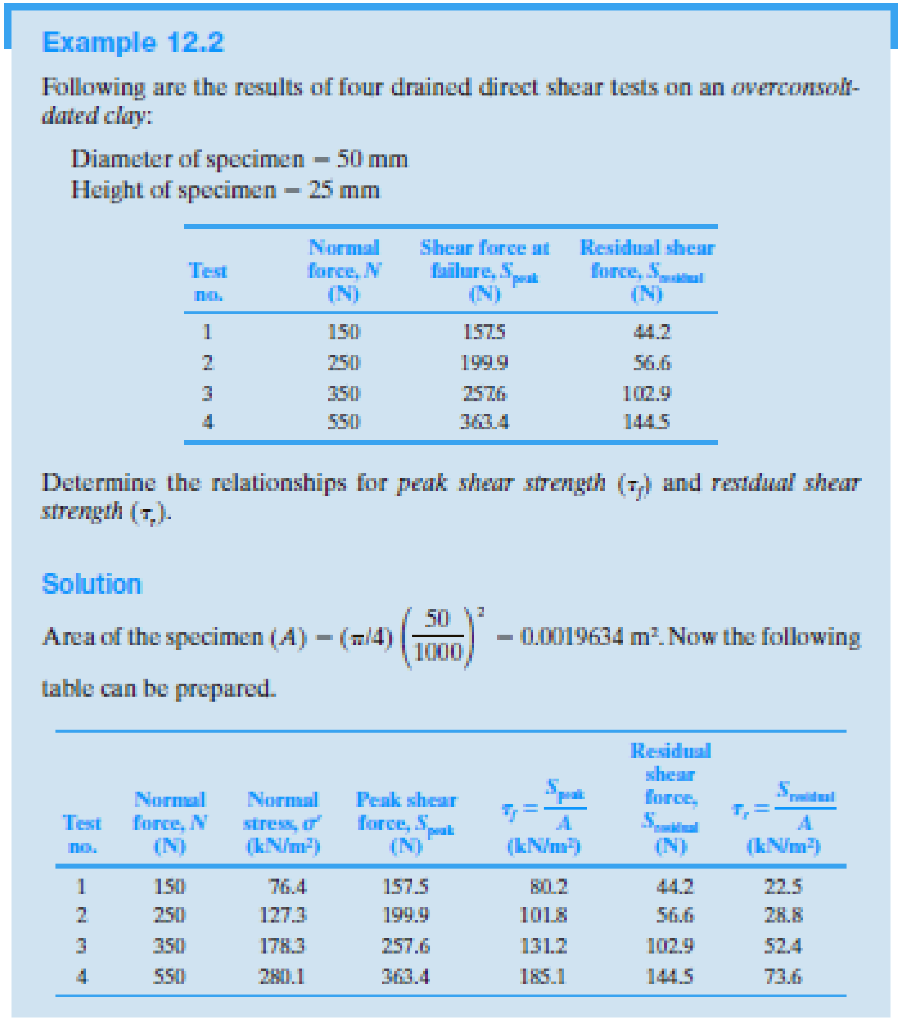
Find the shear stress parameters.

Solution

The shear stresses, τ_f, obtained from the tests are plotted against the normal stresses in Figure 12.18, from which $c' = 0$, $\phi' = 32°$.

Figure 12.18

Example 12.2



Example 12.2

Following are the results of four drained direct shear tests on an *overconsolidated clay*:

Diameter of specimen = 50 mm
Height of specimen = 25 mm

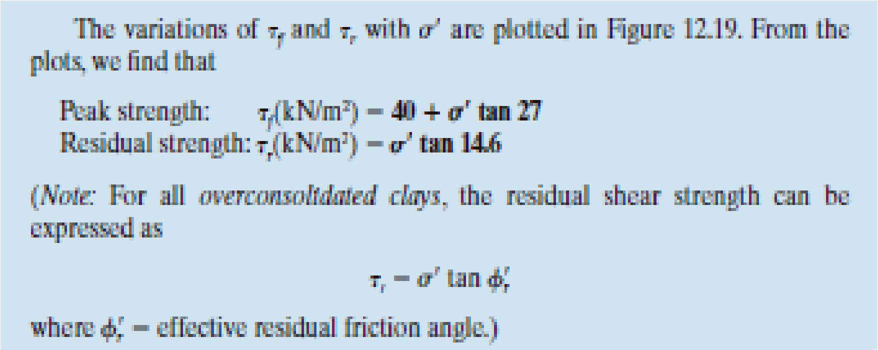
Test no.	Normal force, N (N)	Shear force at failure, S_{peak} (N)	Residual shear force, S_{residual} (N)
1	150	157.5	44.2
2	250	199.9	56.6
3	350	257.6	102.9
4	550	363.4	144.5

Determine the relationships for *peak shear strength* (τ_f) and *residual shear strength* (τ_r).

Solution

Area of the specimen (A) = $(\pi/4)\left(\frac{50}{1000}\right)^2$ = 0.0019634 m². Now the following table can be prepared.

Test no.	Normal force, N (N)	Normal stress, σ' (kN/m²)	Peak shear force, S_{peak} (N)	$\tau_f = \frac{S_{\text{peak}}}{A}$ (kN/m²)	Residual shear force, S_{residual} (N)	$\tau_r = \frac{S_{\text{residual}}}{A}$ (kN/m²)
1	150	76.4	157.5	80.2	44.2	22.5
2	250	127.3	199.9	101.8	56.6	28.8
3	350	178.3	257.6	131.2	102.9	52.4
4	550	280.1	363.4	185.1	144.5	73.6



The variations of τ_f and τ_r with σ' are plotted in Figure 12.19. From the plots, we find that

Peak strength: $\tau_f(\text{kN/m}^2) = \mathbf{40 + \sigma' \tan 27}$
Residual strength: $\tau_r(\text{kN/m}^2) = \mathbf{\sigma' \tan 14.6}$

(*Note:* For all *overconsolidated clays*, the residual shear strength can be expressed as

$$\tau_r = \sigma' \tan \phi'_r$$

where ϕ'_r = effective residual friction angle.)

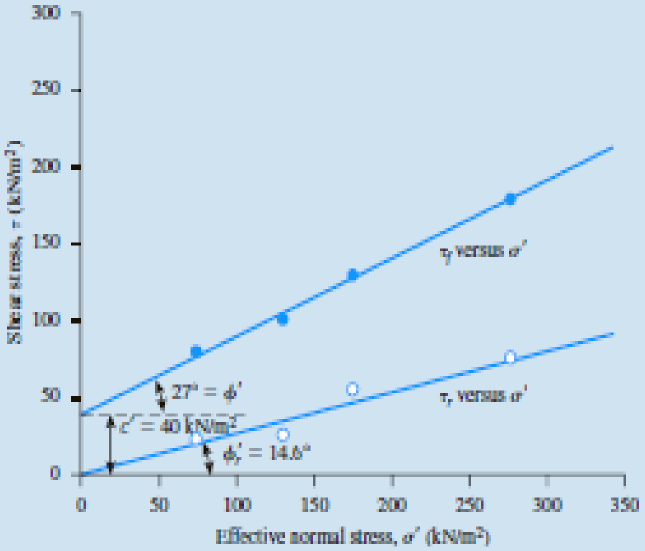


Figure 12.19 Variations of τ_f and τ_r with σ'

Example

For a dry sand specimen in a direct shear test box, the following are given:

- **Size of specimen: 63.5 mm 63.5 mm 31.75 mm (height)**
- **Angle of friction: 33°**
- **Normal stress: 193 kN/m^2.**

Determine the shear force required to cause failure.

τ = 193 tan (33) = 125.33 kPa

Shear force = τ *area

Shear force = 125.33x 0.0635x0.0635 = 0.50538 kN = 505.38 N

Example

A direct shear test is run on a sand sample (c = 0) with the normal stress = 65 kPa.
At failure, the normal stress is 65 kPa and the shear stress is 40 kPa.
Draw the Mohr circle at failure and determine:

- The friction angle
- The major and minor principal stresses at failure
- The orientation of the failure plane
- The orientation of the plane of maximum shear stress at failure