SHEAR STRENGTH OF SOIL

Chapter 10: Sections

10.2

10.3

Chapter 12: All sections except

12.13

12.14

12.15

12.17

12.18

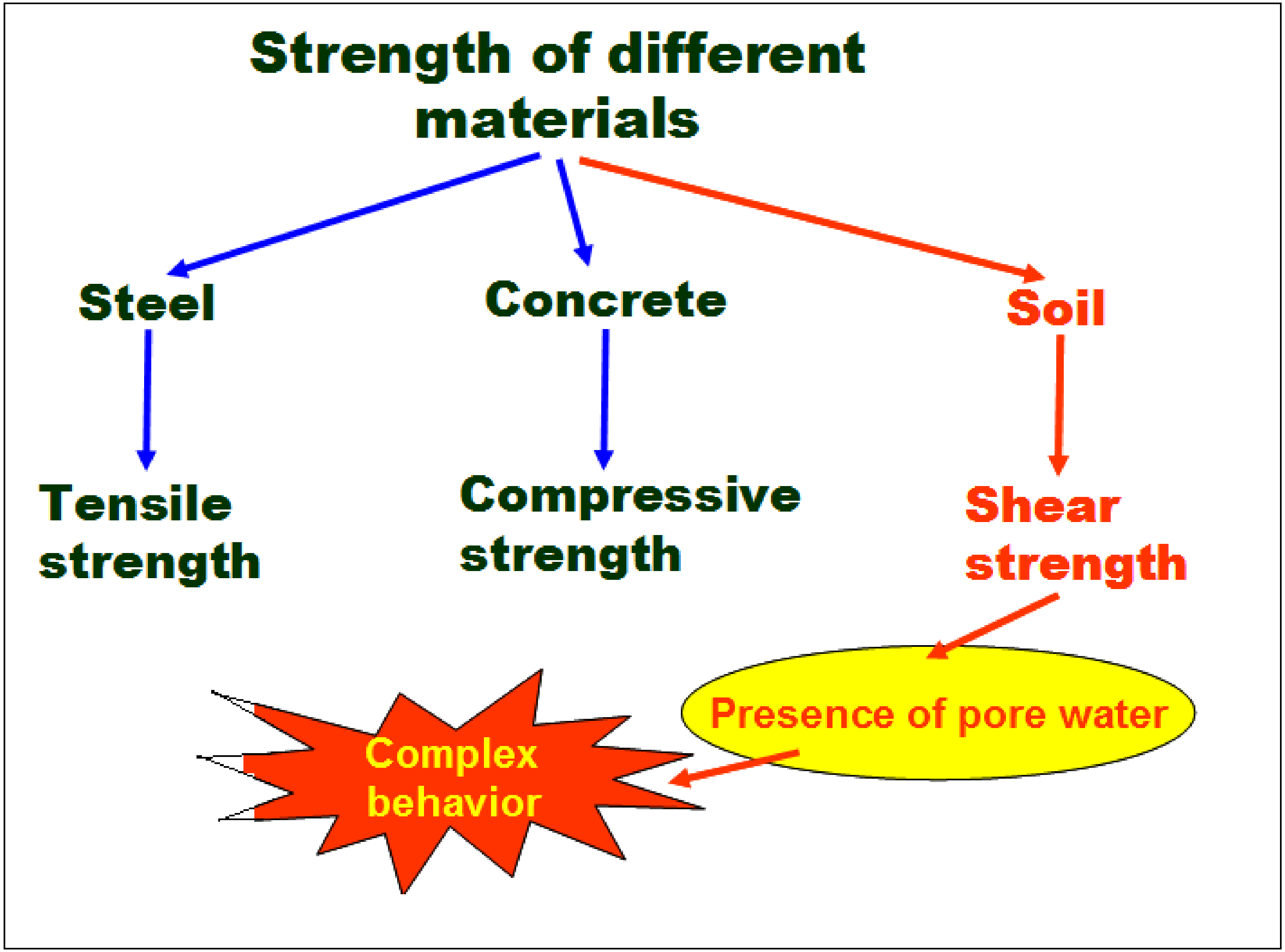
TOPICS

- Introduction
- Components of Shear Strength of Soils
- Normal and Shear Stresses on a Plane
- Mohr-Coulomb Failure Criterion
- Laboratory Shear Strength Testing
  - Direct Shear Test
  - Triaxial Compression Test
  - Unconfined Compression Test
- Field Testing (Vane test)

INTRODUCTION

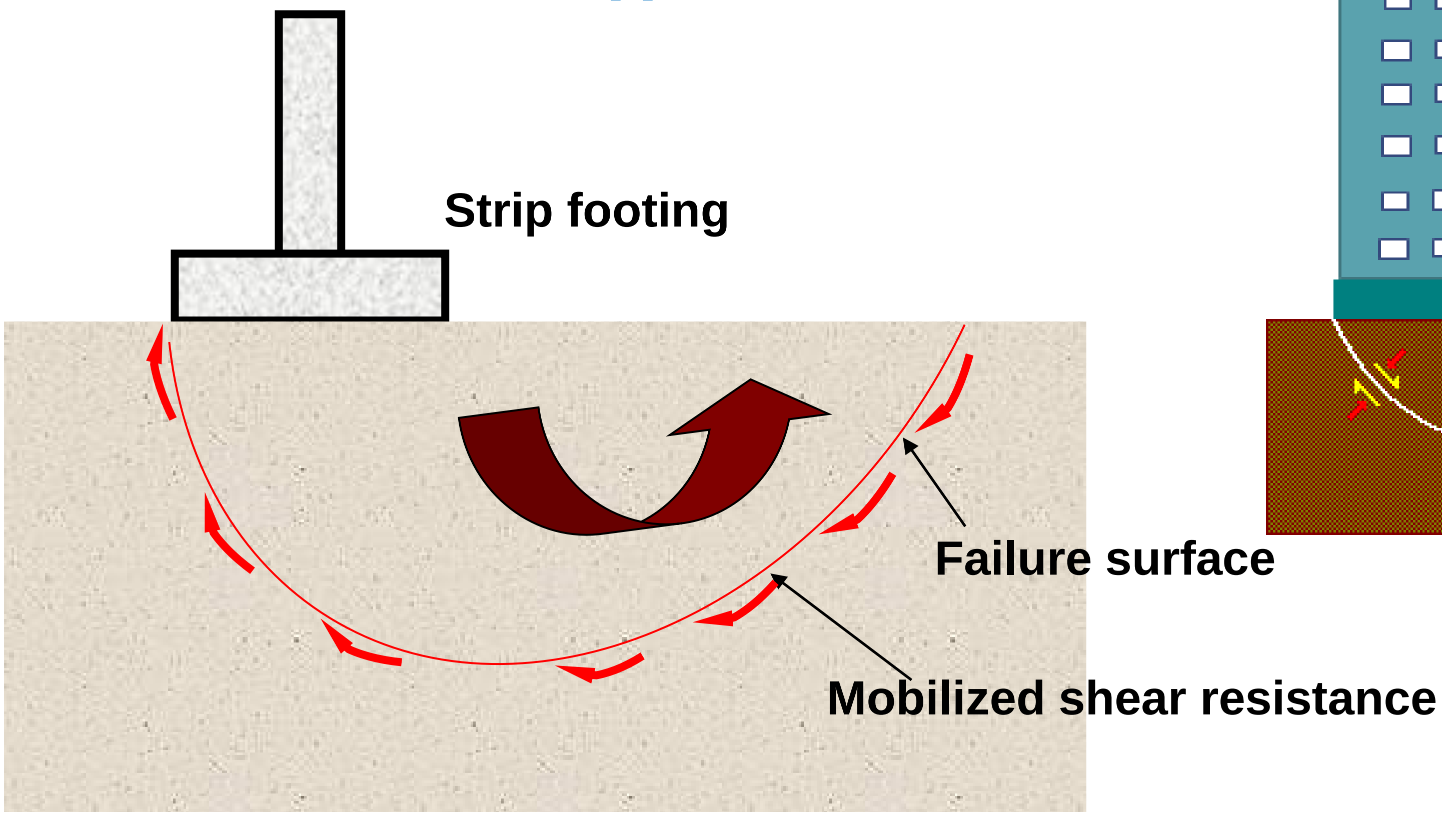
- Soil failure usually occurs in the form of "**shearing**" along internal surface within the soil.
- The ***shear strength*** of a soil mass is the internal resistance per unit area that the soil mass can offer to resist failure and sliding along any plane inside it.
- The safety of any geotechnical structure is dependent on the strength of the soil.
- Shear strength determination is a very important aspect in geotechnical engineering. Understanding shear strength is the basis to analyze soil stability problems like:
  - Bearing capacity.
  - Lateral pressure on earth retaining structures
  - Slope stability

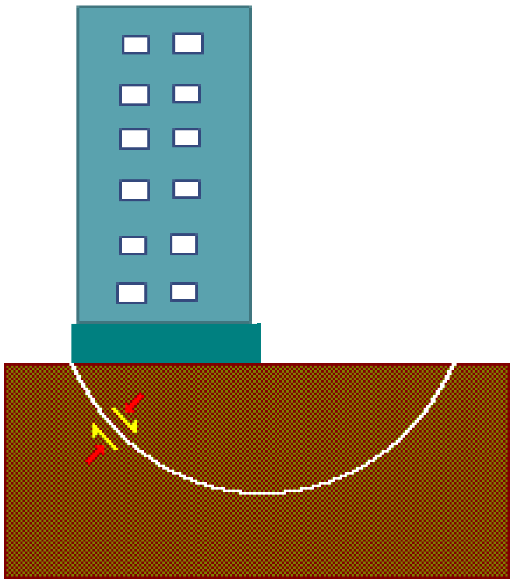
INTRODUCTION



Bearing Capacity Failure

In most foundations and earthwork engineering, failure results from excessive applied shear stresses.

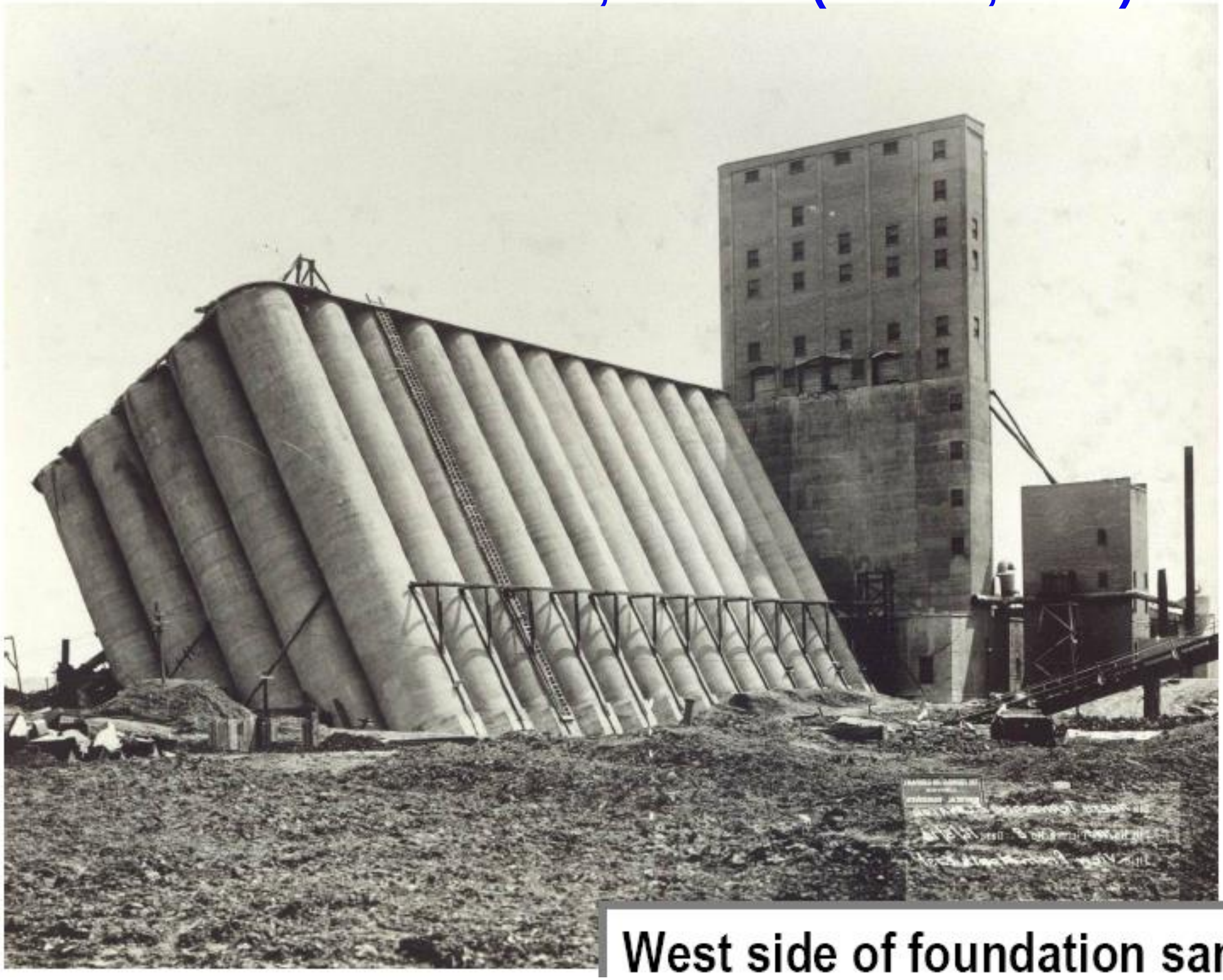




At failure, shear stress along the failure surface (mobilized shear resistance) reaches the shear strength.

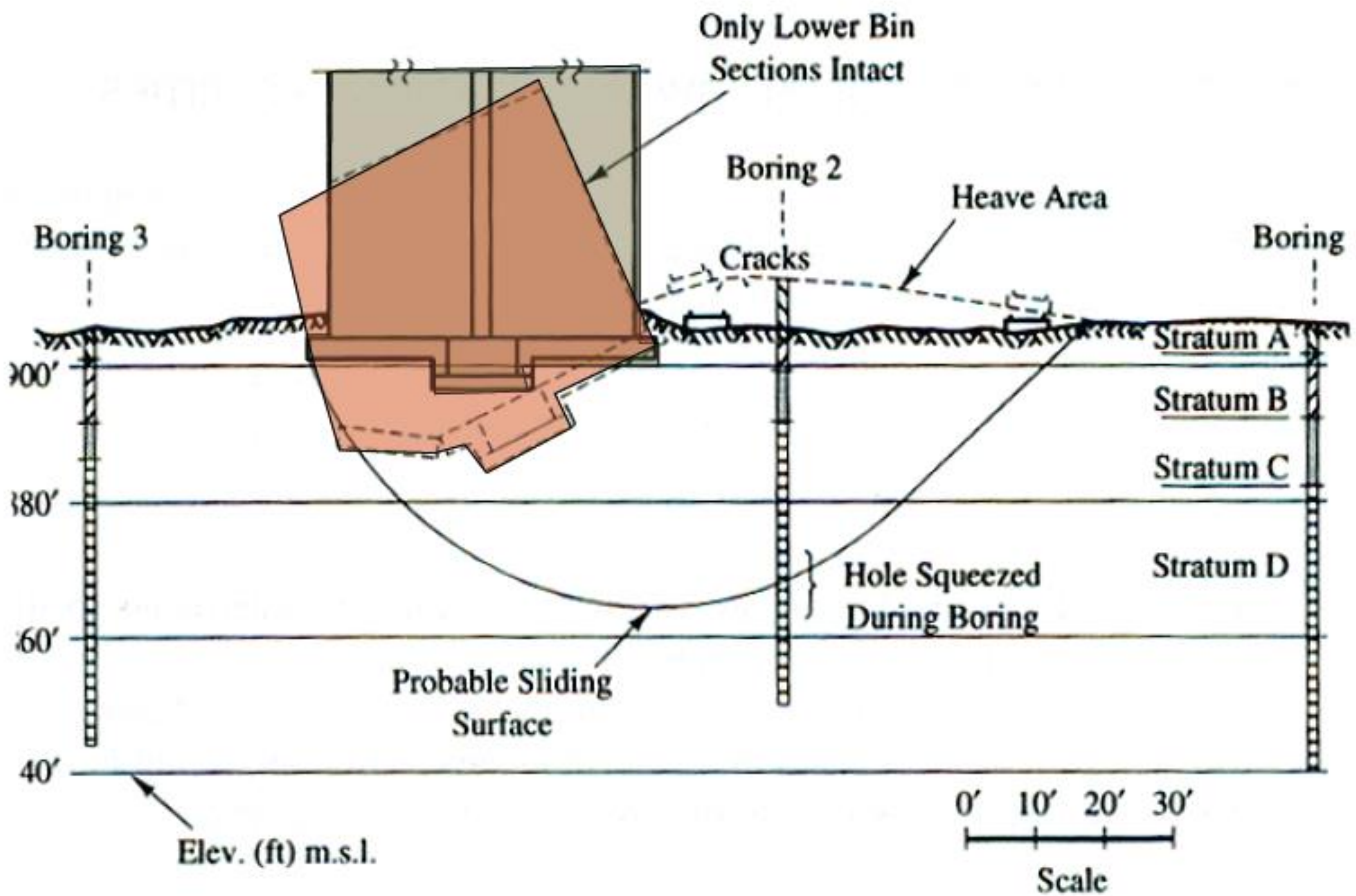
Bearing Capacity Failure

Transcona Grain Elevator, Canada (Oct. 18, 1913)

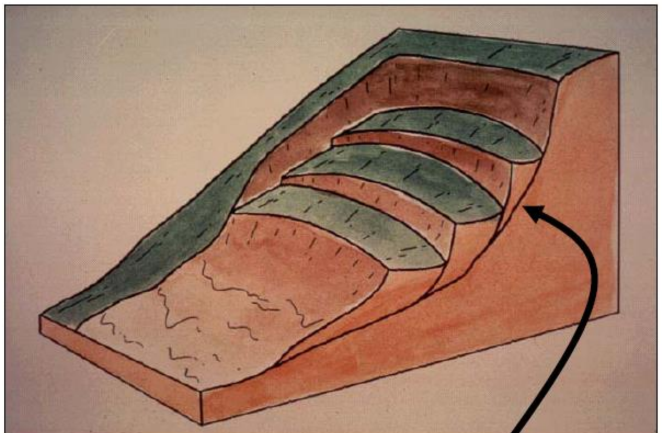


West side of foundation sank 24-ft

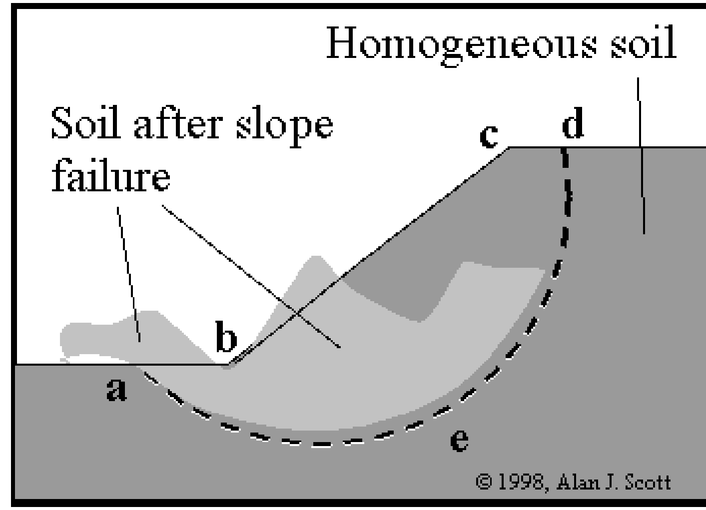
Bearing Capacity Failure

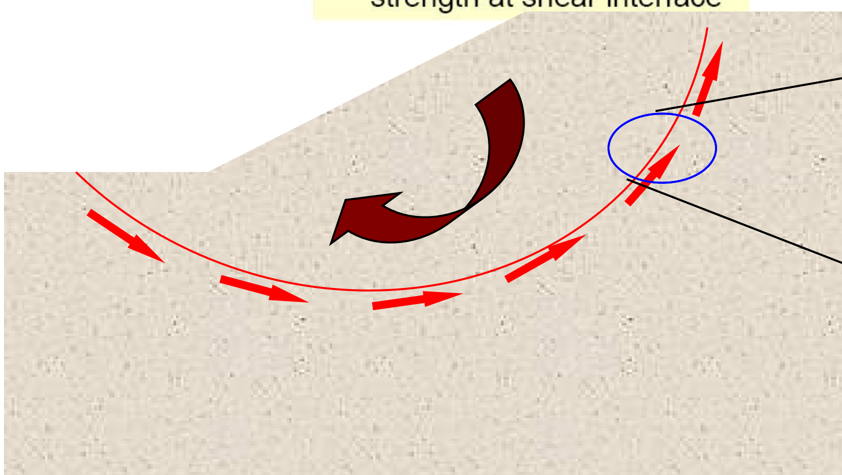


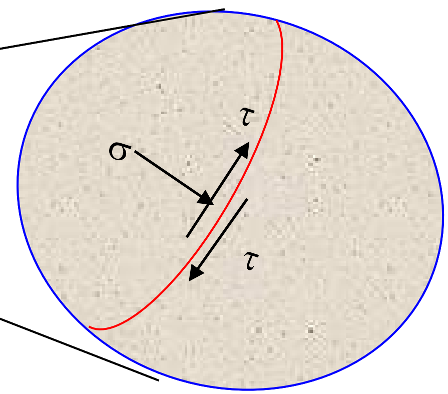
SLOPE FAILURE



Failure due to inadequate strength at shear interface





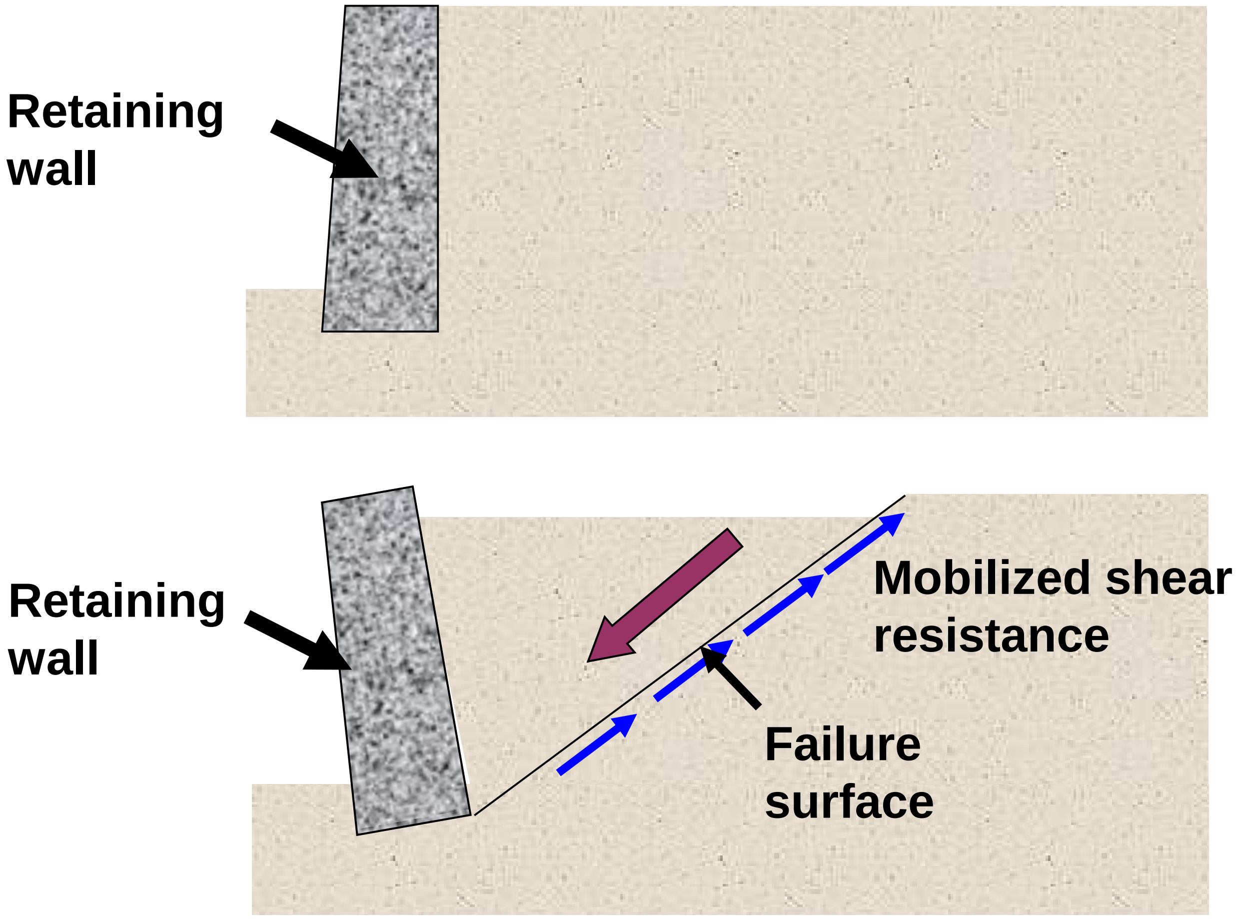


At failure, shear stress along the failure surface (τ) reaches the shear strength (τ_f).

The soil grains slide over each other along the failure surface.

SLOPE FAILURE

Failure of Retaining Walls



At failure, shear stress along the failure surface (mobilized shear resistance) reaches the shear strength.

TOPICS

- Introduction
- **Components of Shear Strength of Soils**
- Normal and Shear Stresses on a Plane
- Mohr-Coulomb Failure Criterion
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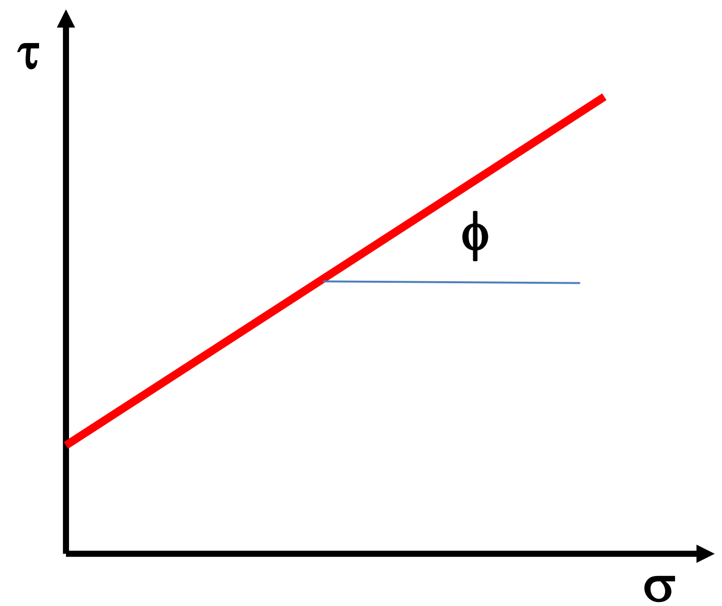
SHEAR STRENGTH OF SOIL

- Coulomb (1776) observed that there was a stress-dependent component of shear strength and a stress-independent component.
- The stress-dependent component is similar to sliding friction in solids described above. The other component is related to the intrinsic COHESION of the material. Coulomb proposed the following equation for shear strength of soil:

$$\tau_f = C + \sigma_n \tan \phi$$

cohesion

Friction



τ_f = shear strength of soil

σ_n = Applied normal stress

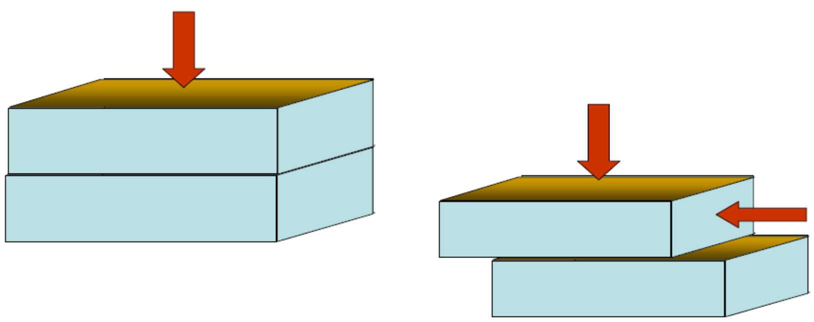
C = Cohesion

ϕ = Angle of internal friction (or angle of shearing resistance)

SHEAR STRENGTH OF SOIL

- **Cohesion (c), is a measure of the forces that cement particles of soils (stress independent).**

- **Internal Friction angle (ϕ), is a measure of the shear strength of soils due to friction (stress dependent).**



- **For granular materials, there is no cohesion between particles**

$$\tau_f = \sigma_n \tan \emptyset$$

SHEAR STRENGTH OF SOIL

Saturated Soils

$$\tau_f = C' + \sigma'_n \tan \emptyset'$$

But from the principle of effective stress

$$\sigma' = \sigma - u$$

Where u is the pore water pressure (p.w.p.)

Then

$$\tau_f = C' + (\sigma_n - u) \tan \emptyset'$$

- **C , ϕ or C' , ϕ' are called strength parameters, and we will discuss various laboratory tests for their determination.**

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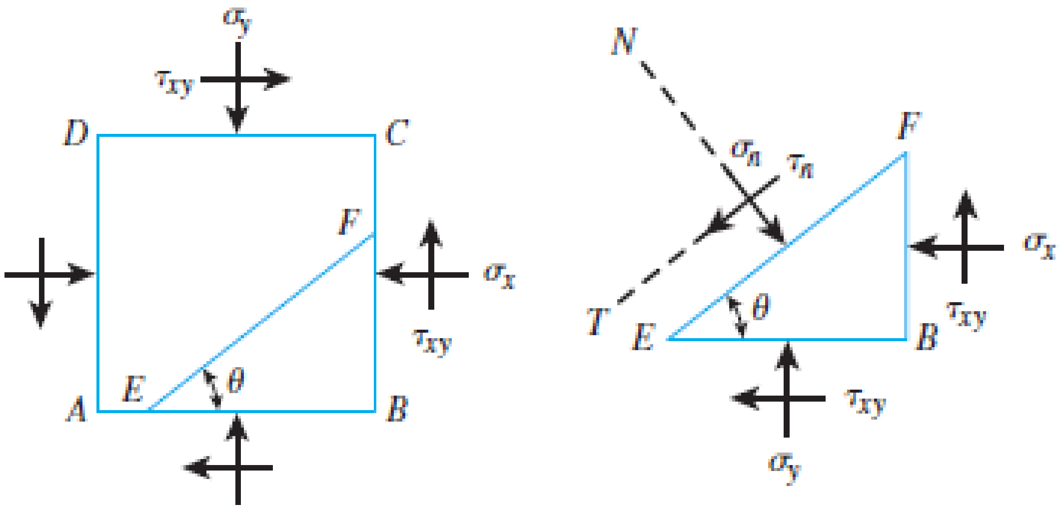
Normal and Shear Stress along a Plane

Chapter 10

- Normal and Shear Stresses along a Plane (Sec. 10.2)
- Pole Method for Finding Stresses along a Plane (Sec. 10.3)

Normal and Shear Stress along a Plane



From geometry

$$\overline{EB} = \overline{EF}\cos\theta$$

$$\overline{FB} = \overline{EF}\sin\theta$$

Sign Convention	Normal Stresses	Shear Stresses
Positive	Compression	Counter clockwise rotation
Negative	Tension	Clockwise rotation

- Note that for convenience our sign convention has **compressive forces and stresses positive** because most normal stresses in geotechnical engineering are compressive.
- **These conventions are the *opposite* of those normally assumed in structural mechanics.**

Normal and Shear Stress along a Plane

$$\sum F_N = 0$$

$$\sigma_n * (\overline{EF}) - \sigma_x \sin\theta * (\overline{EF}\sin\theta) - \sigma_y \cos\theta * (\overline{EF}\cos\theta)$$

$$-\tau_{xy}\cos\theta * (\overline{EF}\sin\theta) - \tau_{xy}\sin\theta * (\overline{EF}\cos\theta) = 0$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

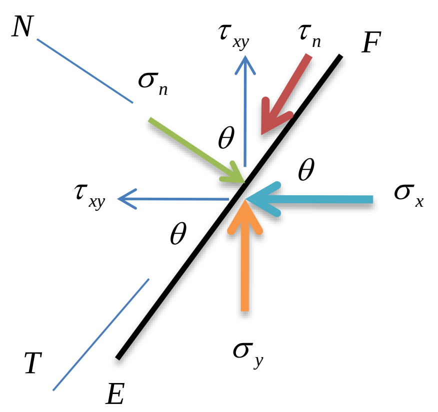
Similarly,

$$\sum F_T = 0$$

$$\tau_n * (\overline{EF}) - \sigma_y \sin\theta * (\overline{EF}\cos\theta) + \sigma_x \cos\theta * (\overline{EF}\sin\theta)$$

$$-\tau_{xy}\sin\theta * (\overline{EF}\sin\theta) + \tau_{xy}\cos\theta * (\overline{EF}\cos\theta) = 0$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$



$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

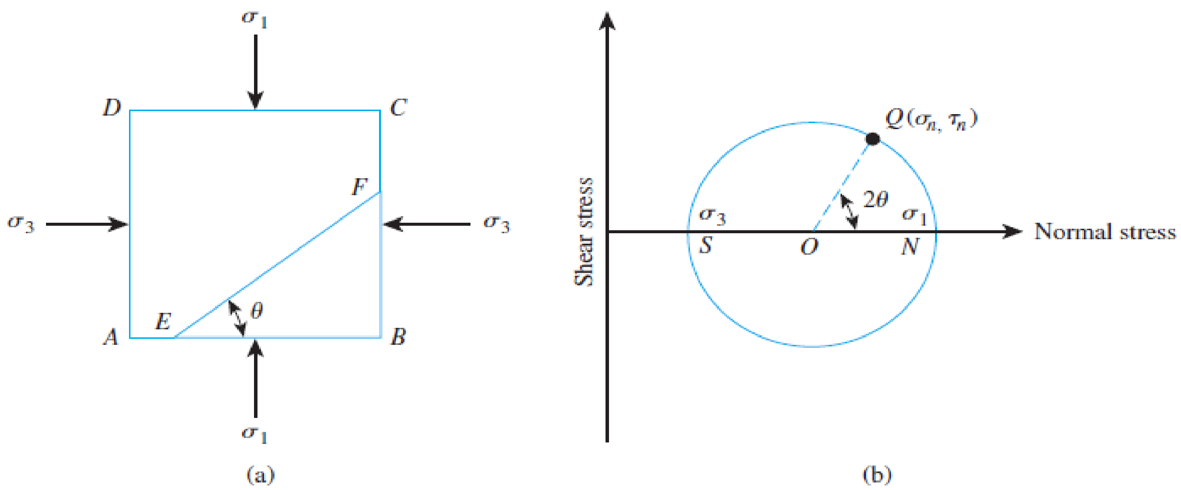
Principal Planes & Principal Stresses

Principal Planes

Planes on which the shear stress is equal to zero

Principal Stresses

Normal stress acting on the principal planes



$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta$$

$$\tau_n = \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta$$

Principal Stresses

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad (1)$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \quad (2)$$

$$For \ \ \tau_n == 0$$

$$0 = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$\tan 2\theta_p = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2\tau_{xy}}{\sigma_y - \sigma_x} \quad (3)$$

For any given values of σ_x , σ_y and τ_{xy}

Equation (3) will give two values of θ

which are 90 degrees apart

Two principal planes 90 degrees apart

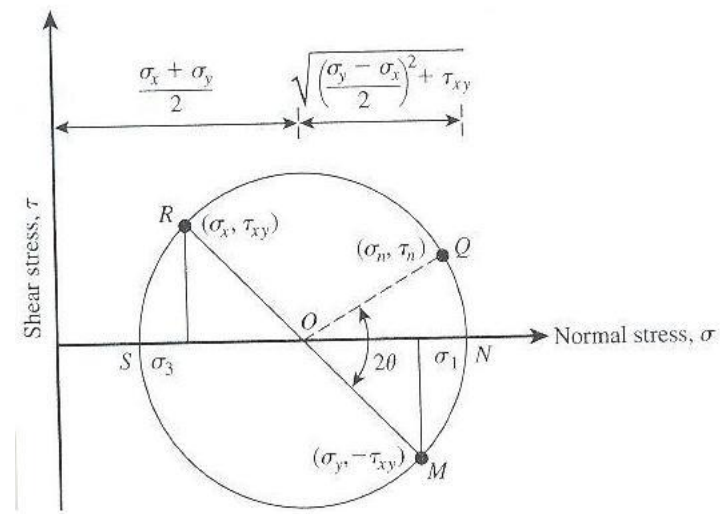
Substitute eq (3) into eq (1)

Major Principal Stress

$$\sigma_n = \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

Minor Principal Stress

$$\sigma_n = \sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$



Example 10.1

Example 10.1

A soil element is shown in Figure 10.4. The magnitudes of stresses are $\sigma_x = 120$ kN/m², $\tau = 40$ kN/m², $\sigma_y = 300$ kN/m², and $\theta = 20°$. Determine

a. Magnitudes of the principal stresses.
b. Normal and shear stresses on plane AB. Use Eqs. (10.3), (10.4), (10.6), and (10.7).

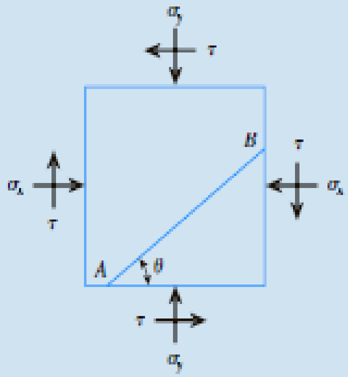


Figure 10.4 Soil element with stresses acting on it

Solution

Part a

From Eqs. (10.6) and (10.7),

$$\left.\begin{matrix}\sigma_3 \\ \sigma_1\end{matrix}\right\} = \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2}$$

$$= \frac{300 + 120}{2} \pm \sqrt{\left[\frac{300 - 120}{2}\right]^2 + (-40)^2}$$

$$\sigma_1 = \mathbf{308.5\ kN/m^2}$$

$$\sigma_3 = \mathbf{111.5\ kN/m^2}$$

Part b

From Eq. (10.3),

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau \sin 2\theta$$

$$= \frac{300 + 120}{2} + \frac{300 - 120}{2}\cos(2 \times 20) + (-40)\sin(2 \times 20)$$

$$= \mathbf{253.23\ kN/m^2}$$

From Eq. (10.4),

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau\cos 2\theta$$

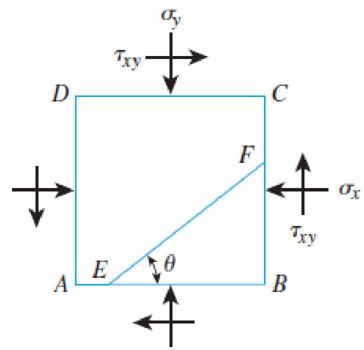
$$= \frac{300 - 120}{2}\sin(2 \times 20) - (-40)\cos(2 \times 20)$$

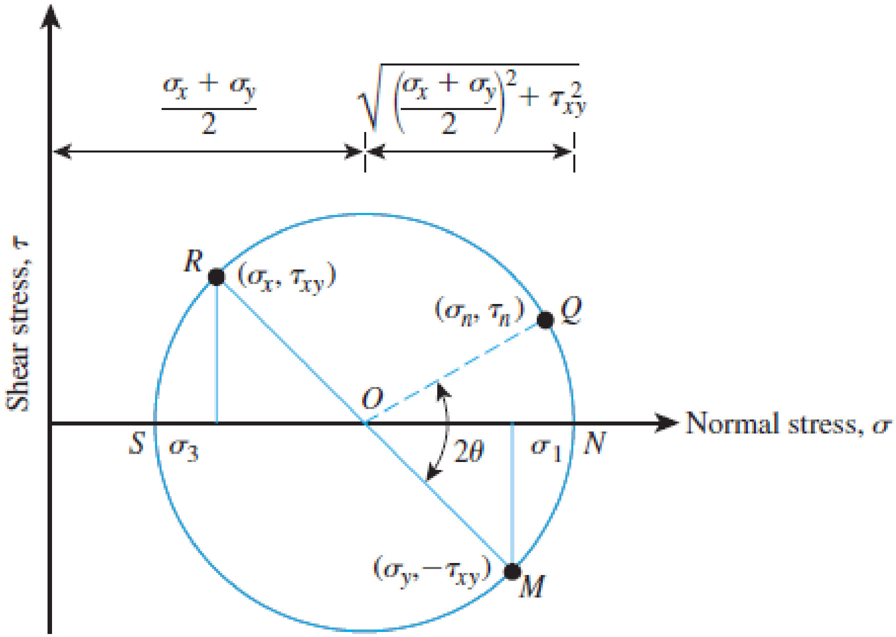
$$= \mathbf{88.40\ kN/m^2}$$

Construction of Mohr's Circle

1. Plot σ_y, τ_{xy} as point M
2. Plot σ_x, τ_{xy} as point R
3. Connect M and R
4. Draw a circle of diameter of the line RM about the point where the line RM crosses the horizontal axis (denote this as point O)

Sign Convention	Normal Stresses	Shear Stresses
Positive	Compression	Counter clockwise rotation
Negative	Tension	Clockwise rotation





- The points R and M in Figure above represent the stress conditions on plane AD and AB, respectively. O is the point of intersection of the normal stress axis with the line RM.

Pole Method for Finding Stresses on a Plane

There is a unique point on the Mohr's circle called the POLE (origin of planes)

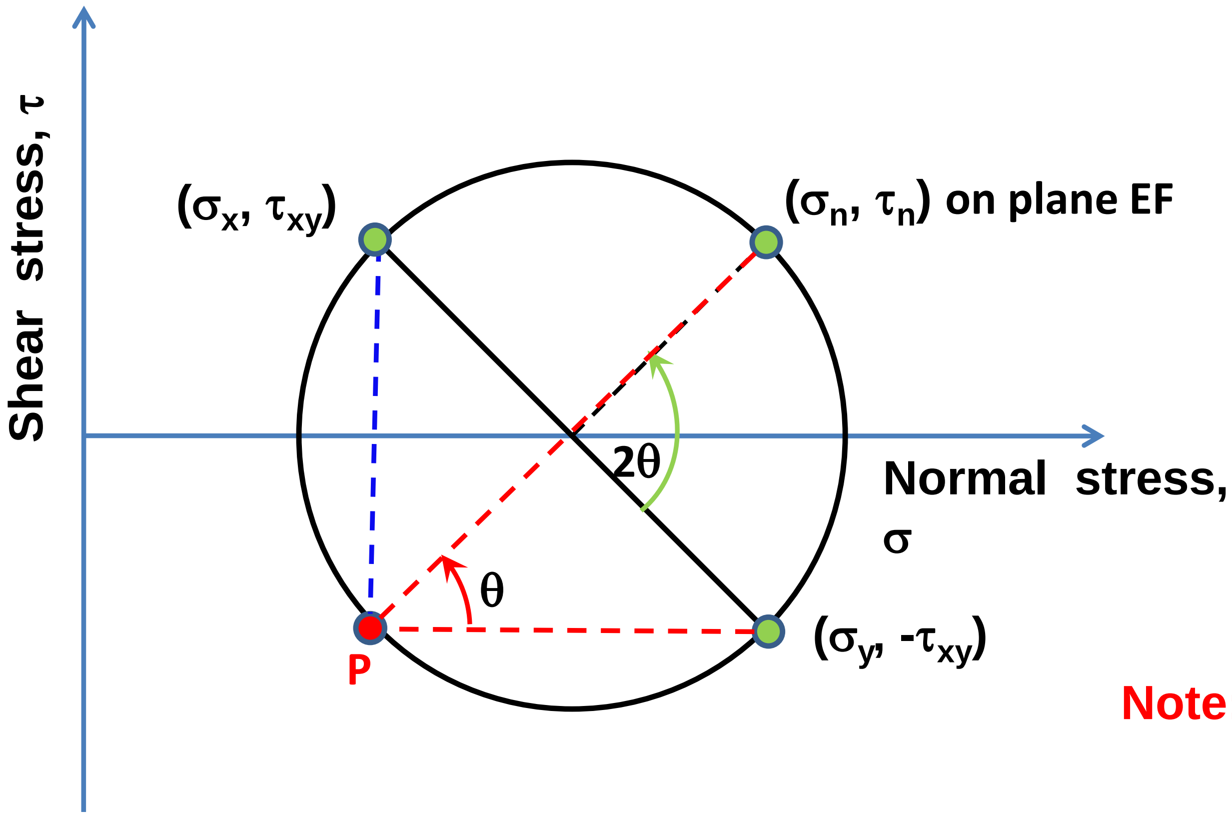
Any straight line drawn through the pole will intersect the Mohr's circle at a point which represents the state of stress on a plane inclined at the same orientation in space as the line.

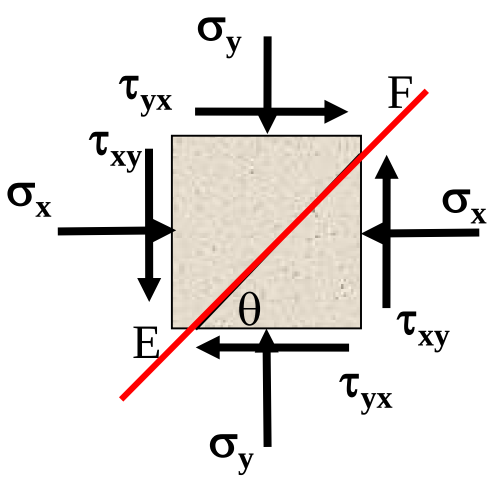
Draw a line parallel to a plane on which you know the stresses, it will intersect the circle in a point (Pole)

Once the pole is known, the stresses on *any plane* can readily be found by simply drawing a line from the pole parallel to that plane; the coordinates of the point of intersection with the Mohr circle determine the stresses on that plane.

Pole Method for Finding Stresses on a Plane

How to determine the location of the Pole?





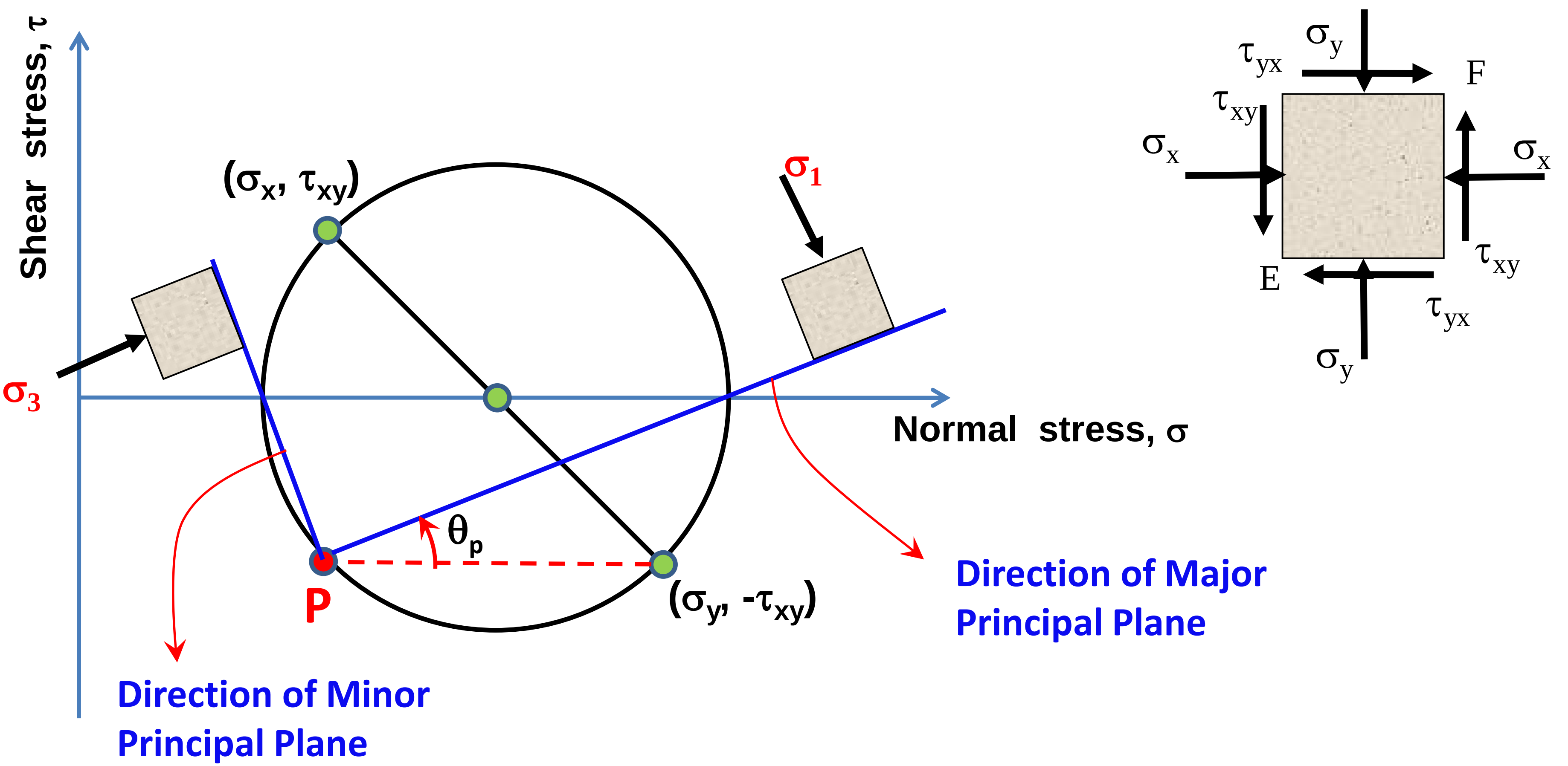
Note: it is assumed that $\sigma_y > \sigma_x$

1. From a point of known stress coordinates and plane orientation, draw a line parallel to the plane where the stress is acting on.

2. The line intersecting the Mohr circle is the pole, P.

Normal and Shear Stress along a Plane

Using the Pole to Determine Principal Planes



Example 10.2

Example 10.2

For the stressed soil element shown in Figure 10.6a, determine

a. Major principal stress
b. Minor principal stress
c. Normal and shear stresses on the plane *DE*

Use the pole method.

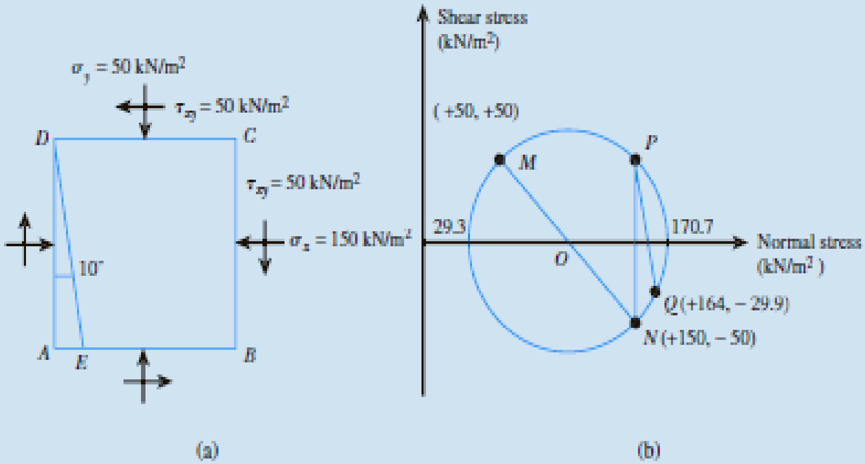


Figure 10.6 (a) Stressed soil element; (b) Mohr's circle for the soil element

Solution

Part a

From Eqs. (10.6) and (10.7),

$$\left.\begin{matrix}\sigma_3\\ \sigma_1\end{matrix}\right\} = \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2}$$

$$= \frac{300 + 120}{2} \pm \sqrt{\left[\frac{300 - 120}{2}\right]^2 + (-40)^2}$$

$$\sigma_1 = \mathbf{308.5\ kN/m^2}$$

$$\sigma_3 = \mathbf{111.5\ kN/m^2}$$

Part b

From Eq. (10.3),

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau \sin 2\theta$$

$$= \frac{300 + 120}{2} + \frac{300 - 120}{2}\cos(2 \times 20) + (-40)\sin(2 \times 20)$$

$$= \mathbf{253.23\ kN/m^2}$$

From Eq. (10.4),

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau \cos 2\theta$$

$$= \frac{300 - 120}{2}\sin(2 \times 20) - (-40)\cos(2 \times 20)$$

$$= \mathbf{88.40\ kN/m^2}$$

Example

For the stresses of the element shown across, determine the normal stress and the shear stress on the plane inclined at α = 35° from the horizontal reference plane.

Solution

- Center of circle $= \dfrac{\sigma_1+\sigma_3}{2} = \dfrac{52+12}{2} = 32$ kPa

- Radius of circle $= \dfrac{\sigma_1-\sigma_3}{2} = \dfrac{52-12}{2} = 20$ kPa

- Plot the Mohr circle to some convenient scale (See the figure across).
- Establish the POLE
- Draw a line through the POLE inclined at angle α = 35° from the horizontal plane it intersects the Mohr circle at point C.

σ_α = 39 kPa

τ_α = 18.6 kPa

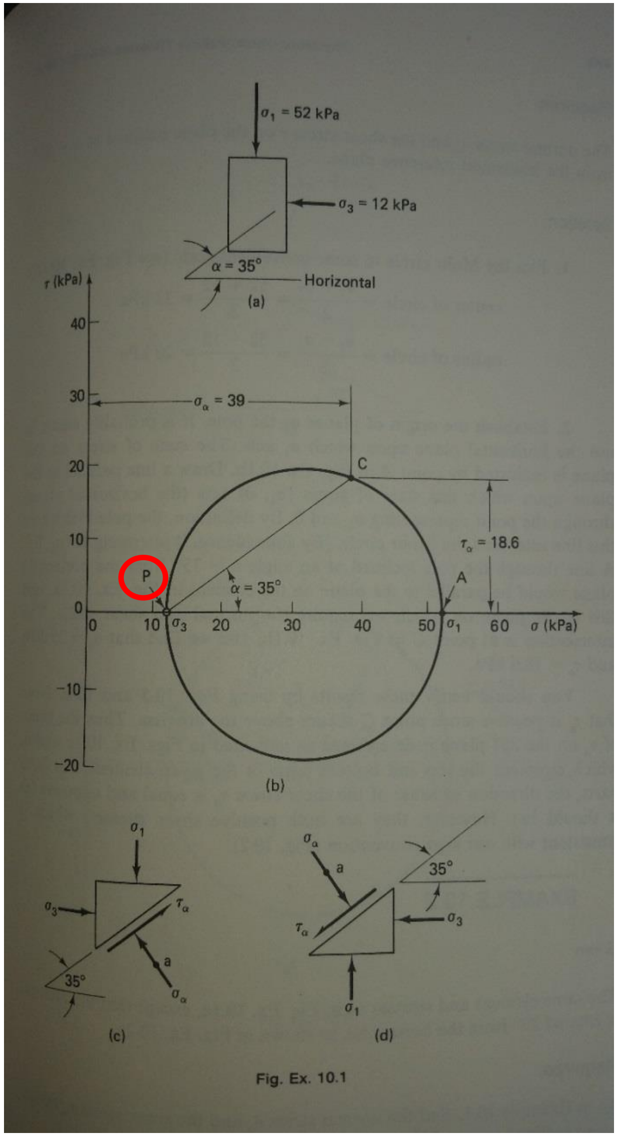


Fig. Ex. 10.1

Example

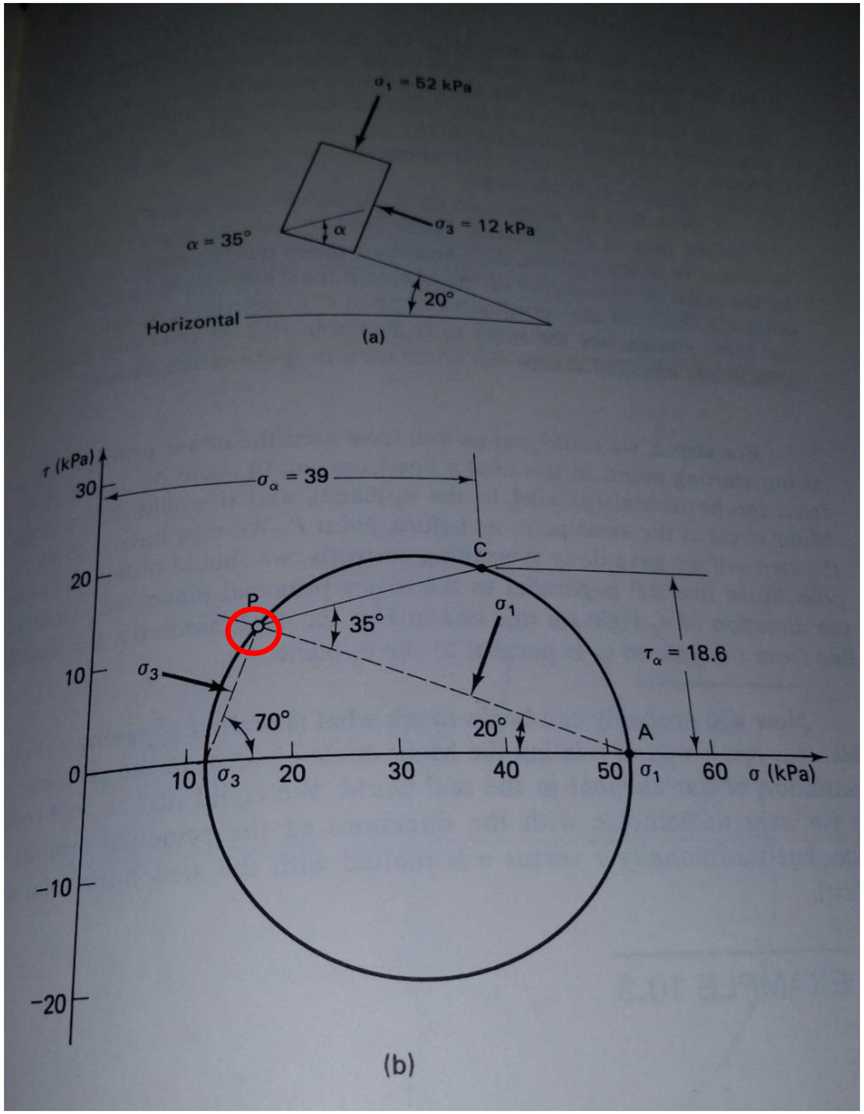
The same element and stresses as in Example 2 except that the element is rotated 20º from the horizontal as shown.

Solution

- **Since the principal stresses are the same, the Mohr circle will be the same as in Example 2.**
- **Establish the POLE.**
- **Draw a line through the POLE inclined at angle $\alpha = 35^o$ from the plane of major principal stress. It intersects the Mohr circle at point C.**
- **The coordinates of point C yields**

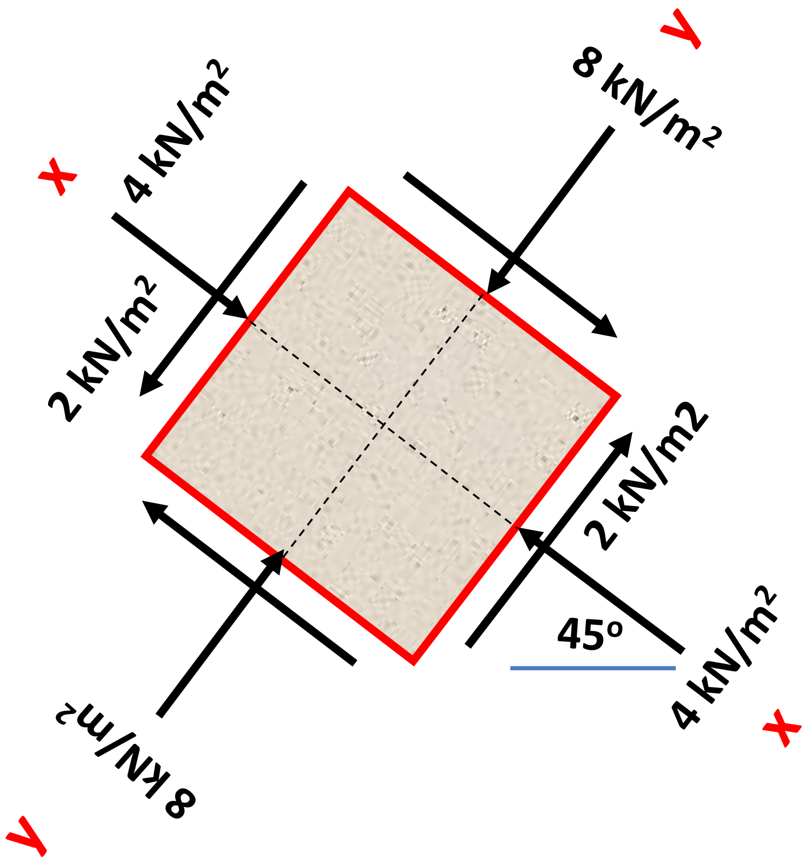
$$\sigma_\alpha = 39 \text{ kPa}$$

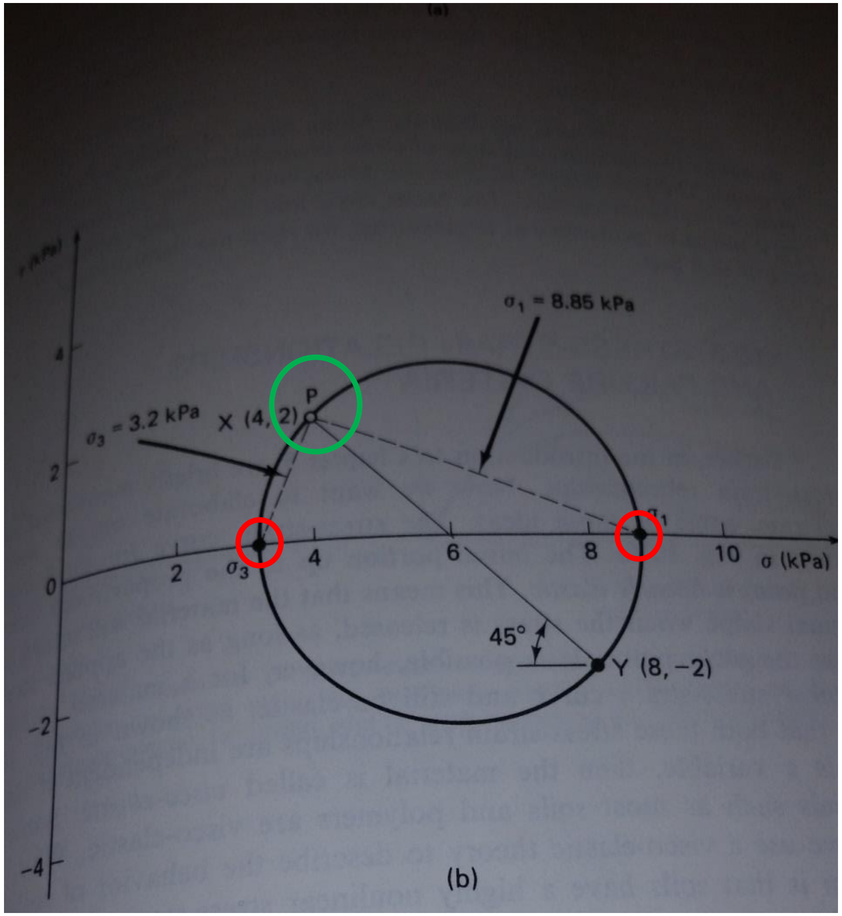
$$\tau_\alpha = 18.6 \text{ kPa}$$



Example

Given the stress shown on the element across. Find the magnitude and direction of the major and minor principal stresses.





Example

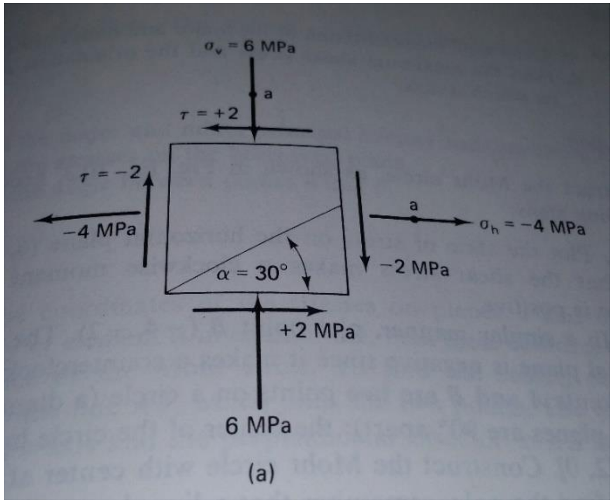
Given the stress shown on the element across. **Required:**

a. Evaluate σ_α and τ_α when $\alpha = 30^o$.

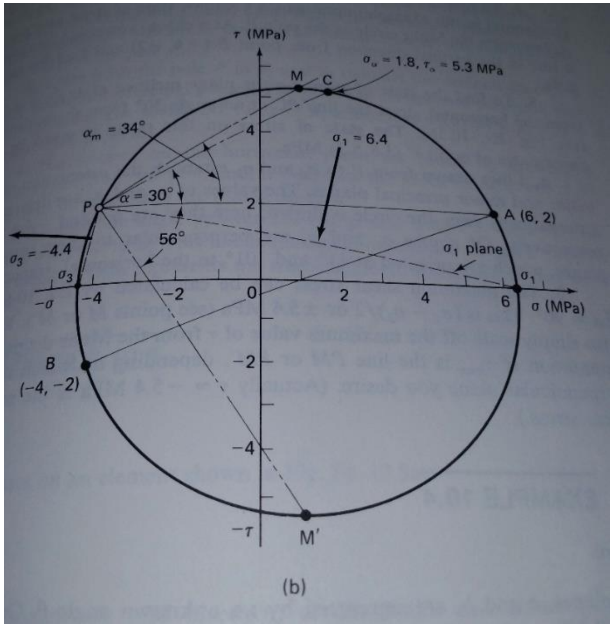
b. Evaluate σ_1 and σ_3.

c. Determine the orientation of the major and minor principal planes.

d. Determine the maximum shear stress and the orientation of the plane on which it acts.

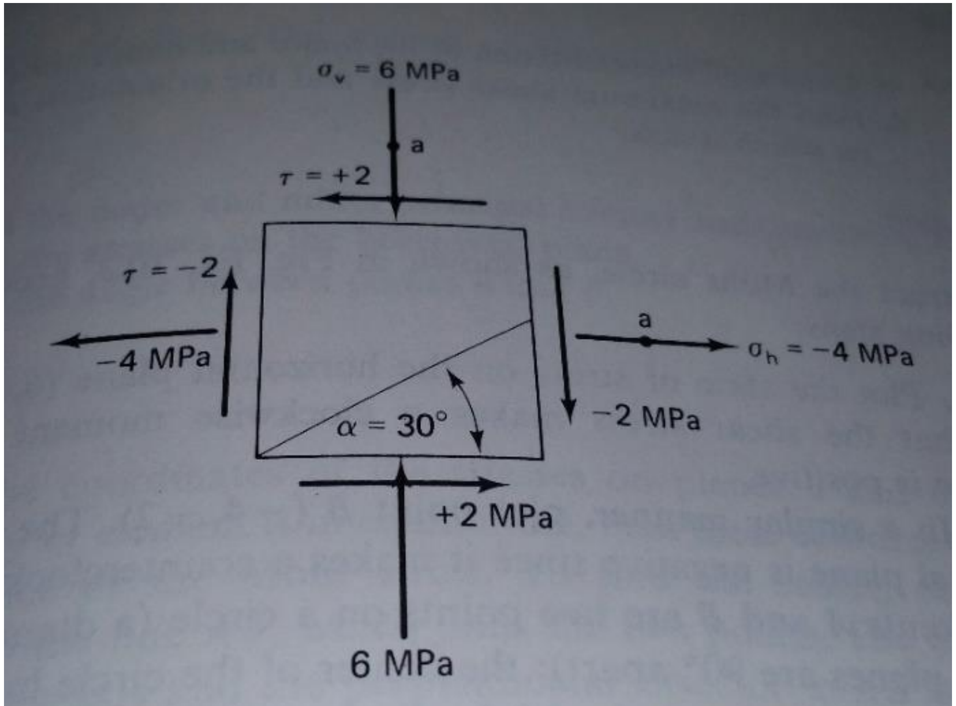


(a)



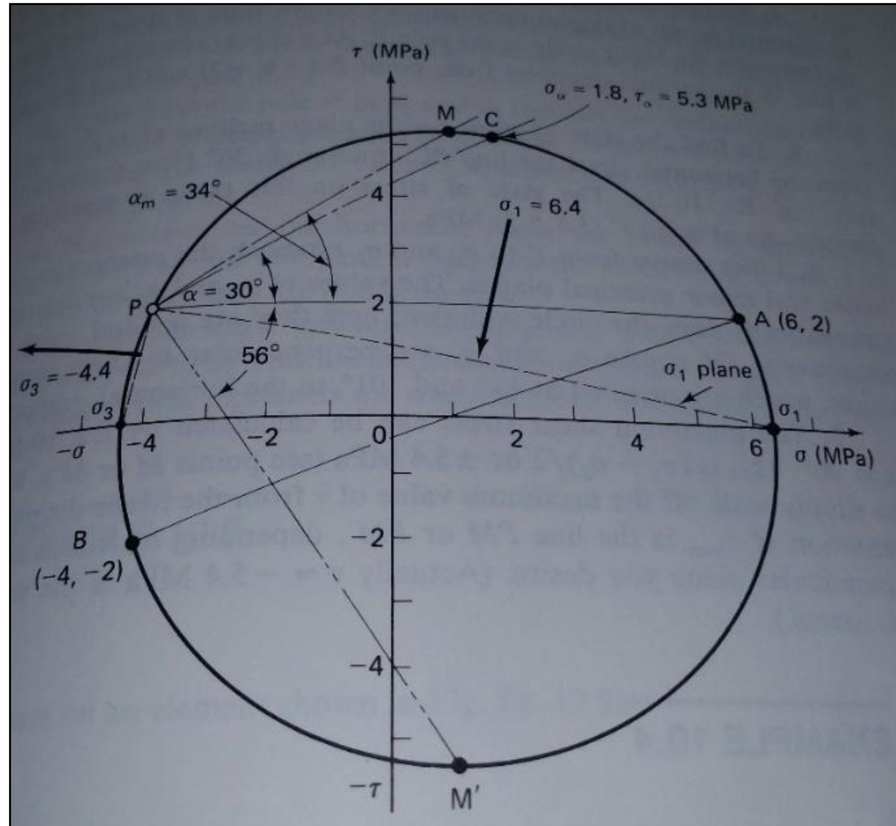
(b)

Example



σv = 6 MPa
a
τ = +2
τ = −2
−4 MPa
a
σh = −4 MPa
−2 MPa
α = 30°
+2 MPa
6 MPa

(a)



τ (MPa)
σα = 1.8, τα = 5.3 MPa
M
C
αm = 34°
σ1 = 6.4
α = 30°
P
A (6, 2)
56°
σ3 = −4.4
σ1 plane
σ3
σ1
−σ
−4
−2
0
2
4
6
σ (MPa)
B
(−4, −2)
−2
−4
−τ
M′

(b)

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FAILURE CRITERIA FOR SOILS

- There are many ways of defining failure in real materials, or put another way, there are many **failure criteria**.
- Various theories are available for different engineering materials. However, no one is general for all materials
- The one generally accepted and used for soil is **Mohr Theory of Failure**.
- According to Coulomb relation for shear strength

$$\tau_f = C + \sigma_n \tan \emptyset$$

Where σ_n is the normal stress on the failure plane.

From the previous slides we new know how to estimate σ_n but we still need to know the plane of failure.

FAILURE CRITERIA FOR SOILS

- **What is failure Plane?**

•Is it the plane of a maximum shear stress **No**

•Is it the plane of a maximum normal stress

• (major principal stress) **No**

•It is the plane when shear stress reaches some unique function of the normal stress on that plane: $\tau_f = f(\sigma_f)$

•It is the plane , where the failure angle measured

•From the plane of the major principal stress is

$$\theta_f = 45 + \frac{\phi}{2}$$

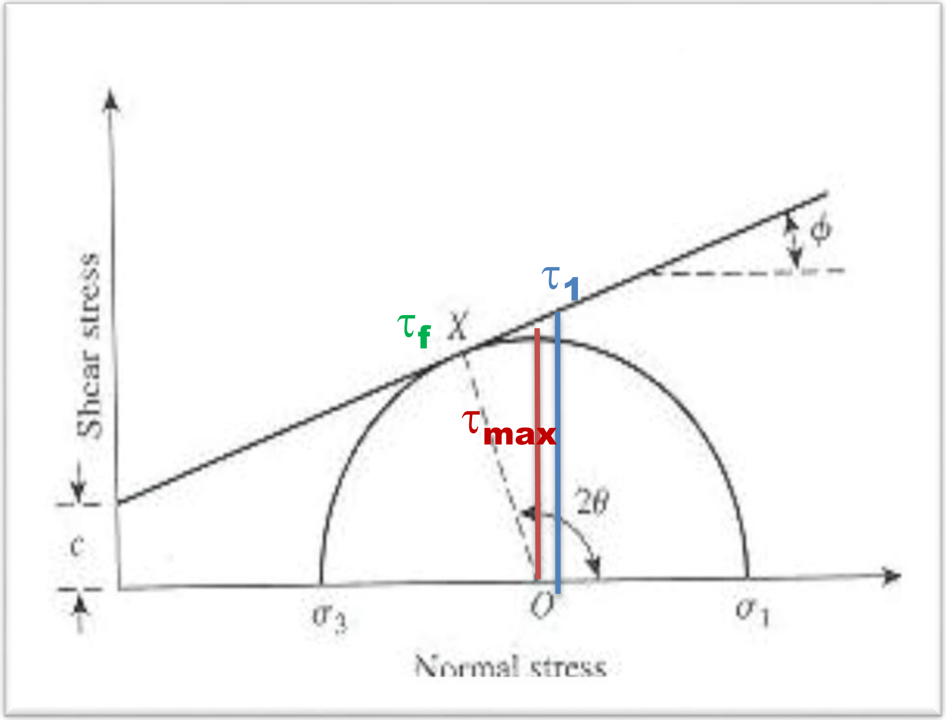
FAILURE CRITERIA FOR SOILS

- **Inclination of failure Plane due to shear**

- τ_f **is not the maximum shear stress ..... Why?**

$$F_s = \frac{\tau_{aviable}}{\tau_{applied}}$$

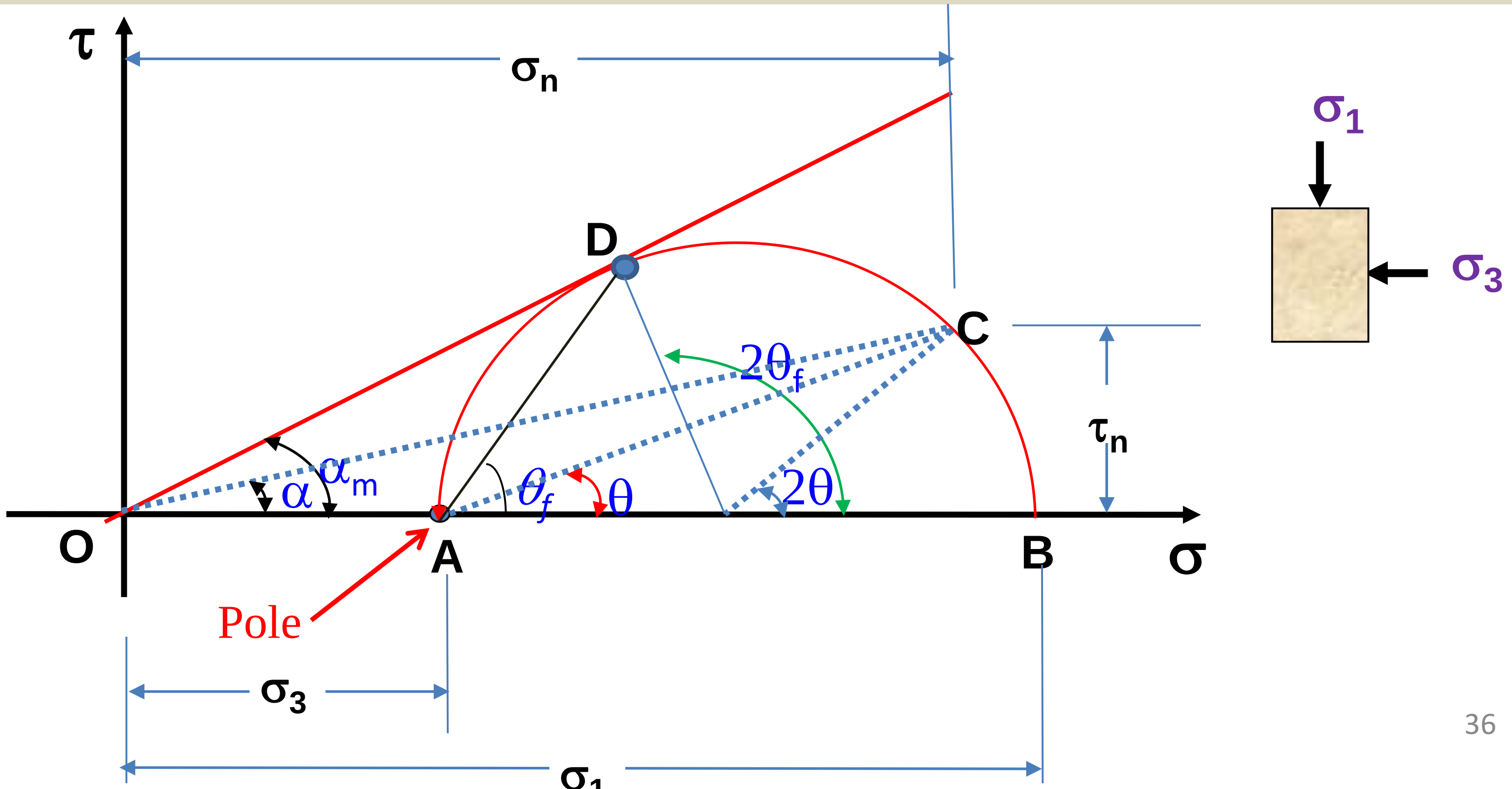
$$F_s = \frac{\tau_1}{\tau_{max}}$$



Mohr Theory of Failure

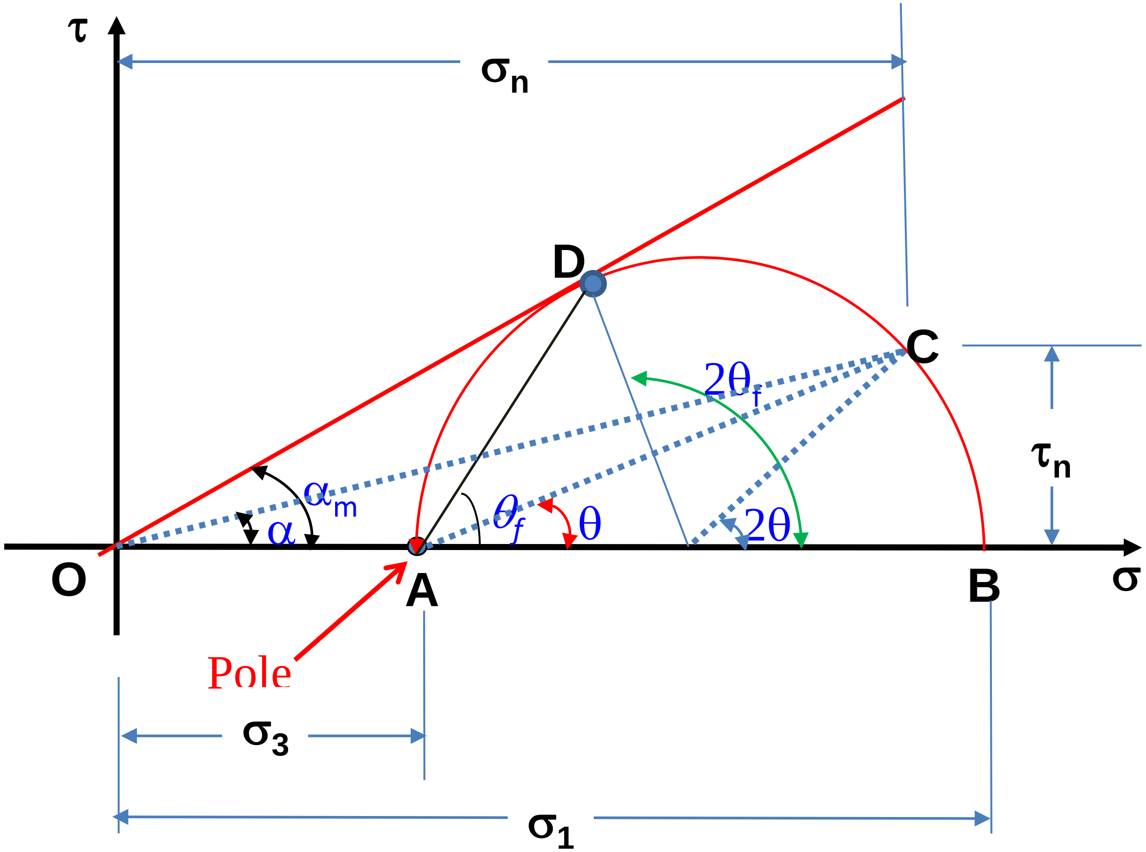
Mohr theory of failure states that:

A material fails along the plane and at the time at which the angle between the RESULTANT of the NORMAL and SHEARING STRESS and the NORMAL STRESS is a maximum; that is , when the combination of NORMAL and SHEARING stresses produces the maximum obliquity angle α_m.



Mohr Theory of Failure

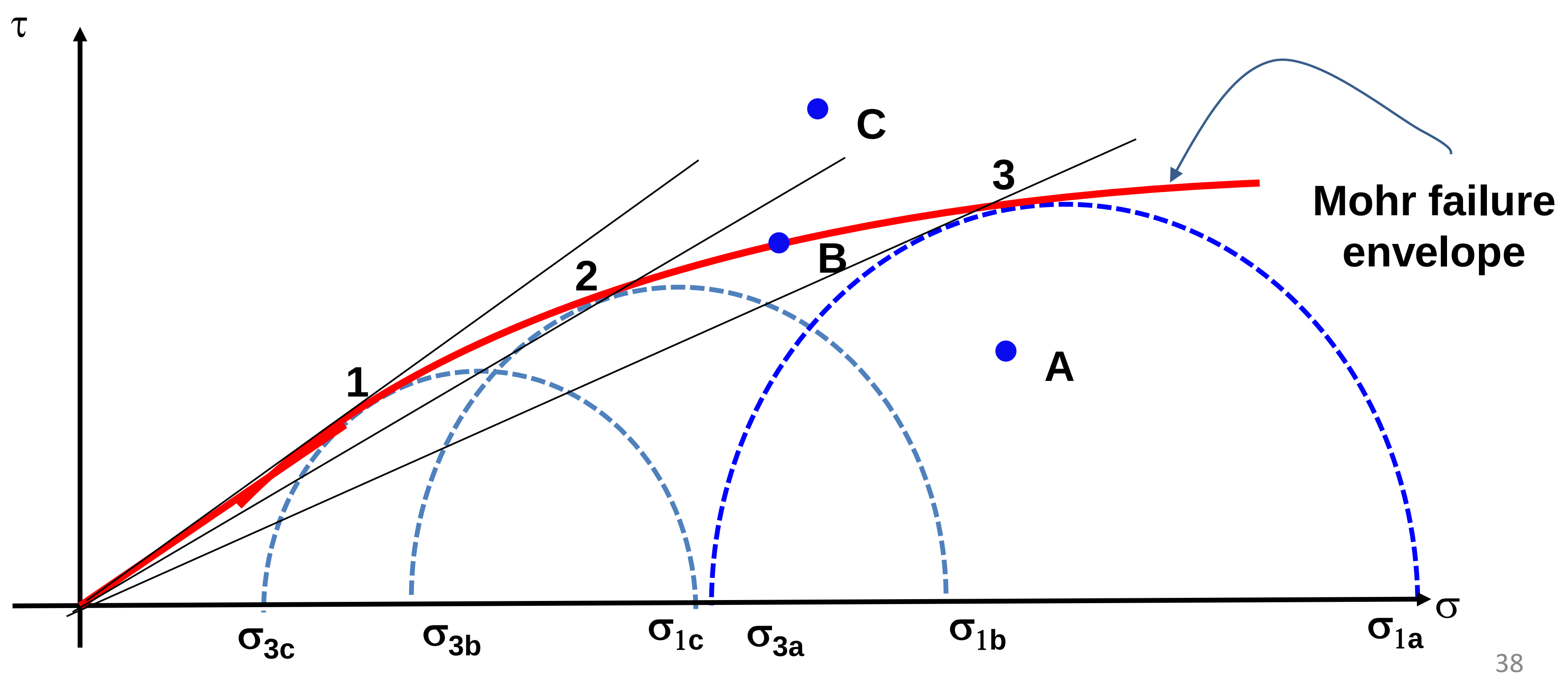
- In the across diagram, it is seen that the optimum stress combination which fulfills Mohr's criterion is that represented by the point D, and the orientation of the failure plane is represented by the line AD which makes an angle θ_f with the maximum principal plane.



- Since, according to the Mohr theory, the tangent line **OD** represents the stress situation at failure, the maximum obliquity angle α_m is equal to the friction angle ϕ, just as indicated in the case of the brick sliding on a horizontal surface.

Mohr Failure Envelope

- By plotting Mohr's circles for different states of stresses and in each case draw a tangent to each circle from the origin we come up with points 1,2,3… etc. If we connect those points we come up with what is called Mohr's failure envelope.



Mohr Failure Envelope

- This envelope separates cases of stresses which cause failure from that which do not.

- For instance if the normal stress and shear stress on a plane in a soil mass are such that they plot as point A , shear failure will not occur along that plane.

- For point B failure takes place, and point C cannot exist, since it plots above failure envelope and shear failure in a soil would have occurred already.

- Therefore, failure occurs only when the combination of shear and normal stress is such that the Mohr circle is TANGENT to the Mohr failure envelope.

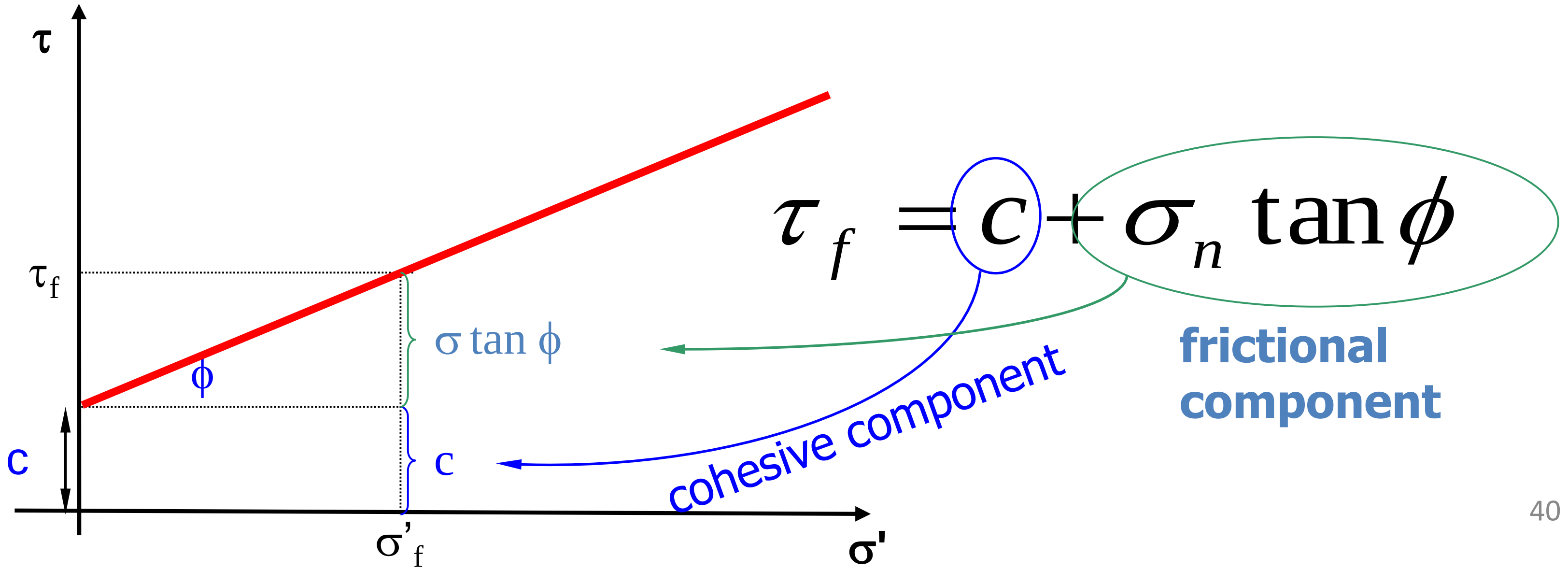
- Then once the point of the tangency is determined the angle of failure plane and the stresses $(\sigma_n, \tau)_f$ can be determined using the POLE method.

Mohr-Coulomb Failure Criterion

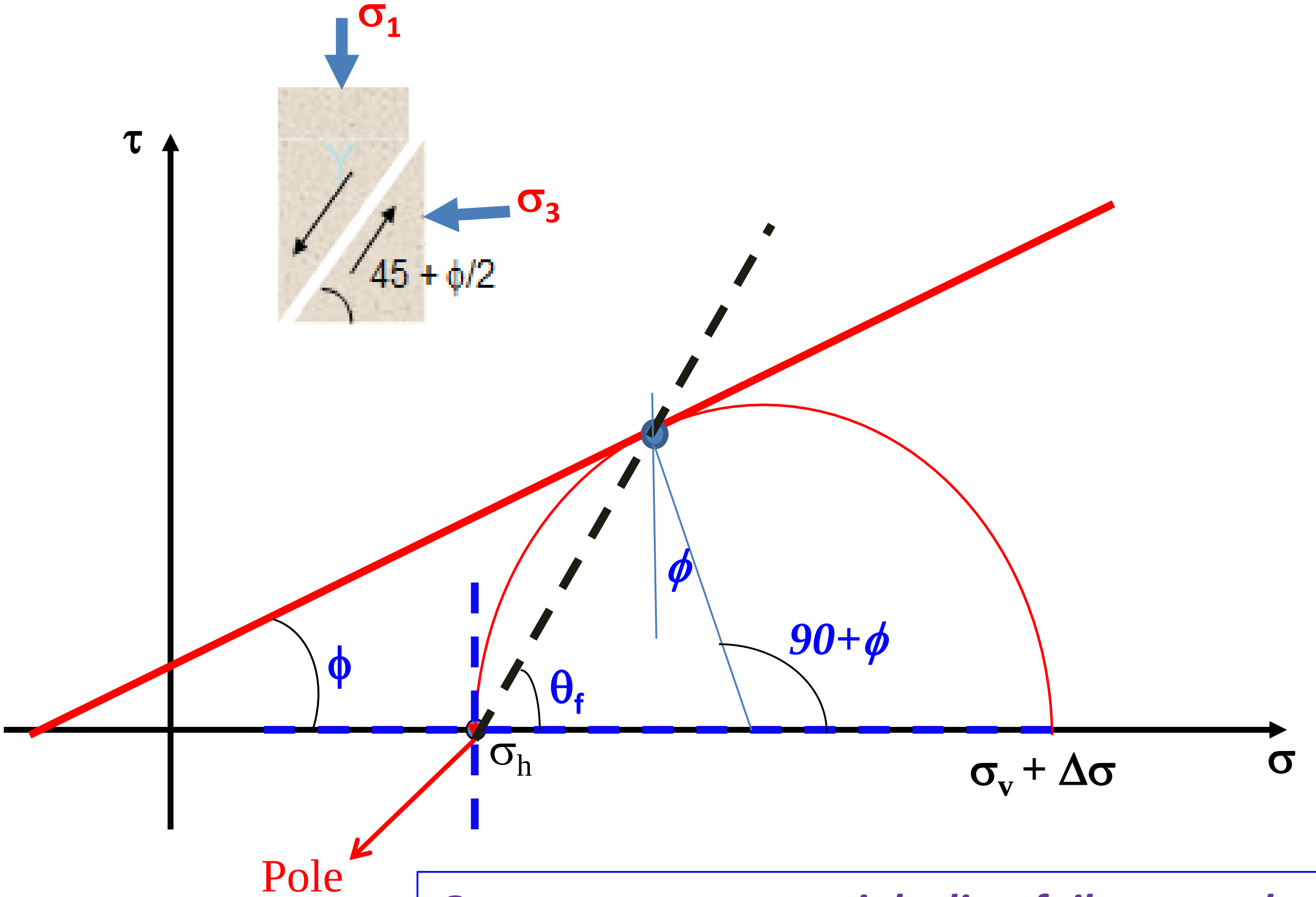
- The disadvantages of Mohr failure envelope is that it is a curved line, and needs a lot of tests to construct and difficult to use.
- It was then approximated to be a straight line, and the equation for the line was written in terms of the Coulomb strength parameters C and ϕ , as

$$\tau_f = C + \sigma_n \tan\emptyset$$

- This gave the birth to MOHR-COULOMB FAILURE CRITERION, which is by far the most popular criterion applied to soils.



Orientation of Failure Plane



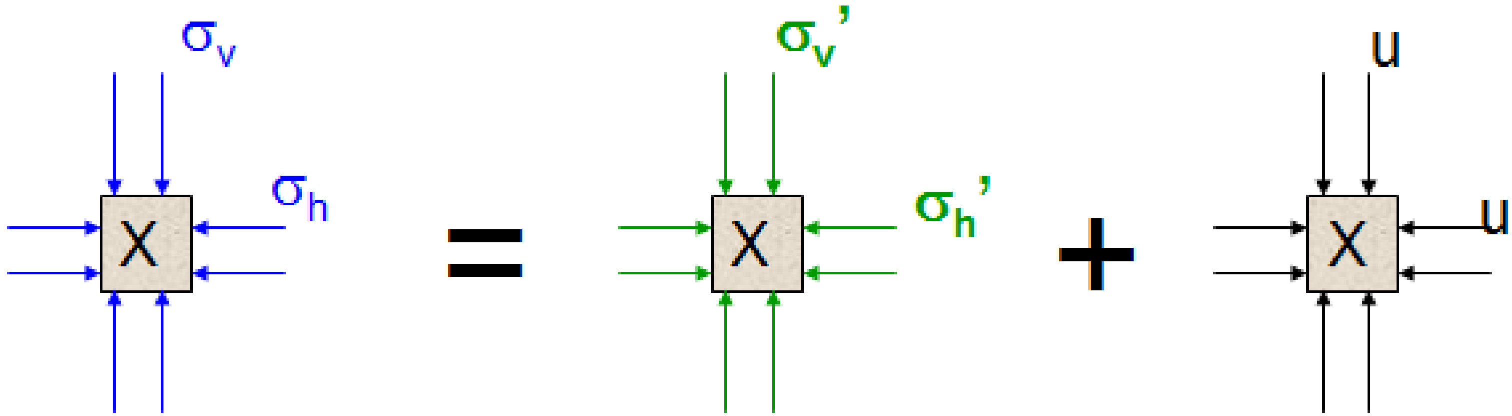
$$2\theta_f = 90 + \phi$$

$$\theta_f = 45 + \phi/2$$

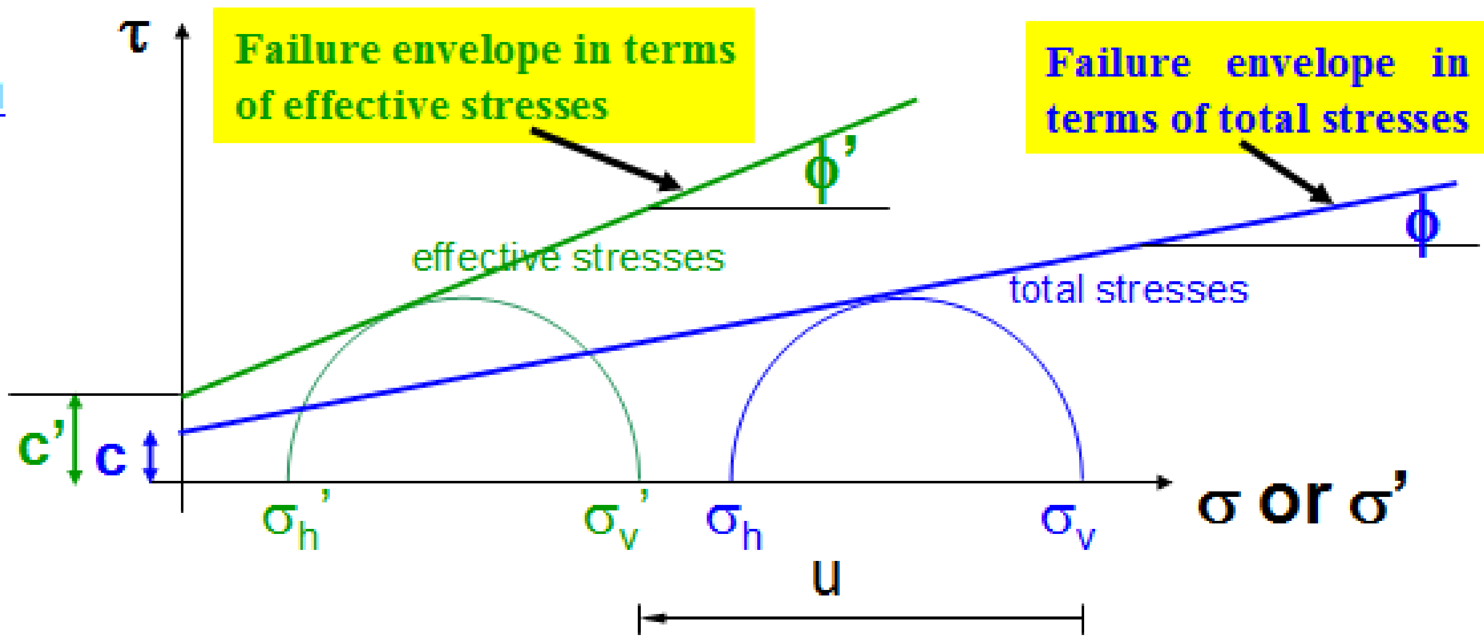
This could be proved analytically (See Das)

Once we assume straight line failure envelope, $\theta_f = 45 + \phi/2$ always and independent of the values of σ_1 and σ_3 (i.e. confinement).

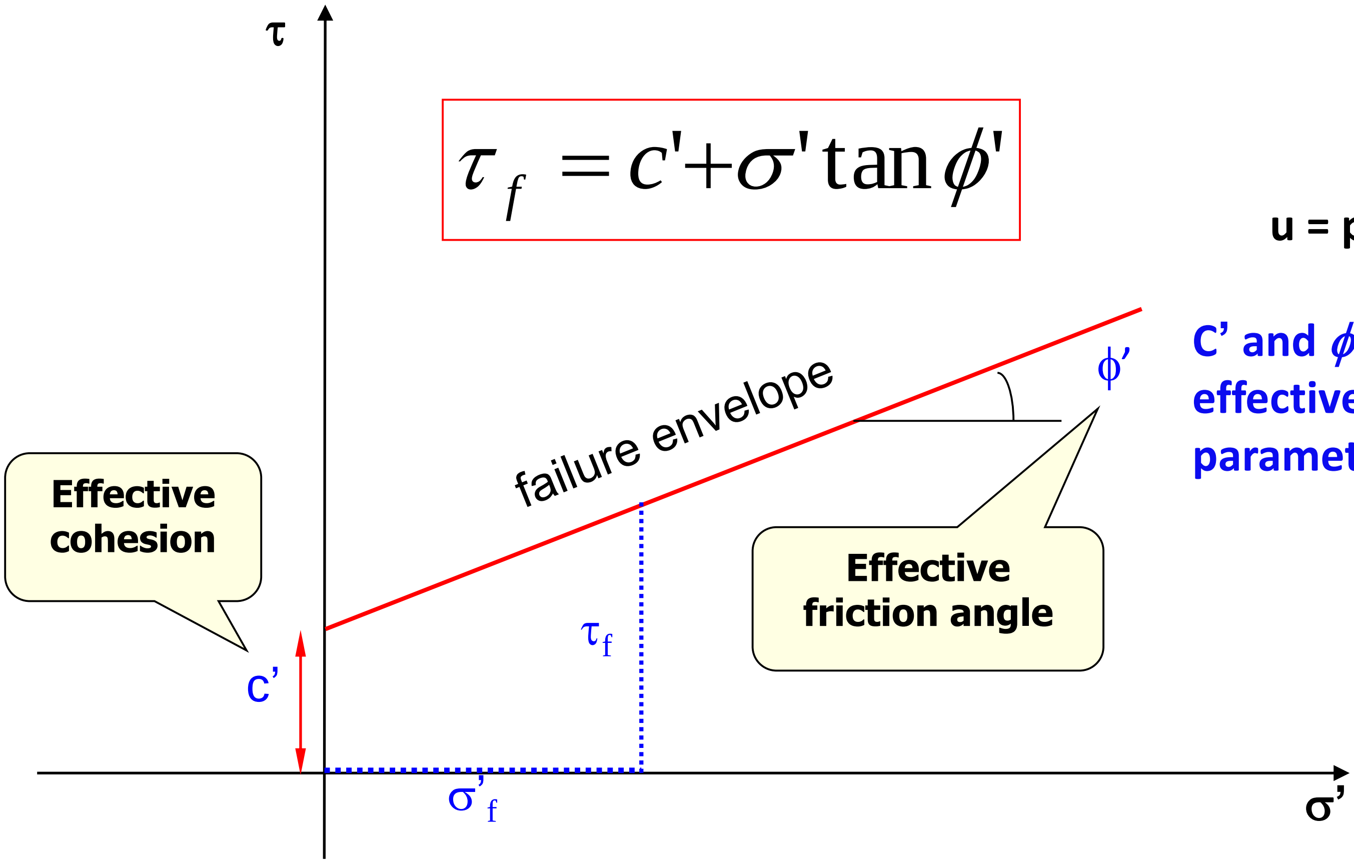
Mohr circles & failure envelope in terms of total and effective stress



If X is on failure



Mohr-Coulomb Failure Criterion in terms of effective stress



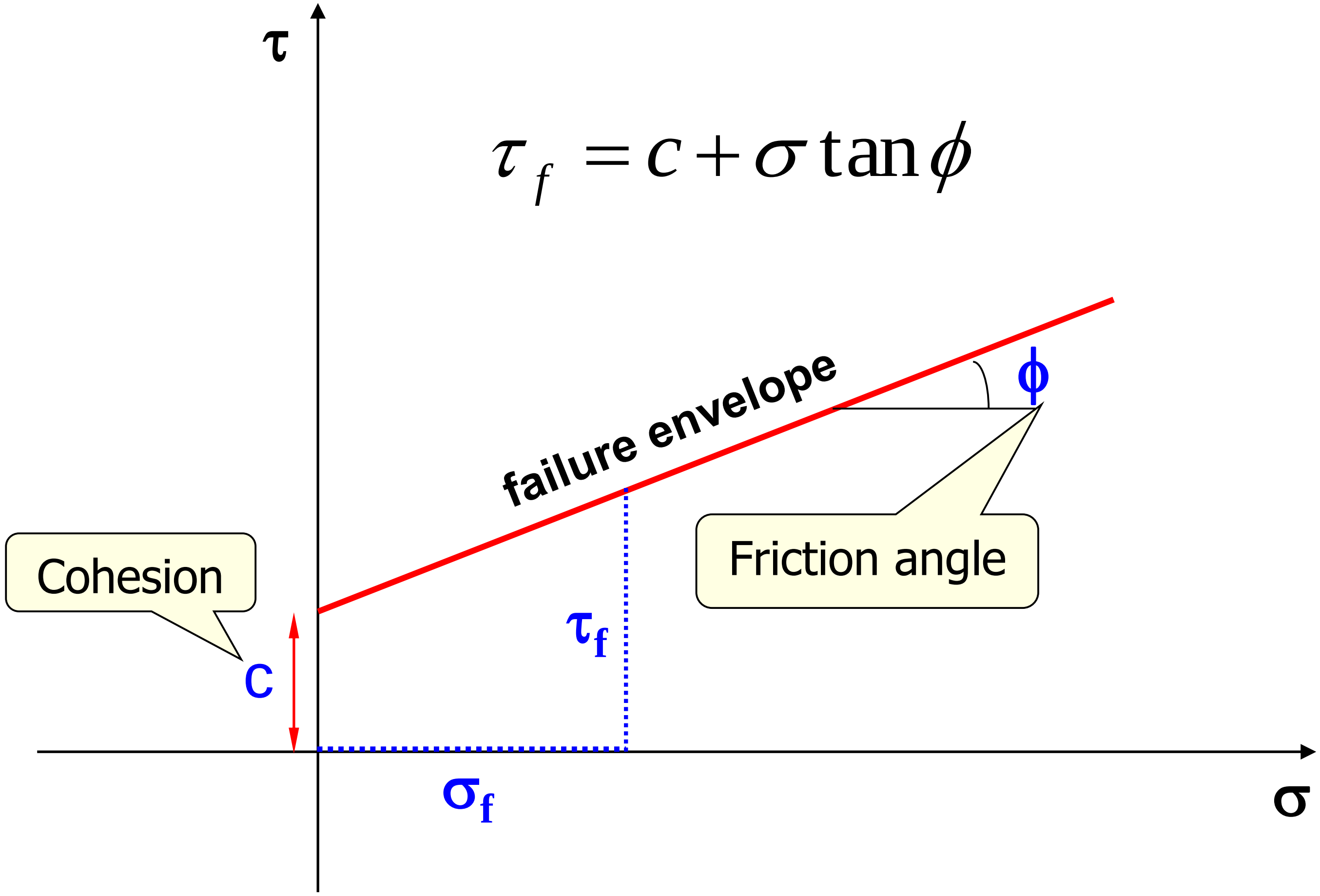
$$\sigma' = \sigma - u$$

u = pore water pressure

C' and ϕ' are called the effective strength parameters.

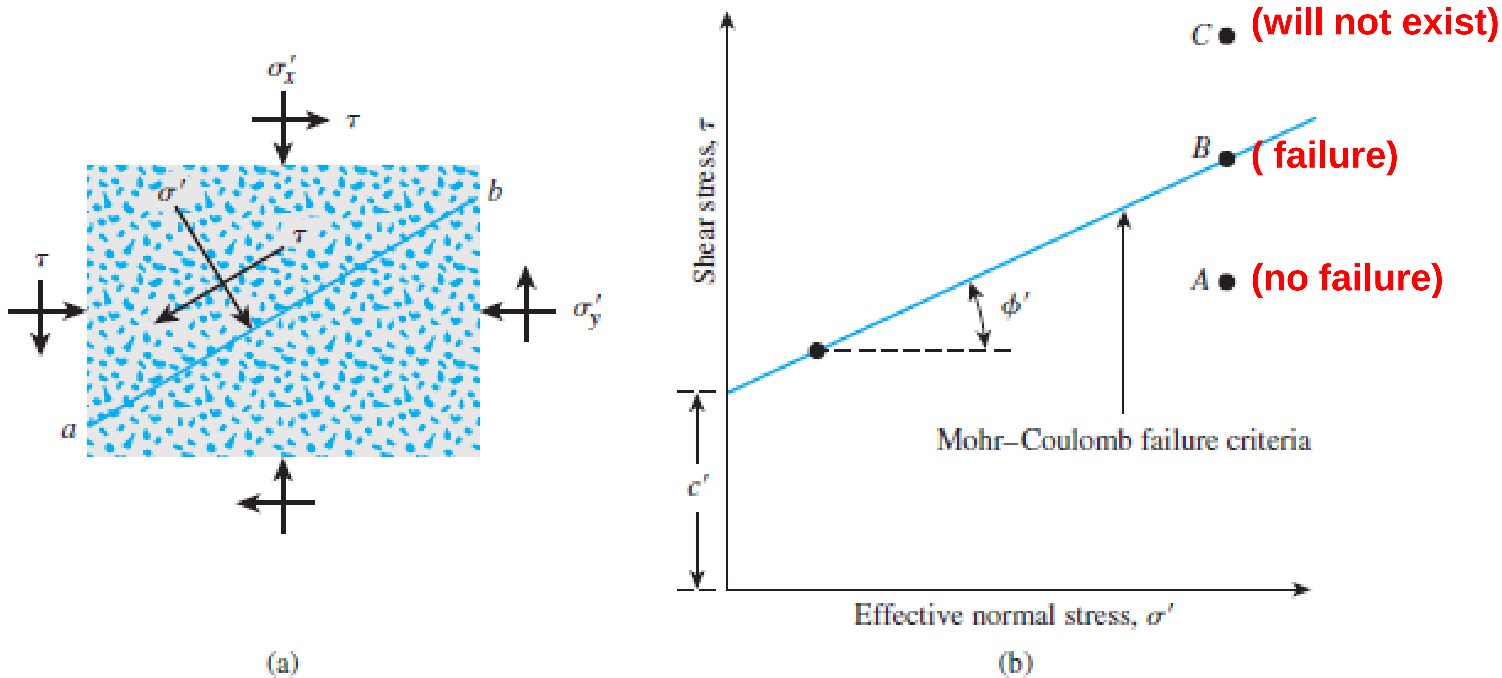
In this case, soil behavior is controlled by effective stresses, and the effective strength parameters are the fundamental strength parameters.

Mohr-Coulomb Failure Criterion in terms of total stress



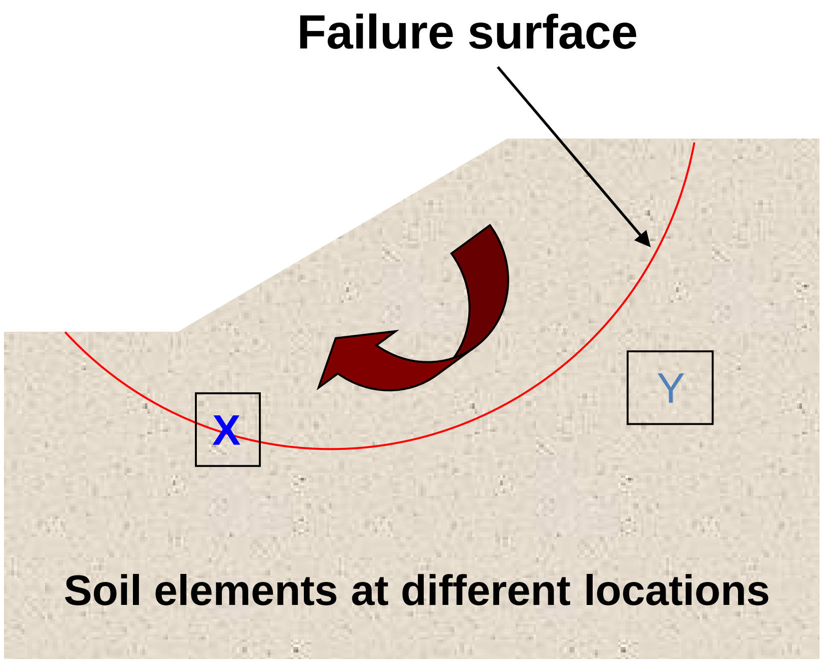
τ_f is the maximum shear stress the soil can take without failure, under normal stress of σ.

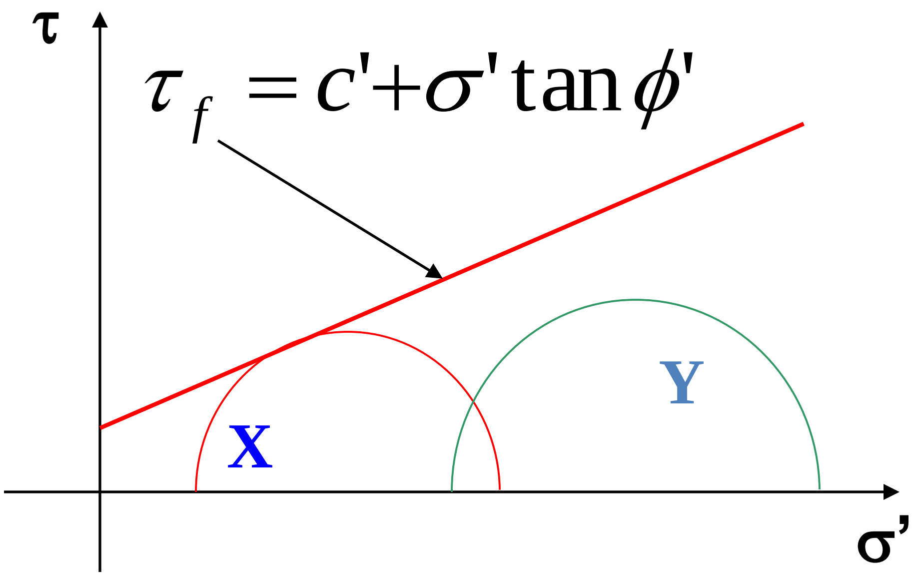
What does the Failure Envelope Signify?



- If the magnitudes of σ and τ on plane ***ab*** are such that they plot as point ***A*** shear failure will not occur along the plane.
- If the effective normal stress and the shear stress on plane ***ab*** plot as point **B** (which falls on the failure envelope), shear failure will occur along that plane.
- A state of stress on a plane represented by point **C** cannot exist, because it plots above the failure envelope, and shear failure in a soil would have occurred already.

Mohr Circles & Failure Envelope



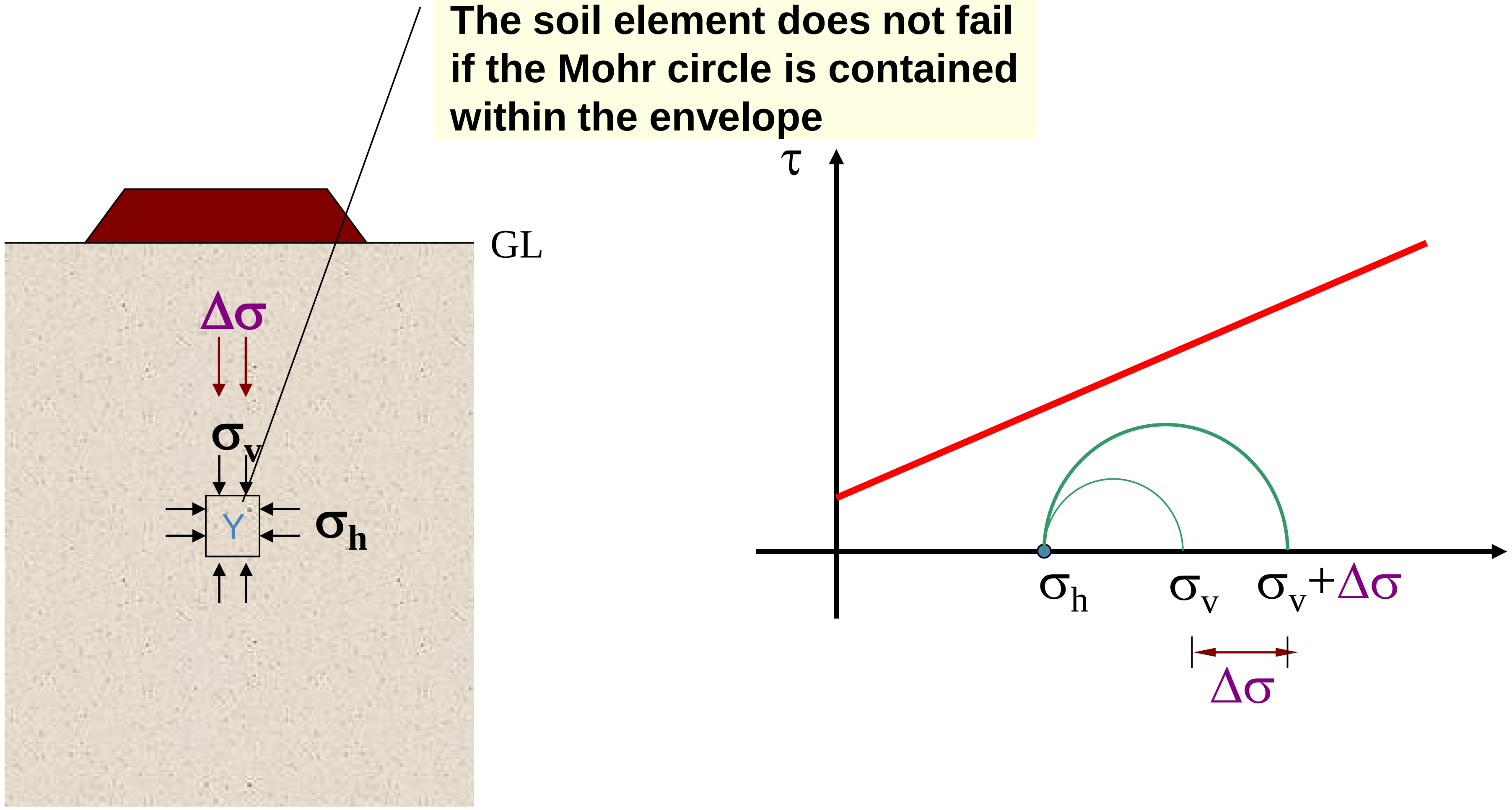


Y ~ **stable**

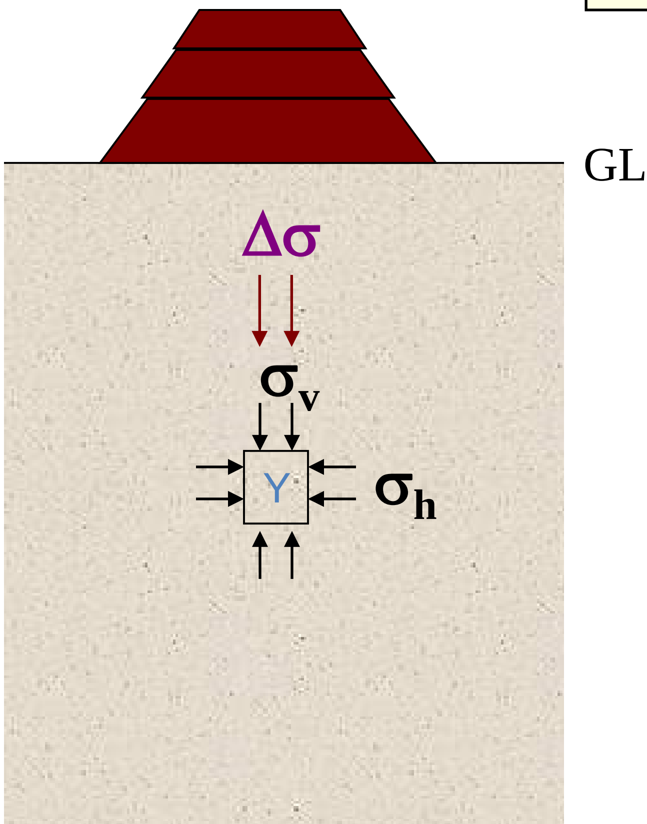
X ~ **failure**

Mohr Circles & Failure Envelope

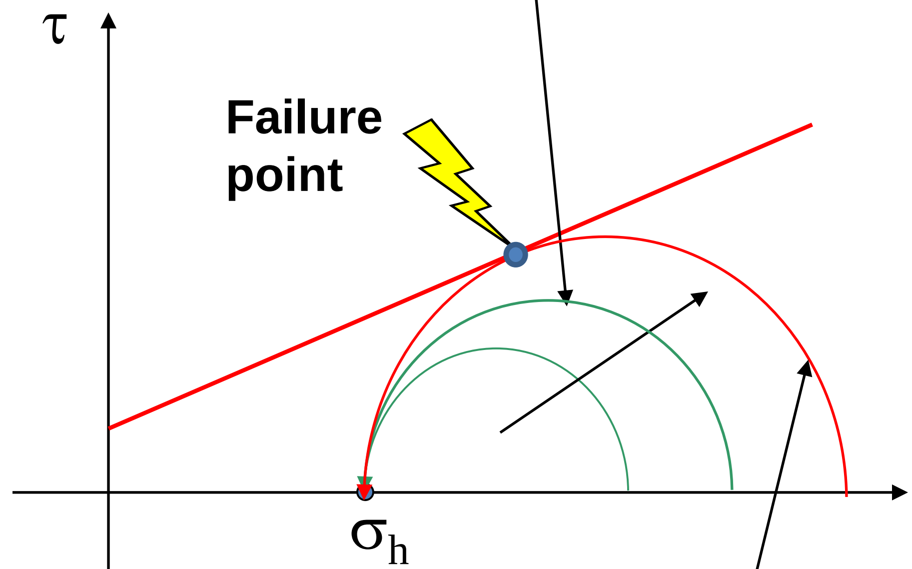
The soil element does not fail if the Mohr circle is contained within the envelope



Mohr Circles & Failure Envelope

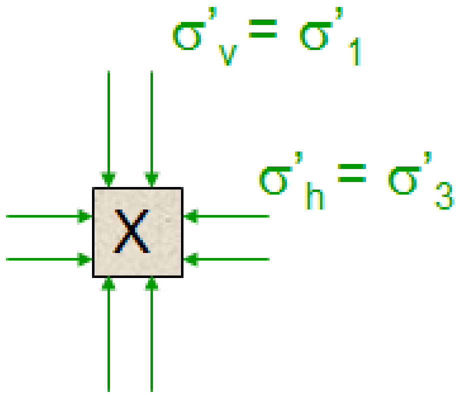


As loading progresses, Mohr circle becomes larger…

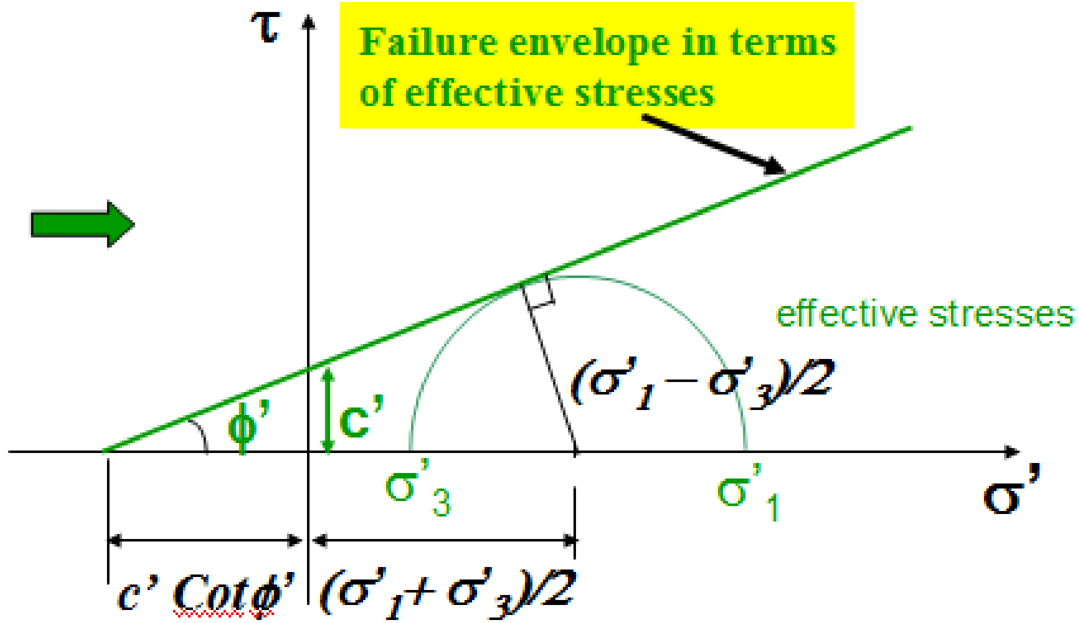


.. and finally failure occurs when Mohr circle touches the envelope

Relationship between principle stresses and shear strength parameters



X is on failure



Therefore,

$$\left[c' Cot\phi' + \left(\frac{\sigma_1' + \sigma_3'}{2}\right)\right] Sin\phi' = \left(\frac{\sigma_1' - \sigma_3'}{2}\right)$$

Relationship between principle stresses and shear strength parameters

$$\left[c'Cot\phi' + \left(\frac{\sigma_1' + \sigma_3'}{2}\right)\right]Sin\phi' = \left(\frac{\sigma_1' - \sigma_3'}{2}\right)$$

$$\left(\sigma_1' - \sigma_3'\right) = \left(\sigma_1' + \sigma_3'\right)Sin\phi' + 2c'Cos\phi'$$

$$\sigma_1'\left(1 - Sin\phi'\right) = \sigma_3'\left(1 + Sin\phi'\right) + 2c'Cos\phi'$$

$$\sigma_1' = \sigma_3'\frac{\left(1 + Sin\phi'\right)}{\left(1 - Sin\phi'\right)} + 2c'\frac{Cos\phi'}{\left(1 - Sin\phi'\right)}$$

$$\sigma_1' = \sigma_3' Tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' Tan\left(45 + \frac{\phi'}{2}\right)$$

REMARKS

- C and ϕ are measures of shear strength and are called shear strength parameters.
- The parameters C, ϕ are in general not soil constants. They depend on the initial state of the soil (e.g. density, water content), and type of loading (drained or undrained).
- In case of using effective stress, C' and ϕ' are called effective shear strength parameters.
- The value of C' for sand and inorganic silt is 0. For normally consolidated clays, C' can be approximated at 0. Overconsolidated clays have values of C' that are greater than 0.

TOPICS

- Introduction
- Components of Shear Strength of Soils
- Normal and Shear Stresses on a Plane
- Mohr-Coulomb Failure Criterion
- **Laboratory Shear Strength Testing**
  - Direct Shear Test
  - Triaxial Compression Test
  - Unconfined Compression Test
- Field Testing (Vane test)

Laboratory Shear Strength Testing

- **The purpose of laboratory testing is to determine the shear strength parameters of soil (C, ϕ or C', ϕ') through the determination of failure envelope.**
- **The shear strength parameters for a particular soil can be determined by means of laboratory tests on specimens taken from representative samples of the in-situ soil.**
- **There are several laboratory methods available to determine the shear strength parameters (i.e. C, ϕ, C', ϕ'). Some of the tests are rather complicated .**
- **For further details you should consult manuals and books on laboratory testing, especially those by the ASTM.**

Determination of shear strength parameters of soils

Determination of shear strength parameters of soils (c, ϕ or c' , ϕ')

Laboratory Tests

Most common laboratory tests to determine the shear strength parameters are,

1. Direct shear test
2. Triaxial shear test

Other laboratory tests include:

- Direct simple shear test
- Torsional or ring shear test
- Hollow cylinder test
- Plane strain triaxial test
- Laboratory vane shear test
- Laboratory fall cone test.

Field Tests

1. Standard penetration test Pressuremeter
2. Vane shear test
3. Pocket penetrometer
4. Static cone penetrometer

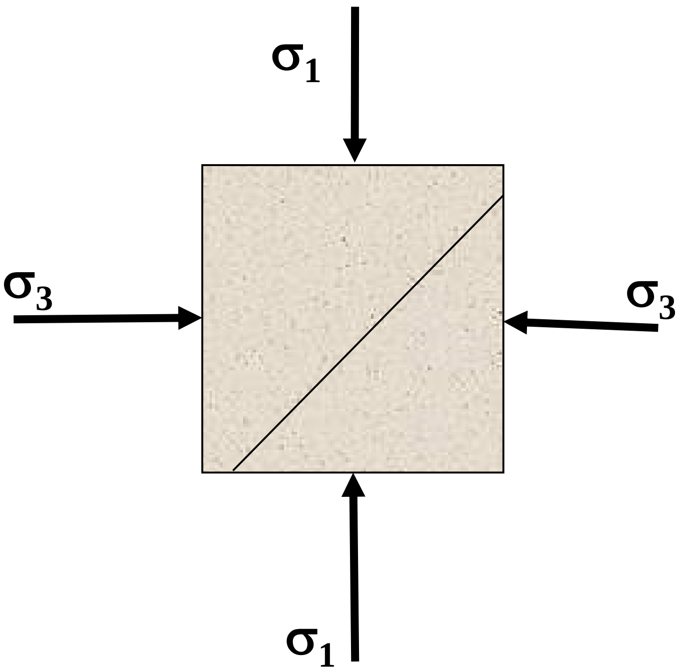
Field test equipment and test methods are described in most textbooks on foundation engineering.

Laboratory Shear Strength Testing

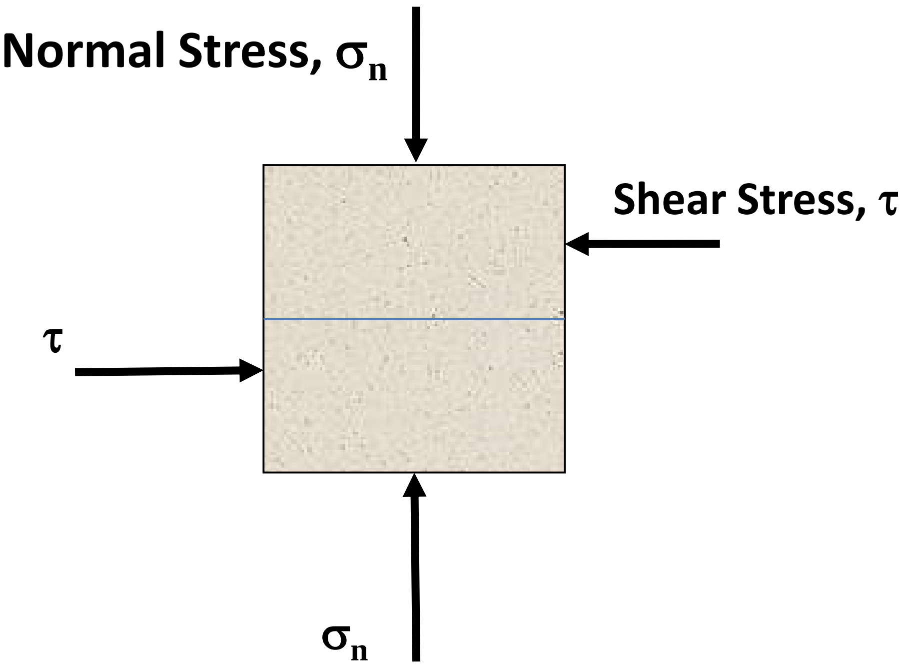
- In laboratory, a soil sample can fail in two ways:

Way 1: Increase normal stress (σ_1) to failure with confining stress (σ_3) constant

Way 2: Normal stress is applied and held constant then shear stress is applied to failure



Triaxial Shear Test

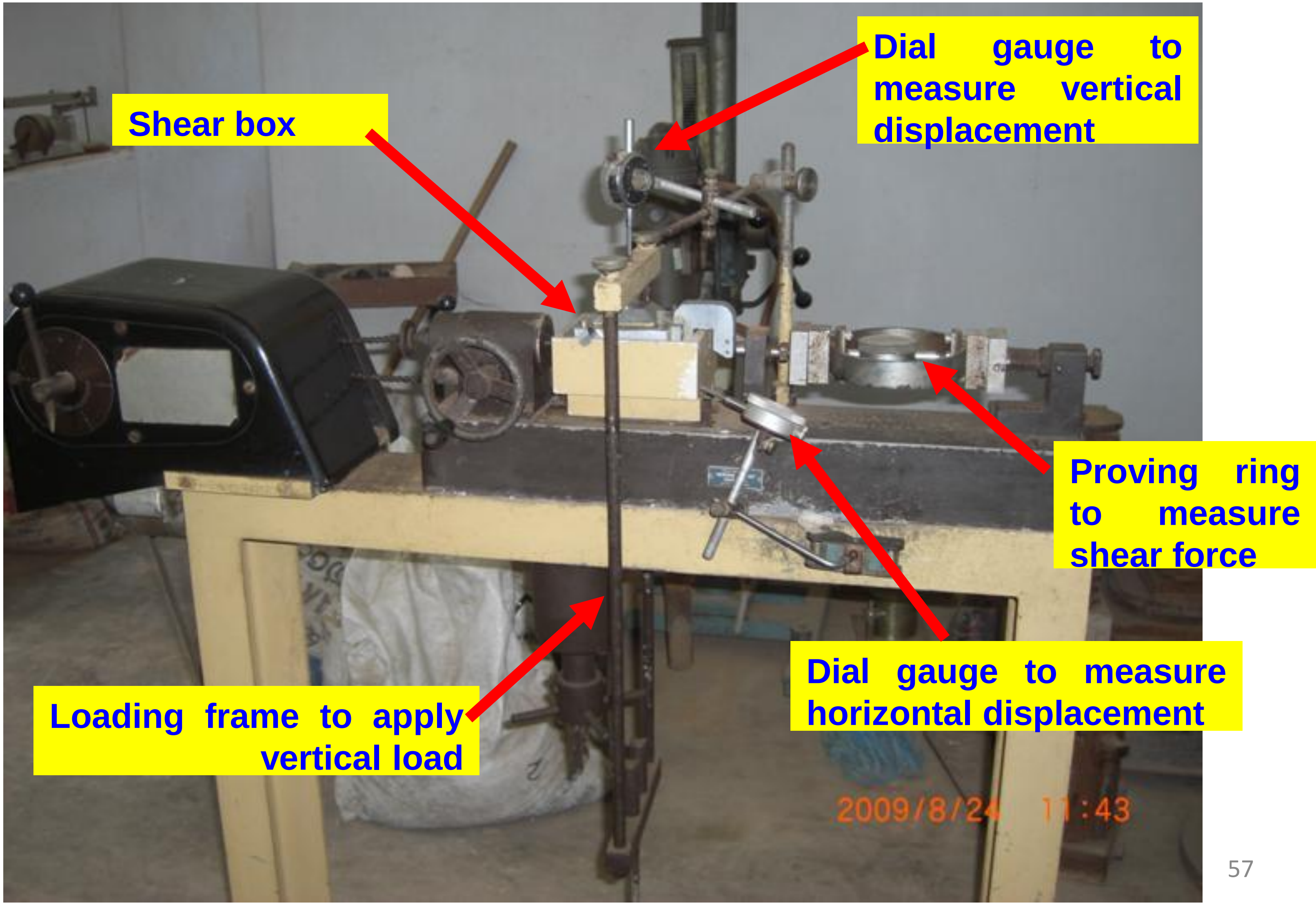


Direct Shear Test

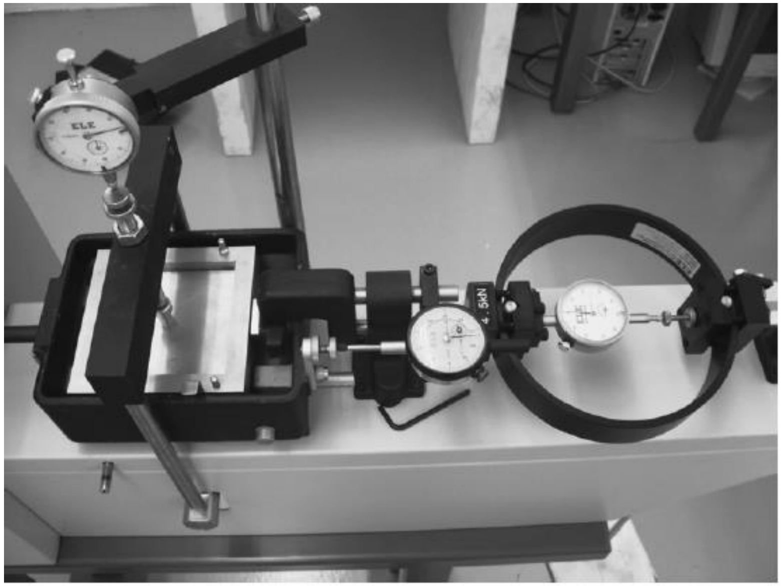
TOPICS

- ❑ Introduction
- ❑ Components of Shear Strength of Soils
- ❑ Normal and Shear Stresses on a Plane
- ❑ Mohr-Coulomb Failure Criterion
- ❑ Laboratory Shear Strength Testing
  - Direct Shear Test
  - Triaxial Compression Test
  - Unconfined Compression Test
- ❑ Field Testing (Vane test)

Direct Shear Test

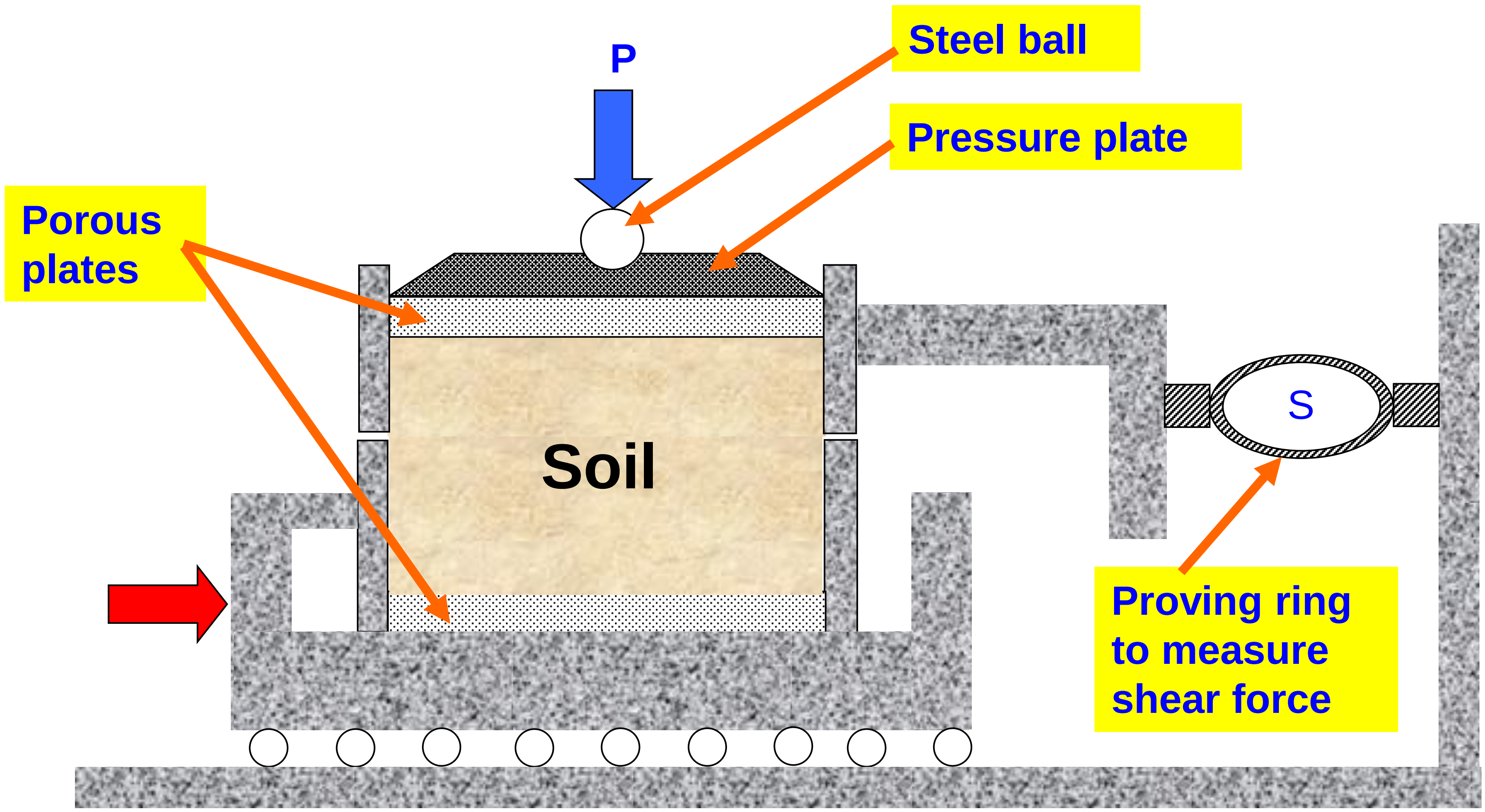


Direct Shear Test

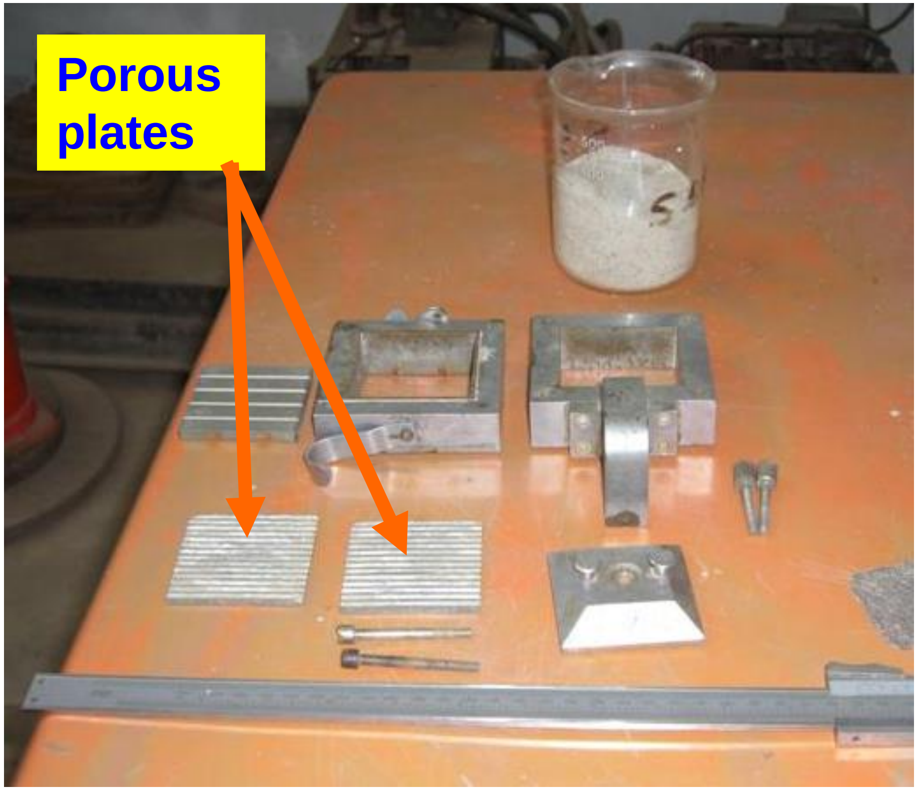


- The advantage of the strain-controlled tests is that in the case of dense sand, peak shear resistance (that is, at failure) as well as lesser shear resistance (that is, at a point after failure called ultimate strength) can be observed and plotted.
- Compared with strain-controlled tests, stress-controlled tests probably model real field situations better.

Direct Shear Test



Preparation of a sand specimen



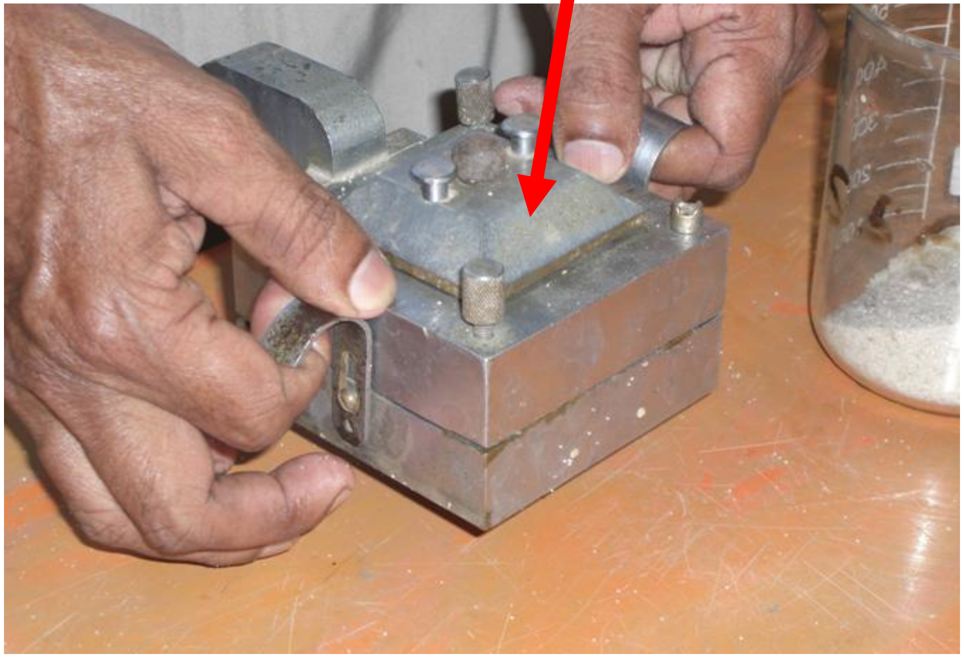
Components of the shear box

Preparation of a sand specimen

Preparation of a sand specimen

Leveling the top surface of specimen

Pressure plate



Specimen preparation completed

Test Procedure

- A constant vertical force (normal stress) is applied through a metal platen.
- Shear force is applied by moving one half of the box relative to the other and increased to cause failure in the soil sample.
- The tests are repeated on similar specimens at various normal stresses
- The normal stresses and the corresponding values of τ_f obtained from a number of tests are plotted on a graph from which the shear strength parameters are determined.

TEST RESULTS

Normal Load : __________ kg

Area of Sample: __________ cm^2

Horizontal Dial Reading (mm)	Vertical Dial Reading (mm)	Horizontal Shear Force (N)	Shear Stress (kPa)

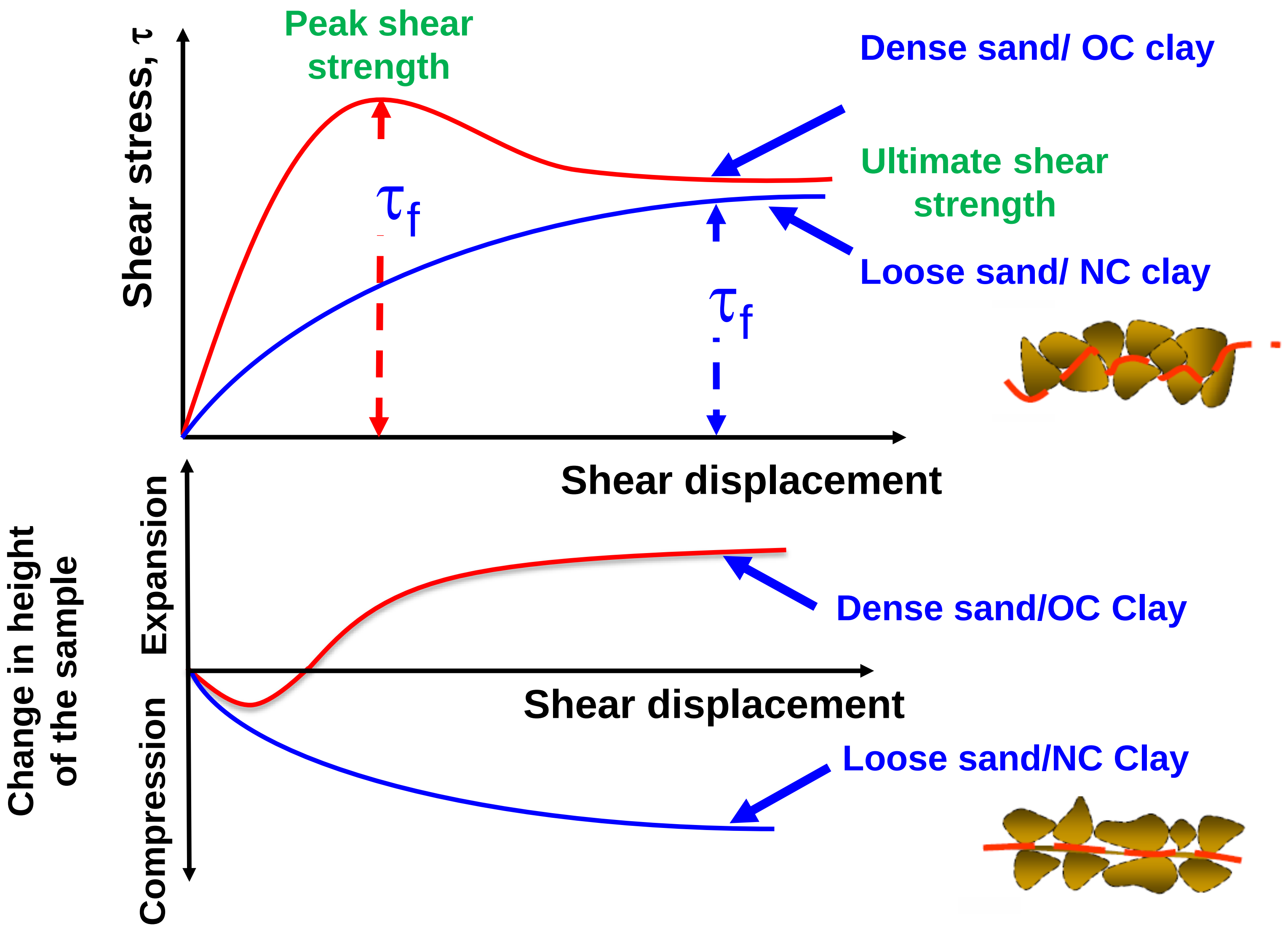
For a given test, the normal stress can be calculated as

$$\sigma = \text{Normal stress} = \frac{\text{Normal force}}{\text{Cross-sectional area of the specimen}}$$

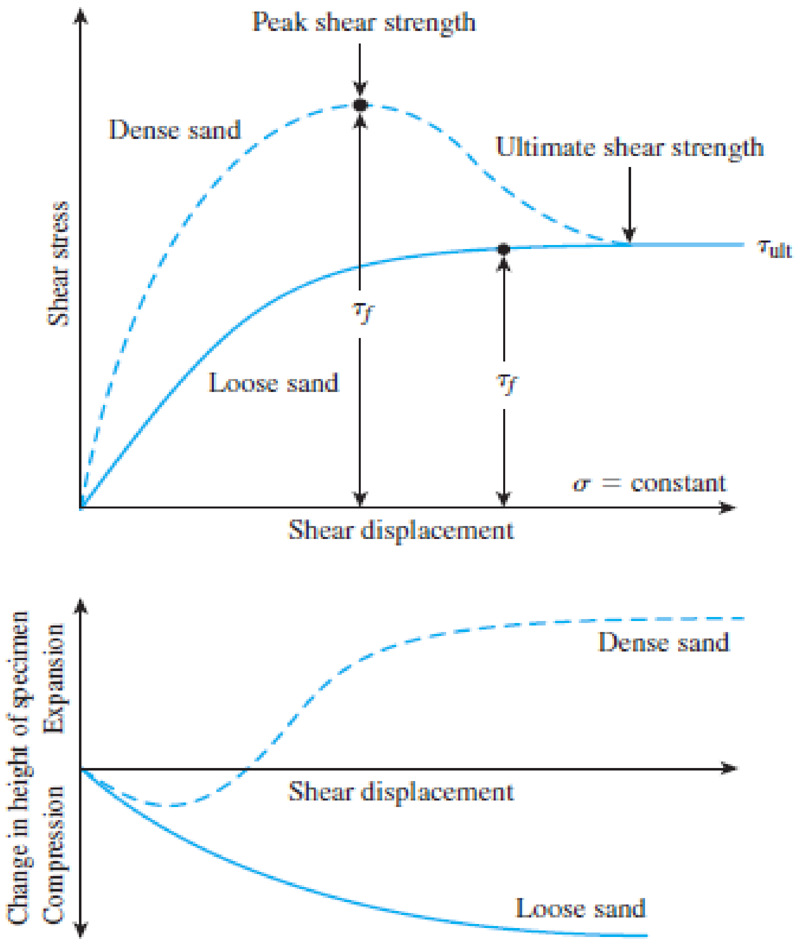
The resisting shear stress for any shear displacement can be calculated as

$$\tau = \text{Shear stress} = \frac{\text{Resisting shear force}}{\text{Cross-sectional area of the specimen}}$$

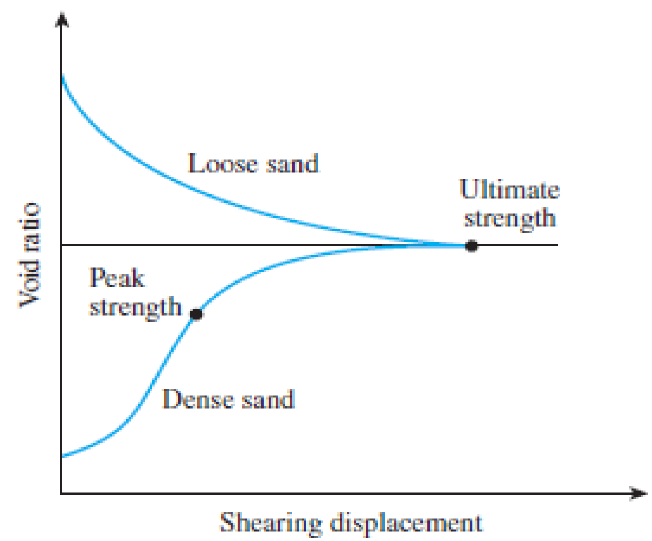
Stress-Strain Relationship



Stress-Strain Relationship



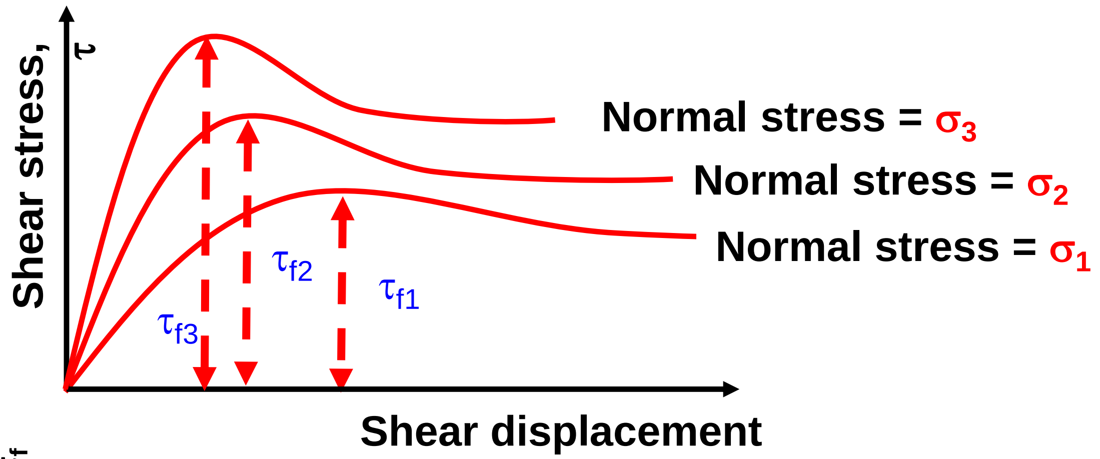
Plot of shear stress and change in height of specimen against shear displacement for loose and dense dry sand (direct shear test)

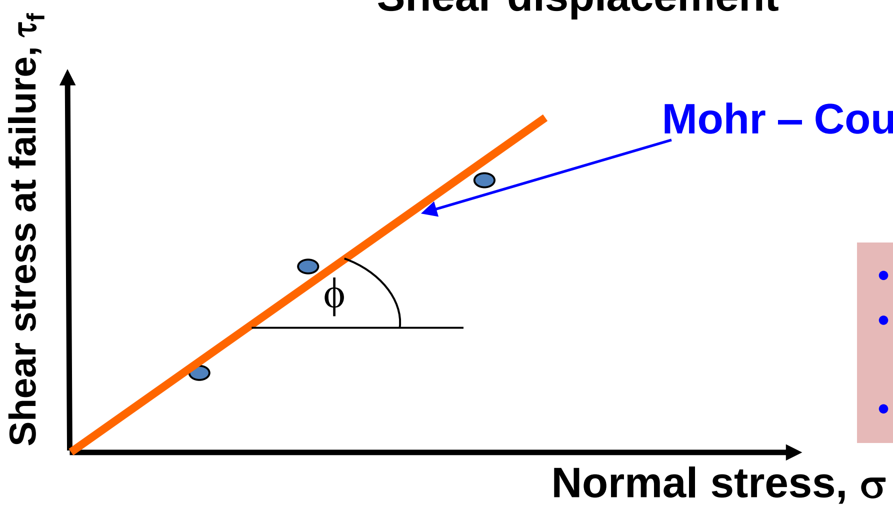


Nature of variation of void ratio with shearing displacement

Determining strength parameters C and ϕ

An example of testing three samples of a sand at the same relative density just before shearing.



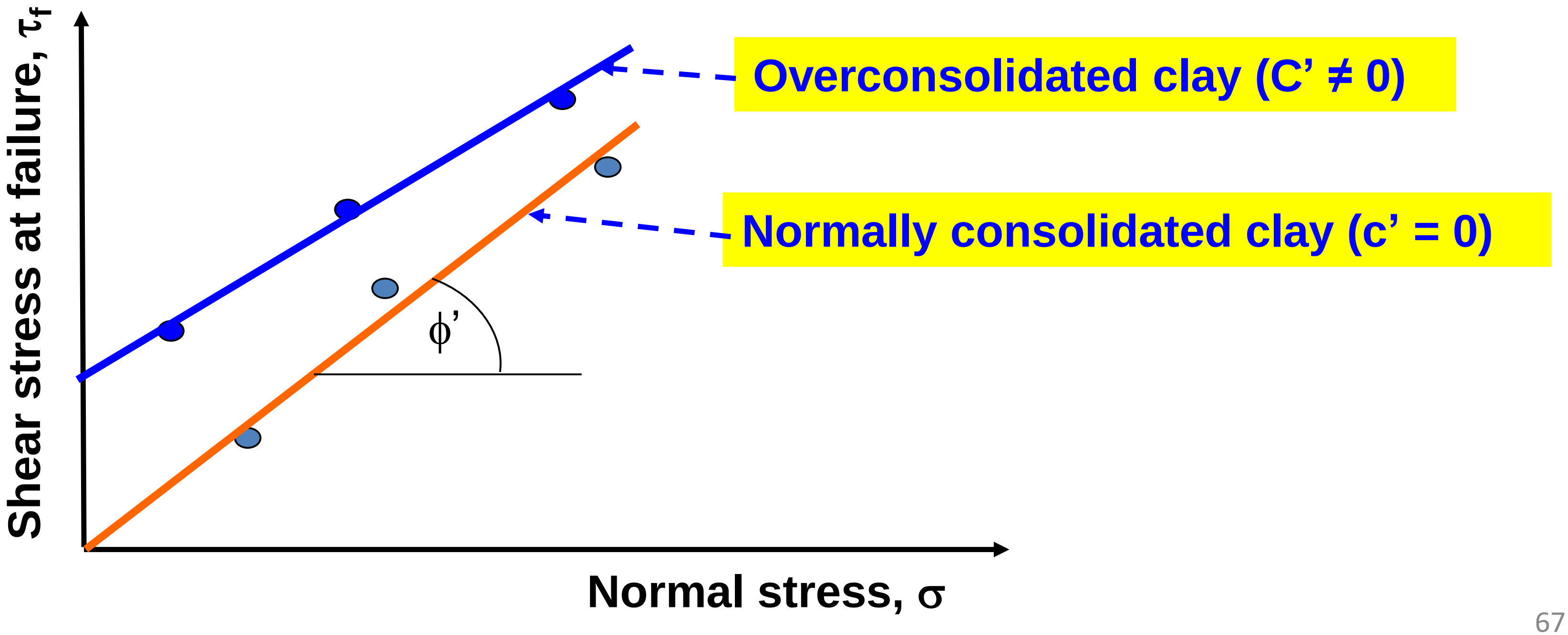


- Sand is cohesionless, then C = 0
- Direct shear tests are drained and pore water pressures are dissipated, then u = 0
- Therefore, $\phi = \phi'$

Determining strength parameters C and ϕ

In case of clay, horizontal displacement should be applied at a very slow rate to allow dissipation of pore water pressure (therefore, one test would take several days to finish)

Failure envelopes for clay from drained direct shear tests



Notes on Direct Shear Test

- **This test is probably the oldest strength test because Coulomb used a type of shear box test more than two centuries ago to determine the necessary parameters for his strength equation.**
- **The test is quick and inexpansive and common in practice.**
- **Used to determine the shear strength of both cohesive as well as non-cohesive soils.**
- **The test equipment consists of a metal box in which the soil specimen is placed. The box is split horizontally into two halves.**
- **The shear test can be either stress controlled or strain controlled.**
- **Tests on sands and gravels are usually performed dry. Water does not significantly affect the (drained) strength.**

Notes on Direct Shear Test

- Usually only relatively slow drained tests are performed in shear box apparatus. For clays rate of shearing must be chosen to prevent excess pore pressures building up. For sands and gravels tests can be performed quickly.
- If there are no excess pore pressures and as the pore pressure is approximately zero the total and effective stresses will be identical.
- The failure stresses thus define an effective stress failure envelope from which the effective (drained) strength parameters C', ϕ' can be determined.
- Normally consolidated clays (OCR = 1) and loose sands do not show separate peak and ultimate failure loci, and for soils in these states $C' = 0$.
- Overconsolidated clays and dense sands have peak strengths with $C' > 0$.

Advantages of direct shear test

- Inexpensive, fast, and simple, especially for granular materials.
- Easiness of sample preparation in case of sand.
- Due to the smaller thickness of the sample, rapid drainage can be achieved
- Large deformations can be achieved by reversing shear direction. This is useful for determining the residual strength of a soil.
- Samples may be sheared along predetermined planes. This is useful when the shear strengths along fissures or an interface is required.

Disadvantages of direct shear test

- Failure occurs along a predetermined failure plane which may not be the weakest plane.
- Non-uniform of shear stresses along failure surface in the specimen. There are rather stress concentrations at the sample boundaries, which lead to highly nonuniform stress conditions within the test specimen itself.
- There is no means of estimating pore pressures, so effective stresses cannot be determined and only the total normal stress can be determined.
- It is very difficult if not impossible to control drainage, especially for fine-grained soils. Consequently, the test is not suitable for other than completely drained conditions.

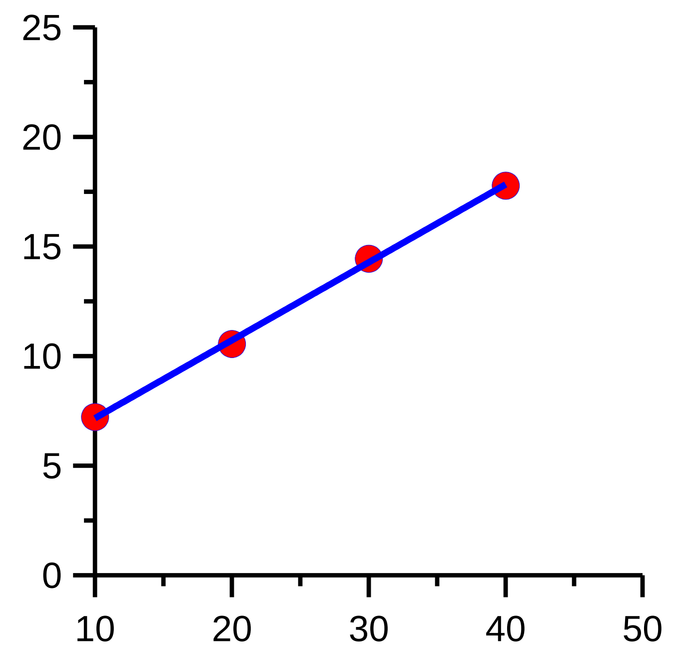
Example

- A direct shear test was performed on a clay sample. The cross section are of the device is 6 cm x 6 cm.

Normal Load (kg)	Shear Load (kg)
360	260
720	380
1080	520
1440	640

- Determine the shear envelope and
- shear strength parameters for the clay.

Normal Load (kg)	Normal Stress (kg/cm2)	Shear Load at failure (kg)	Shear Stress at failure (kg/cm2)
360	10	260	7.22
720	20	380	10.56
1080	30	520	14.44
1440	40	640	17.78



Example 12.1

Example 12.1

Direct shear tests were performed on a dry, sandy soil. The size of the specimen was 2 in. × 2 in. × 0.75 in. Test results are as follows:

Test no.	Normal force (lb)	Normal[a] stress $\sigma = \sigma'$ (lb/ft^2)	Shear force at failure (lb)	Shear stress[b] at failure τ_f (lb/ft^2)
1	20	720	12.0	432.0
2	30	1080	18.3	658.8
3	70	2520	42.1	1515.6
4	100	3600	60.1	2163.6

[a] $\sigma' = \dfrac{\text{normal force}}{\text{area of specimen}} = \dfrac{(\text{normal force}) \times 144}{(2\text{ in.})(2\text{ in.})}$

[b] $\tau_f = \dfrac{\text{shear force}}{\text{area of specimen}} = \dfrac{(\text{shear force}) \times 144}{(2\text{ in.})(2\text{ in.})}$

Find the shear stress parameters.

Solution

The shear stresses, τ_f, obtained from the tests are plotted against the normal stresses in Figure 12.18, from which $c' = 0$, $\phi' = 32°$.

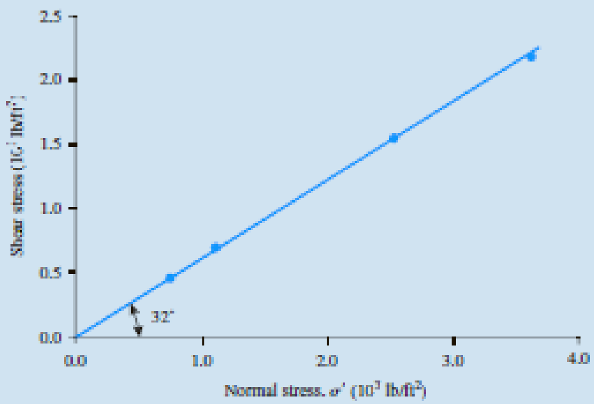


Figure 12.18

Example 12.2

Example 12.2

Following are the results of four drained direct shear tests on an *overconsolidated clay*:

Diameter of specimen = 50 mm
Height of specimen = 25 mm

Test no.	Normal force, N (N)	Shear force at failure, S_{peak} (N)	Residual shear force, $S_{residual}$ (N)
1	150	157.5	44.2
2	250	199.9	56.6
3	350	257.6	102.9
4	550	363.4	144.5

Determine the relationships for *peak shear strength* (τ_f) and *residual shear strength* (τ_r).

Solution

Area of the specimen (A) = $(\pi/4)\left(\frac{50}{1000}\right)^2$ = 0.0019634 m². Now the following table can be prepared.

Test no.	Normal force, N (N)	Normal stress, σ' (kN/m²)	Peak shear force, S_{peak} (N)	$\tau_f = \frac{S_{peak}}{A}$ (kN/m²)	Residual shear force, $S_{residual}$ (N)	$\tau_r = \frac{S_{residual}}{A}$ (kN/m²)
1	150	76.4	157.5	80.2	44.2	22.5
2	250	127.3	199.9	101.8	56.6	28.8
3	350	178.3	257.6	131.2	102.9	52.4
4	550	280.1	363.4	185.1	144.5	73.6

Example 12.2

The variations of τ_f and τ_r with σ' are plotted in Figure 12.19. From the plots, we find that

Peak strength: $\tau_f(\text{kN/m}^2) = \mathbf{40 + \sigma' \tan 27}$
Residual strength: $\tau_r(\text{kN/m}^2) = \mathbf{\sigma' \tan 14.6}$

(*Note*: For all *overconsolidated clays*, the residual shear strength can be expressed as

$$\tau_r = \sigma' \tan \phi'_r$$

where ϕ'_r = effective residual friction angle.)

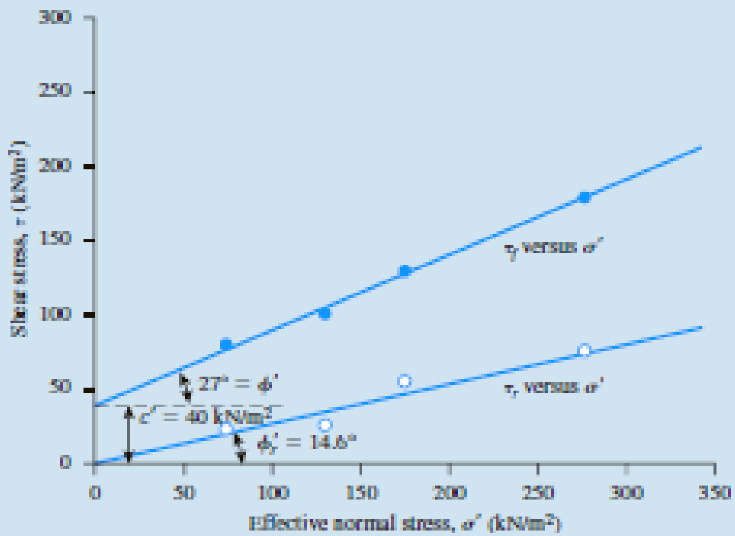


Figure 12.19 Variations of τ_f and τ_r with σ'

Example

For a dry sand specimen in a direct shear test box, the following are given:

- **Size of specimen: 63.5 mm 63.5 mm 31.75 mm (height)**
- **Angle of friction: 33°**
- **Normal stress: 193 kN/m^2.**

Determine the shear force required to cause failure.

τ = 193 tan (33) = 125.33 kPa

Shear force = τ *area

Shear force = 125.33x 0.0635x0.0635 = 0.50538 kN = 505.38 N

The end