Chapter 11

Compressibility of Soil

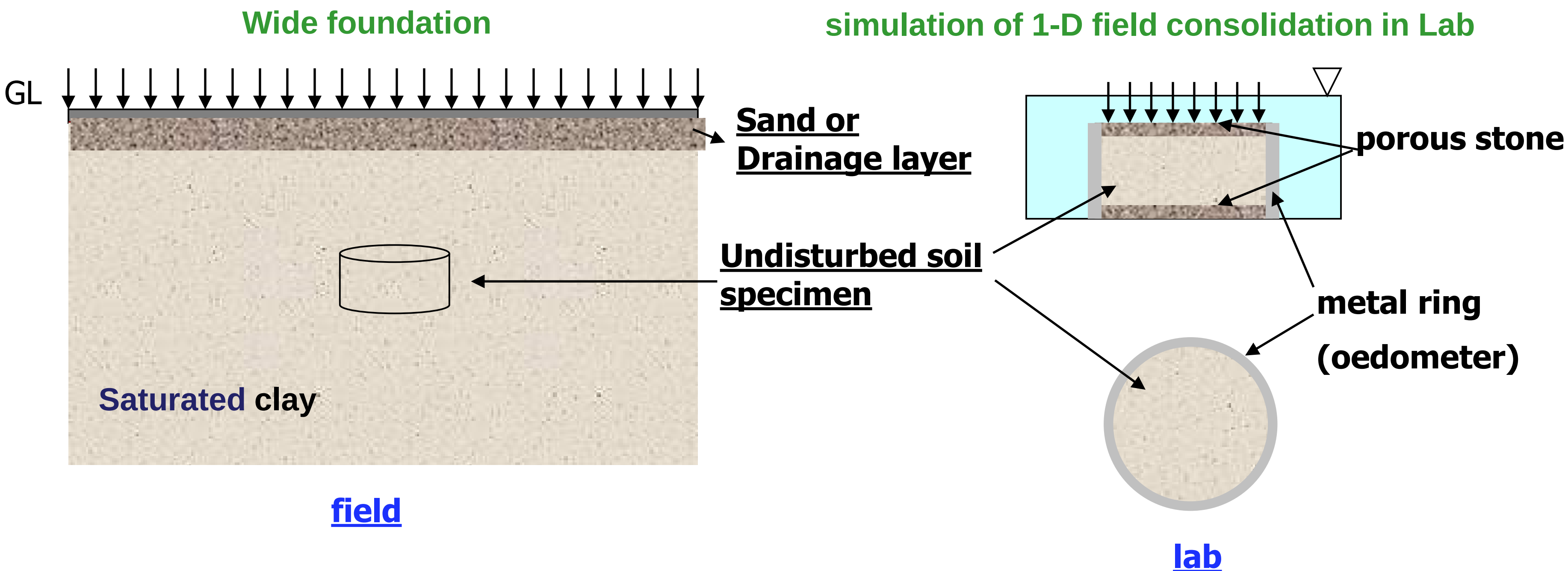
TOPICS

- ❑ INTRODUCTION
- ❑ ELASTIC SETTLEMENT
- ❑ CONSOLIDATION SETTLEMENT
  - Fundamentals of consolidation
  - One-dimensional Laboratory Consolidation Test
  - Calculation of Settlement from 1-D Primary Consolidation
  - Stress distribution in soil masses
- ❑ PRIMARY CONSOLIDATION SETTLEMENT
- ❑ SECONDARY CONSOLIDATION SETTLEMENT
- ❑ TIME RATE OF CONSOLIDATION SETTLEMENT
  - 1-D theory of consolidation

One-dimensional Laboratory Consolidation Test

One-dimensional Laboratory Consolidation Test

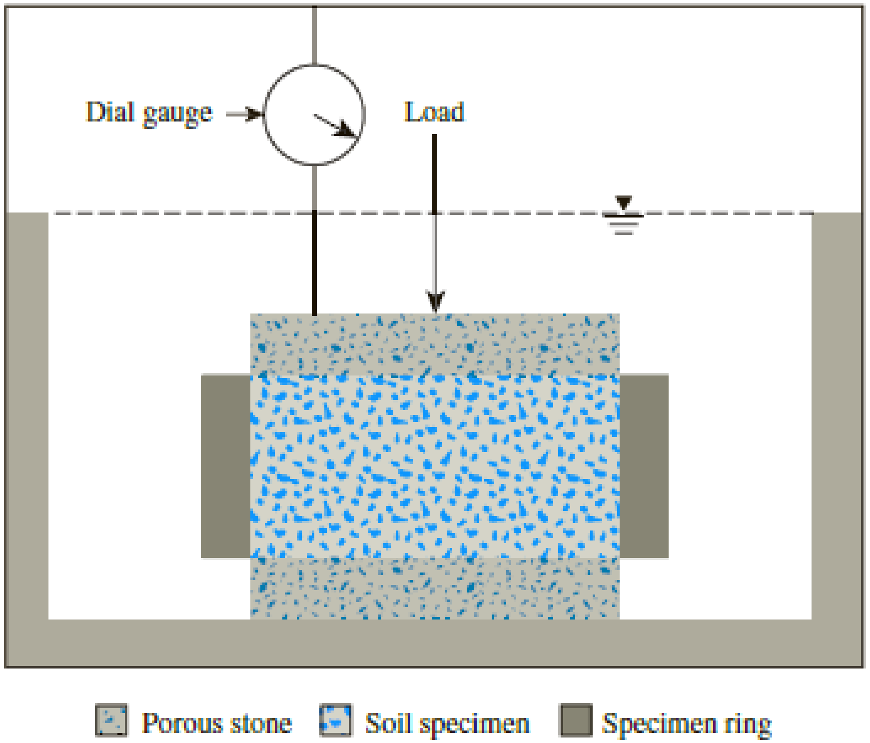
- **1-D field consolidation can be simulated in laboratory.**
- **Data obtained from laboratory testing can be used to predict magnitude of consolidation settlement reasonably, but rate is often poorly estimated.**



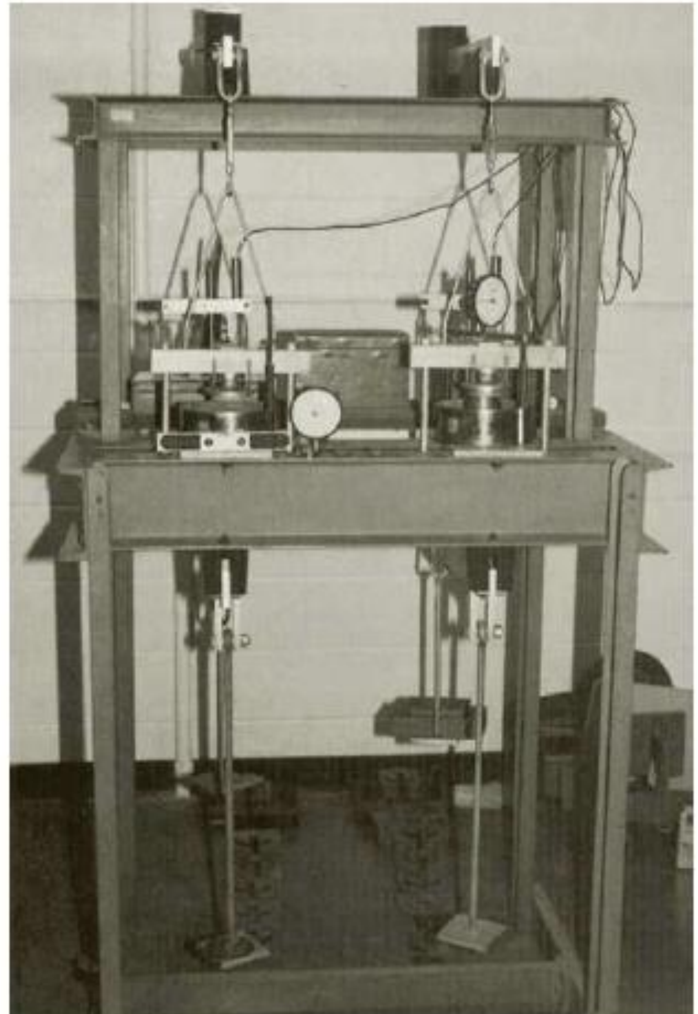
One-dimensional Laboratory Consolidation Test

- The one-dimensional consolidation test was first suggested by Terzaghi. It is performed in a consolidometer (sometimes referred to as oedometer). The schematic diagram of a consolidometer is shown below.

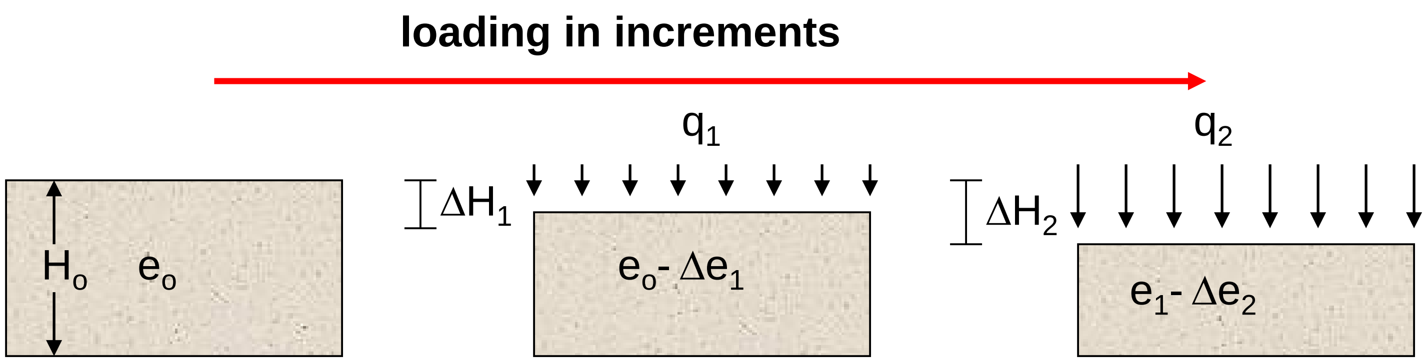
- The complete procedures and discussion of the test was presented in CE 380.



One-dimensional Laboratory Consolidation Test



Incremental Loading



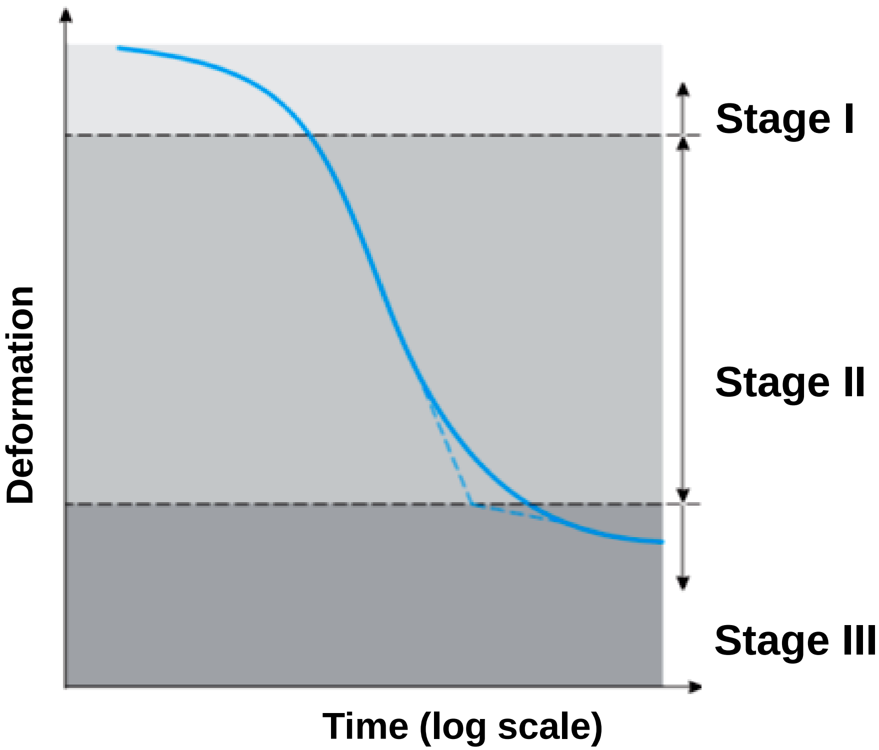
Load increment ratio (LIR) = $\Delta q/q = 1$ (i.e., double the load)

- **Allow full consolidation before next increment (24 hours)**
- **Record compression during and at the end of each increment using dial gauge.**
- **Example of time sequence: (10 sec, 30 sec, 1 min, 2, 4, 8, 15, 30, 1 hr, 2, 4, 8, 16, 24)**
- **The procedure is repeated for additional doublings of applied pressure until the applied pressure is in excess of the total stress to which the clay layer is believed to be subjected to when the proposed structure is built.**
- **The total pressure includes effective overburden pressure and net additional pressure due to the structure.**
- **Example of load sequence (25, 50, 100, 200, 400, 800, 1600, … kPa)**

Presentation of Results

- The results of the consolidation tests can be summarized in the following plots:
- Rate of consolidation curves (dial reading vs. log time or dial reading vs. square root time)
- Void ratio-pressure plots (Consolidation curve)

  $e - \sigma_v'$ plot or $e - \log \sigma_v'$ plot
- The plot of deformation of the specimen against time for a given load increment can observe three distinct stages:



Stage I: Initial compression, which is caused mostly by preloading.

Stage II: Primary consolidation, during which excess pore water pressure gradually is transferred into effective stress because of the expulsion of pore water.

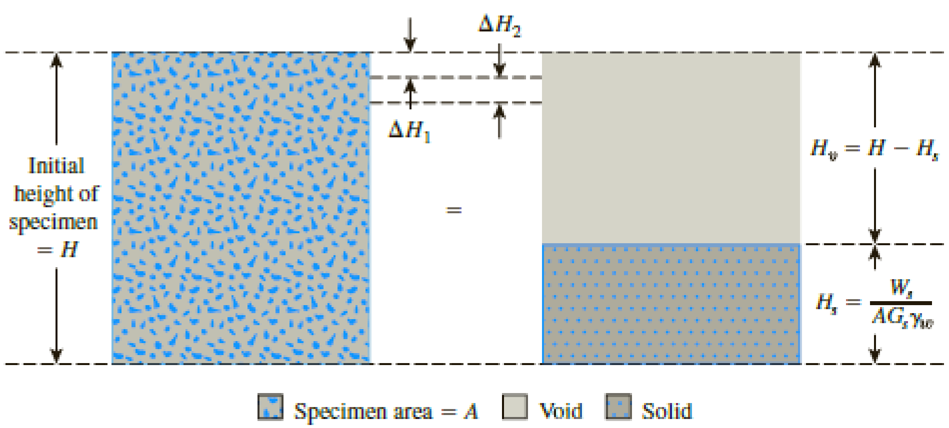
Stage III: Secondary consolidation, which occurs after complete dissipation of the excess pore water pressure, caused by plastic readjustment of soil fabric.

Void Ratio–Pressure Plots

Step 1. Calculate the height of solids, H_s, in the soil specimen using the equation

$$H_s = \frac{W_s}{AG_s\gamma_w} = \frac{M_s}{AG_s\rho_w}$$

where W_s = dry weight of the specimen
M_s = dry mass of the specimen
A = area of the specimen
G_s = specific gravity of soil solids
γ_w = unit weight of water
ρ_w = density of water



Void Ratio–Pressure Plots

Step 2. Calculate the initial height of voids as

$$H_v = H - H_s$$

where H = initial height of the specimen.

Step 3. Calculate the initial void ratio, e_o, of the specimen, using the equation

$$e_o = \frac{V_v}{V_s} = \frac{H_v}{H_s}\frac{A}{A} = \frac{H_v}{H_s}$$

Step 4. For the first incremental loading, σ_1 (total load/unit area of specimen), which causes a deformation ΔH_1, calculate the change in the void ratio as

$$\Delta e_1 = \frac{\Delta H_1}{H_s}$$

(ΔH_1 is obtained from the initial and the final dial readings for the loading).

It is important to note that, at the end of consolidation, total stress σ_1 is equal to effective stress σ'_1.

Step 5. Calculate the new void ratio after consolidation caused by the pressure increment as

$$e_1 = e_o - \Delta e_1$$

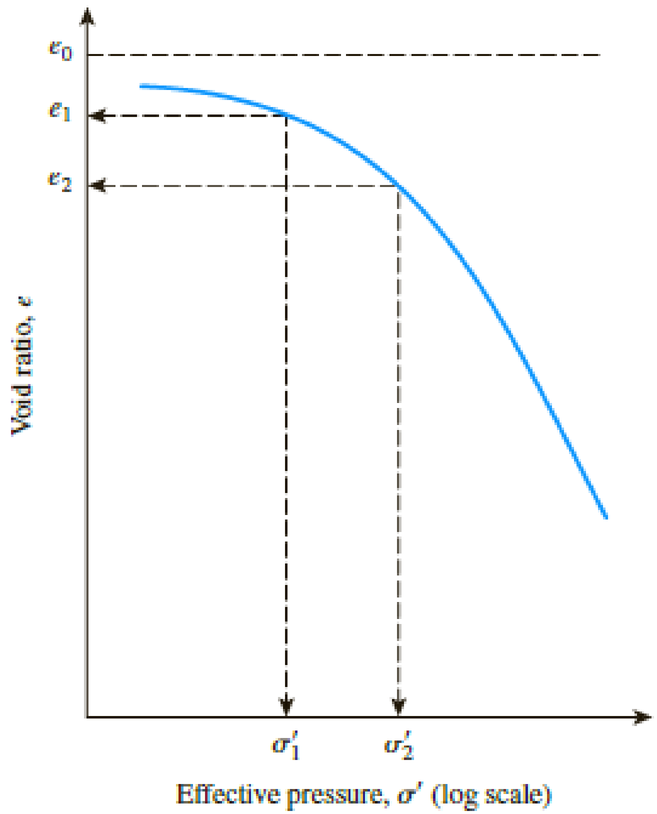
Void Ratio–Pressure Plots

For the next loading, σ_2 (*note:* σ_2 equals the cumulative load per unit area of specimen), which causes additional deformation ΔH_2, the void ratio at the end of consolidation can be calculated as

$$e_2 = e_1 - \frac{\Delta H_2}{H_s}$$

At this time, σ_2 – effective stress, σ'_2. Proceeding in a similar manner, one can obtain the void ratios at the end of the consolidation for all load increments.

The effective stress σ' and the corresponding void ratios (e) at the end of consolidation are plotted on semilogarithmic graph paper. The typical shape of such a plot is shown in Figure



EXAMPLE 11.3

Example 11.3

Following are the results of a laboratory consolidation test on a soil specimen obtained from the field: Dry mass of specimen = 128 g, height of specimen at the beginning of the test = 2.54 cm, G_s = 2.75, and area of the specimen = 30.68 cm^2.

Effective pressure, σ' (ton/ft^2)	Final height of specimen at the end of consolidation (cm)
0	2.540
0.5	2.488
1	2.465
2	2.431
4	2.389
8	2.324
16	2.225
32	2.115

Make necessary calculations and draw an e versus log σ' curve.

EXAMPLE 11.3

Solution

From Eq. (11.20),

$$H_s = \frac{W_s}{AG_s\gamma_w} = \frac{M_s}{AG_s\rho_w} = \frac{128 \text{ g}}{(30.68 \text{ cm}^2)(2.75)(1 \text{g/cm}^3)} = 1.52 \text{ cm}$$

Now the following table can be prepared.

Effective pressure, σ' (ton/ft^2)	Height at the end of consolidation, H (cm)	$H_v = H - H_s$ (cm)	$e = H_v/H_s$
0	2.540	1.02	0.671
0.5	2.488	0.968	0.637
1	2.465	0.945	0.622
2	2.431	0.911	0.599
4	2.389	0.869	0.572
8	2.324	0.804	0.529
16	2.225	0.705	0.464
32	2.115	0.595	0.390

The e versus log σ' plot is shown in Figure 11.15.

EXAMPLE 11.3

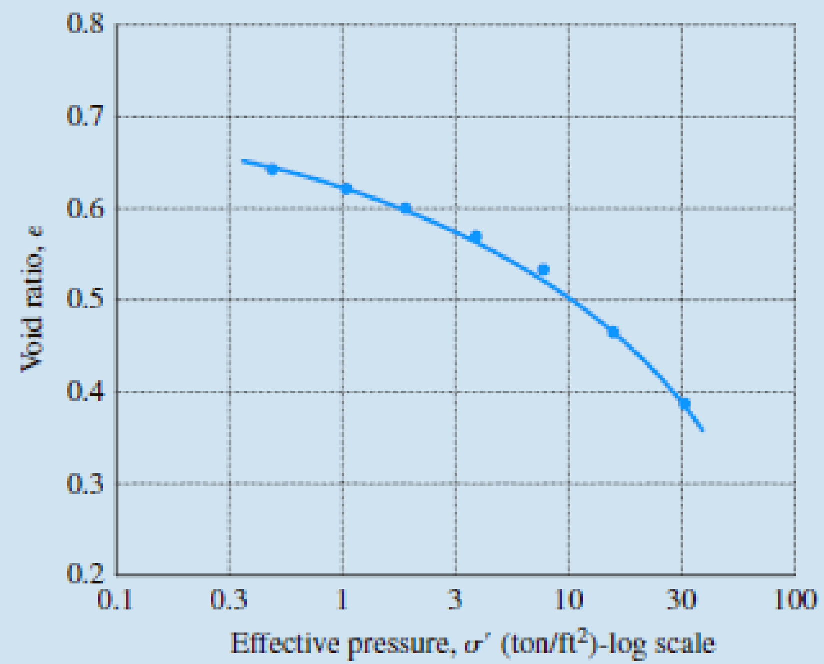
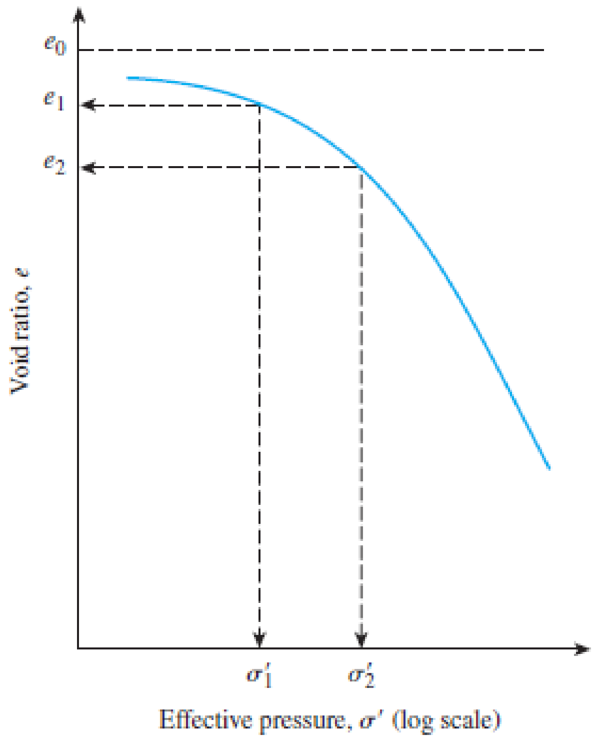


Figure 11.15 Variation of void ratio with effective pressure

Normally Consolidated and Overconsolidated Clays

The upper part of the e – log σ' plot is as shown below somewhat curved with a flat slope, followed by a linear relationship having a steeper slope.

This can be explained as follows:



- A soil in the field at some depth has been subjected to a certain maximum effective past pressure in its geologic history.

- This maximum effective past pressure may be equal to or less than the existing effective overburden pressure at the time of sampling.

- The reduction of effective pressure may be due to natural geological processes or human processes.

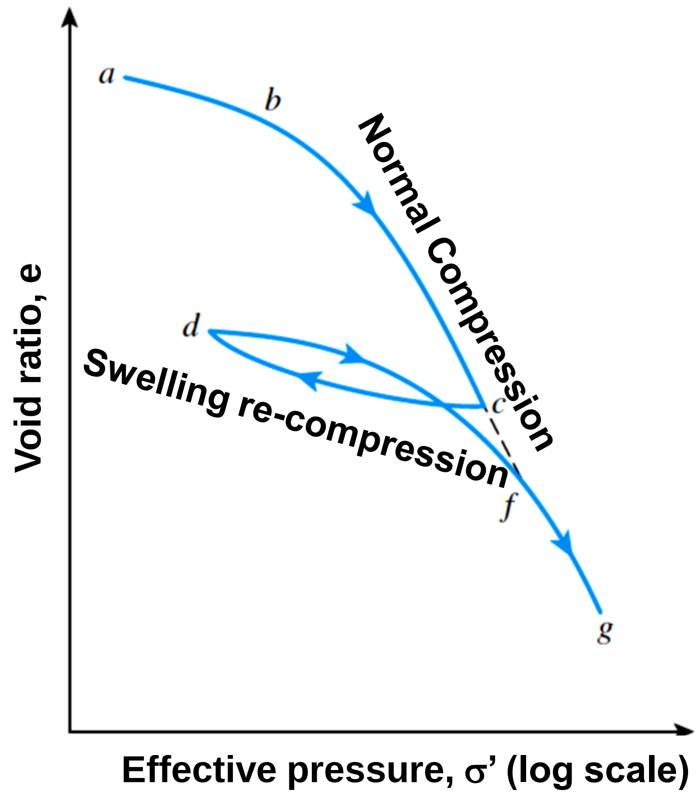
- During the soil sampling, the existing effective overburden pressure is also released, which results in some expansion.

Normally Consolidated and Overconsolidated Clays

- The soil will show relatively small decrease of e with load up until the point of the maximum effective stress to which the soil was subjected to in the past.

(Note: this could be the overburden pressure if the soil has not been subjected to any external load other than the weight of soil above that point concerned).

- This can be verified in the laboratory by loading, unloading and reloading a soil sample as shown across.



Normally Consolidated and Overconsolidated Clays

- **Normally Consolidated Clay (N.C. Clay)**

  A soil is NC if the present effective pressure to which it is subjected is the maximum pressure the soil has ever been subjected to.

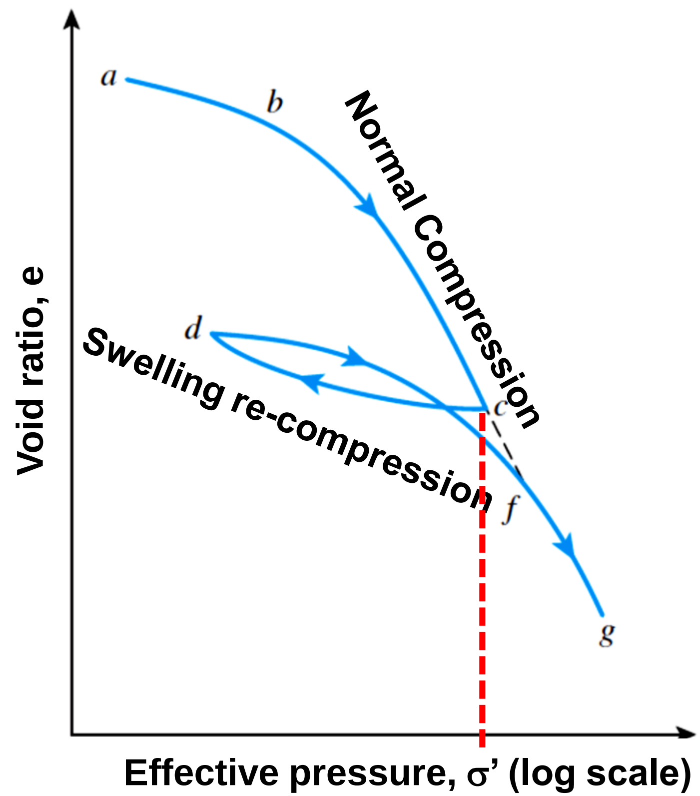
  The branches bc and fg are NC state of a soil.

- **Over Consolidated Clays (O.C. Clay)**

  A soil is OC if the present effective pressure to which it is subjected to is less than the maximum pressure to which the soil was subjected to in the past

  The branches ab, cd, df, are the OC state of a soil.

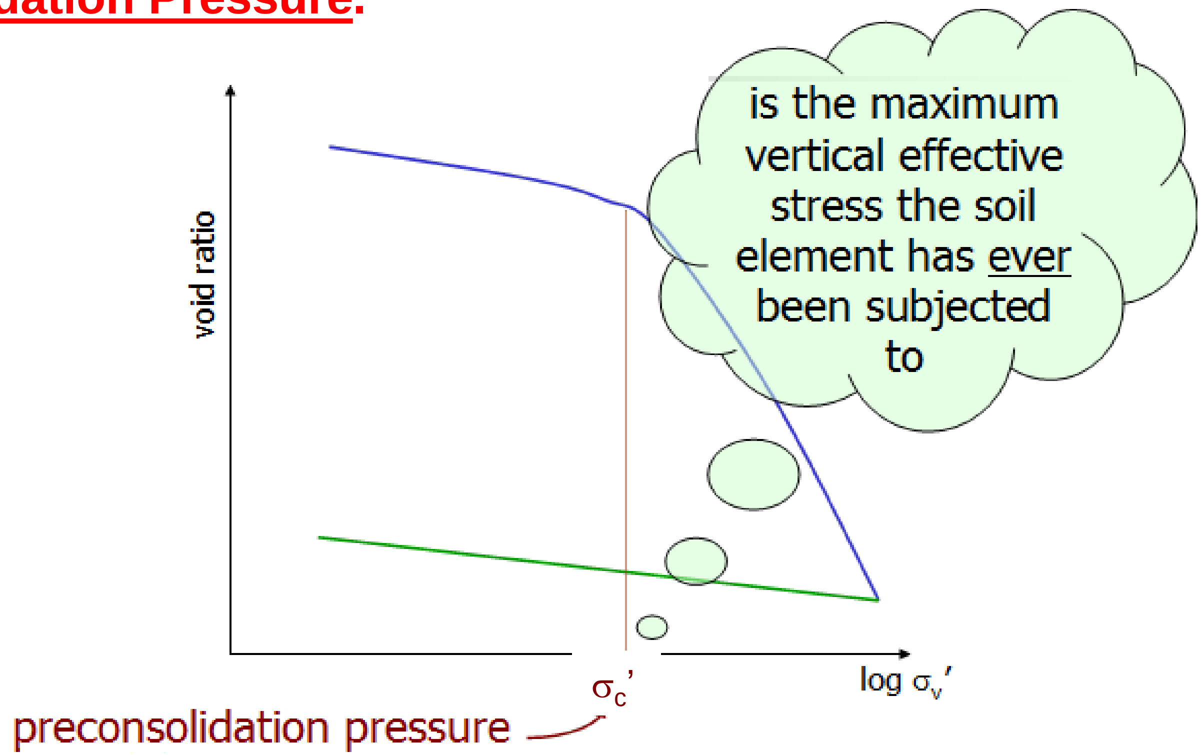
  The maximum effective past pressure is called the preconsolidation pressure.



Preconsolidation Pressure

- The stress at which the transition or "break" occurs in the curve of e vs. log σ' is an indication of the maximum vertical overburden stress that a particular soil sample has sustained in the past.

- This stress is very important in geotechnical engineering and is known as Preconsolidation Pressure.



Determination of Preconsolidation Pressure

<u>Casagrande</u> (1936) suggested a simple graphic construction to determine the preconsolidation pressure σ'_c from the laboratory e –log σ' plot.

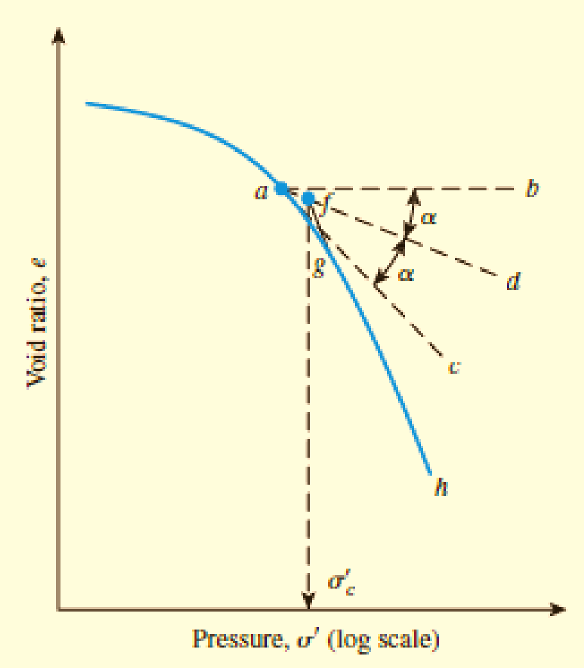
Step 1. By visual observation, establish point *a*, at which the *e*-log σ' plot has a minimum radius of curvature.

Step 2. Draw a horizontal line *ab*.

Step 3. Draw the line *ac* tangent at *a*.

Step 4. Draw the line *ad*, which is the bisector of the angle *bac*.

Step 5. Project the straight-line portion *gh* of the *e*-log σ' plot back to intersect line *ad* at *f*. The abscissa of point *f* is the preconsolidation pressure, σ'_c.

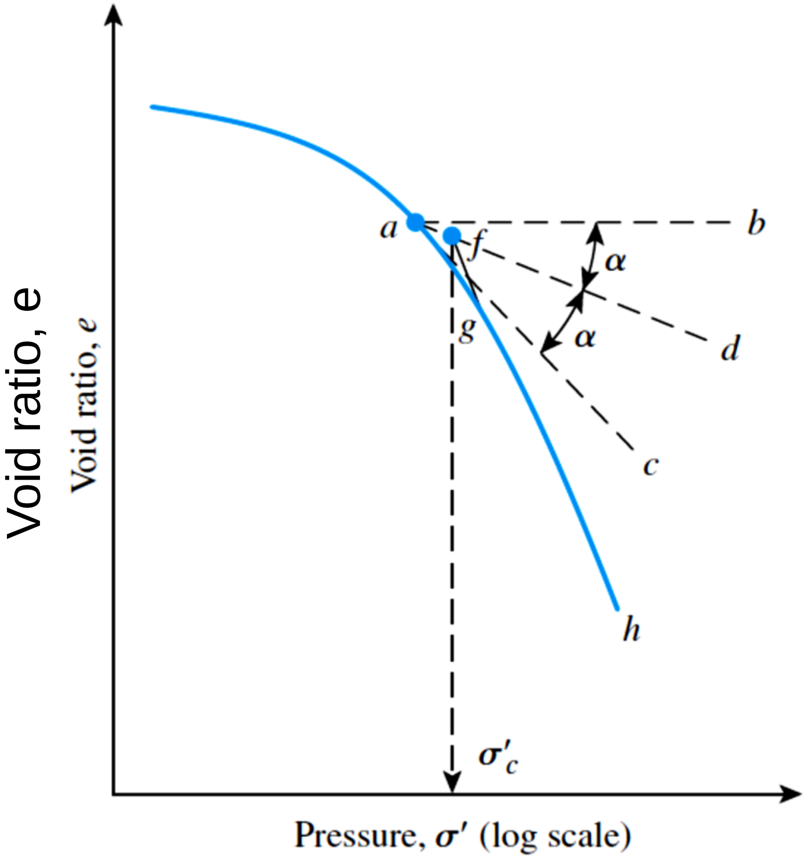


Overconsolidation Ratio (OCR)

- In general the overconsolidation ratio (OCR) for a soil can be defined as:

$$\boxed{OCR = \frac{\sigma'_c}{\sigma'}}$$

where σ' is the present effective vertical pressure.



- From the definition of NC soils, they always have OCR=1.

- To calculate OCR the preconsolidation pressure σ_c' should be known from the consolidation test and σ' is the effective stress in the field.

Preconsolidation Pressure

In the literature, some empirical relationships are available to predict the preconsolidation pressure.

- Stas and Kulhawy (1984):

$$\frac{\sigma'_c}{p_a} = 10^{[1.11 - 1.62(LI)]}$$

where p_a = atmospheric pressure ($\approx$100 kN/m^2)
LI = liquidity index

- Hansbo (1957)

$$\sigma'_c = \alpha_{(\mathrm{VST})}\, c_{u(\mathrm{VST})}$$

where $\alpha_{(\mathrm{VST})}$ = an empirical coefficient = $\dfrac{222}{LL(\%)}$

$c_{u(\mathrm{VST})}$ = undrained shear strength obtained from vane shear test

In any case, these above relationships may change from soil to soil. They may be taken as an initial approximation.

Factors Affecting the Determination of σ_c'

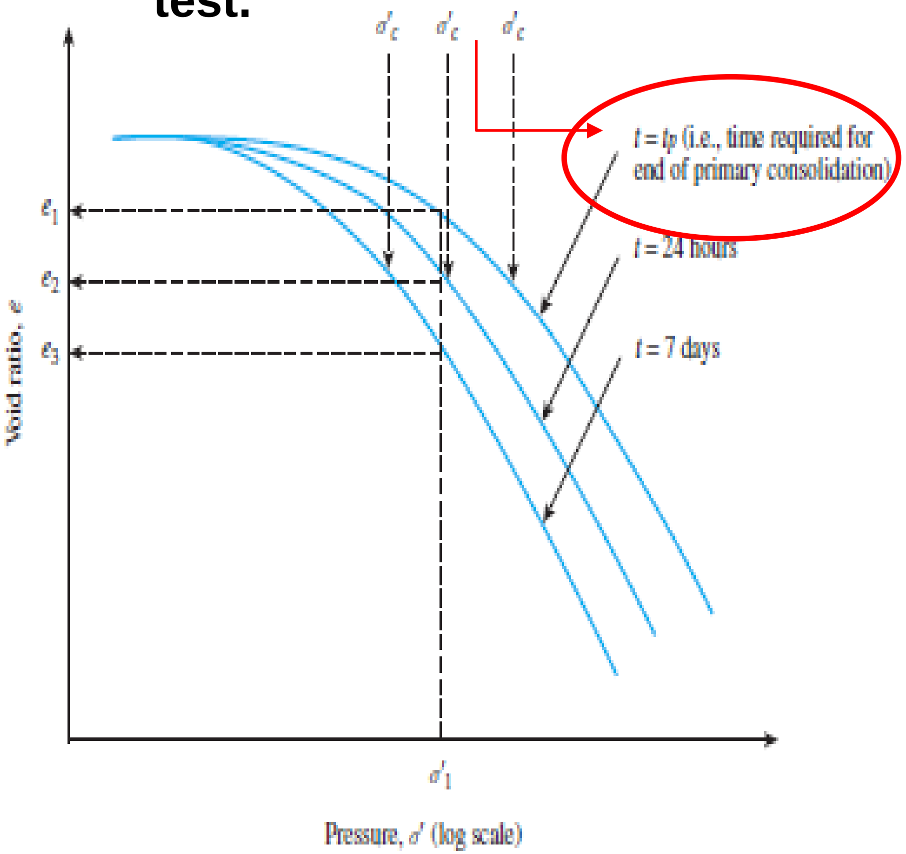
1. Duration of load increment

- When the duration of load maintained on a sample is increased the e vs. log σ' gradually moves to the left.

- The reason for this is that as time increased the amount of secondary consolidation of the sample is also increased. This will tend to reduce the void ratio e.

- The value of σ_c ' will increase with the decrease of t.

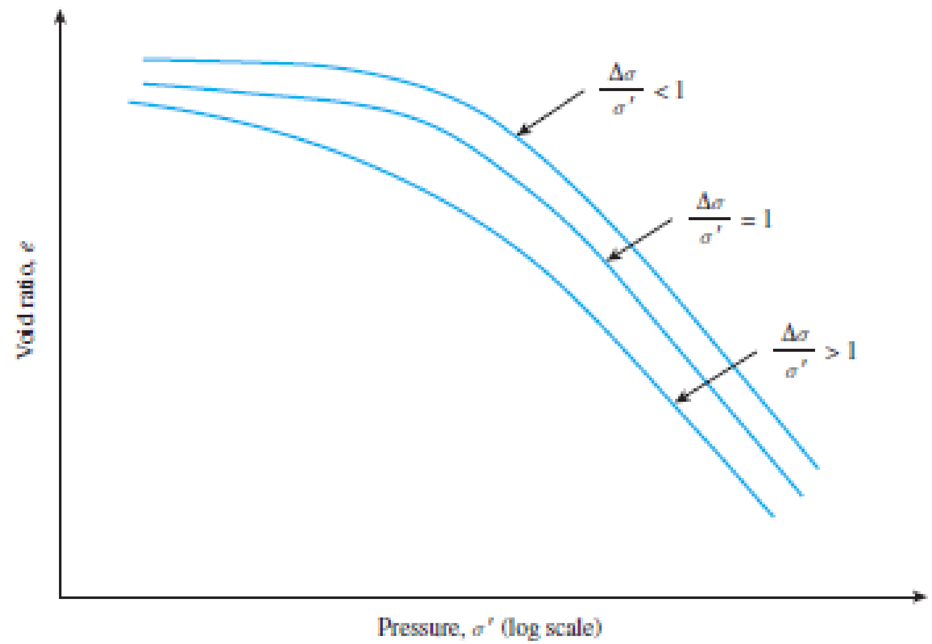
t_p is to be known from either plotting of deformation vs. time or excess p.w.p. if it is being monitored during the test.



Factors Affecting the Determination of σ_c'

2. Load Increment Ratio (LIR)

- **LIR is defined as the change in pressure of the pressure increment divided by the initial pressure before the load is applied.**
- **LIR =1, means the load is doubled each time, this results in evenly spaced data points on e vs. log σ' curve**
- **When LIR is gradually increased, the e vs. log σ' curve gradually moves to the left.**



EXAMPLE 11.4

Example 11.4

Following are the results of a laboratory consolidation test.

Pressure, σ' (kN/m^2)	Void ratio, e
50	0.840
100	0.826
200	0.774
400	0.696
800	0.612
1000	0.528

Using Casagrande's procedure, determine the preconsolidation pressure σ'_c.

EXAMPLE 11.4

Solution

Figure 11.18 shows the e-log σ' plot. In this plot, a is the point where the radius of curvature is minimum. The preconsolidation pressure is determined using the procedure shown in Figure 11.17. From the plot, $\boldsymbol{\sigma'_c = 160}$ **kN/m²**.

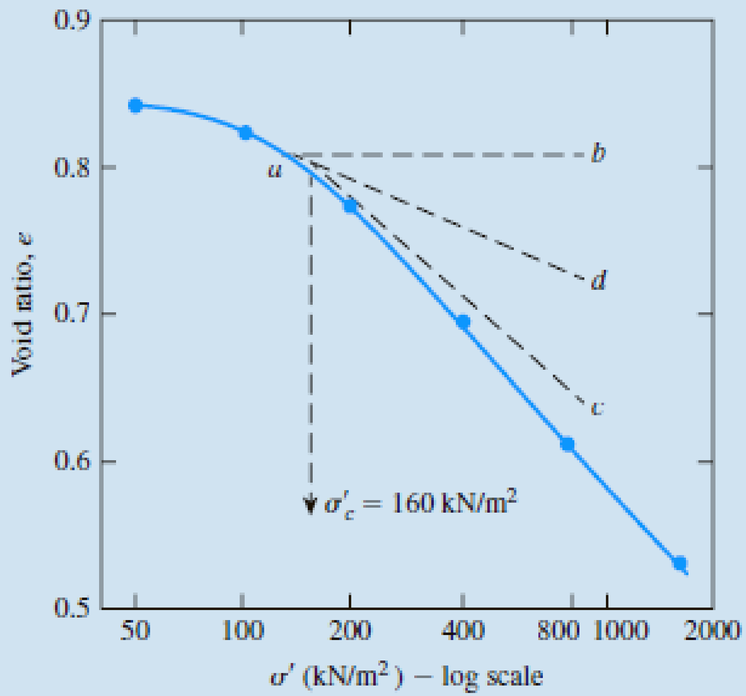


Figure 11.18

EXAMPLE 11.5

Example 11.5

A soil profile is shown in Figure 11.19. Using Eq. (11.27), estimate the preconsolidation pressure σ'_c and overconsolidation ratio OCR at point A.

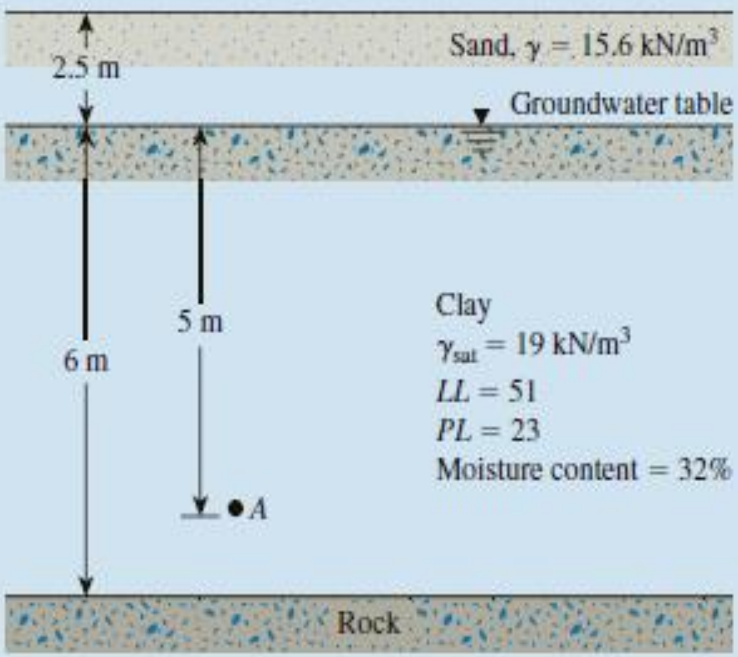


Figure 11.19

EXAMPLE 11.5

Solution

Liquidity index:

$$LI = \frac{w - PL}{LL - PL} = \frac{32 - 23}{51 - 23} = 0.32$$

From Eq. (11.27),

$$\sigma'_c = (p_a)10^{[1.11 - 1.62(LI)]} = (100)10^{[1.11 - (1.62)(0.32)]} = \mathbf{390.48\ kN/m^2}$$

At A, the present effective pressure is

$$\sigma' = (2.5)(15.6) + (5)(19 - 9.81) = 84.95\ \text{kN/m}^2$$

So,

$$OCR = \frac{390.48}{84.95} \approx \mathbf{4.6}$$

Field Compression Curve

- Due to soil disturbance, even with high-quality sampling and testing the actual compression curve has a SLOPE which is somewhat LESS than the slope of the field VIRGIN COMPRESSION CURVE. The "break" in the curve becomes less sharp with increasing disturbance.

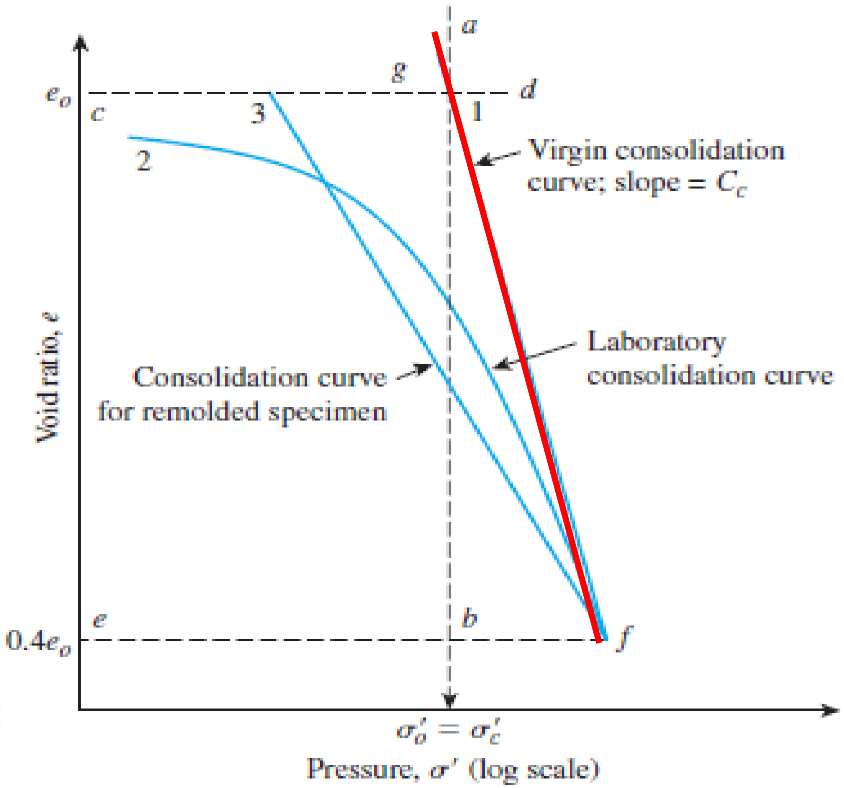
Sources of disturbance:

- Sampling
- Transportation
- Storage
- Preparation of the specimen (like trimming)

Normally consolidated and overconsolidated clays

- We know the present effective overburden σ'_0 and void ratio e_0.
- We should know from the beginning whether the soil is NC or OC by comparing σ'_0 and σ'_C . $\sigma'_0 = \gamma\, z$, σ'_C we find it through the procedures presented in a previous slides.

Graphical procedures to evaluate the slope of the field compression curve



- **Determine from Curve 2 (Laboratory test) the preconsolidation pressure $\sigma'_c = \sigma'_o$**
- **Draw a vertical line *ab***
- **Calculate the void ratio in the field e_o**

$$e_o = \frac{V_v}{V_s} = \frac{H_v A}{H_s A} = \frac{H_v}{H_s} \qquad H_s = \frac{W_s}{AG_s\gamma_w} \qquad H_v = H - H_s$$

- **Draw a horizontal line *cd***
- **Calculate $0.4e_o$ Draw a horizontal line *ef***
- **Join Points *f* and *g***

This is the virgin compression curve

Normally consolidated clays

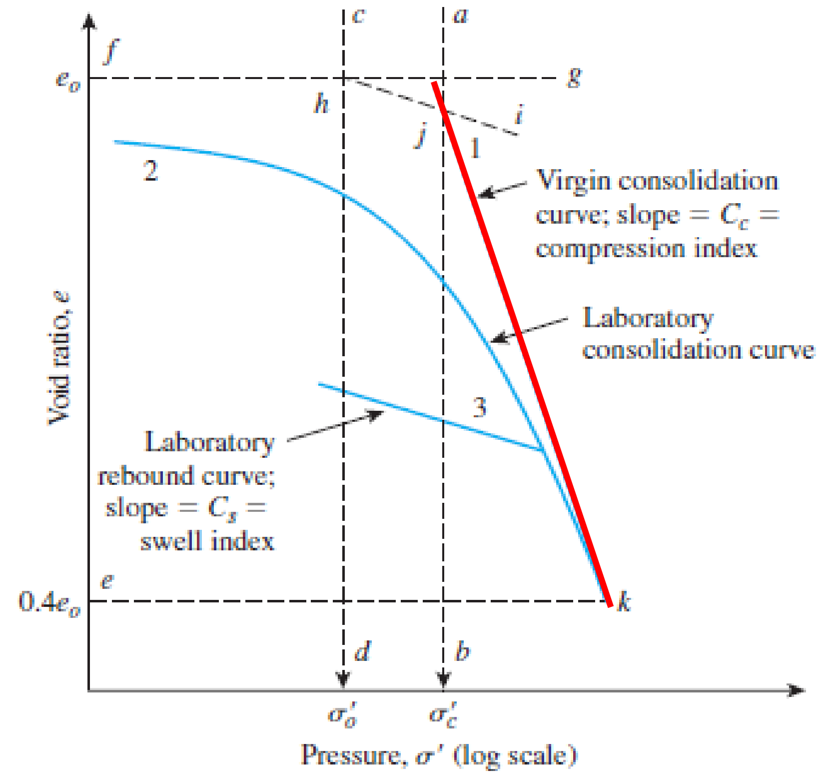
Graphical procedures to evaluate the slope of the field compression curve

- **Determine from Curve 2 (Laboratory test) the preconsolidation pressure σ'_C**
- **Draw a vertical line *ab***
- **Determine the field effective overburden pressure σ'_o Draw a vertical line *cd***
- **Calculate the void ratio in the field e_o**

$$e_o = \frac{V_v}{V_s} = \frac{H_v\,A}{H_s\,A} = \frac{H_v}{H_s} \qquad H_s = \frac{W_s}{AG_s\gamma_w} \qquad H_v = H - H_s$$

- **Draw a horizontal line *fg***
- **Calculate $0.4e_o$ Draw a horizontal line *ek***
- **Draw a line *hi* parallel to curve 3**
- **Join Points *k* and *j***

This is the virgin compression curve



Overconsolidated clays

Calculation of 1-D Consolidation Settlement

Calculation of 1-D Consolidation Settlement

$$\Delta V = V_0 - V_1 = HA - (H - S_c)A = \underline{S_c\, A}$$

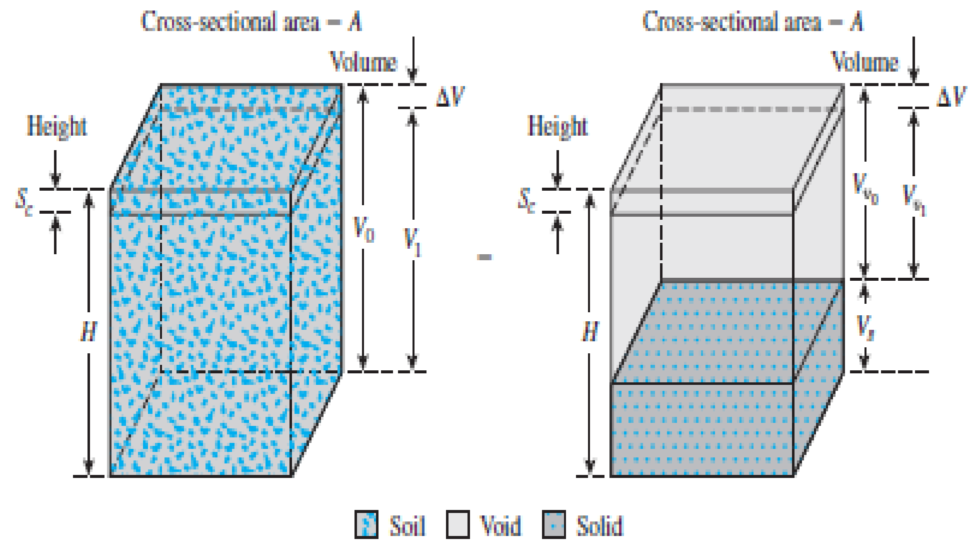
$$\Delta V = S_c A = V_{v_0} - V_{v_1} = \underline{\Delta V_v}$$

$$\Delta V_v = \underline{\Delta e V_s}$$

$$V_s = \frac{V_0}{1 + e_o} = \frac{AH}{1 + e_o}$$

$$\Delta V = S_c A = \Delta e V_s = \frac{AH}{1 + e_o} \Delta e$$

$$\boxed{S_c = H \frac{\Delta e}{1 + e_o}}$$



The consolidation settlement can be determined knowing:

- **Initial void ratio e_0.**
- **Thickness of layer H**
- **Change of void ratio Δe**

It only requires the evaluation of Δe

Calculation of 1-D Consolidation Settlement

Settlement Calculation

$$S_c = \Delta H = H_o - H_f = (h_1 - h_2)$$

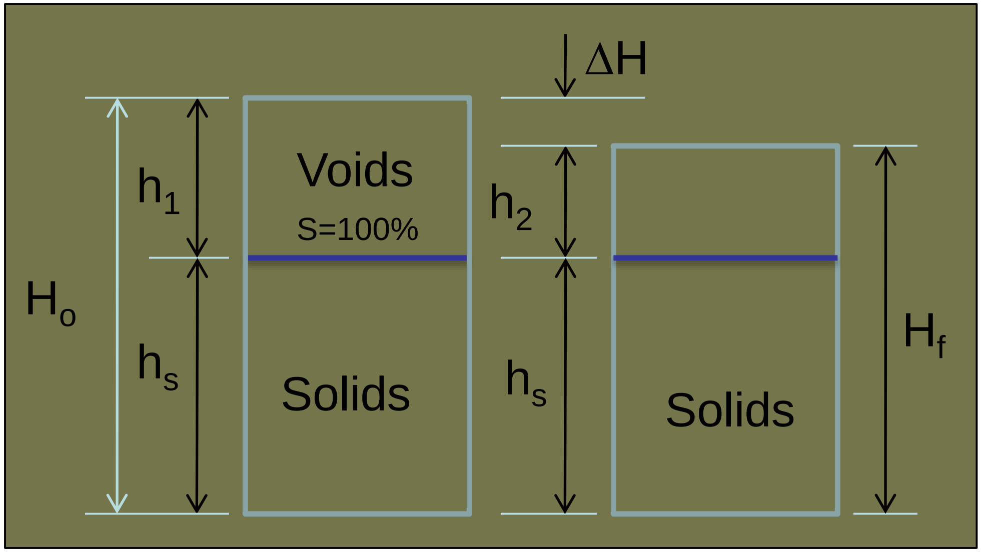
$$S_c = (h_1 - h_2)\frac{H_o}{H_o}$$

$$S_c = \left(\frac{h_1 - h_2}{h_s + h_1}\right)H$$

$$S_c = \left(\frac{(h_1 - h_2)/h_s}{(h_s + h_1)/h_s}\right)H$$

$$S_c = \left(\frac{e_o - e_f}{1 + e_o}\right)H$$

$$\boxed{S_c = \frac{\Delta e}{1 + e_o}H}$$



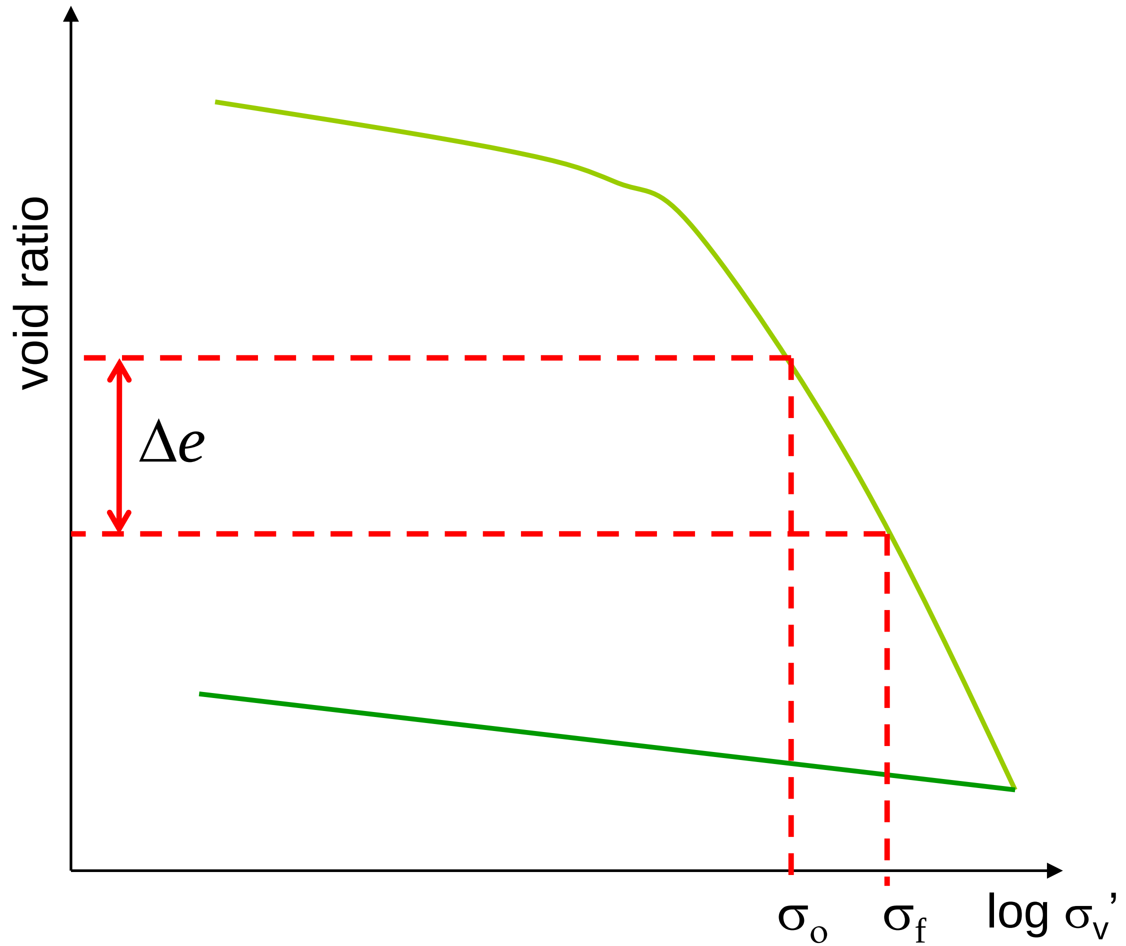
It only requires the evaluation of Δe

Calculation of Primary Consolidation Settlement

I) Using e - log σ_v

If the e-log σ' curve is given, Δe can simply be picked off the plot for the appropriate range and pressures.

$$S_c = \frac{\Delta e}{1+e_o} H$$



Calculation of Primary Consolidation Settlement

II) Using m_v

$$S_C = m_v . H . \Delta\sigma$$

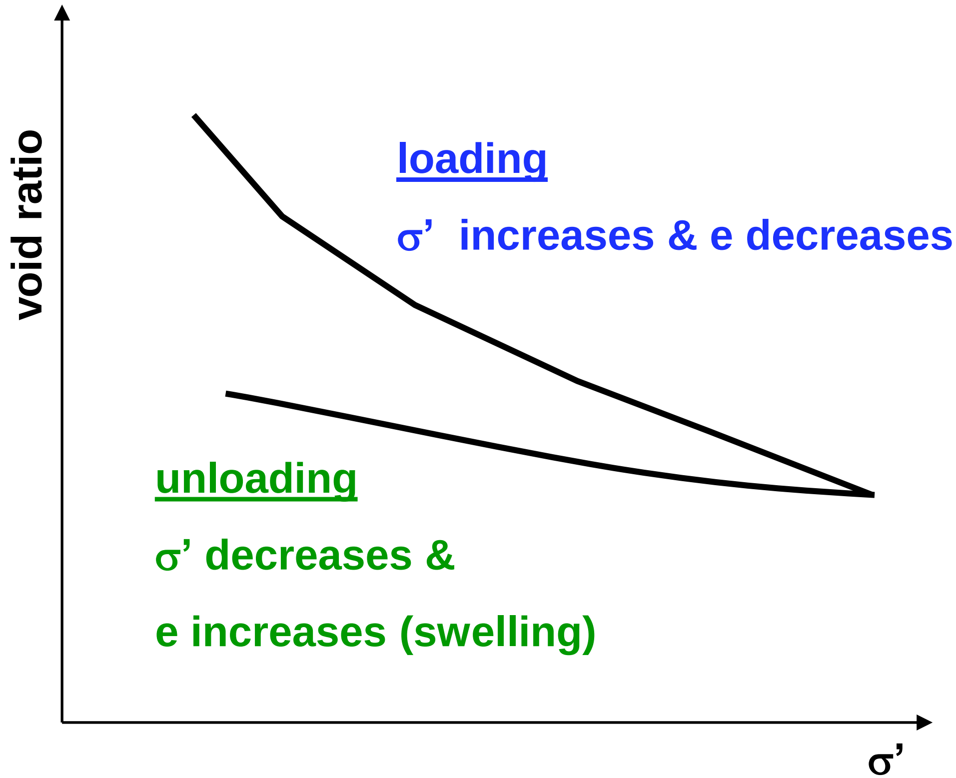
$$m_v = \frac{\Delta e}{\Delta\sigma(1+e_0)}$$

Disadvantage

m_v is obtained from e vs. $\Delta\sigma$ which is nonlinear and m_v is stress level dependent. This is on contrast to C_c which is constant for a wide range of stress level.

Presentation of Results

e – σ’ plot



- **The figure above is usually termed the compressibility curve__, where compressibility is the term applied to 1-D volume change that occurs in cohesive soils that are subjected to compressive loading.**

- **Note: It is more convenient to express the stress-stain relationship for soil in consolidation studies in terms of void ratio and unit pressure instead of unit strain and stress used in the case of most other engineering materials.**

Coefficient of Volume Compressibility [m_v]

- **m_v is defined as the volume change per unit volume per unit increase in effective stress**

~ denoted by m_v

~ is the volumetric strain in a clay element per unit increase in stress

$$= \frac{\text{change in volume}}{\text{original volume}}$$

no units

i.e.,

$$m_v = \frac{\Delta V / V}{\Delta \sigma}$$

$$\frac{\Delta e}{1+e_o}$$

kPa^{-1} or MPa^{-1}

kPa or MPa

$$m_v = \frac{\Delta e}{\Delta \sigma'(1+e_o)}$$

- **m_v is also known as Coefficient of Volume Change.**

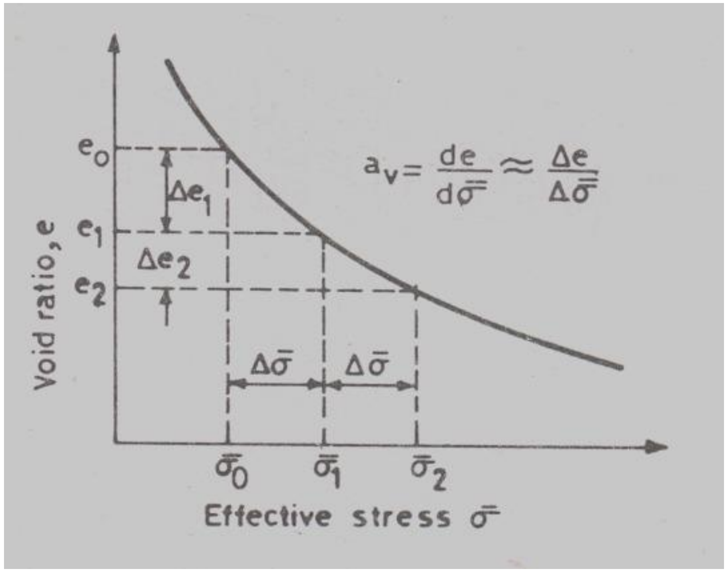
- **The value of m_v for a particular soil is <u>not constant</u> but depends on the stress range over which it is calculated.**

Coefficient of Compressibility a_v

- a_v is the slope of e-σ' plot, or $a_v = -de/d\sigma'$ (m^2/kN)
- Within a narrow range of pressures, there is a linear relationship between the decrease of the voids ratio *e* and the increase in the pressure (stress). Mathematically,

$$a_v = \frac{\Delta e}{\Delta \sigma'}$$

$$m_v = \frac{a_v}{1+e_o}$$



- a_v decreases with increases in effective stress
- Because the slope of the curve e-σ' is constantly changing, it is somewhat difficult to use a_v in a mathematical analysis, as is desired in order to make settlement calculations.

Calculation of Primary Consolidation Settlement

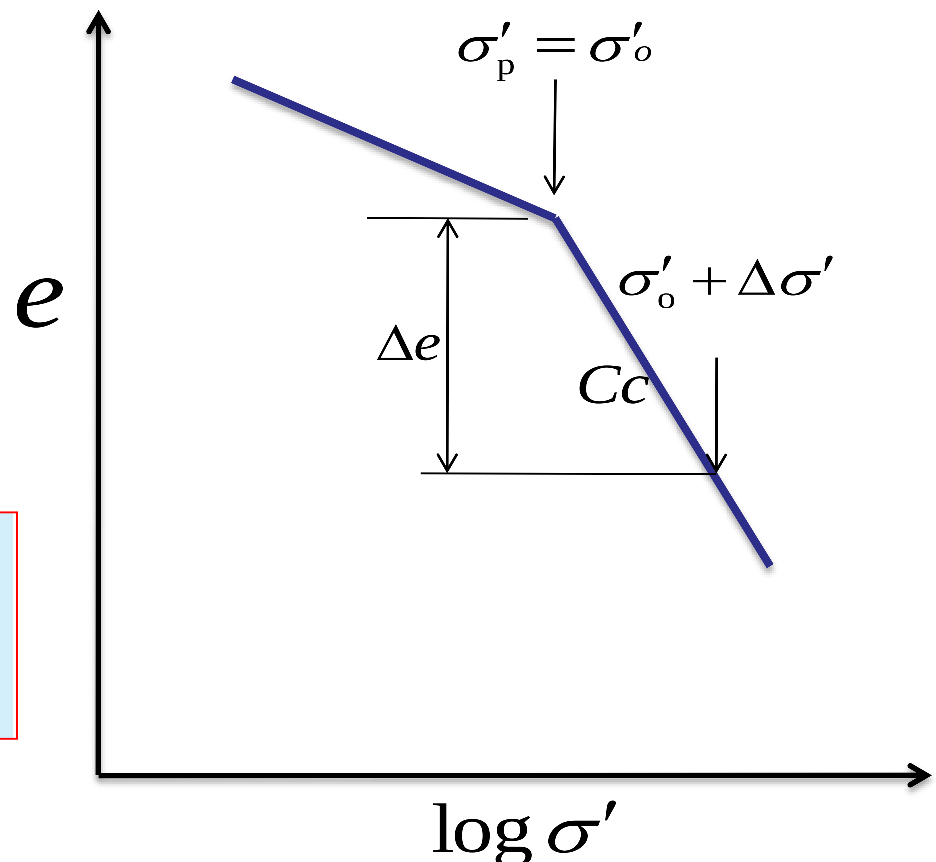
III) Using Compression and Swelling Indices

a) Normally Consolidated Clay ($\sigma'_0 = \sigma_c'$)

$$S_c = \frac{\Delta e}{1+e_o} H$$

$$\Delta e = C_c \log\left(\frac{\sigma'_o + \Delta\sigma}{\sigma'_o}\right)$$

$$\boxed{S_c = \frac{C_c H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)}$$



Calculation of Primary Consolidation Settlement

b) Overconsolidated Clays

$$S_c = \frac{\Delta e}{1+e_o} H$$

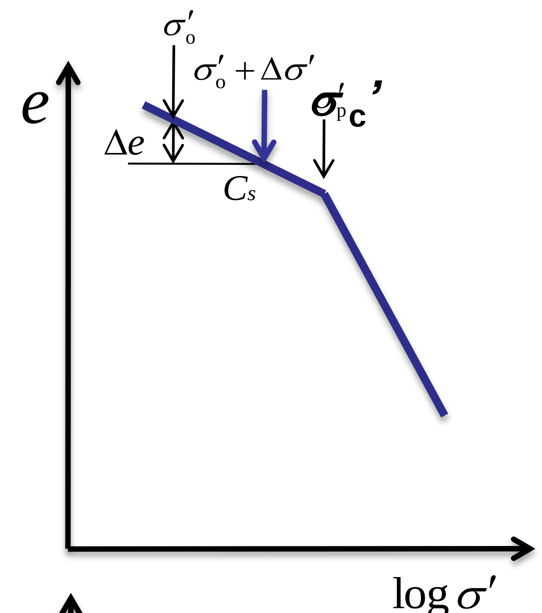
Case I: $\sigma'_0 + \Delta\sigma' \le \sigma_c'$

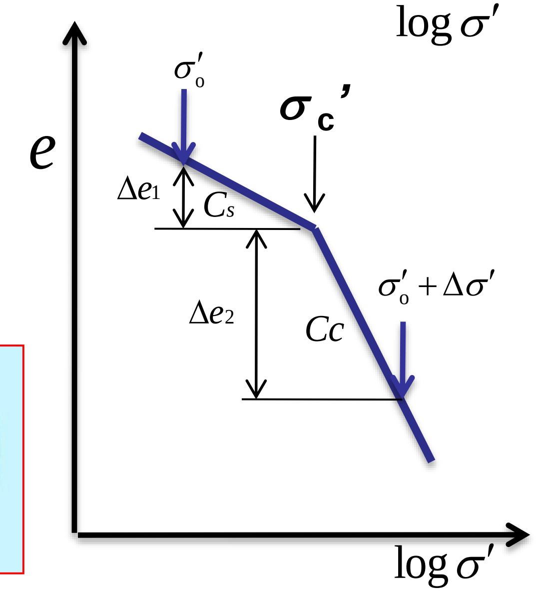
$$\Delta e = C_s[\log(\sigma'_o + \Delta\sigma') - \log \sigma'_o]$$

$$\boxed{S_c = \frac{C_s H}{1+e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)}$$

Case II: $\sigma'_0 + \Delta\sigma' > \sigma_c'$

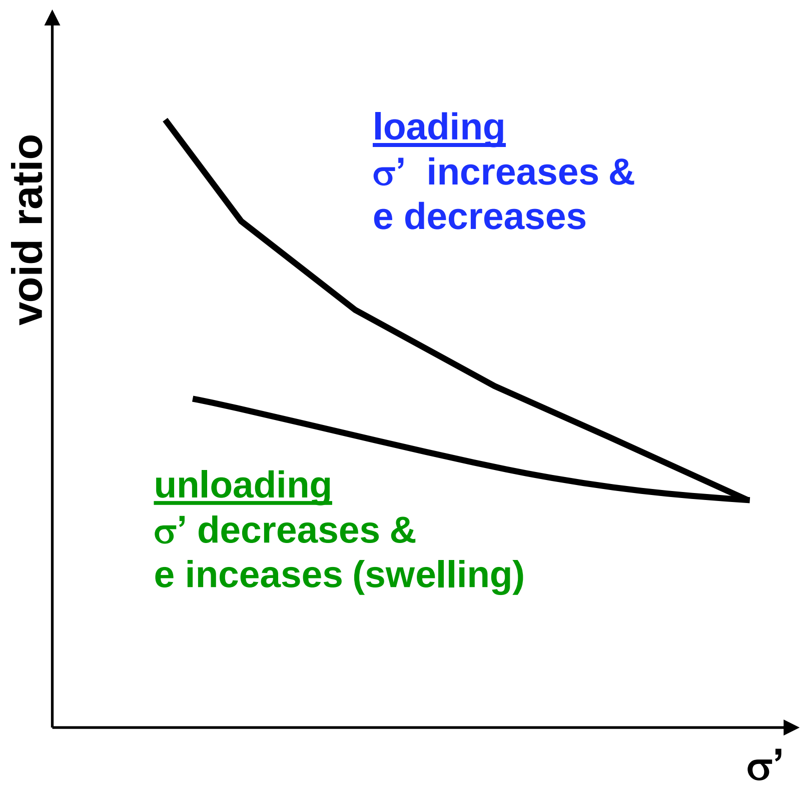
$$\boxed{S_c = \frac{C_s H}{1+e_o} \log\frac{\sigma'_c}{\sigma'_o} + \frac{C_c H}{1+e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right)}$$



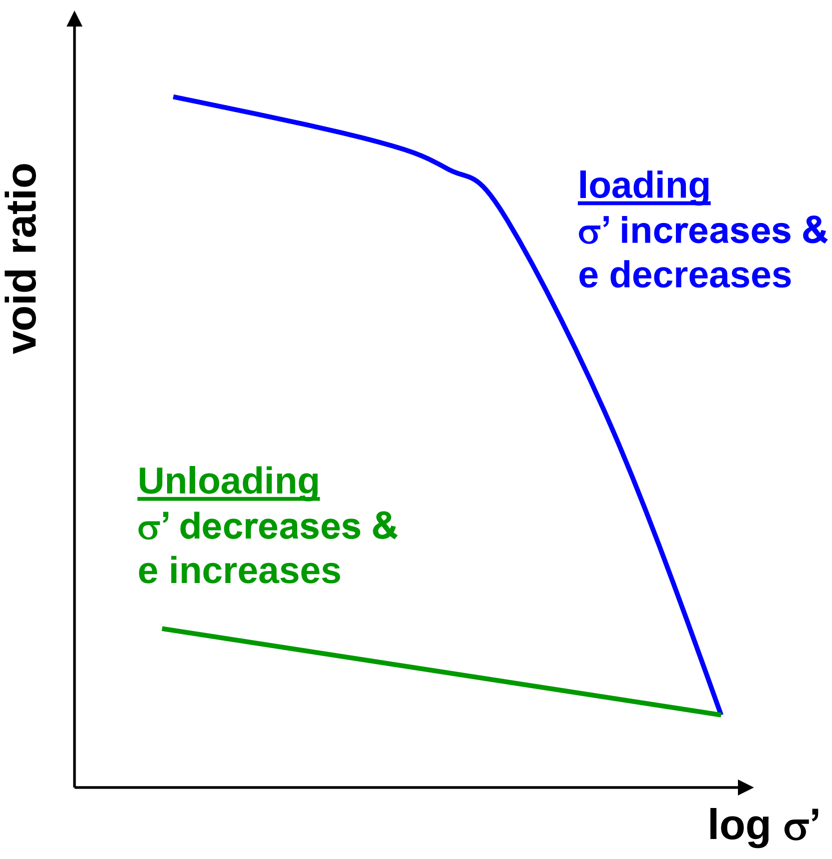


Presentation of Results

e – σ' plot

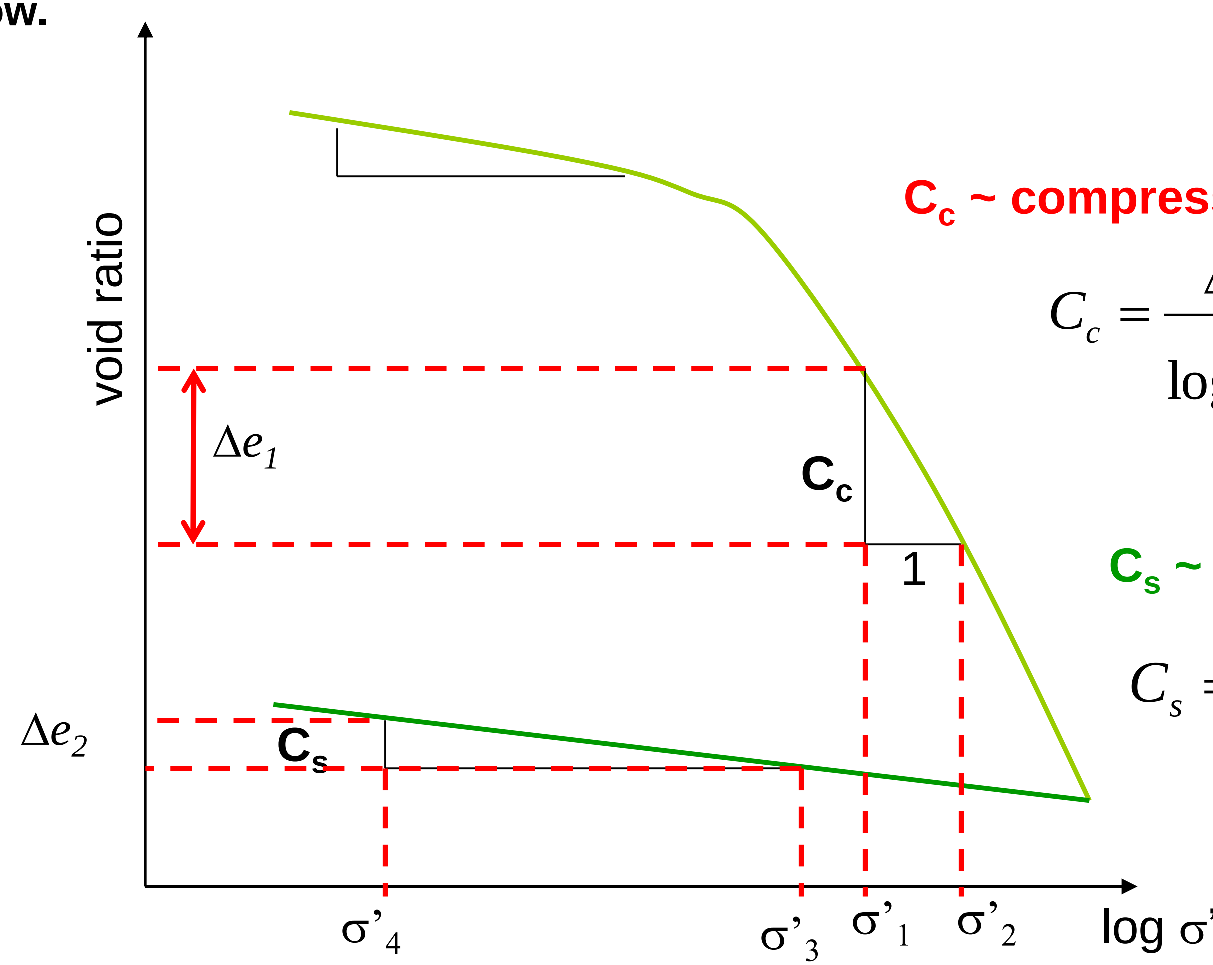


e – log σ' plot



Compression and Swell Indices

As we said earlier, the main limitation of using a_v and m_v in describing soil compressibility is that they are not constant. To overcome this shortcoming the relationship between e and σ_v' is usually plotted in a semi logarithmic plot as shown below.



C_c ~ compression index

$$C_c = \frac{\Delta e_1}{\log \dfrac{\sigma_2'}{\sigma_1'}}$$

C_s ~ Swell index

$$C_s = \frac{\Delta e_2}{\log \dfrac{\sigma_3'}{\sigma_4'}}$$

Correlations for Compression Index, c_c

- This index is best determined by the laboratory test results for void ratio, e, and pressure σ' (as shown above).
- Because conducting compression (consolidation) test is relatively time consuming (usually 2 weeks), C_c is usually related to other index properties like:

$$C_c = 0.009(LL - 10)$$

$$C_c = 0.141 G_s^{1.2}\left(\frac{1 + e_o}{G_s}\right)^{2.38}$$

$$C_c = 0.2343\left[\frac{LL(\%)}{100}\right]G_s$$

$$C_c \approx 0.5 G_s \frac{[PI(\%)]}{100}$$

G_S: Specific Gravity
e_0 : in situ void ratio
PI: Plasticity Index
LL: Liquid Limit

Correlations for Compression Index, c_c

Table 11.6 Correlations for Compression Index, C_c*

Equation	Reference	Region of applicability
$C_c = 0.007(LL - 7)$	Skempton (1944)	Remolded clays
$C_c = 0.01w_N$		Chicago clays
$C_c = 1.15(e_o - 0.27)$	Nishida (1956)	All clays
$C_c = 0.30(e_o - 0.27)$	Hough (1957)	Inorganic cohesive soil: silt, silty clay, clay
$C_c = 0.0115w_N$		Organic soils, peats, organic silt, and clay
$C_c = 0.0046(LL - 9)$		Brazilian clays
$C_c = 0.75(e_o - 0.5)$		Soils with low plasticity
$C_c = 0.208e_o + 0.0083$		Chicago clays
$C_c = 0.156e_o + 0.0107$		All clays

*After Rendon-Herrero, 1980. With permission from ASCE.
Note: e_o = in situ void ratio; w_N = in situ water content.

Correlations for Swell Index, c_s

$$C_s \simeq \frac{1}{5} \text{ to } \frac{1}{10} C_c$$

$$C_s = 0.0463\left[\frac{LL(\%)}{100}\right]G_s$$

$$C_s \approx \frac{PI}{370}$$

EXAMPLE 11.6

Example 11.6

The following are the results of a laboratory consolidation test:

Pressure, σ' (kN/m^2)	Void ratio, e	Remarks	Pressure, σ' (kN/m^2)	Void ratio, e	Remarks
25	0.93	Loading	800	0.61	Loading
50	0.92		1600	0.52	
100	0.88		800	0.535	Unloading
200	0.81		400	0.555	
400	0.69		200	0.57	

a. Calculate the compression index and the ratio of C_s/C_c.

b. On the basis of the average e-log σ' plot, calculate the void ratio at $\sigma'_o = 1000$ kN/m^2.

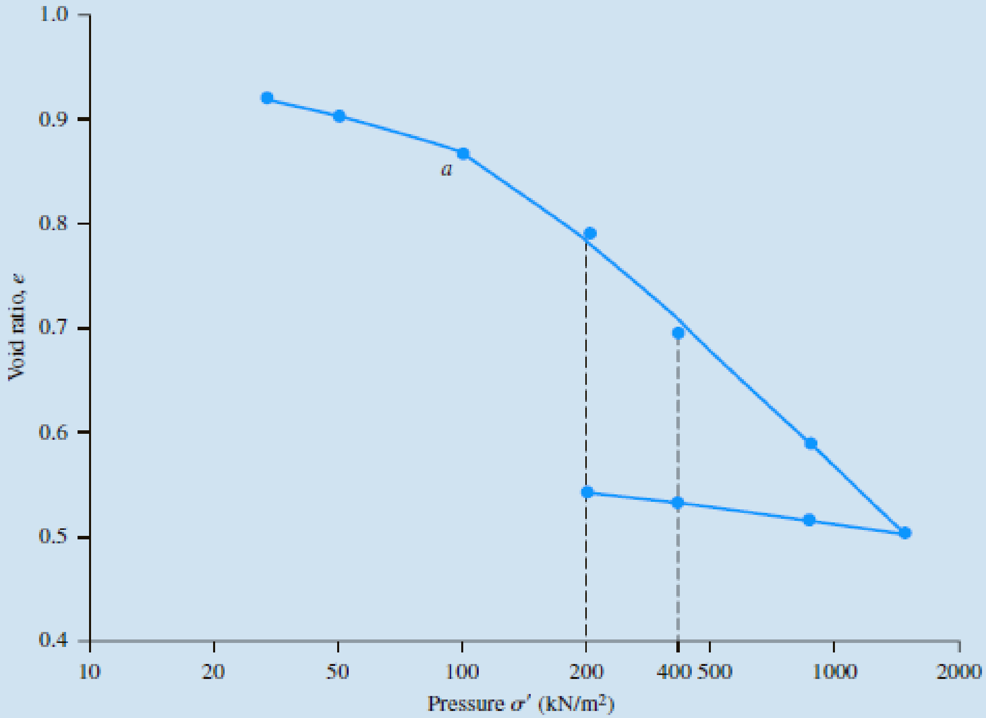
EXAMPLE 11.6

Solution

Part a

The e versus log σ' plot is shown in Figure 11.23. From the average e-log σ' plot, for the loading and unloading branches, the following values can be determined:

Branch	e	σ'_o (kN/m^2)
Loading	0.8	200
	0.7	400
Unloading	0.57	200
	0.555	400



EXAMPLE 11.6

From the loading branch,

$$C_c = \frac{e_1 - e_2}{\log \dfrac{\sigma'_2}{\sigma'_1}} = \frac{0.8 - 0.7}{\log\left(\dfrac{400}{200}\right)} = \mathbf{0.33}$$

From the unloading branch,

$$C_s = \frac{e_1 - e_2}{\log \dfrac{\sigma'_2}{\sigma'_1}} = \frac{0.57 - 0.555}{\log\left(\dfrac{400}{200}\right)} = 0.0498 \approx 0.05$$

$$\frac{C_s}{C_c} = \frac{0.05}{0.33} = \mathbf{0.15}$$

Part b

$$C_c = \frac{e_1 - e_3}{\log \dfrac{\sigma'_3}{\sigma'_1}}$$

We know that $e_1 = 0.8$ at $\sigma'_1 = 200$ kN/m^2 and that $C_c = 0.33$ [part (a)]. Let $\sigma'_3 - 1000$ kN/m^2. So,

$$0.33 = \frac{0.8 - e_3}{\log\left(\dfrac{1000}{200}\right)}$$

$$e_3 = 0.8 - 0.33 \log\left(\frac{1000}{200}\right) \approx \mathbf{0.57}$$

EXAMPLE 11.7

Example 11.7

For a given clay soil in the field, given: $G_s = 2.68$, $e_o = 0.75$. Estimate C_c based on Eqs. (11.40) and (11.44).

Solution

From Eq. (11.40),

$$C_c = 0.141 G_s^{1.2} \left(\frac{1 + e_o}{G_s} \right)^{2.38} = (0.141)(2.68)^{1.2} \left(\frac{1 + 0.75}{2.68} \right)^{2.38} \approx \mathbf{0.167}$$

From Eq. (11.44),

$$C_c = \frac{n_o}{371.747 - 4.275 n_o}$$

$$n_o = \frac{e_o}{1 + e_o} = \frac{0.75}{1 + 0.75} = 0.429$$

$$C_c = \frac{(0.429)(100)}{371.747 - (4.275)(0.429 \times 100)} = \mathbf{0.228}$$

Note: It is important to know that the empirical correlations are approximations only and may deviate from one soil to another.

Summary of calculation procedure

1. Calculate σ'_o at the middle of the clay layer
2. Determine σ'_c from the e-log σ' plot (if not given)
3. Determine whether the clay is N.C. or O.C.
4. Calculate $\Delta\sigma$
5. Use the appropriate equation

- If N.C.

$$S_c = \frac{C_c H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

- If O.C.

$$S_c = \frac{C_s H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

$$\underline{\text{If } \sigma'_o + \Delta\sigma \le \sigma'_c}$$

$$S_c = \frac{C_s H}{1 + e_o} \log\frac{\sigma'_c}{\sigma'_o} + \frac{C_c H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right)$$

$$\underline{\text{If } \sigma'_o + \Delta\sigma > \sigma'_c}$$

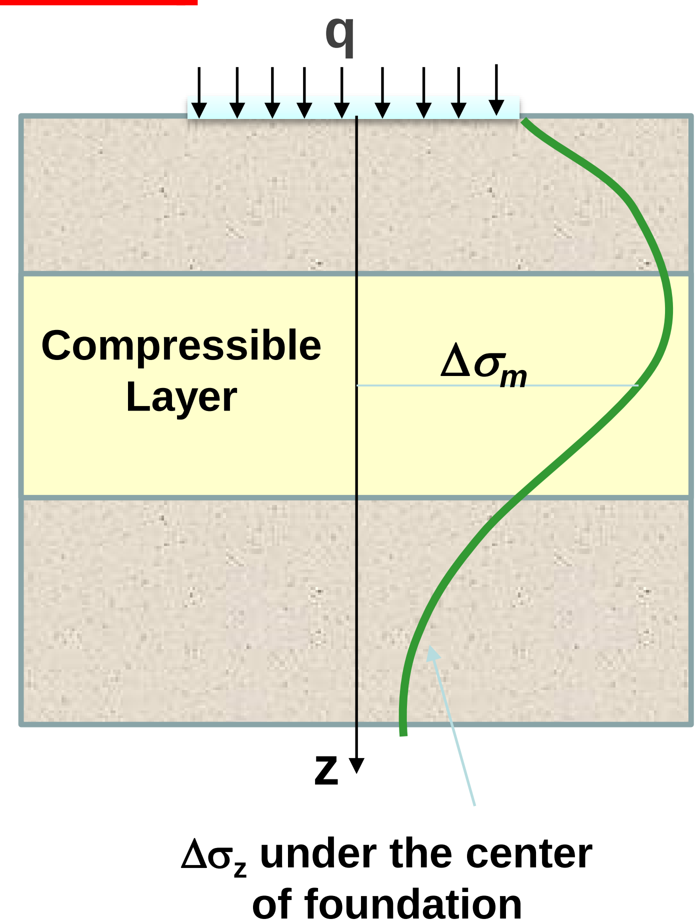
Nonlinear pressure increase

Approach 1: Middle of layer (midpoint rule)

- For settlement calculation, the pressure increase $\Delta\sigma_z$ can be approximated as :

$$\boxed{\Delta\sigma_z = \Delta\sigma_m}$$

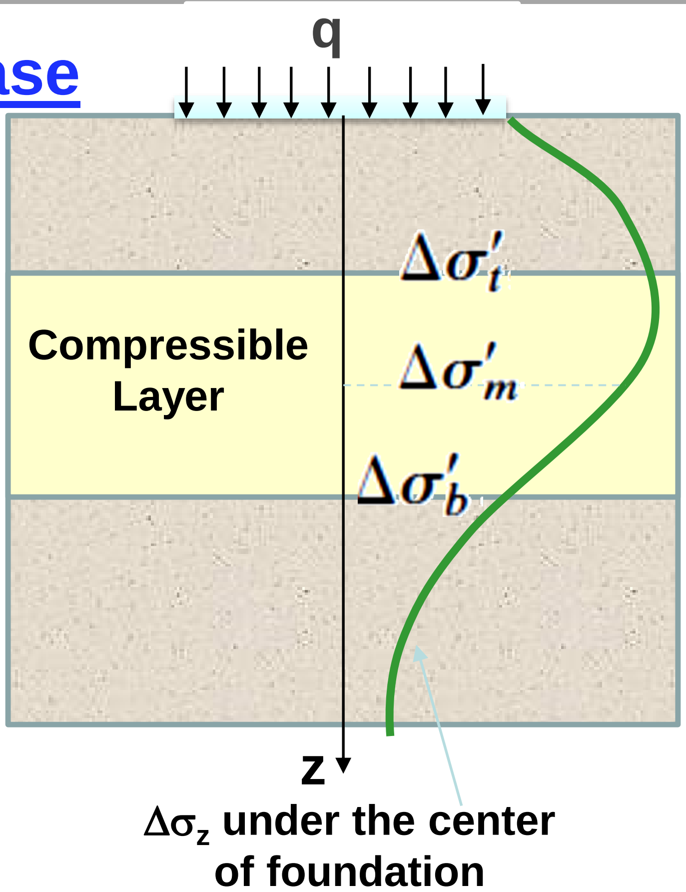
where $\Delta\sigma_m$ represent the increase in the effective pressure in the middle of the layer.



Nonlinear pressure increase

Approach 2: Average pressure increase

- **For settlement calculation we will use the average pressure increase $\Delta\sigma_{av}$, using weighted average method (Simpson's rule):**

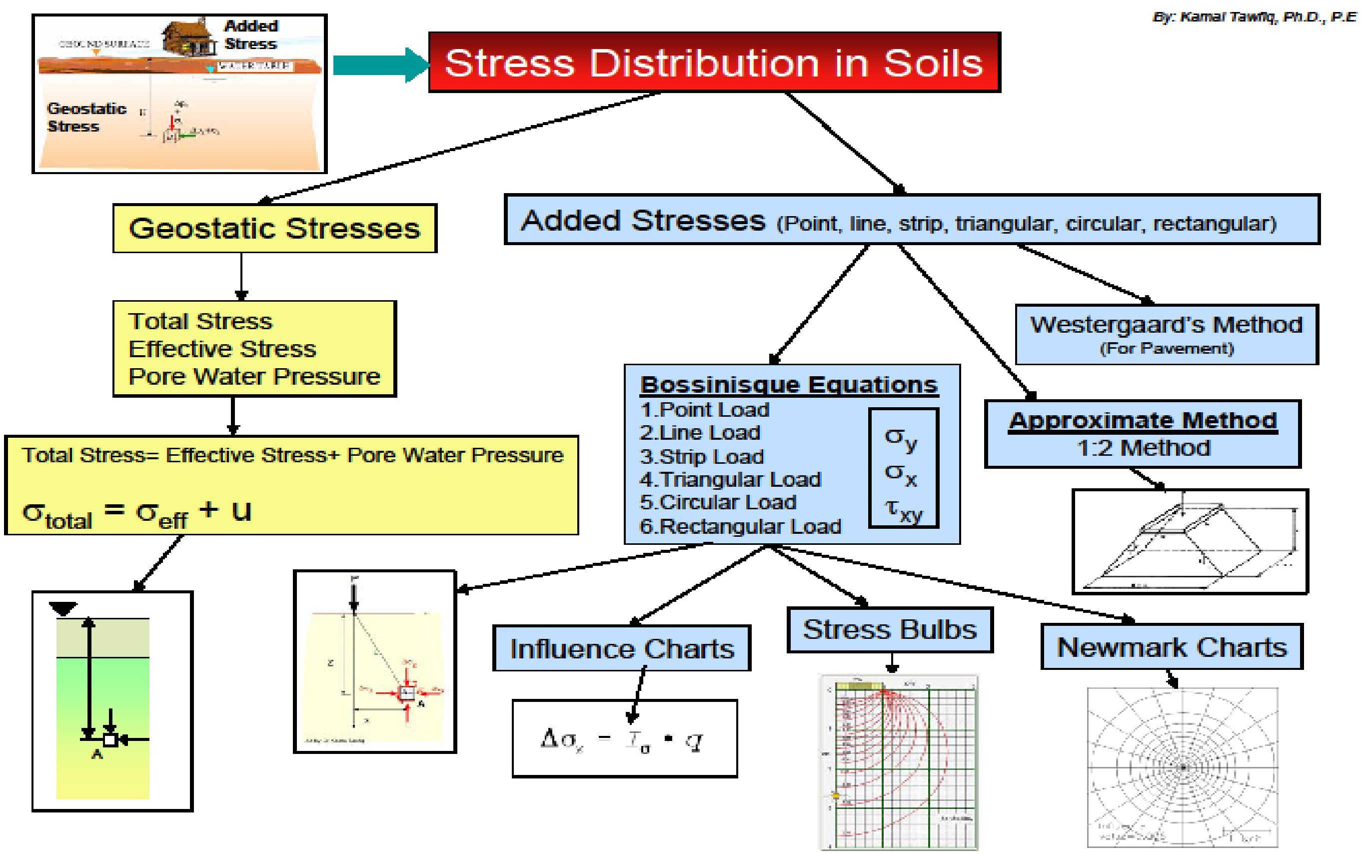


$$\boxed{\Delta\sigma'_{av} = \frac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}}$$

where $\Delta\sigma_t$, $\Delta\sigma_m$ and $\Delta\sigma_b$ represent the increase in the pressure at the top, middle, and bottom of the clay, respectively, under the center of the footing.

Stresses Distribution in Soils

Stresses Distribution in Soils



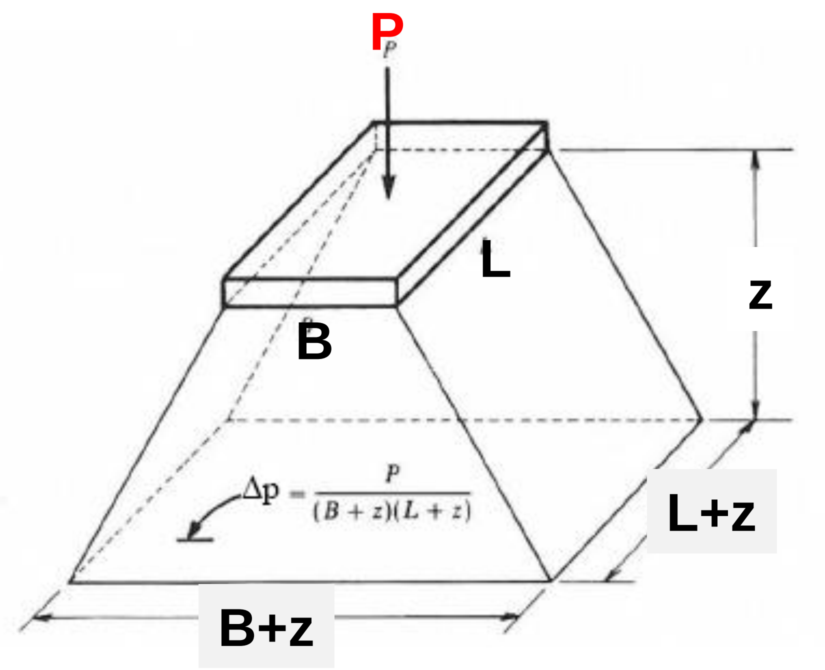
Stress Increase Due to Added Loads

I. Stresses From Approximate Methods

2:1 Method

- **In this method it is assumed that the STRESSED AREA is larger than the corresponding dimension of the loaded area by an amount equal to the depth of the subsurface area.**

$$\boxed{\sigma_z = \frac{P}{(B+z)(L+z)}}$$



Stress Increase Due to Added Loads

There are solutions available for different cases depending on the following conditions:

- **Load**: - point
  - distributed
- **Loaded area**: - Rectangular
  - Square
  - Circular
- **Stiffness**: - Flexible
  - Rigid
- **Soil**: - Cohesive
  - Cohesionless
- **Medium**: - Finite
  - Infinite
  - Layered

- **These conditions are the same as these discussed at the time when we presented stresses in soil mass from theory of elasticity in CE 382.**
- **One of the well-known and used formula is that for the vertical settlement of the surface of an elastic half space uniformly loaded.**

Stress Increase Due to Added Loads

In CE 382, the relationships for determining the increase in stress were based on the following assumptions:

- ✓ **The load is applied at the ground surface.**
- ✓ **The loaded area is *flexible.***
- ✓ **The soil medium is homogeneous, elastic, isotropic, and extends to a great depth.**

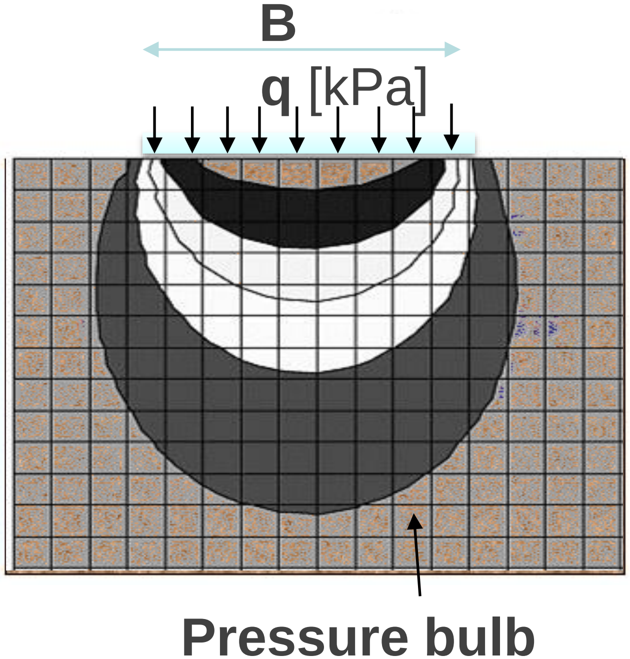
Stress Distribution in Soil Masses

- Settlement is caused by stress increase, therefore for settlement calculations, we first need vertical stress increase, $\Delta\sigma$, in soil mass imposed by a net load, q, applied at the foundation level.

- CE 382 and Chapter 10 in the textbook present many methods based on Theory of Elasticity to estimate the stress in soil imposed by foundation loadings.

- Since we consider only vertical settlement we limit ourselves to vertical stress distribution.

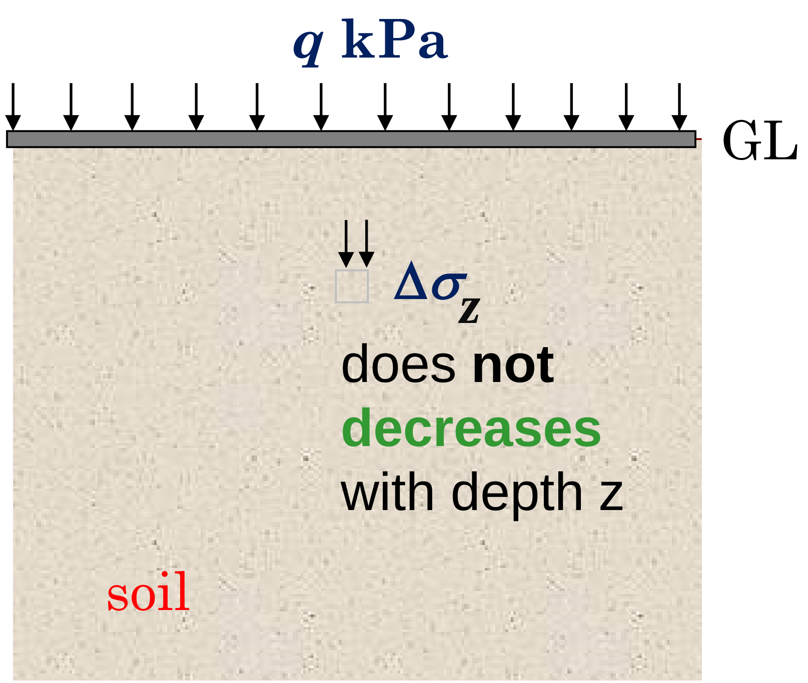
- Since mostly we have distributed load we will not consider point or line load.



Wide Uniformly Distributed Load

For **wide uniformly distributed load**, **such as for vey wide embankment fill, the stress increase at any depth, z, can be given as:**

$$\boxed{\Delta\sigma_z = q}$$



II. Stresses From Theory of Elasticity

- There are a number of solutions which are based on the theory of elasticity. Most of them assume the following assumptions:
  - The soil is homogeneous
  - The soil is isotropic
  - The soil is perfectly elastic infinite or semi-finite medium
- Tens of solutions for different problems are now available in the literature. It is enough to say that a whole book (Poulos and Davis) is now available for the elastic solutions of various problems.

ELASTIC SOLUTIONS
FOR
SOIL AND ROCK MECHANICS

by
H.G. Poulos
and
E.H. Davis

The book contains a comprehensive collection of graphs, tables and explicit solutions of problems in elasticity relevant to soil and rock mechanics.

Vertical Stress Below the Center of a Uniformly Loaded Circular Area

$$\Delta\sigma_z = q\left\{1 - \frac{1}{[(R/z)^2 + 1]^{3/2}}\right\}$$

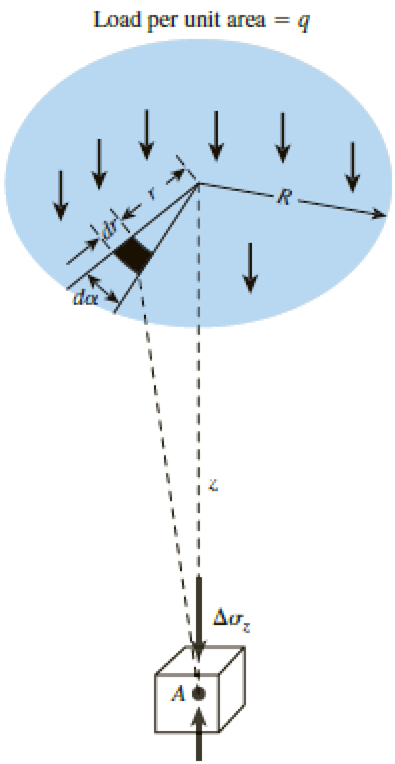
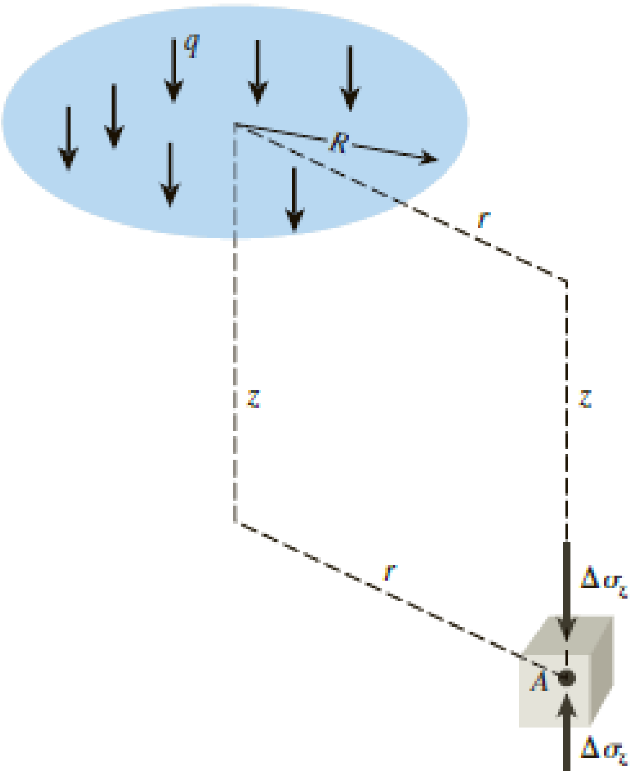


Table 10.7 Variation of $\Delta\sigma_z/q$ with z/R

z/R	$\Delta\sigma_z/q$	z/R	$\Delta\sigma_z/q$
0	1	1.0	0.6465
0.02	0.9999	1.5	0.4240
0.05	0.9998	2.0	0.2845
0.10	0.9990	2.5	0.1996
0.2	0.9925	3.0	0.1436
0.4	0.9488	4.0	0.0869
0.5	0.9106	5.0	0.0571
0.8	0.7562		

Vertical Stress Below any point of a Uniformly Loaded Circular Area

$$\Delta\sigma_Z = q(A' + B')$$



Vertical Stress Below any point of a Uniformly Loaded Circular Area

$$\Delta\sigma_Z = q(A' + B')$$

Table 10.8 Variation of A' with z/R and r/R*

z/R	r/R = 0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	1.0	1.0	1.0	1.0	1.0	0.5	0	0	0
0.1	0.90050	0.89748	0.88679	0.86126	0.78797	0.43015	0.09645	0.02787	0.00856
0.2	0.80388	0.79824	0.77884	0.73483	0.63014	0.38269	0.15433	0.05251	0.01680
0.3	0.71265	0.70518	0.68316	0.62690	0.52081	0.34375	0.17964	0.07199	0.02440
0.4	0.62861	0.62015	0.59241	0.53767	0.44329	0.31048	0.18709	0.08593	0.03118
0.5	0.55279	0.54403	0.51622	0.46448	0.38390	0.28156	0.18556	0.09499	0.03701
0.6	0.48550	0.47691	0.45078	0.40427	0.33676	0.25588	0.17952	0.10010	
0.7	0.42654	0.41874	0.39491	0.35428	0.29833	0.21727	0.17124	0.10228	0.04558
0.8	0.37531	0.36832	0.34729	0.31243	0.26581	0.21297	0.16206	0.10236	
0.9	0.33104	0.32492	0.30669	0.27707	0.23832	0.19488	0.15253	0.10094	
1	0.29289	0.28763	0.27005	0.24697	0.21468	0.17868	0.14329	0.09849	0.05185
1.2	0.23178	0.22795	0.21662	0.19890	0.17626	0.15101	0.12570	0.09192	0.05260
1.5	0.16795	0.16552	0.15877	0.14804	0.13436	0.11892	0.10296	0.08048	0.05116
2	0.10557	0.10453	0.10140	0.09647	0.09011	0.08269	0.07471	0.06275	0.04496
2.5	0.07152	0.07098	0.06947	0.06698	0.06373	0.05974	0.05555	0.04880	0.03787
3	0.05132	0.05101	0.05022	0.04886	0.04707	0.04487	0.04241	0.03839	0.03150
4	0.02986	0.02976	0.02907	0.02802	0.02832	0.02749	0.02651	0.02490	0.02193
5	0.01942	0.01938				0.01835			0.01573
6	0.01361					0.01307			0.01168
7	0.01005					0.00976			0.00894
8	0.00772					0.00755			0.00703
9	0.00612					0.00600			0.00566
10								0.00477	0.00465

*Source: From Ahlvin, R. G., and H. H. Ulery. Tabulated Values for Determining the Complete Pattern of Stresses, Strains, and Deflections Beneath a Uniform Circular Load on a Homogeneous Half Space. In Highway Research Bulletin 342, Highway Research Board, National Research Council, Washington, D.C., 1962, Tables 1 and 2, p. 3. Reproduced with permission of the Transportation Research Board.

Vertical Stress Below any point of a Uniformly Loaded Circular Area

$$\Delta\sigma_Z = q(A' + B')$$

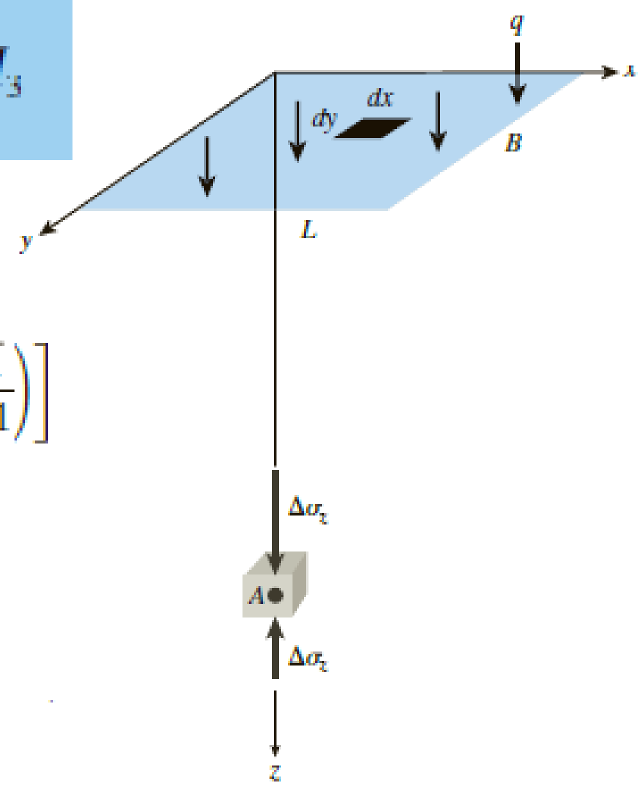
Table 10.9 Variation of B' with z/R and r/R*

	r/R								
z/R	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	0	0	0	0	0	0	0	0	0
0.1	0.09852	0.10140	0.11138	0.13424	0.18796	0.05388	−0.07899	−0.02672	−0.00845
0.2	0.18857	0.19306	0.20772	0.23524	0.25983	0.08513	−0.07759	−0.04448	−0.01593
0.3	0.26362	0.26787	0.28018	0.29483	0.27257	0.10757	−0.04316	−0.04999	−0.02166
0.4	0.32016	0.32259	0.32748	0.32273	0.26925	0.12404	−0.00766	−0.04535	−0.02522
0.5	0.35777	0.35752	0.35323	0.33106	0.26236	0.13591	0.02165	−0.03455	−0.02651
0.6	0.37831	0.37531	0.36308	0.32822	0.25411	0.14440	0.04457	−0.02101	
0.7	0.38487	0.37962	0.36072	0.31929	0.24638	0.14986	0.06209	−0.00702	−0.02329
0.8	0.38091	0.37408	0.35133	0.30699	0.23779	0.15292	0.07530	0.00614	
0.9	0.36962	0.36275	0.33734	0.29299	0.22891	0.15404	0.08507	0.01795	
1	0.35355	0.34553	0.32075	0.27819	0.21978	0.15355	0.09210	0.02814	−0.01005
1.2	0.31485	0.30730	0.28481	0.24836	0.20113	0.14915	0.10002	0.04378	0.00023
1.5	0.25602	0.25025	0.23338	0.20694	0.17368	0.13732	0.10193	0.05745	0.01385
2	0.17889	0.18144	0.16644	0.15198	0.13375	0.11331	0.09254	0.06371	0.02836
2.5	0.12807	0.12633	0.12126	0.11327	0.10298	0.09130	0.07869	0.06022	0.03429
3	0.09487	0.09394	0.09099	0.08635	0.08033	0.07325	0.06551	0.05354	0.03511
4	0.05707	0.05666	0.05562	0.05383	0.05145	0.04773	0.04532	0.03995	0.03066
5	0.03772	0.03760				0.03384			0.02474
6	0.02666					0.02468			0.01968
7	0.01980					0.01868			0.01577
8	0.01526					0.01459			0.01279
9	0.01212					0.01170			0.01054
10								0.00924	0.00879

* *Source:* From Ahlvin, R. G., and H. H. Ulery. Tabulated Values for Determining the Complete Pattern of Stresses, Strains, and Deflections Beneath a Uniform Circular Load on a Homogeneous Half Space. In Highway Research Bulletin 342, Highway Research Board, National Research Council, Washington, D.C., 1962, Tables 1 and 2, p. 3. Reproduced with permission of the Transportation Research Board.

Vertical Stress Below the Corner of a Uniformly Loaded Rectangular Area

$$\Delta\sigma_z = \int d\sigma_z = \int_{y=0}^{B}\int_{x=0}^{L} \frac{3qz^3(dx\,dy)}{2\pi(x^2+y^2+z^2)^{5/2}} = qI_3$$



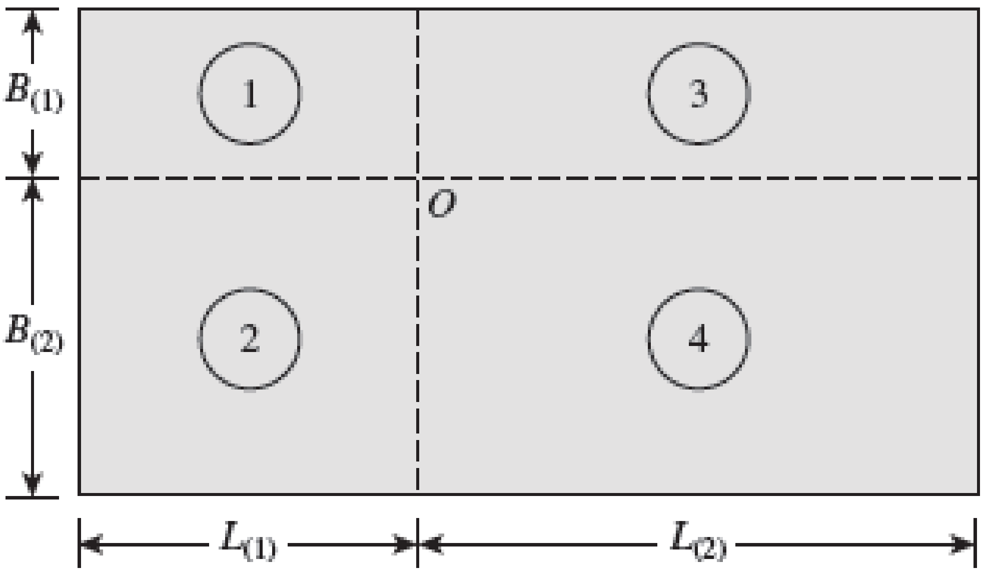
$$I_3 = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right]$$

$$m = \frac{B}{z}$$

$$n = \frac{L}{z}$$

I_3 is a dimensionless factor and represents the influence of a surcharge covering a rectangular area on the vertical stress at a point located at a depth z below one of its corner.

Vertical Stress Below the Corner of a Uniformly Loaded Rectangular Area

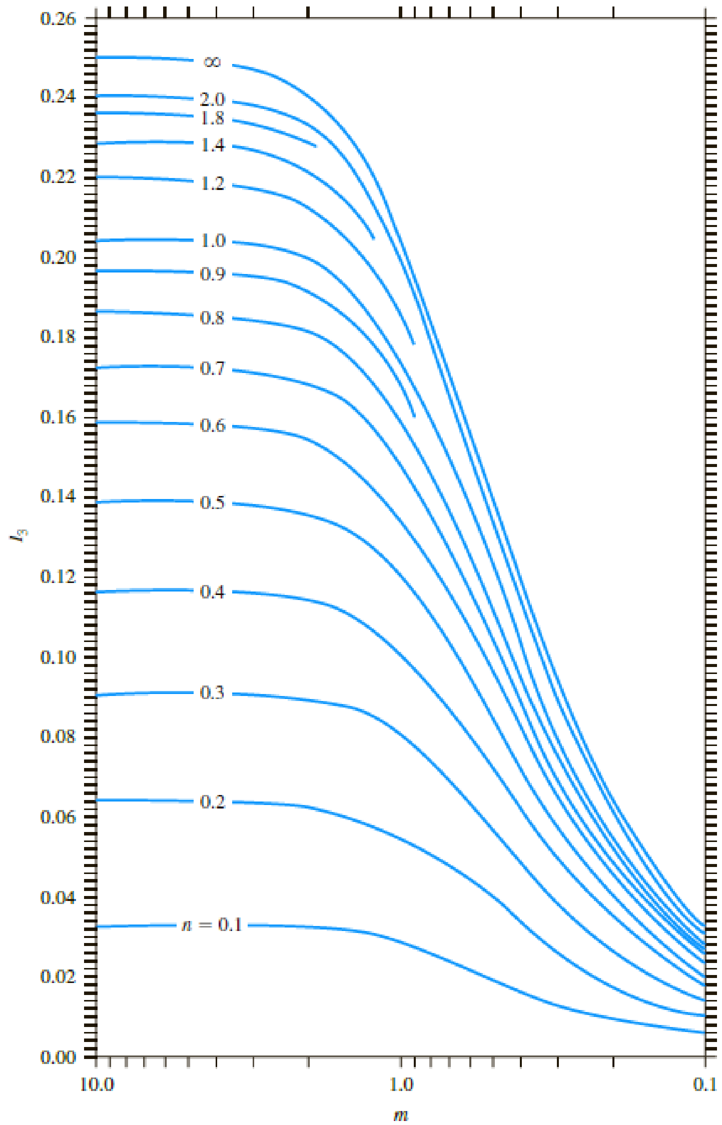


Vertical Stress Below the Corner of a Uniformly Loaded Rectangular Area

Table 10.10 Variation of I_3 with m and n

	m									
n	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
0.1	0.0047	0.0092	0.0132	0.0168	0.0198	0.0222	0.0242	0.0258	0.0270	0.0279
0.2	0.0092	0.0179	0.0259	0.0328	0.0387	0.0435	0.0474	0.0504	0.0528	0.0547
0.3	0.0132	0.0259	0.0374	0.0474	0.0559	0.0629	0.0686	0.0731	0.0766	0.0794
0.4	0.0168	0.0328	0.0474	0.0602	0.0711	0.0801	0.0873	0.0931	0.0977	0.1013
0.5	0.0198	0.0387	0.0559	0.0711	0.0840	0.0947	0.1034	0.1104	0.1158	0.1202
0.6	0.0222	0.0435	0.0629	0.0801	0.0947	0.1069	0.1168	0.1247	0.1311	0.1361
0.7	0.0242	0.0474	0.0686	0.0873	0.1034	0.1169	0.1277	0.1365	0.1436	0.1491
0.8	0.0258	0.0504	0.0731	0.0931	0.1104	0.1247	0.1365	0.1461	0.1537	0.1598
0.9	0.0270	0.0528	0.0766	0.0977	0.1158	0.1311	0.1436	0.1537	0.1619	0.1684
1.0	0.0279	0.0547	0.0794	0.1013	0.1202	0.1361	0.1491	0.1598	0.1684	0.1752
1.2	0.0293	0.0573	0.0832	0.1063	0.1263	0.1431	0.1570	0.1684	0.1777	0.1851
1.4	0.0301	0.0589	0.0856	0.1094	0.1300	0.1475	0.1620	0.1739	0.1836	0.1914
1.6	0.0306	0.0599	0.0871	0.1114	0.1324	0.1503	0.1652	0.1774	0.1874	0.1955
1.8	0.0309	0.0606	0.0880	0.1126	0.1340	0.1521	0.1672	0.1797	0.1899	0.1981
2.0	0.0311	0.0610	0.0887	0.1134	0.1350	0.1533	0.1686	0.1812	0.1915	0.1999
2.5	0.0314	0.0616	0.0895	0.1145	0.1363	0.1548	0.1704	0.1832	0.1938	0.2024
3.0	0.0315	0.0618	0.0898	0.1150	0.1368	0.1555	0.1711	0.1841	0.1947	0.2034
4.0	0.0316	0.0619	0.0901	0.1153	0.1372	0.1560	0.1717	0.1847	0.1954	0.2042
5.0	0.0316	0.0620	0.0901	0.1154	0.1374	0.1561	0.1719	0.1849	0.1956	0.2044
6.0	0.0316	0.0620	0.0902	0.1154	0.1374	0.1562	0.1719	0.1850	0.1957	0.2045

Vertical Stress Below the Corner of a Uniformly Loaded Rectangular Area



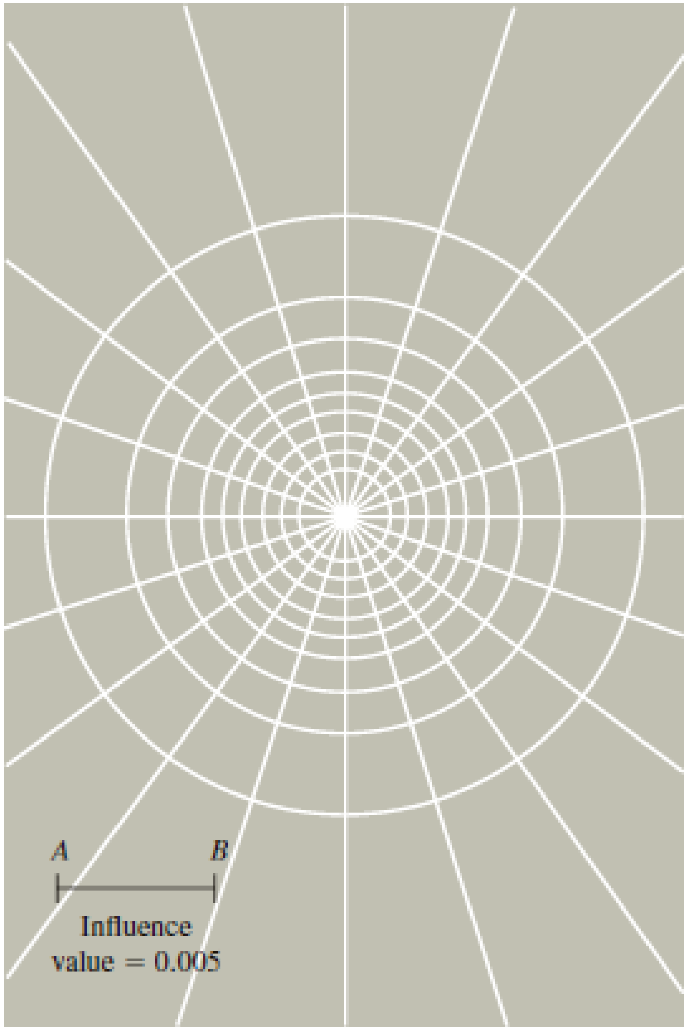
Newmark's Influence Chart

$$\frac{R}{z} = \sqrt{\left(1 - \frac{\Delta\sigma_z}{q}\right)^{2/3} - 1}$$

Table 10.12 Values of R/z for Various Pressure Ratios [Eq. (10.41)]

$\Delta\sigma_z/q$	R/z	$\Delta\sigma_z/q$	R/z
0	0	0.55	0.8384
0.05	0.1865	0.60	0.9176
0.10	0.2698	0.65	1.0067
0.15	0.3383	0.70	1.1097
0.20	0.4005	0.75	1.2328
0.25	0.4598	0.80	1.3871
0.30	0.5181	0.85	1.5943
0.35	0.5768	0.90	1.9084
0.40	0.6370	0.95	2.5232
0.45	0.6997	1.00	∞
0.50	0.7664		

Newmark's Influence Chart



Newmark's Influence Chart

The procedure for obtaining vertical pressure at any point below a loaded area is as follows:

Step 1. Determine the depth z below the uniformly loaded area at which the stress increase is required.

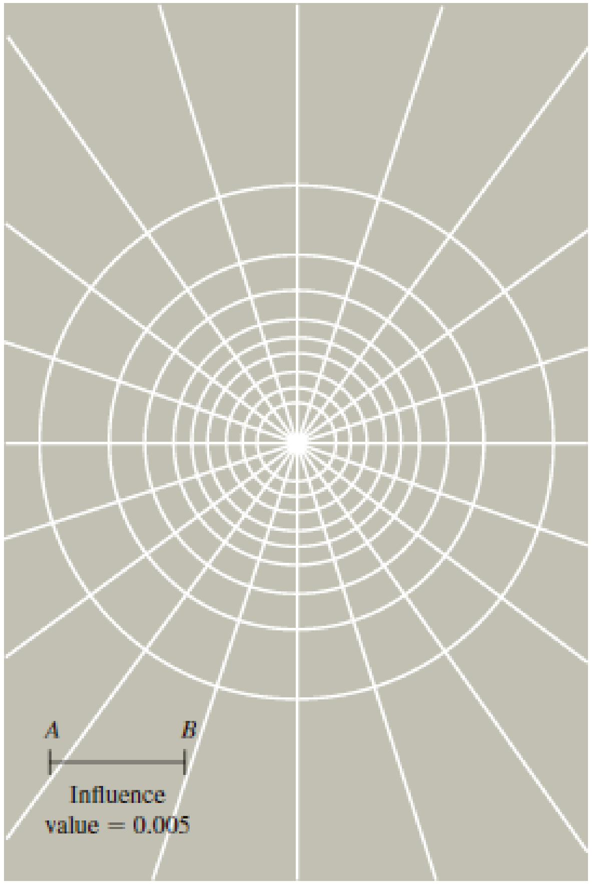
Step 2. Plot the plan of the loaded area with a scale of z equal to the unit length of the chart ($\overline{AB}$).

Step 3. Place the plan (plotted in step 2) on the influence chart in such a way that the point below which the stress is to be determined is located at the center of the chart.

Step 4. Count the number of elements (M) of the chart enclosed by the plan of the loaded area.

$$\Delta\sigma_z = (IV)qM$$

where IV = influence value
q = pressure on the loaded area



EXAMPLE 11.8

Example 11.8

A soil profile is shown in Figure 11.24. If a uniformly distributed load, $\Delta\sigma$, is applied at the ground surface, what is the settlement of the clay layer caused by primary consolidation if

a. The clay is normally consolidated

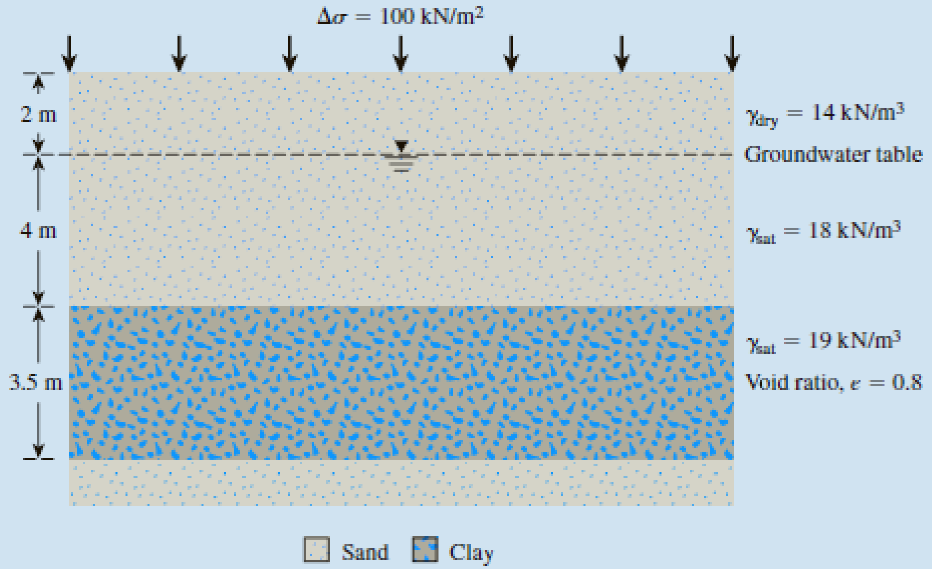


Figure 11.24

b. The preconsolidation pressure, $\sigma'_c = 200\ \text{kN/m}^2$

c. $\sigma'_c = 150\ \text{kN/m}^2$

Use $C_s \approx \dfrac{1}{5} C_c$; and Eq. (11.40).

EXAMPLE 11.8

Solution

Part a

The average effective stress at the middle of the clay layer is

$$\sigma'_o = 2\gamma_{dry} + 4[\gamma_{sat(sand)} - \gamma_w] + \frac{3.5}{2}[\gamma_{sat(clay)} - \gamma_w]$$

$$\sigma'_o = (2)(14) + 4(18 - 9.81) + 1.75(19 - 9.81) = 76.08 \text{ kN/m}^2$$

$$\gamma_{sat(clay)} = 19 \text{ kN/m}^3 = \frac{(G_s + e)\gamma_w}{1 + e} = \frac{(G_s + 0.8)(9.81)}{1 + 0.8}; G_s = 2.686$$

From Eq. (11.40),

$$C_c = 0.141 G_s^{1.2}\left(\frac{1 + e_o}{G_s}\right)^{2.38} = (0.141)(2.686)^{1.2}\left(\frac{1 + 0.8}{2.686}\right)^{2.38} = 0.178$$

From Eq. (11.35),

$$S_c = \frac{C_c H}{1 + e_o}\log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

So,

$$S_c = \frac{(0.178)(3.5)}{1 + 0.8}\log\left(\frac{76.08 + 100}{76.08}\right) = 0.126 \text{ m} = \mathbf{126\ mm}$$

EXAMPLE 11.8

Part b

$$\sigma'_o + \Delta\sigma' = 76.08 + 100 = 176.08 \text{ kN/m}^2$$

$$\sigma'_c = 200 \text{ kN/m}^2$$

Because $\sigma'_o + \Delta\sigma' < \sigma'_c$, use Eq. (11.37):

$$S_c = \frac{C_s H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

$$C_s = \frac{C_c}{5} = \frac{0.178}{5} = 0.0356$$

$$S_c = \frac{(0.0356)(3.5)}{1 + 0.8} \log\left(\frac{76.08 + 100}{76.08}\right) = 0.025 \text{ m} = \mathbf{25\ mm}$$

EXAMPLE 11.8

Part c

$$\sigma'_o = 76.08 \text{ kN/m}^2$$

$$\sigma'_o + \Delta\sigma' = 176.08 \text{ kN/m}^2$$

$$\sigma'_c = 150 \text{ kN/m}^2$$

Because $\sigma'_o < \sigma'_c < \sigma'_o + \Delta\sigma'$, use Eq. (11.38):

$$S_c - \frac{C_s H}{1 + e_o} \log \frac{\sigma'_c}{\sigma'_o} + \frac{C_c H}{1 + e_o} \log \left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c} \right)$$

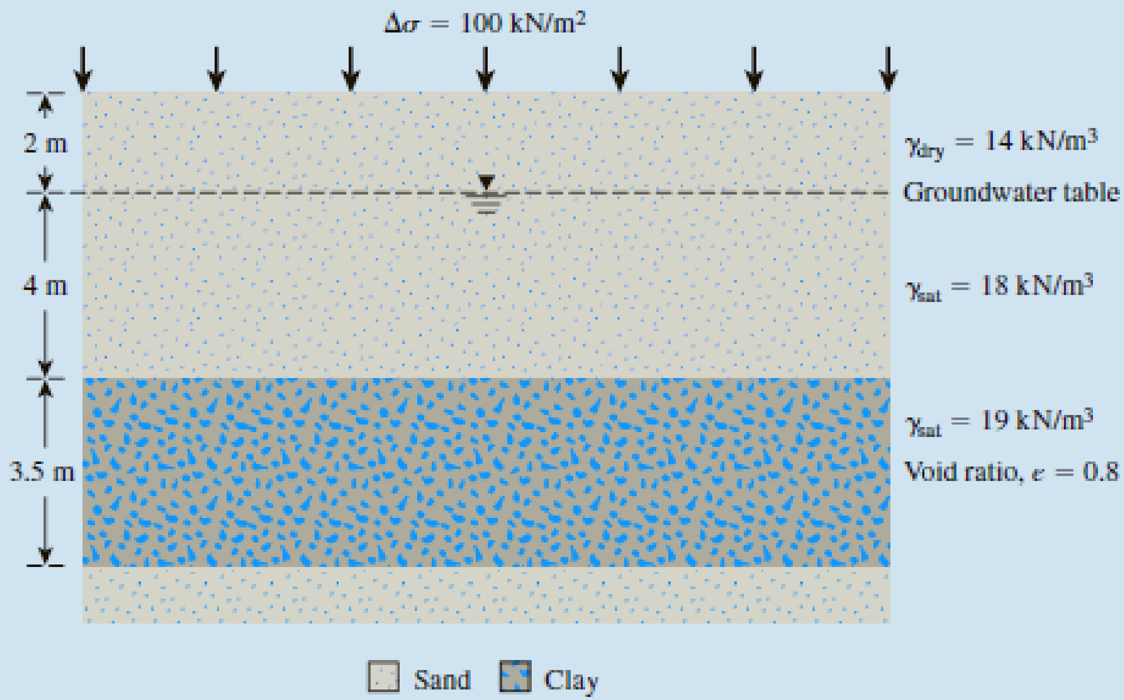
$$= \frac{(0.0356)(3.5)}{1.8} \log \left(\frac{150}{76.08} \right) + \frac{(0.178)(3.5)}{1.8} \log \left(\frac{176.08}{150} \right)$$

$$\approx 0.0445 \text{ m} = \mathbf{44.5 \text{ mm}}$$

EXAMPLE 11.9

Example 11.9

Refer to Example 11.8. For each part, calculate and plot a graph of e vs. σ' at the beginning and end of consolidation.



EXAMPLE 11.9

Solution

For each part, $e = 0.8$ at the beginning of consolidation. For e at the end of consolidation, the following calculations can be made.

Part a

$$e = 0.8 - C_c \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right) = 0.8 - 0.178 \log\left(\frac{176.08}{76.08}\right) = \mathbf{0.735}$$

Part b

$$e = 0.8 - C_s \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right) = 0.8 - 0.0356 \log\left(\frac{176.08}{76.08}\right) = \mathbf{0.787}$$

Part c

$$e = 0.8 - \left[C_s \log\left(\frac{\sigma'_c}{\sigma'_o}\right) + C_c \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right)\right]$$

$$= 0.8 - \left[0.0356 \log\left(\frac{150}{76.08}\right) + 0.178 \log\left(\frac{176.08}{150}\right)\right]$$

$$= 0.8 - 0.0105 - 0.0124 = \mathbf{0.771}$$

EXAMPLE 11.9

A plot of e versus log σ' is shown in Figure 11.25.

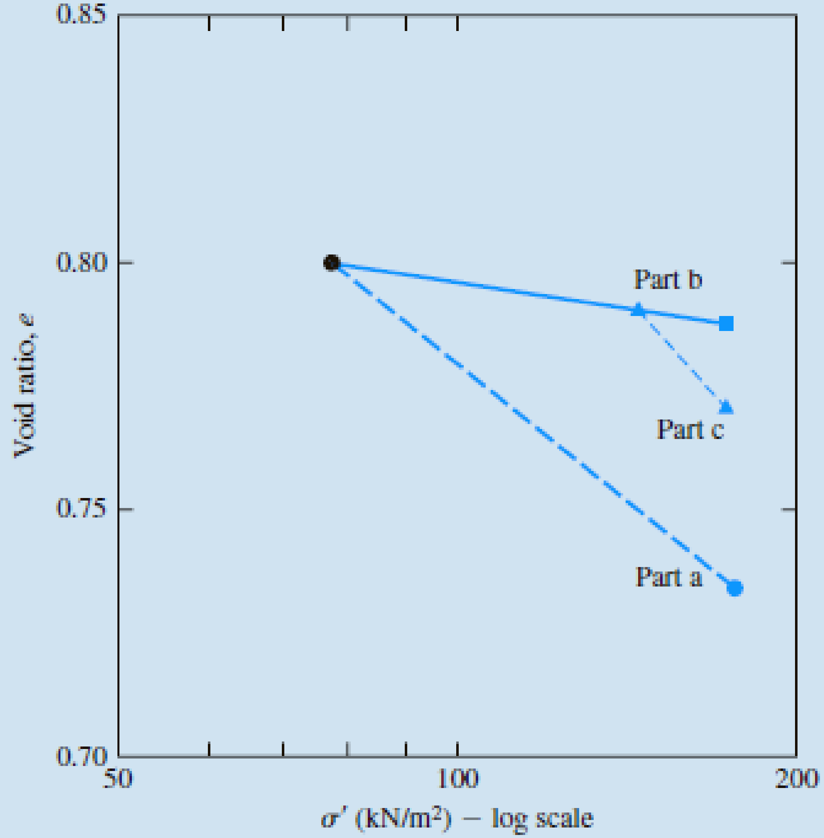
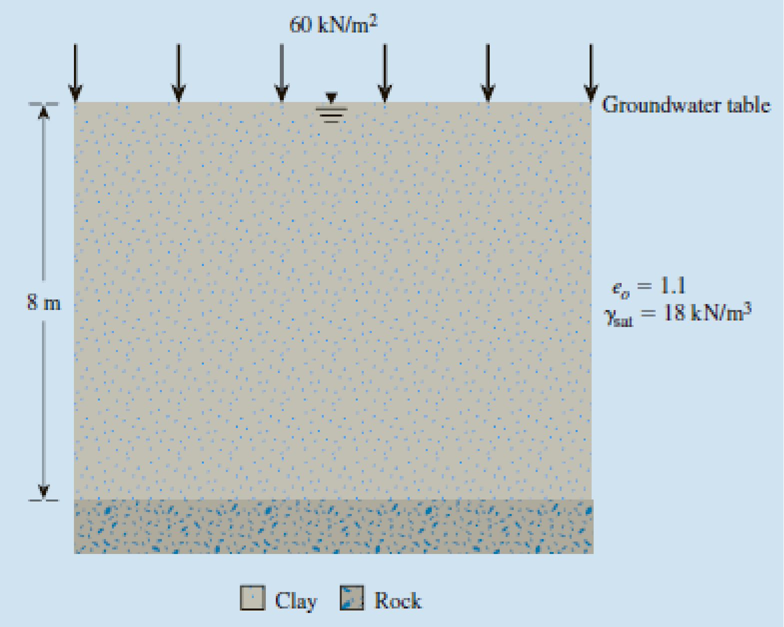


Figure 11.25

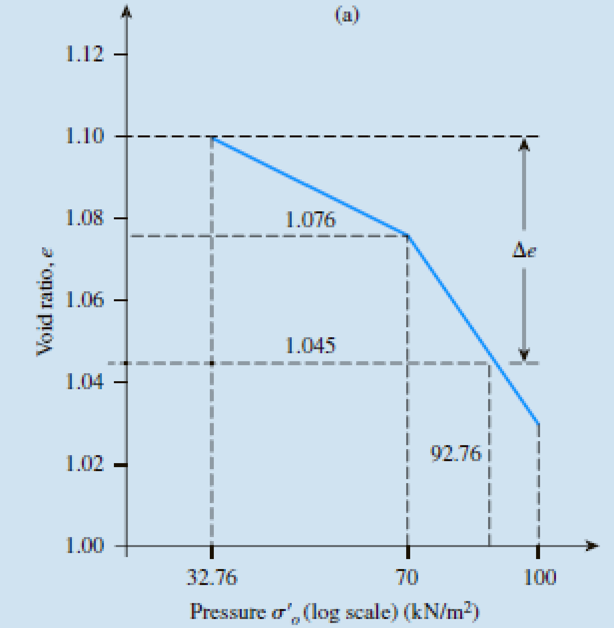
EXAMPLE 11.10

Example 11.10

A soil profile is shown in Figure 11.26a. Laboratory consolidation tests were conducted on a specimen collected from the middle of the clay layer. The field consolidation curve interpolated from the laboratory test results is shown in Figure 11.26b. Calculate the settlement in the field caused by primary consolidation for a surcharge of 60 kN/m^2 applied at the ground surface.



Soil profile



field consolidation curve

EXAMPLE 11.10

Solution

$$\sigma'_o = (4)(\gamma_{sat} - \gamma_w) = 4(18.0 - 9.81)$$
$$= 32.76 \text{ kN/m}^2$$
$$e_o = 1.1$$
$$\Delta\sigma' = 60 \text{ kN/m}^2$$
$$\sigma_O' + \Delta\sigma' = 32.76 + 60 = 92.76 \text{ kN/m}^2$$

The void ratio corresponding to 92.76 kN/m^2 (see Figure 11.26b) is 1.045. Hence, $\Delta e = 1.1 - 1.045 = 0.055$. We have

$$\text{Settlement, } S_c = H\frac{\Delta e}{1 + e_o} \qquad \text{[Eq. (11.33)]}$$

So,

$$S_c = 8\frac{(0.055)}{1 + 1.1} = 0.21 \text{ m} = \mathbf{210 \text{ mm}}$$

Secondary Consolidation Settlement

Secondary Consolidation Settlement

- In some soils (especially recent organic soils) the compression continues under constant loading after all of the excess pore pressure has dissipated, i.e. after primary consolidation has ceased.
- This is called secondary compression or creep, and it is due to plastic adjustment of soil fabrics.
- Secondary compression is different from primary consolidation in that it takes place at a constant effective stress.
- This settlement can be calculated using the secondary compression index, C_α.
- The Log-Time plot (of the consolidation test) can be used to estimate the coefficient of secondary compression C_α as the slope of the straight line portion of e vs. log time curve which occurs after primary consolidation is complete.

Secondary Consolidation Settlement

The magnitude of the secondary consolidation can be calculated as:

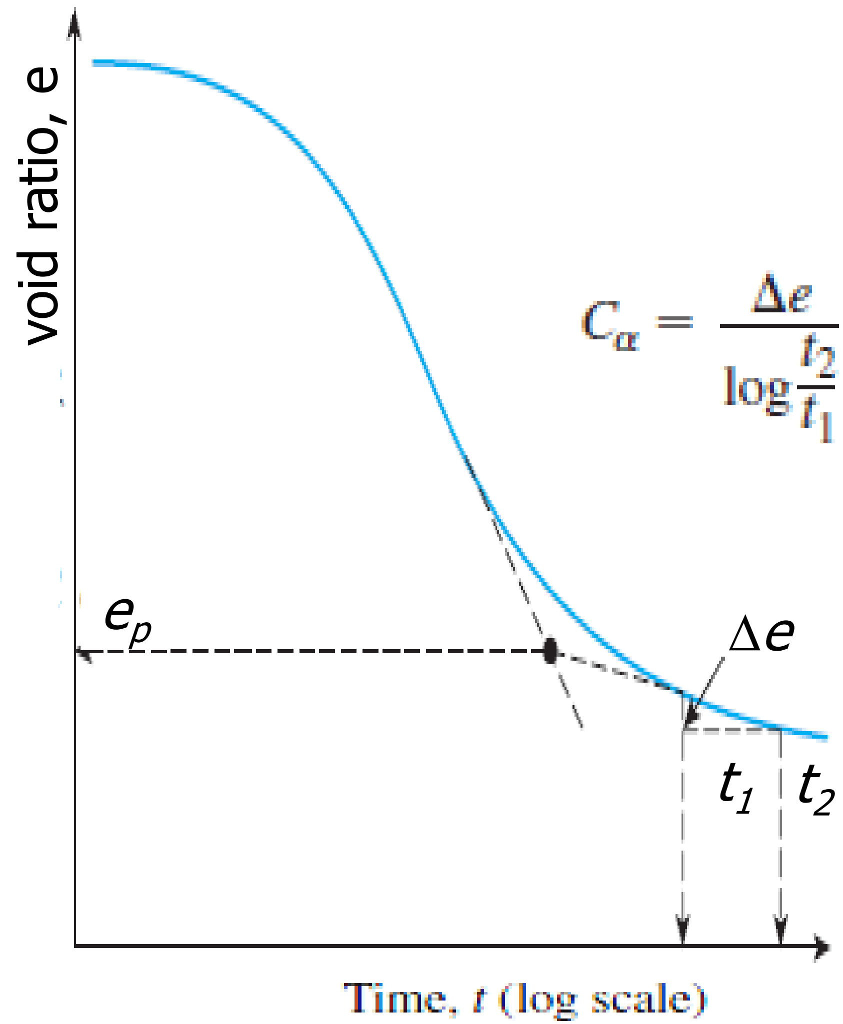
$$\boxed{S_s = \frac{H}{1 + e_p}\Delta e}$$

e_p void ratio at the end of primary consolidation,
H thickness of clay layer.

$$\Delta e = C_\alpha log\,(t_2/t_1)$$

C_α = coefficient of secondary compression

$$\boxed{S_s = \frac{C_\alpha H}{1 + e_p} log\left(\frac{t_2}{t_1}\right)}$$



Secondary Consolidation Settlement

$$\boxed{S_s = \frac{C_\alpha H}{1 + e_p} log\left(\frac{t_2}{t_1}\right)}$$

$$C_\alpha = \frac{\Delta e}{\log t_2 - \log t_1} = \frac{\Delta e}{\log (t_2/t_1)}$$

$$\boxed{S_s = C'_\alpha H \log\left(\frac{t_2}{t_1}\right)}$$

$$C'_\alpha = \frac{C_\alpha}{1 + e_p}$$

Secondary Consolidation Settlement

Remarks

- **Causes** of secondary settlement are not fully understood but is attributed to:
  - Plastic adjustment of soil fabrics
  - Compression of the bonds between individual clay particles and domains
- **Factors** that might affect the magnitude of S_s are not fully understood. In general secondary consolidation is **large** for:
  - Soft soils
  - Organic soils
  - Smaller ratio of induced stress to effective overburden pressure.

Coefficient of Secondary Compression

The general magnitudes of C'_α as observed in various natural deposits are as follows:

- Overconsolidated clays = 0.001 or less
- Normally consolidated clays = 0.005 to 0.03
- Organic soil = 0.04 or more

Mersri and Godlewski (1977) compiled the ratio of C_α/C_c for a number of natural clays. From this study, it appears that C_α/C_c for

- Inorganic clays and silts $\approx 0.04 \pm 0.01$
- Organic clays and silts $\approx 0.05 \pm 0.01$
- Peats $\approx 0.075 \pm 0.01$

Example

For a normally consolidated clay layer in the field, the following values are given:

- Thickness of clay layer = 2.6 m
- Void ratio (e_o) = 0.8
- Compression index (C_c) = 0.28
- Average effective pressure on the clay layer (σ'_o) = 127 kN/m^2
- $\Delta\sigma'$ = 46.5 kN/m^2
- Secondary compression index (C_α) = 0.02

What is the total consolidation settlement of the clay layer five years after the completion of primary consolidation settlement? (*Note:* Time for completion of primary settlement = 1.5 years.)

Example

$$C'_\alpha = \frac{C_\alpha}{1 + e_p}$$

The value of e_p can be calculated as

$$e_p = e_o - \Delta e_{\text{primary}}$$

$$\Delta e = C_c \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right) = 0.28 \log\left(\frac{127 + 46.5}{127}\right)$$

$$= 0.038$$

Example

Primary consolidation, $S_c = \dfrac{\Delta e H}{1 + e_o} = \dfrac{(0.038)(2.6 \times 1000)}{1 + 0.8} = 54.9$ mm

It is given that $e_o = 0.8$, and thus,

$$e_p = 0.8 - 0.038 = 0.762$$

Hence,

$$C'_\alpha = \frac{0.02}{1 + 0.762} = 0.011$$

$$S_s = C'_\alpha H \log\left(\frac{t_2}{t_1}\right) = (0.011)(2.6 \times 1000) \log\left(\frac{5}{1.5}\right) \approx 14.95 \text{ mm}$$

Total consolidation settlement = primary consolidation (S_c) + secondary settlement (S_s). So

Total consolidation settlement = 54.9 + 14.95 = **69.85 mm**

The end