SHEAR STRENGTH OF SOIL

Chapter 10: Sections

10.2

10.3

Chapter 12: All sections except

12.13

12.14

12.15

12.17

12.18

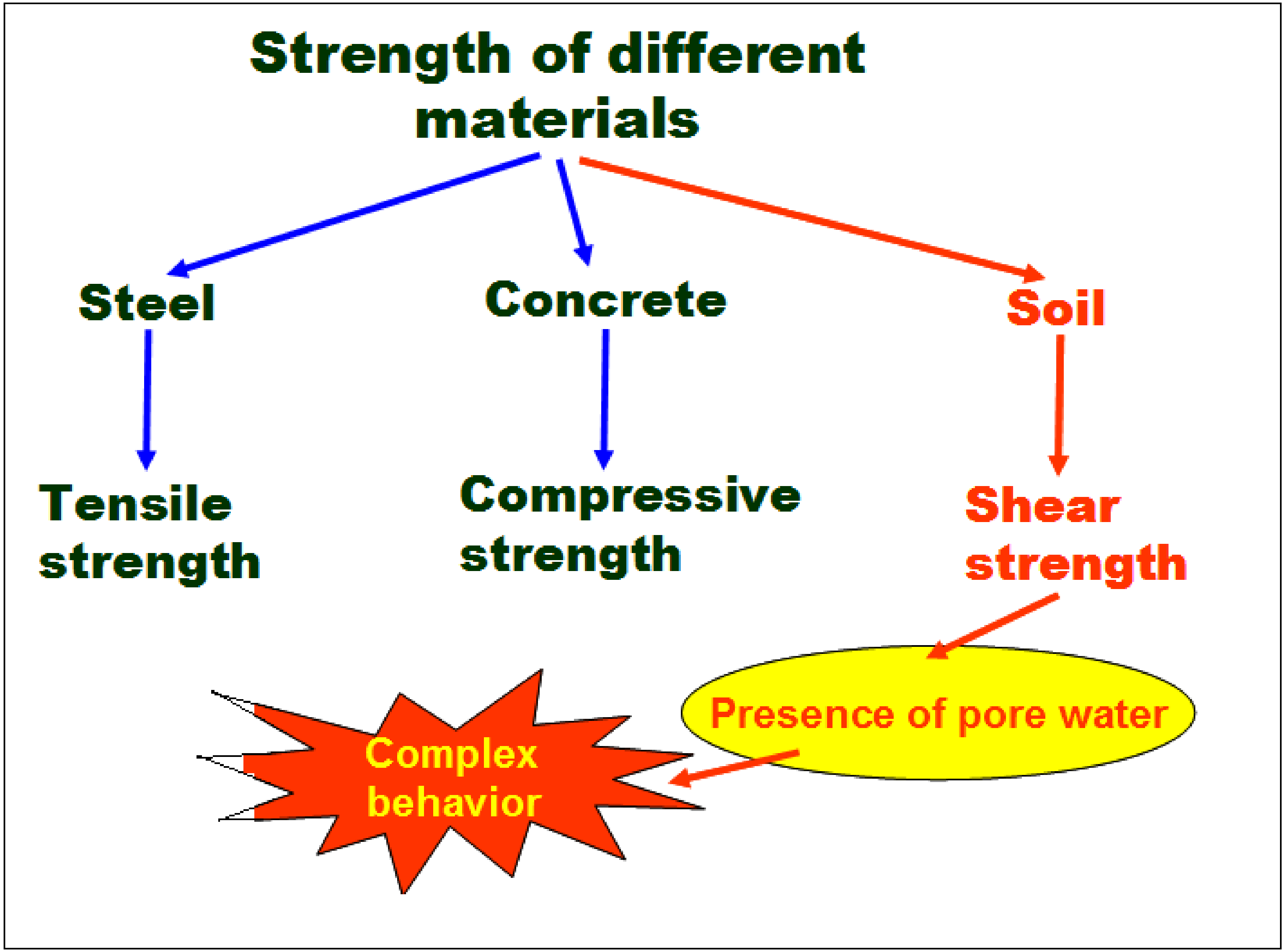
TOPICS

- Introduction
- Components of Shear Strength of Soils
- Normal and Shear Stresses on a Plane
- Mohr-Coulomb Failure Criterion
- Laboratory Shear Strength Testing
    - Direct Shear Test
    - Triaxial Compression Test
    - Unconfined Compression Test
- Field Testing (Vane test)

INTRODUCTION

- Soil failure usually occurs in the form of "**shearing**" along internal surface within the soil.
- The ***shear strength*** of a soil mass is the internal resistance per unit area that the soil mass can offer to resist failure and sliding along any plane inside it.
- The safety of any geotechnical structure is dependent on the strength of the soil.
- Shear strength determination is a very important aspect in geotechnical engineering. Understanding shear strength is the basis to analyze soil stability problems like:
  - Bearing capacity.
  - Lateral pressure on earth retaining structures
  - Slope stability

INTRODUCTION



SHEAR STRENGTH OF SOIL

- Coulomb (1776) observed that there was a stress-dependent component of shear strength and a stress-independent component.
- The stress-dependent component is similar to sliding friction in solids described above. The other component is related to the intrinsic COHESION of the material. Coulomb proposed the following equation for shear strength of soil:

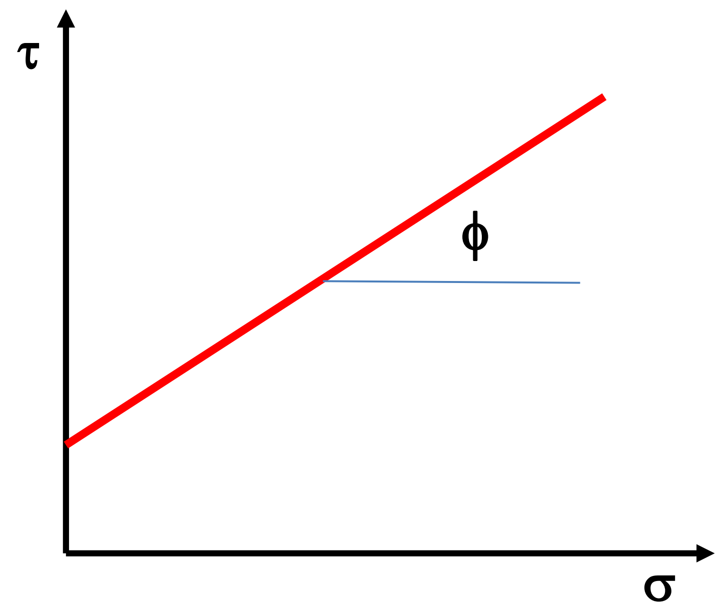
$$\tau_f = C + \sigma_n \tan \phi$$

τ_f = shear strength of soil

σ_n = Applied normal stress

C = Cohesion

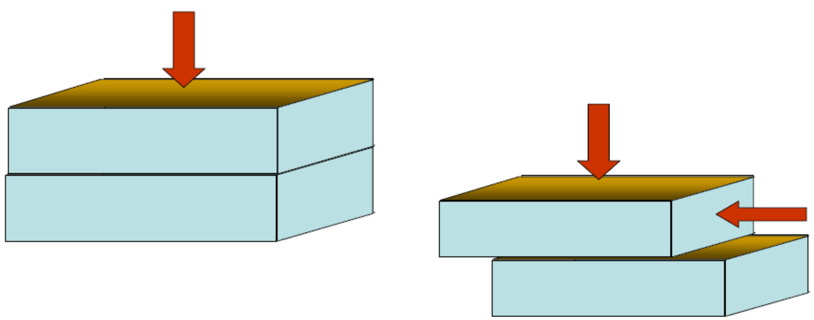
ϕ = Angle of internal friction (or angle of shearing resistance)



SHEAR STRENGTH OF SOIL

- **Cohesion (c), is a measure of the forces that cement particles of soils (stress independent).**

- **Internal Friction angle (ϕ), is a measure of the shear strength of soils due to friction (stress dependent).**



- **For granular materials, there is no cohesion between particles**

$$\tau_f = \sigma_n \tan \emptyset$$

SHEAR STRENGTH OF SOIL

Saturated Soils

$$\tau_f = C' + \sigma'_n \tan \phi'$$

But from the principle of effective stress

$$\sigma' = \sigma - u$$

Where u is the pore water pressure (p.w.p.)

Then

$$\tau_f = C' + (\sigma_n - u) \tan \phi'$$

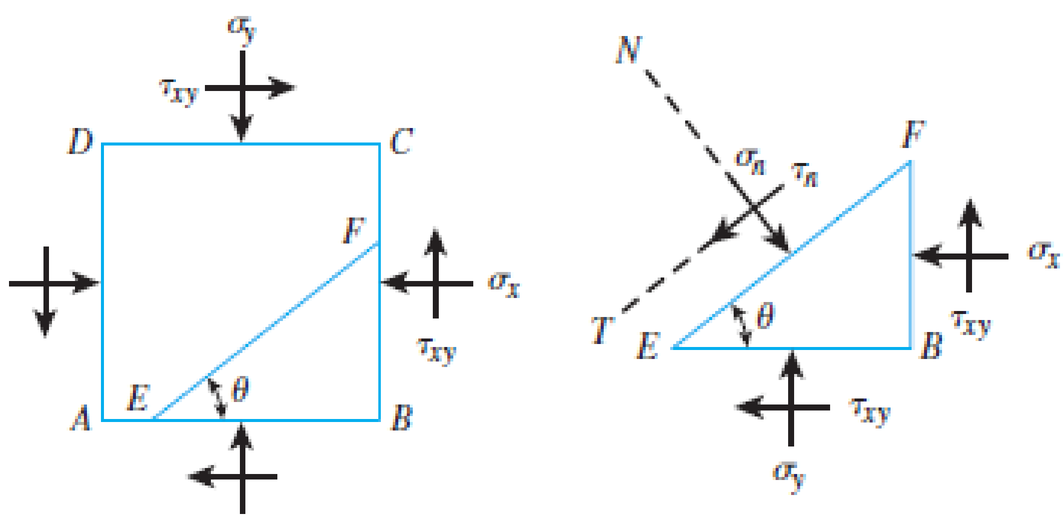
- **C , ϕ or C' , ϕ' are called strength parameters, and we will discuss various laboratory tests for their determination.**

Normal and Shear Stress along a Plane

Chapter 10

- Normal and Shear Stresses along a Plane (Sec. 10.2)
- Pole Method for Finding Stresses along a Plane (Sec. 10.3)

Normal and Shear Stress along a Plane



From geometry

$$\overline{EB} = \overline{EF}\cos\theta$$

$$\overline{FB} = \overline{EF}\sin\theta$$

Sign Convention	Normal Stresses	Shear Stresses
Positive	Compression	Counter clockwise rotation
Negative	Tension	Clockwise rotation

- **Note that for convenience our sign convention has compressive forces and stresses positive because most normal stresses in geotechnical engineering are compressive.**
- **These conventions are the *opposite* of those normally assumed in structural mechanics.**

Normal and Shear Stress along a Plane

$$\sum F_N = 0$$

$$\sigma_n * (\overline{EF}) - \sigma_x \sin\theta * (\overline{EF}\sin\theta) - \sigma_y \cos\theta * (\overline{EF}\cos\theta)$$

$$-\tau_{xy}\cos\theta * (\overline{EF}\sin\theta) - \tau_{xy}\sin\theta * (\overline{EF}\cos\theta) = 0$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

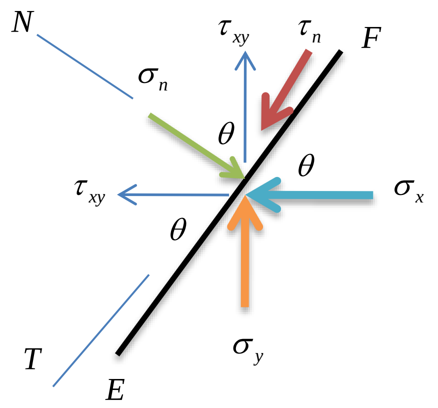
Similarly,

$$\sum F_T = 0$$

$$\tau_n * (\overline{EF}) - \sigma_y \sin\theta * (\overline{EF}\cos\theta) + \sigma_x \cos\theta * (\overline{EF}\sin\theta)$$

$$-\tau_{xy}\sin\theta * (\overline{EF}\sin\theta) + \tau_{xy}\cos\theta * (\overline{EF}\cos\theta) = 0$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$



$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

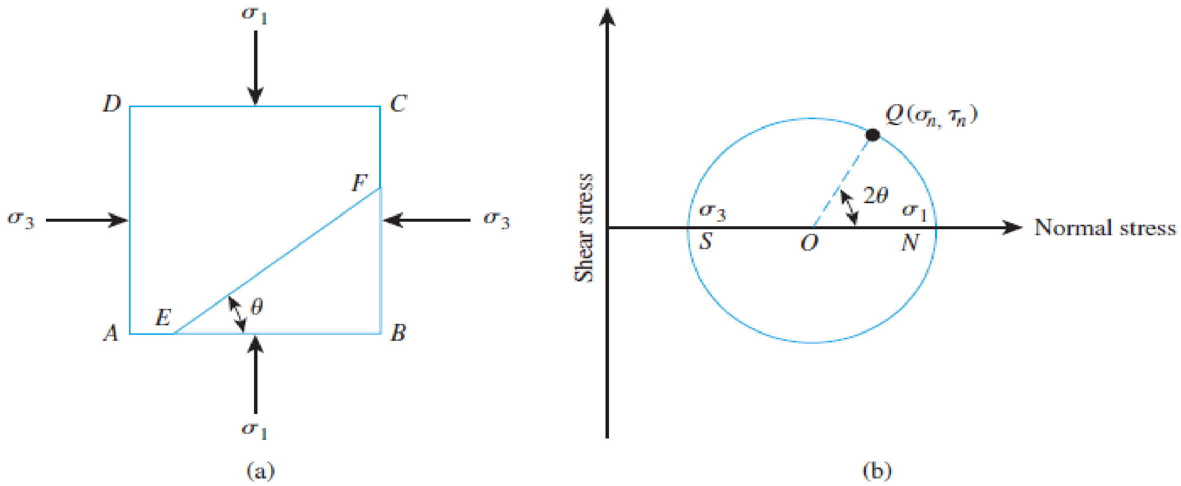
Principal Planes & Principal Stresses

Principal Planes

Planes on which the shear stress is equal to zero

Principal Stresses

Normal stress acting on the principal planes



$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$$

$$\tau_n = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$

Principal Stresses

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad (1)$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \quad (2)$$

$$For \quad \tau_n == 0$$

$$0 = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$\tan 2\theta_p = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2\tau_{xy}}{\sigma_y - \sigma_x} \quad (3)$$

For any given values of σ_x, σ_y and τ_{xy}

Equation (3) will give two values of θ

which are 90 degrees apart

Two principal planes 90 degrees apart

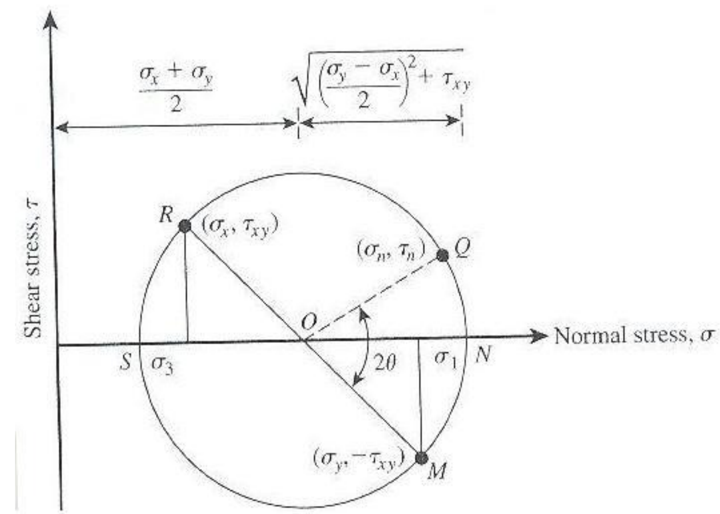
Substitute eq (3) into eq (1)

Major Principal Stress

$$\sigma_n = \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

Minor Principal Stress

$$\sigma_n = \sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$



Example 10.1

Example 10.1

A soil element is shown in Figure 10.4. The magnitudes of stresses are $\sigma_x = 120$ kN/m², $\tau = 40$ kN/m², $\sigma_y = 300$ kN/m², and $\theta = 20°$. Determine

a. Magnitudes of the principal stresses.
b. Normal and shear stresses on plane AB. Use Eqs. (10.3), (10.4), (10.6), and (10.7).

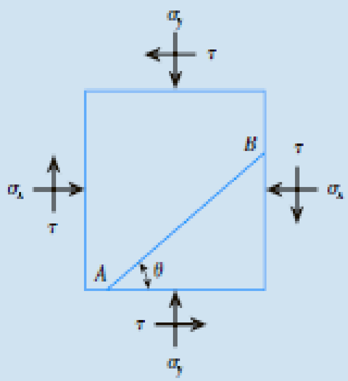


Figure 10.4 Soil element with stresses acting on it

Solution

Part a

From Eqs. (10.6) and (10.7),

$$\left.\begin{array}{l}\sigma_3\\ \sigma_1\end{array}\right\} = \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2}$$

$$= \frac{300 + 120}{2} \pm \sqrt{\left[\frac{300 - 120}{2}\right]^2 + (-40)^2}$$

$$\sigma_1 = \mathbf{308.5\ kN/m^2}$$

$$\sigma_3 = \mathbf{111.5\ kN/m^2}$$

Part b

From Eq. (10.3),

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau \sin 2\theta$$

$$= \frac{300 + 120}{2} + \frac{300 - 120}{2}\cos(2 \times 20) + (-40)\sin(2 \times 20)$$

$$= \mathbf{253.23\ kN/m^2}$$

From Eq. (10.4),

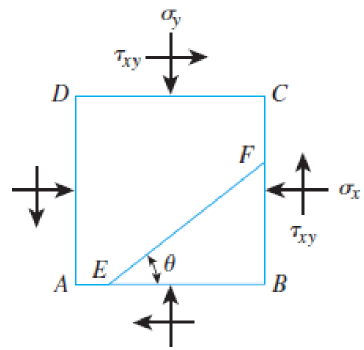
$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau\cos 2\theta$$

$$= \frac{300 - 120}{2}\sin(2 \times 20) - (-40)\cos(2 \times 20)$$

$$= \mathbf{88.40\ kN/m^2}$$

Construction of Mohr's Circle

1. Plot σ_y, τ_{xy} as point M
2. Plot σ_x, τ_{xy} as point R
3. Connect M and R
4. Draw a circle of diameter of the line RM about the point where the line RM crosses the horizontal axis (denote this as point O)



Sign Convention	Normal Stresses	Shear Stresses
Positive	Compression	Counter clockwise rotation
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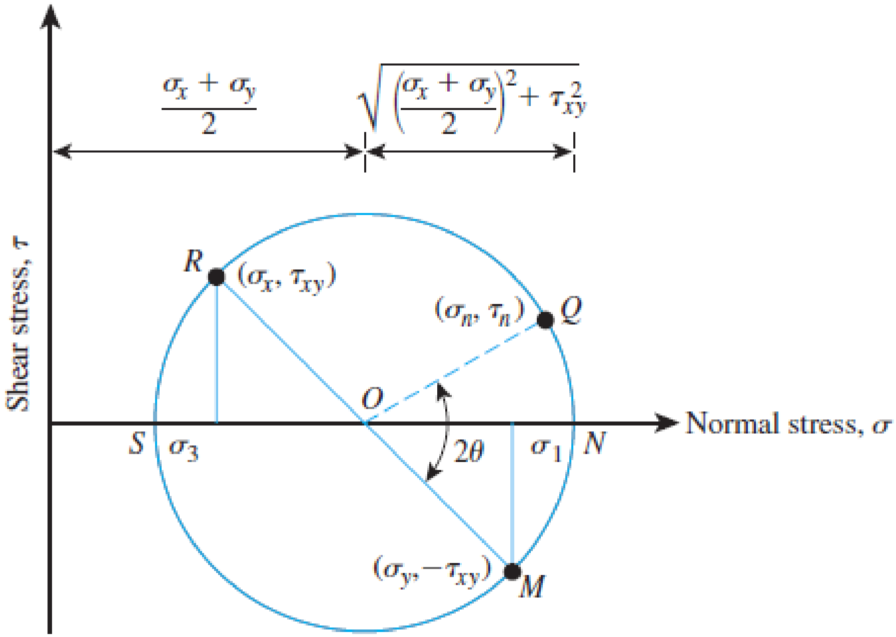


- The points R and M in Figure above represent the stress conditions on plane AD and AB, respectively. O is the point of intersection of the normal stress axis with the line RM.

Pole Method for Finding Stresses on a Plane

There is a unique point on the Mohr's circle called the POLE (origin of planes)

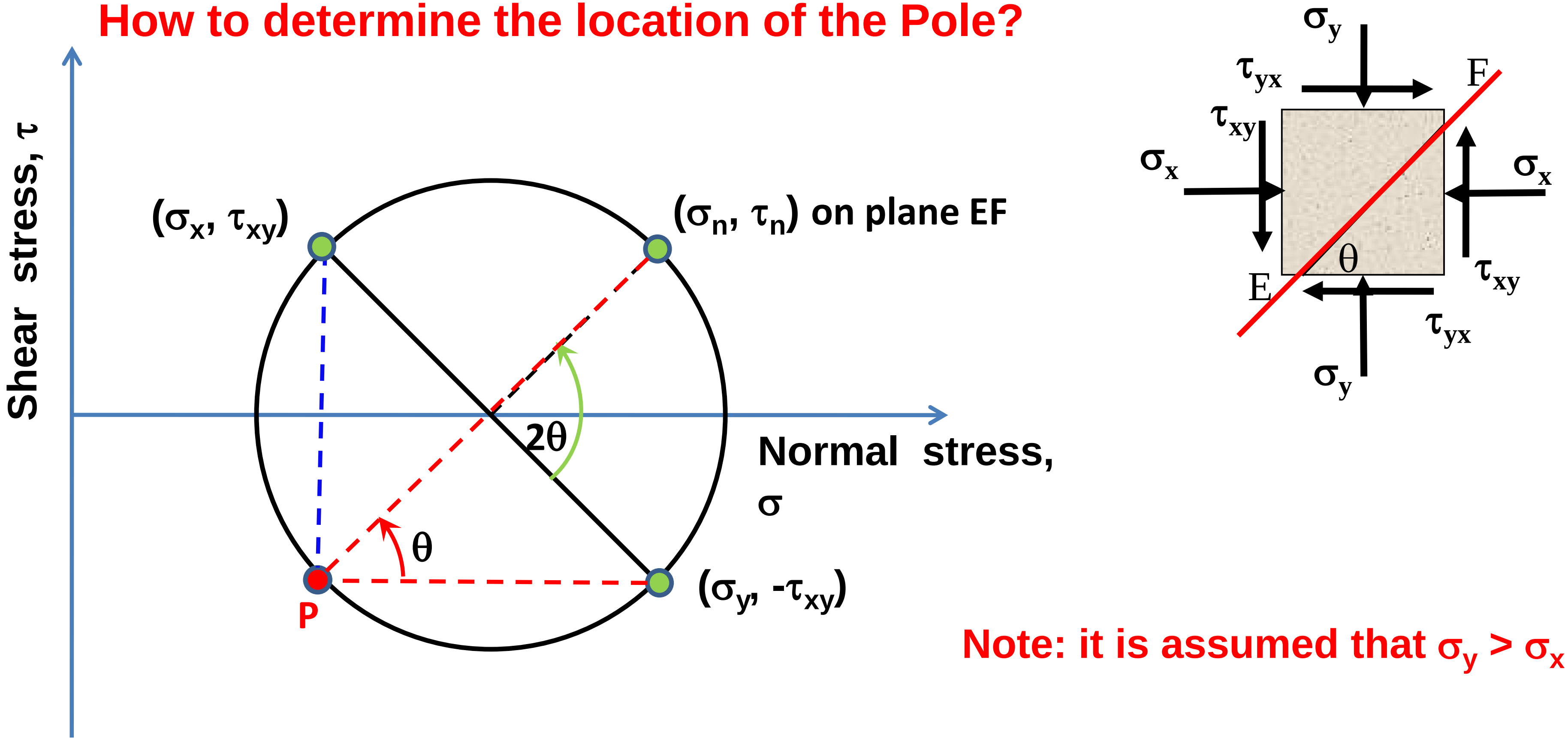
Any straight line drawn through the pole will intersect the Mohr's circle at a point which represents the state of stress on a plane inclined at the same orientation in space as the line.

Draw a line parallel to a plane on which you know the stresses, it will intersect the circle in a point (Pole)

Once the pole is known, the stresses on *any plane* can readily be found by simply drawing a line from the pole parallel to that plane; the coordinates of the point of intersection with the Mohr circle determine the stresses on that plane.

Pole Method for Finding Stresses on a Plane

How to determine the location of the Pole?



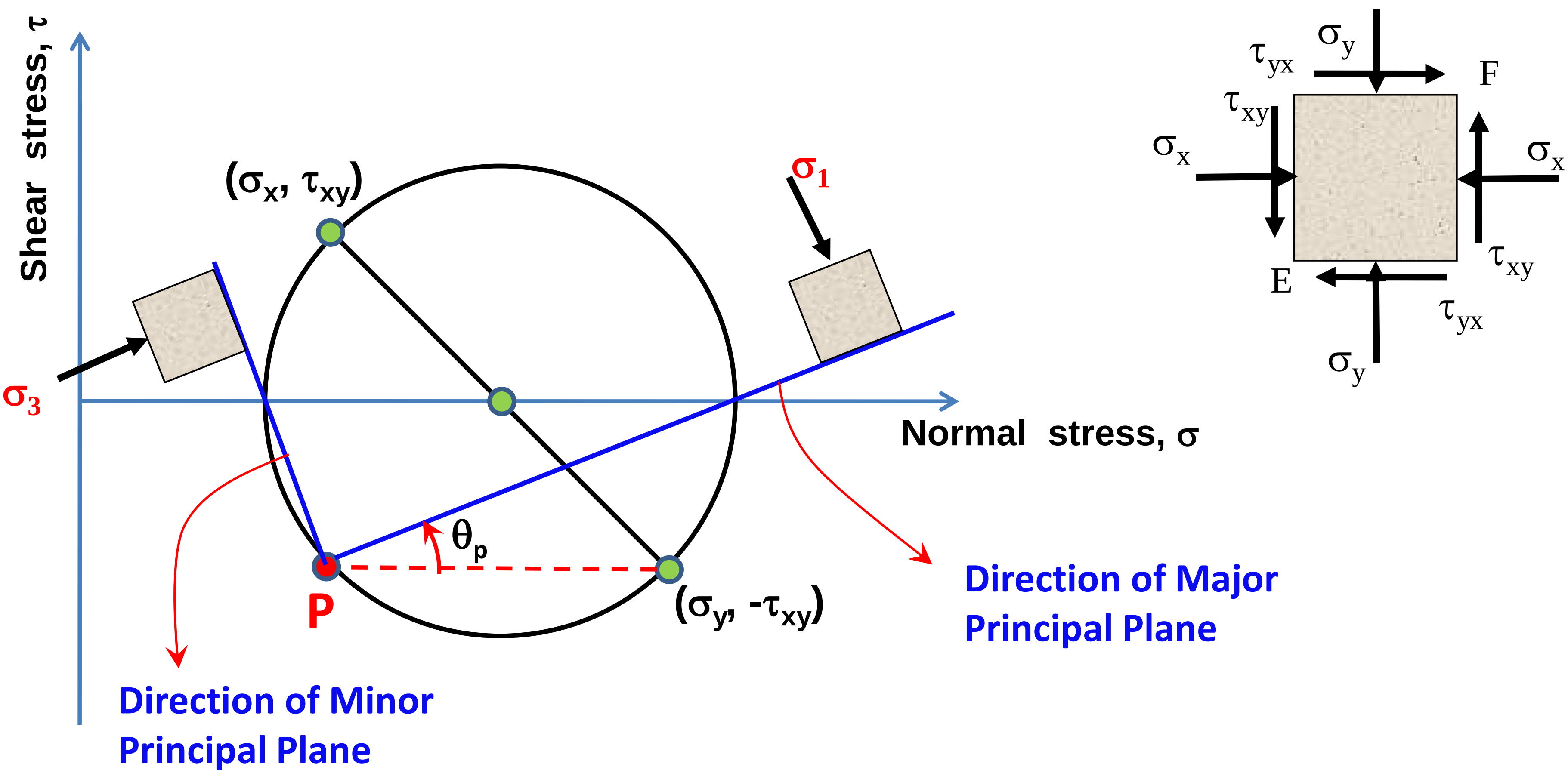
Note: it is assumed that $\sigma_y > \sigma_x$

1. From a point of known stress coordinates and plane orientation, draw a line parallel to the plane where the stress is acting on.

2. The line intersecting the Mohr circle is the pole, P.

Normal and Shear Stress along a Plane

Using the Pole to Determine Principal Planes



Example 10.2

Example 10.2

For the stressed soil element shown in Figure 10.6a, determine

a. Major principal stress
b. Minor principal stress
c. Normal and shear stresses on the plane *DE*

Use the pole method.

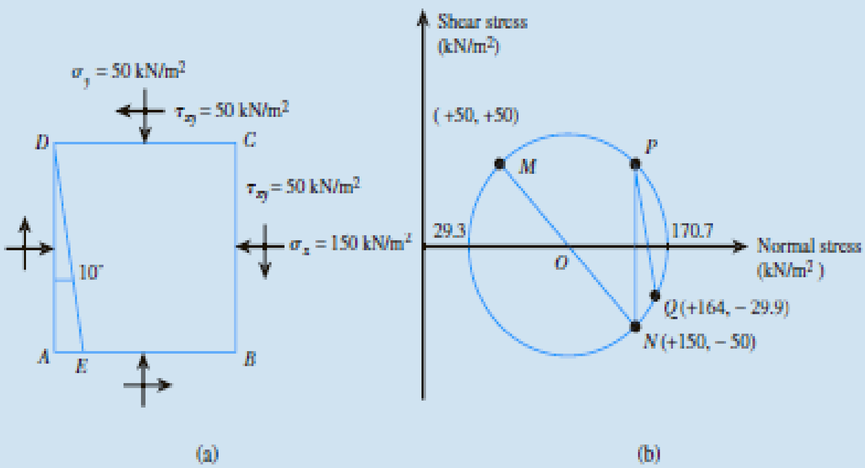


Figure 10.6 (a) Stressed soil element; (b) Mohr's circle for the soil element

Solution

Part a

From Eqs. (10.6) and (10.7),

$$\left.\begin{matrix}\sigma_3\\ \sigma_1\end{matrix}\right\} = \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2}$$

$$= \frac{300 + 120}{2} \pm \sqrt{\left[\frac{300 - 120}{2}\right]^2 + (-40)^2}$$

$$\sigma_1 = \mathbf{308.5\ kN/m^2}$$

$$\sigma_3 = \mathbf{111.5\ kN/m^2}$$

Part b

From Eq. (10.3),

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau \sin 2\theta$$

$$= \frac{300 + 120}{2} + \frac{300 - 120}{2}\cos(2 \times 20) + (-40)\sin(2 \times 20)$$

$$= \mathbf{253.23\ kN/m^2}$$

From Eq. (10.4),

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau \cos 2\theta$$

$$= \frac{300 - 120}{2}\sin(2 \times 20) - (-40)\cos(2 \times 20)$$

$$= \mathbf{88.40\ kN/m^2}$$

Example

For the stresses of the element shown across, determine the normal stress and the shear stress on the plane inclined at α = 35º from the horizontal reference plane.

Solution

- Center of circle $= \frac{\sigma_1+\sigma_3}{2} = \frac{52+12}{2} = 32$ kPa

- Radius of circle $= \frac{\sigma_1-\sigma_3}{2} = \frac{52-12}{2} = 20$ kPa

- Plot the Mohr circle to some convenient scale (See the figure across).
- Establish the POLE
- Draw a line through the POLE inclined at angle α = 35º from the horizontal plane it intersects the Mohr circle at point C.

σ_α = 39 kPa

τ_α = 18.6 kPa

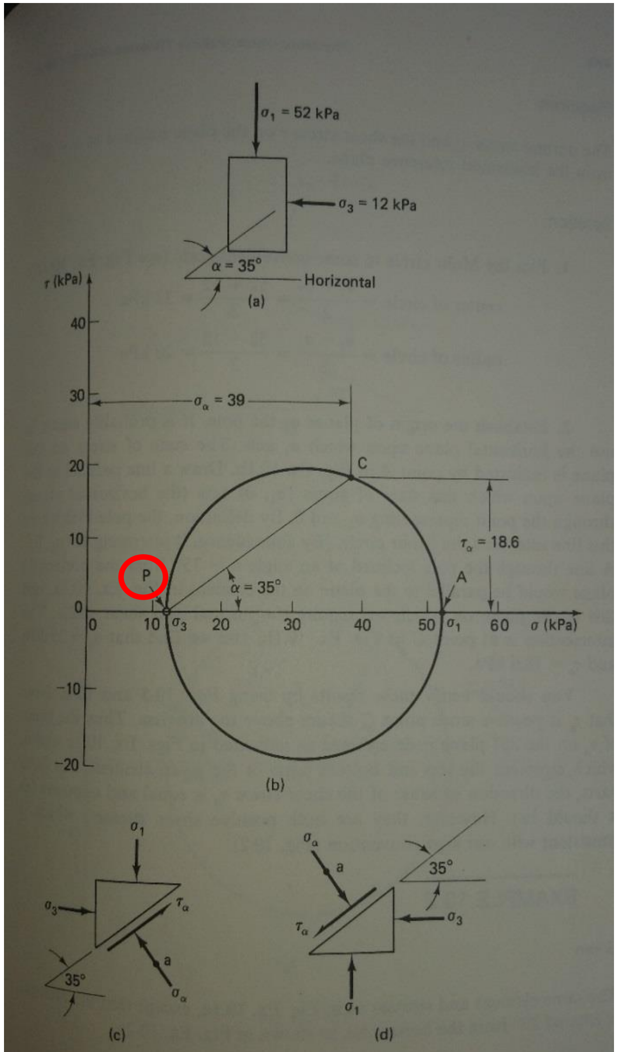


Fig. Ex. 10.1

Example

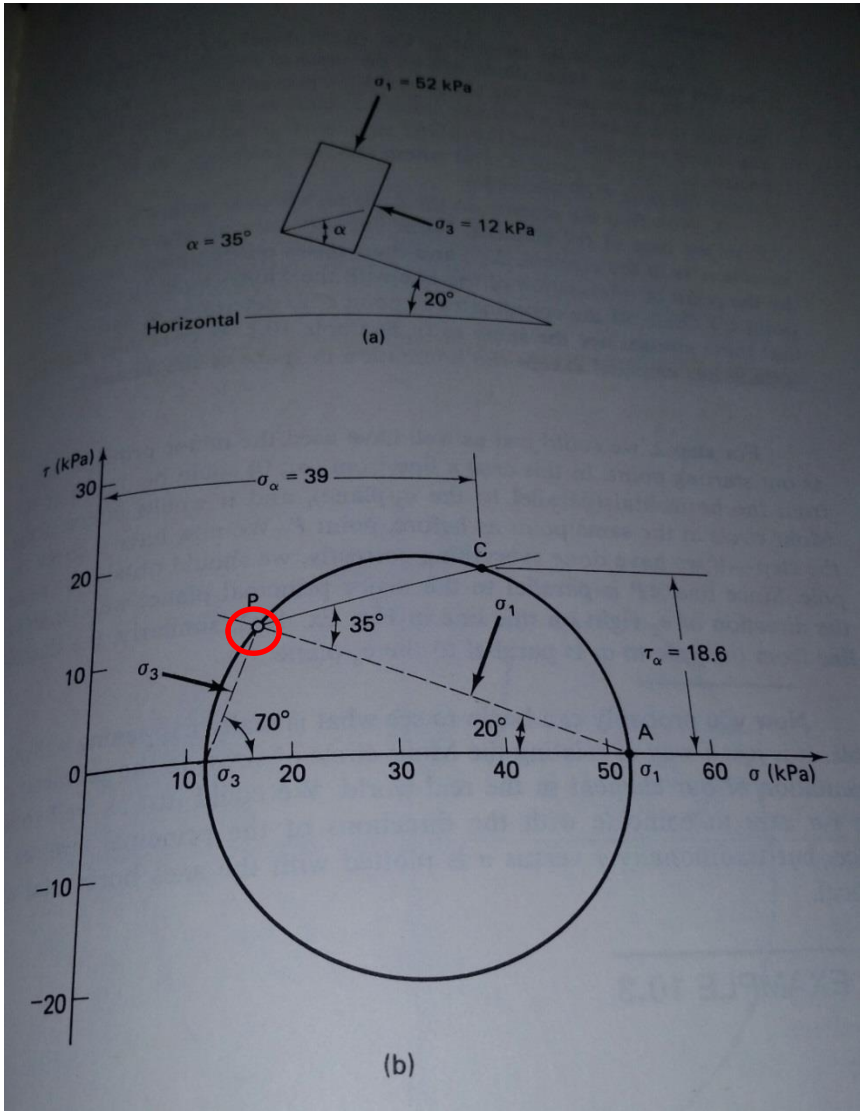
The same element and stresses as in Example 2 except that the element is rotated 20º from the horizontal as shown.

Solution

- **Since the principal stresses are the same, the Mohr circle will be the same as in Example 2.**
- **Establish the POLE.**
- **Draw a line through the POLE inclined at angle $\alpha = 35^o$ from the plane of major principal stress. It intersects the Mohr circle at point C.**
- **The coordinates of point C yields**

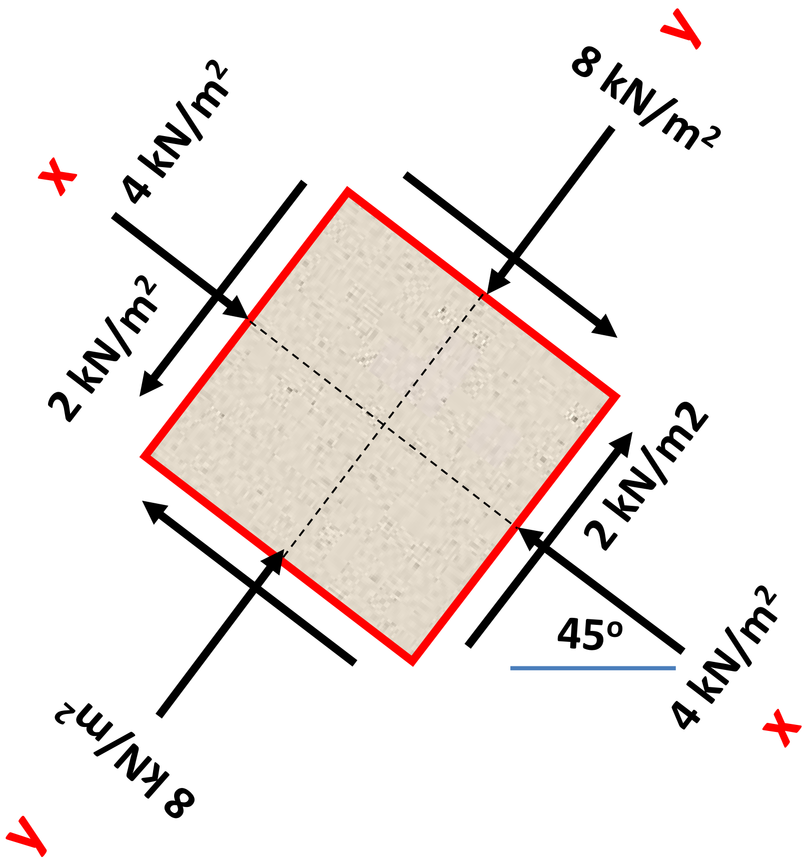
$$\sigma_\alpha = 39 \text{ kPa}$$

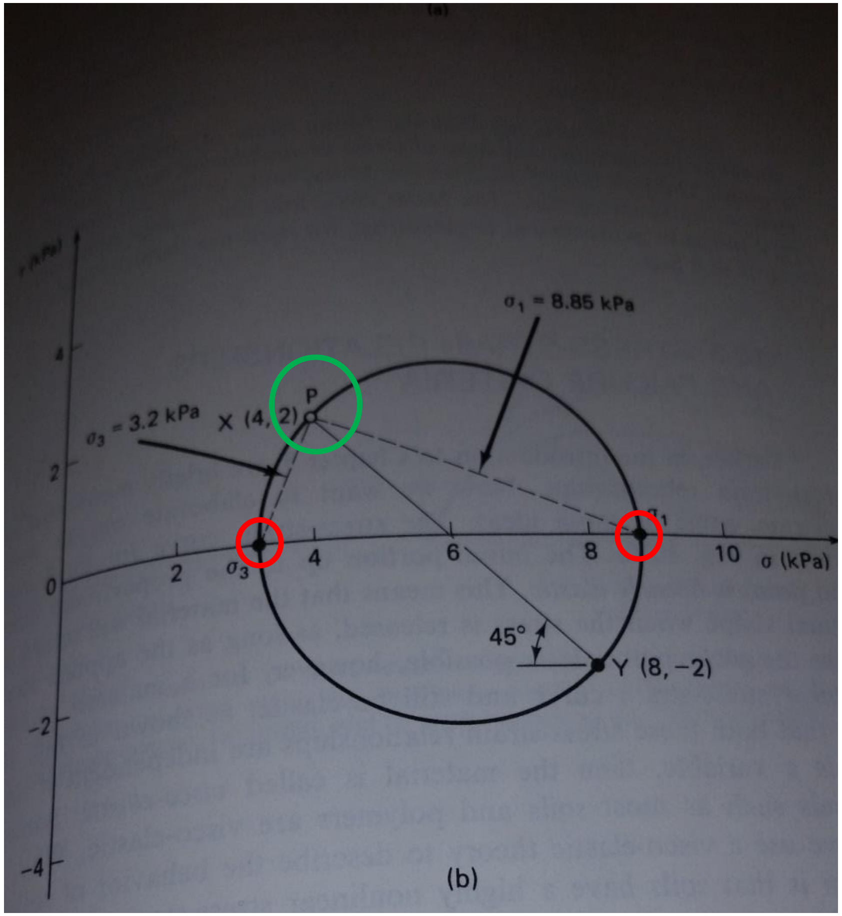
$$\tau_\alpha = 18.6 \text{ kPa}$$



Example

Given the stress shown on the element across. Find the magnitude and direction of the major and minor principal stresses.

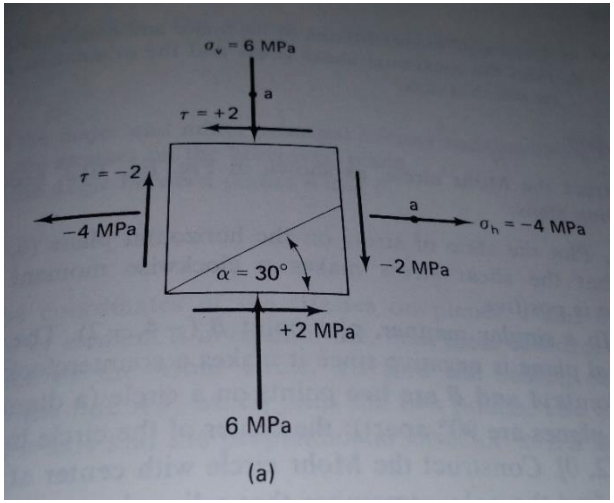




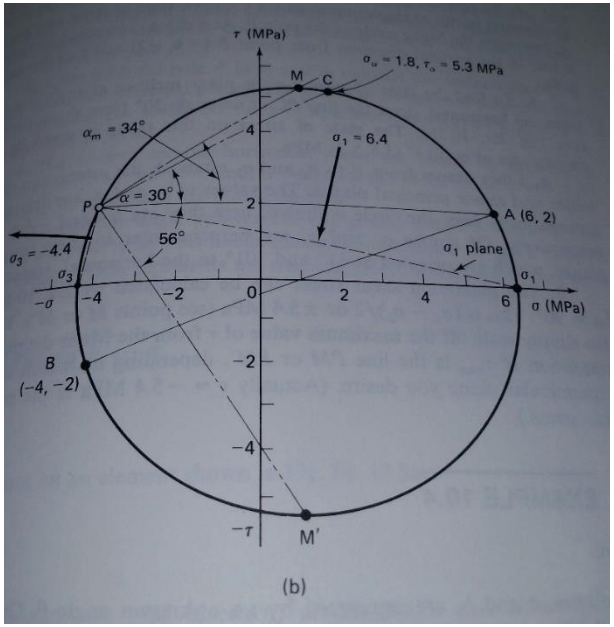
Example

Given the stress shown on the element across.
Required:

a. Evaluate σ_α and τ_α when $\alpha = 30^o$.
b. Evaluate σ_1 and σ_3.
c. Determine the orientation of the major and minor principal planes.
d. Determine the maximum shear stress and the orientation of the plane on which it acts.

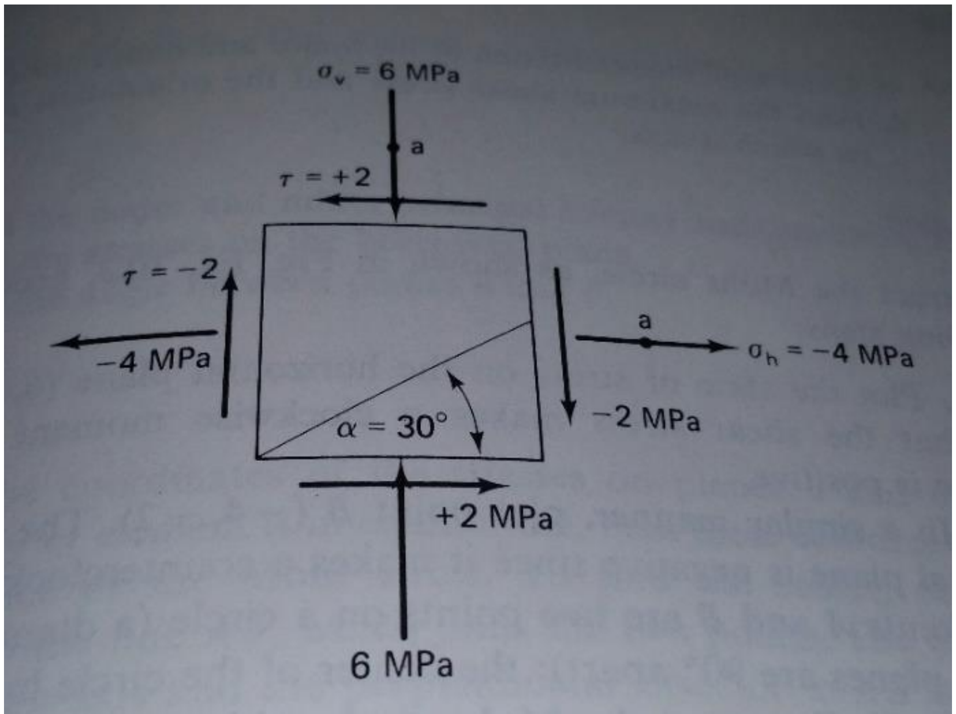


(a)



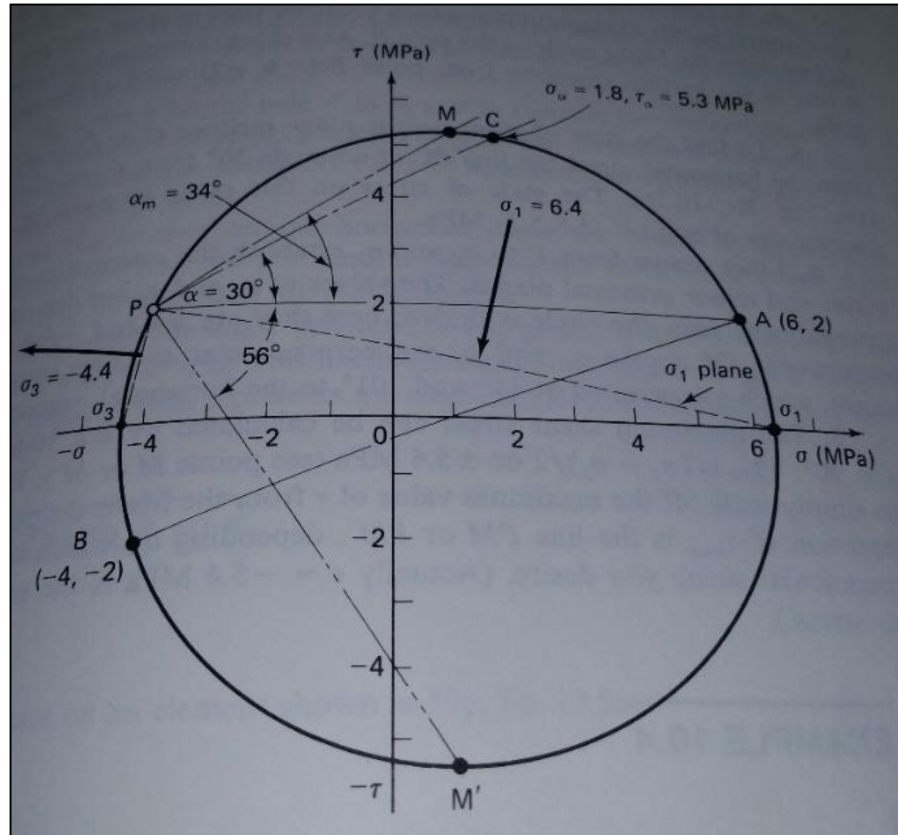
(b)

Example



σv = 6 MPa
a
τ = +2
τ = −2
−4 MPa
a
σh = −4 MPa
−2 MPa
α = 30°
+2 MPa
6 MPa

(a)



τ (MPa)
σα = 1.8, τα = 5.3 MPa
M
C
αm = 34°
σ1 = 6.4
α = 30°
P
A (6, 2)
56°
σ3 = −4.4
σ1 plane
σ3
σ1
−σ
−4
−2
0
2
4
6
σ (MPa)
B
(−4, −2)
−τ
M′

(b)

The end