# Chapter 11

# Compressibility of Soil

# TOPICS

- ☐ INTRODUCTION
- ☐ ELASTIC SETTLEMENT
- ☐ CONSOLIDATION SETTLEMENT
  - Fundamentals of consolidation
  - Calculation of 1-D Consolidation Settlement
  - One-dimensional Laboratory Consolidation Test
  - Calculation of Settlement from 1-D Primary Consolidation
  - Stress distribution in soil masses
- ☐ TIME RATE OF CONSOLIDATION SETTLEMENT

1-D theory of consolidation

- ☐ SECONDARY CONSOLIDATION SETTLEMENT

# INTRODUCTION

## Why should soil compressibility be studied?

**Ignoring soil compressibility may lead to unfavorable settlement and other engineering problems.**

**Embankment and building constructed on soft ground (highly compressible soil)**

**Settlement is one of the aspects that control the design of structures.**

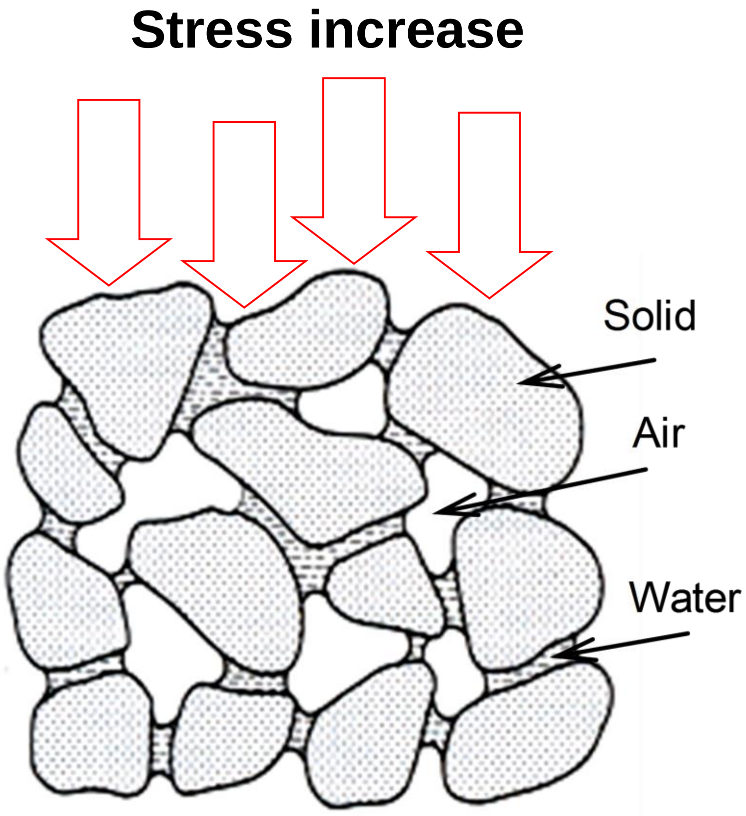
# Why soils compressed?

- Every material undergoes a certain amount of strain when a stress is applied.
- A steel rod lengthens when it is subjected to tensile stress, and a concrete column shortens when a compressive load is applied.
- The same thing holds true for soils which undergo compressive strains upon loading. Compressive strains are responsible for settlement of the structure.
- What distinguish soils from other civil engineering materials is the fact that the deformation of soils is largely unrecoverable (i.e. permanent). Therefore simple elasticity theory like elasticity cannot be applied to soils.

# What makes soil compressed?

In soils voids exist between particles and the voids may be filled with a liquid, usually water, or gas , usually air. As a result, soils are often referred to as a three-phase material or system (solid, liquid and gas).

- Solid (mineral particles)
- Gas (air),
- Liquid (usually water)



# Causes of Settlement

# Causes of Settlement

Settlement of a structure resting on soil may be caused by two distinct kinds of action within the foundation soils:-

## I. Settlement Due to Shear Stress (Distortion Settlement)

In the case the applied load caused shearing stresses to develop within the soil mass which are greater than the shear strength of the material, then the soil fails by sliding downward and laterally, and the structure settle and may tip of vertical alignment. This will be discussed in CE483 Foundation Engineering. This is what we referred to as BEARING CAPACITY.

## II. Settlement Due to Compressive Stress (Volumetric Settlement)

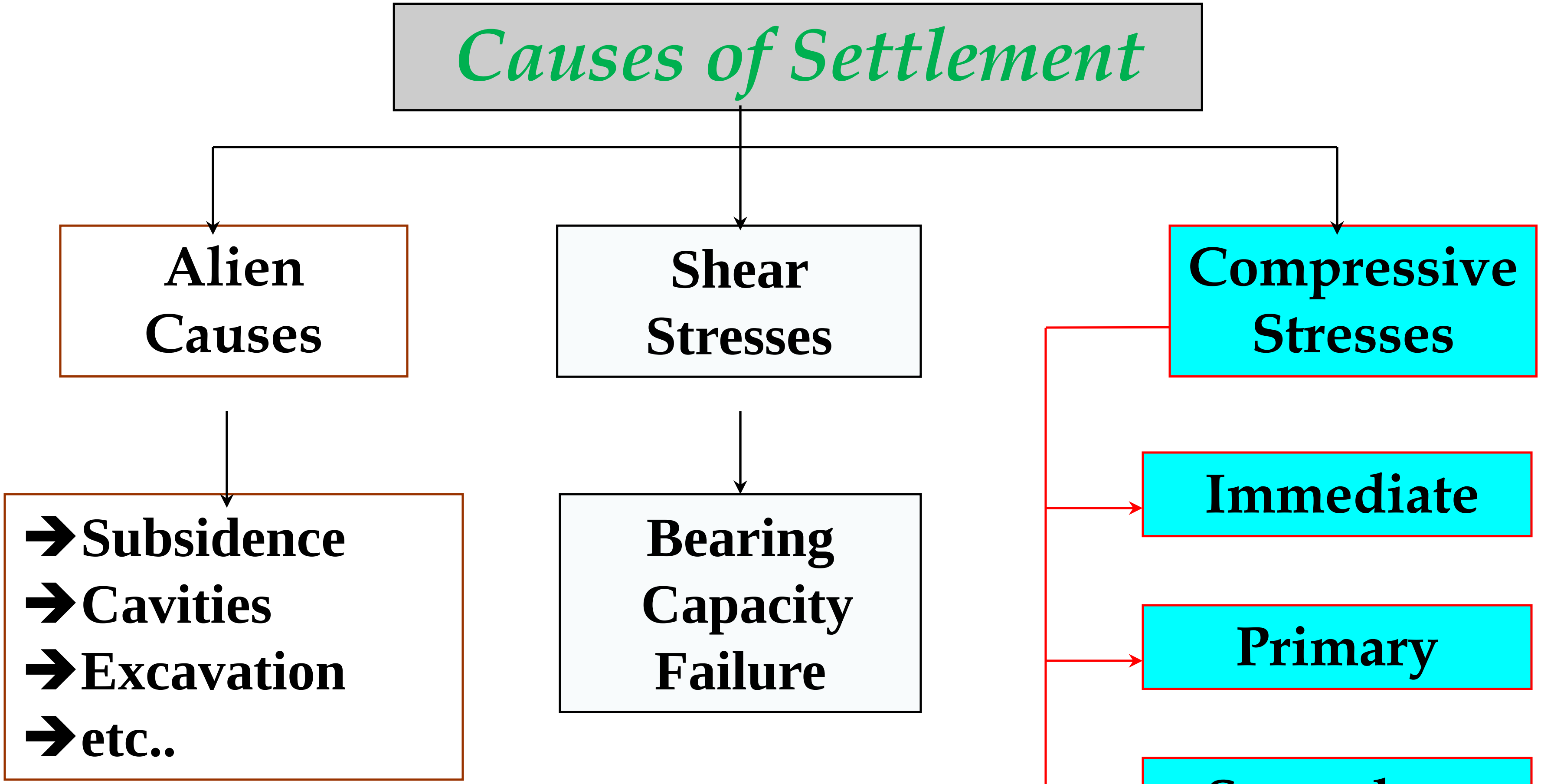
As a result of the applied load a compressive stress is transmitted to the soil leading to compressive strain. Due to the compressive strain the structure settles. This is important only if the settlement is excessive otherwise it is not dangerous.

# Causes of Settlement

- **However, in certain structures, like for example foundation for RADAR or telescope, even small settlement is not allowed since this will affect the function of the equipment.**

- **This type of settlement is what we will consider in this chapter and this course. In the following sections we will discuss its components and ways for their evaluation. We will consider only the simplest case where settlement is one-dimensional and a condition of zero lateral strain is assumed.**

# Causes of Settlement



*Causes of Settlement*

**Alien Causes**

- ➔ **Subsidence**
- ➔ **Cavities**
- ➔ **Excavation**
- ➔ **etc..**

**Shear Stresses**

**Bearing Capacity Failure**

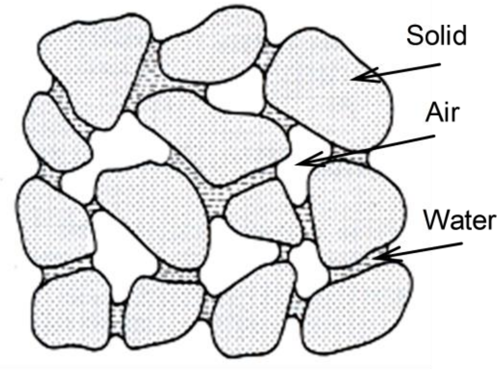
**Compressive Stresses**

- **Immediate**
- **Primary**
- **Secondary**

# Mechanisms of Compression

**Compression of soil is due to a number of mechanisms:**

- **Deformation of soil particles or grains**
- **Relocations of soil particles**
- **Expulsion of water or air from the void spaces**



# Components of Settlement

# Components of Settlement

Settlement of a soil layer under applied load is the sum of two broad components or categories:

1. Elastic settlement (or immediate) settlements
2. Consolidation settlement

## 1. Elastic settlement (or immediate) settlements

Elastic or immediate settlement takes place instantly at the moment of the application of load due to the distortion (but no bearing failure) and bending of soil particles (mainly clay). It is not generally elastic although theory of elasticity is applied for its evaluation. It is predominant in coarse-grained soils.

# Consolidation Settlement

**Consolidation settlement is the sum of two parts or types:**

## A. Primary consolidation settlement

**In this the compression of clay is due to expulsion of water from pores. The process is referred to as PRIMARY CONSOLIDATION and the associated settlement is termed PRIMARY CONSOLIDATION SETTLEMENT. Commonly they are referred to simply as CONSOLIDATION AND CONSOLIDATION SETTLEMENT.**

## B. Secondary consolidation settlement

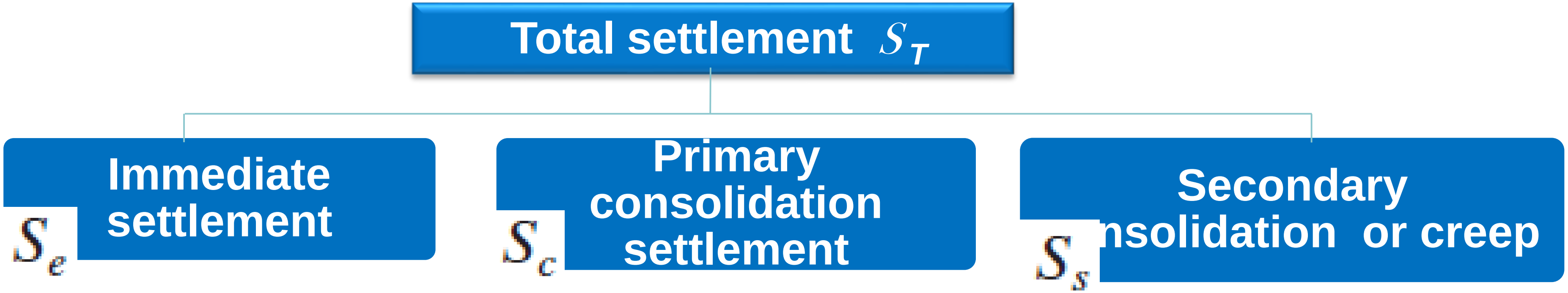
**The compression of clay soil due to plastic readjustment of soil grains and progressive breaking of clayey particles and their interparticles bonds is known as SECONDARY CONSOLIDATION OR SECONDARY COMPRESSION, and the associated settlement is called SECONDARY CONSOLIDATION SETTLEMENT or SECONDARY COMPRESSION.**

# Components of Settlement

The total settlement of a foundation can be expressed as:

$$S_T = S_e + S_c + S_s$$

$S_T$ = Total settlement
$S_e$ = Elastic or immediate settlement
$S_c$ = Primary consolidation settlement
$S_s$= Secondary consolidation settlement



Total settlement $S_T$

Immediate settlement $S_e$

Primary consolidation settlement $S_c$

Secondary nsolidation or creep $S_s$

- ☐ It should be mentioned that $S_c$ and $S_s$ overlap each other and impossible to detect which certainly when one type ends and the other begins. However, for simplicity they are treated separately and secondary consolidation is usually assumed to begin at the end of primary consolidation.

# Components of Settlement

**The total soil settlement $S_T$ may contain one or more of these types:**

**Immediate settlement** $S_e$

- Due to distortion or elastic deformation with no change in water content
- Occurs rapidly during the application of load
- Quite small quantity in dense sands, gravels and stiff clays

**Primary consolidation settlement** $S_c$

- Decrease in voids volume due to squeeze of pore-water out of the soil
- Occurs in saturated fine grained soils (low coefficient of permeability)
  Time dependent
- Only significant in clays and silts

**Secondary consolidation or creep** $S_s$

- Due to gradual changes in the particulate structure of the soil
- Occurs very slowly, long after the primary consolidation is completed
  Time dependent
- Most significant in saturated soft clayey and organic soils and peats

# CONSOLIDATION

**A gradual reduction in volume change of a fully saturated soils of low permeability due to the drainage of pore water from soil voids**

# Rates of Drainage

| soil type | coeff. of permeability (k) | seepage rate |
|---|---|---|
| Gravel | $> 10^{-2}$ m/sec | very quick |
| Sand | $10^{-2} \sim 10^{-5}$ | quick |
| Silt | $10^{-5} \sim 10^{-8}$ | slow |
| Clay | $< 10^{-8}$ | very slow |

**For design purposes it is common to assume:**

- **Quick drainage in coarse soils (Sand and Gravel)**
- **Slow drainage in fine soils (Clay and Silt).**

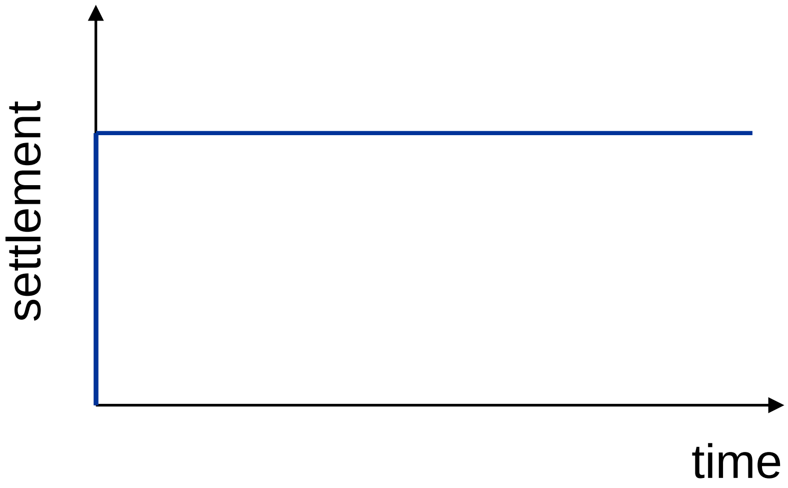
coarse soils

fine soils

# Rates of Drainage

## For coarse grained soils…

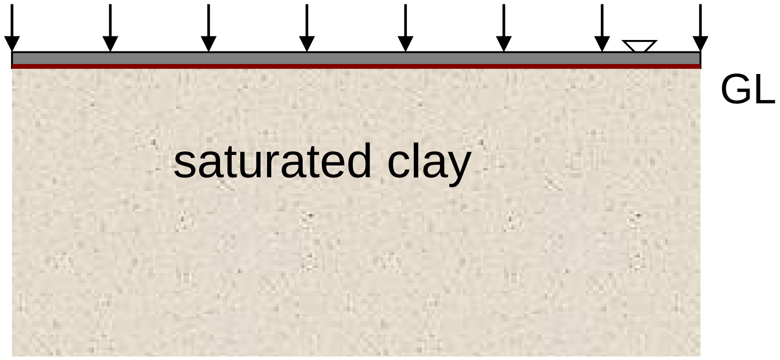
**Granular soils are freely drained, and thus the settlement is instantaneous.**

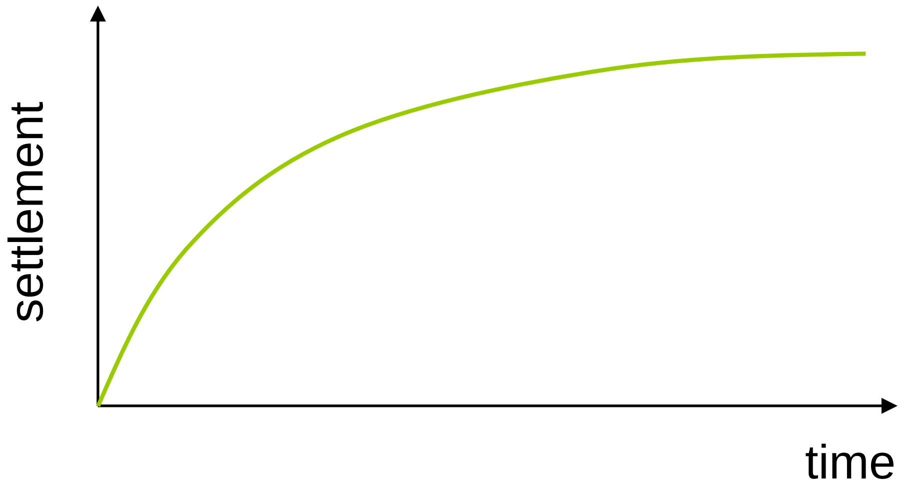


$$S_T = S_e + \cancelto{0}{S_c} + \cancelto{0}{S_s}$$

# Rates of Drainage

## For Fine grained soils…





**When a saturated clay is loaded externally, the water is squeezed out of the clay over a long time (due to low permeability of the clay).**

**This leads to settlements occurring over a long time…..which could be several years**

$$S_t = \cancel{S_e} + S_c + S_s$$

negligible

# ELASTIC SETTLEMENT

# ELASTIC SETTLEMENT

$$S_T = S_e + S_c + S_s$$

This type of settlement occur immediately after the application of load. It is predominant in coarse-grained soil (i.e. gravel, sand). Analytical evaluation of this settlement is a problem which requires satisfaction of the same set of conditions as the determination of stresses in continuous media.

In fact we could view the process as one of :

- Determining the stresses at each point in the medium
- Evaluating the vertical strains
- Integrating these vertical strains over the depth of the material.
- Theory of elasticity is used to determine the immediate settlement. This is to a certain degree reasonable in cohesive soils but not reasonable for cohesionless soils.

# Contact Pressure and Settlement Profile

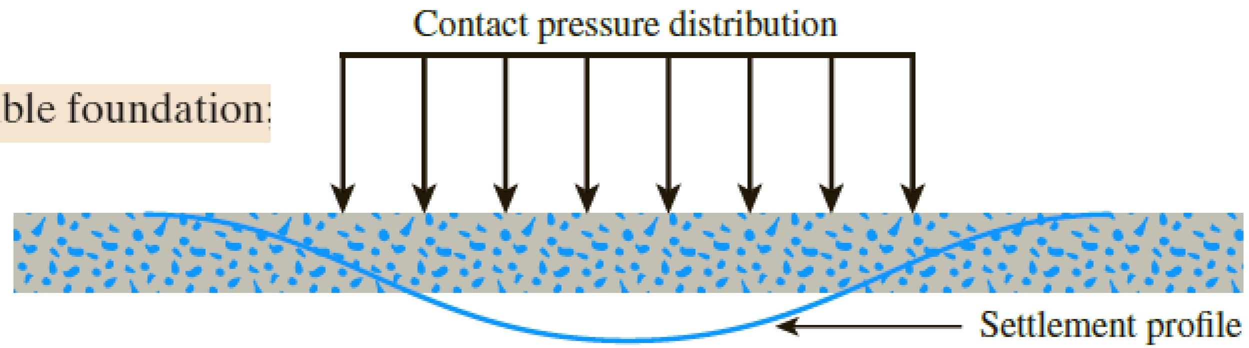
**The contact pressure distribution and settlement profile under the foundation will depend on:**

- **Flexibility of the foundation (flexible or rigid).**
- **Type of soil (clay, silt, sand, or gravel).**

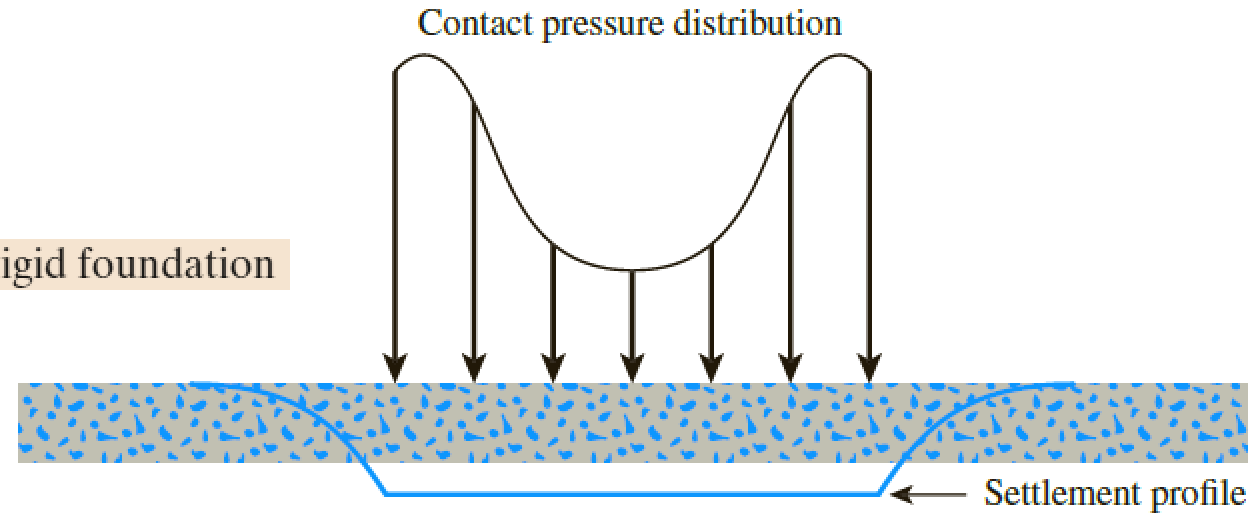
# Contact Pressure and Settlement Profile

## CLAY

flexible foundation:



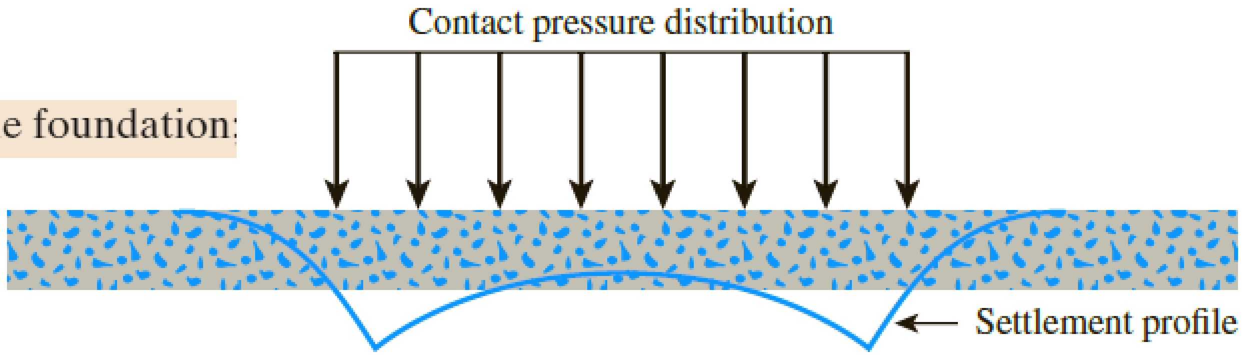
rigid foundation



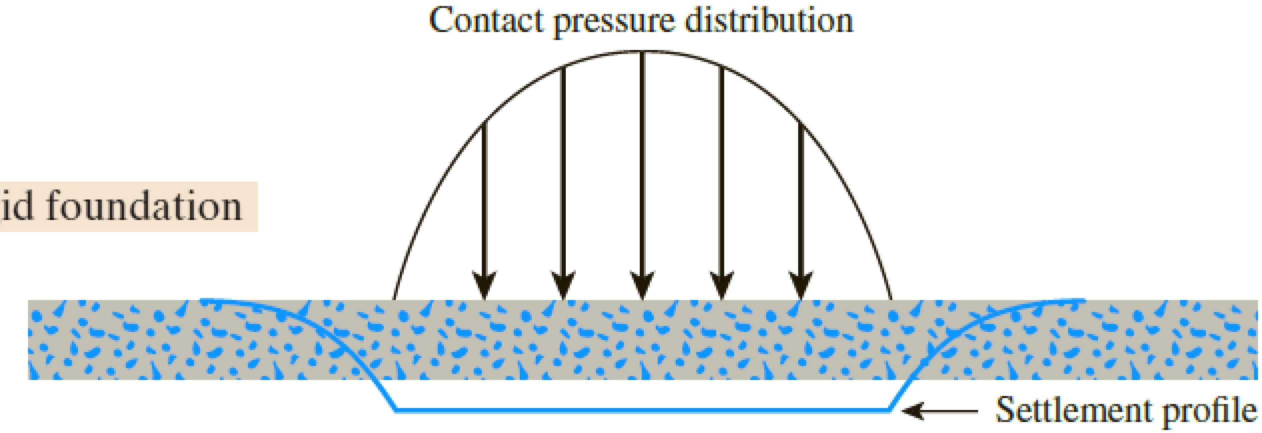
# Contact Pressure and Settlement Profile

## SAND

flexible foundation:



rigid foundation



# Elastic Settlement Calculation

# Elastic Settlement Calculation

**For shallow foundation subjected to a net force per unit area equal to $\Delta\sigma$ and if the foundation is perfectly flexible, the settlement may be expressed as:**

$$S_e = \Delta\sigma(\alpha B')\frac{1-\mu_s^2}{E_s} I_s I_f$$

$\Delta\sigma$ = net applied pressure on the foundation

$\mu_s$ = Poisson's ratio of soil

$E_s$ = average modulus of elasticity of the soil under the foundation measured from $z = 0$ to about $z = 5B$

$B'$ = $B/2$ for center of foundation

= $B$ for corner of foundation

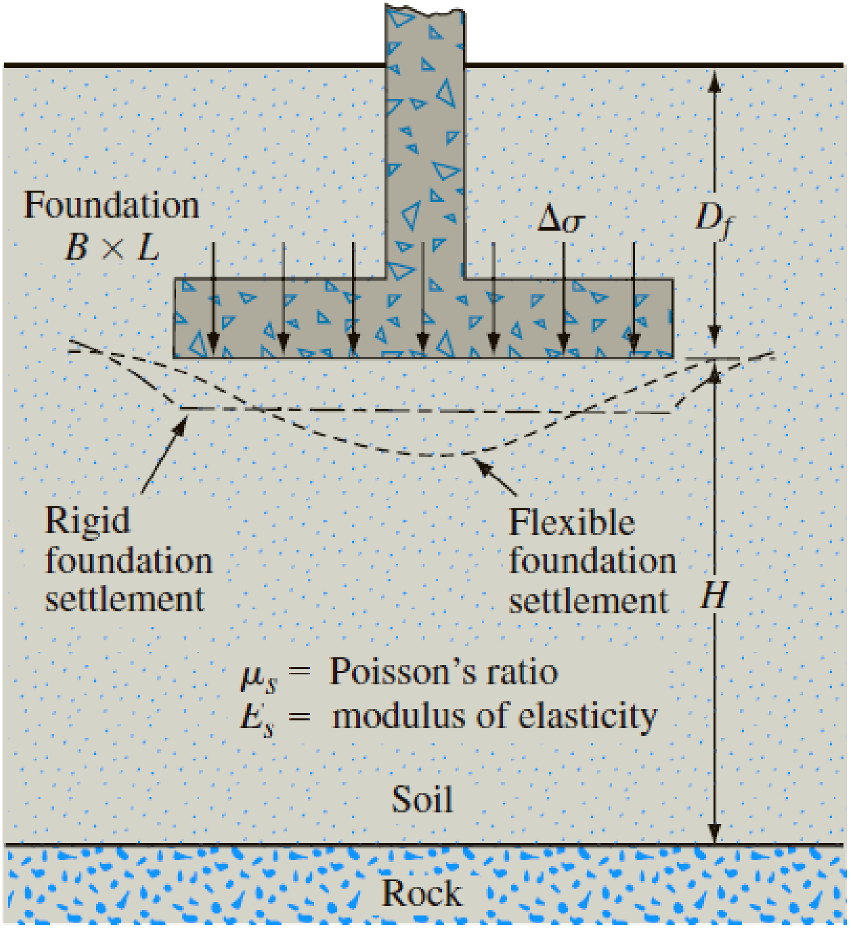
$I_s$ = shape factor (Steinbrenner, 1934)

$I_f$ = depth factor (Fox, 1948)

$$S_{e(\text{rigid})} \approx 0.93 S_{e(\text{flexible, center})}$$

# Elastic Settlement Calculation

$$S_e = \Delta\sigma(\alpha B')\frac{1-\mu_s^2}{E_s} I_s I_f$$



# Elastic Settlement Calculation

$$I_s = \text{shape factor (Steinbrenner, 1934)}$$

$$= F_1 + \frac{1 - 2\mu_s}{1 - \mu_s} F_2$$

$$F_1 = \frac{1}{\pi}(A_0 + A_1)$$

$$F_2 = \frac{n'}{2\pi} \tan^{-1} A_2$$

$$A_0 = m' \ln \frac{\left(1 + \sqrt{m'^2 + 1}\right)\sqrt{m'^2 + n'^2}}{m'\left(1 + \sqrt{m'^2 + n'^2 + 1}\right)}$$

$$A_1 = \ln \frac{\left(m' + \sqrt{m'^2 + 1}\right)\sqrt{1 + n'^2}}{m' + \sqrt{m'^2 + n'^2 + 1}}$$

$$A_2 = \frac{m'}{n'\sqrt{m'^2 + n'^2 + 1}}$$

# Elastic Settlement Calculation

$\alpha$ = factor that depends on the location on the foundation where settlement is being calculated

- For calculation of settlement at the *center* of the foundation:

$$\alpha = 4$$

$$m' = \frac{L}{B}$$

$$n' = \frac{H}{\left(\frac{B}{2}\right)}$$

$$S_e = \Delta\sigma(\alpha B')\frac{1-\mu_s^2}{E_s}I_s I_f$$

- For calculation of settlement at a *corner* of the foundation:

$$\alpha = 1$$

$$m' = \frac{L}{B}$$

$$n' = \frac{H}{B}$$

# Elastic Settlement Calculation

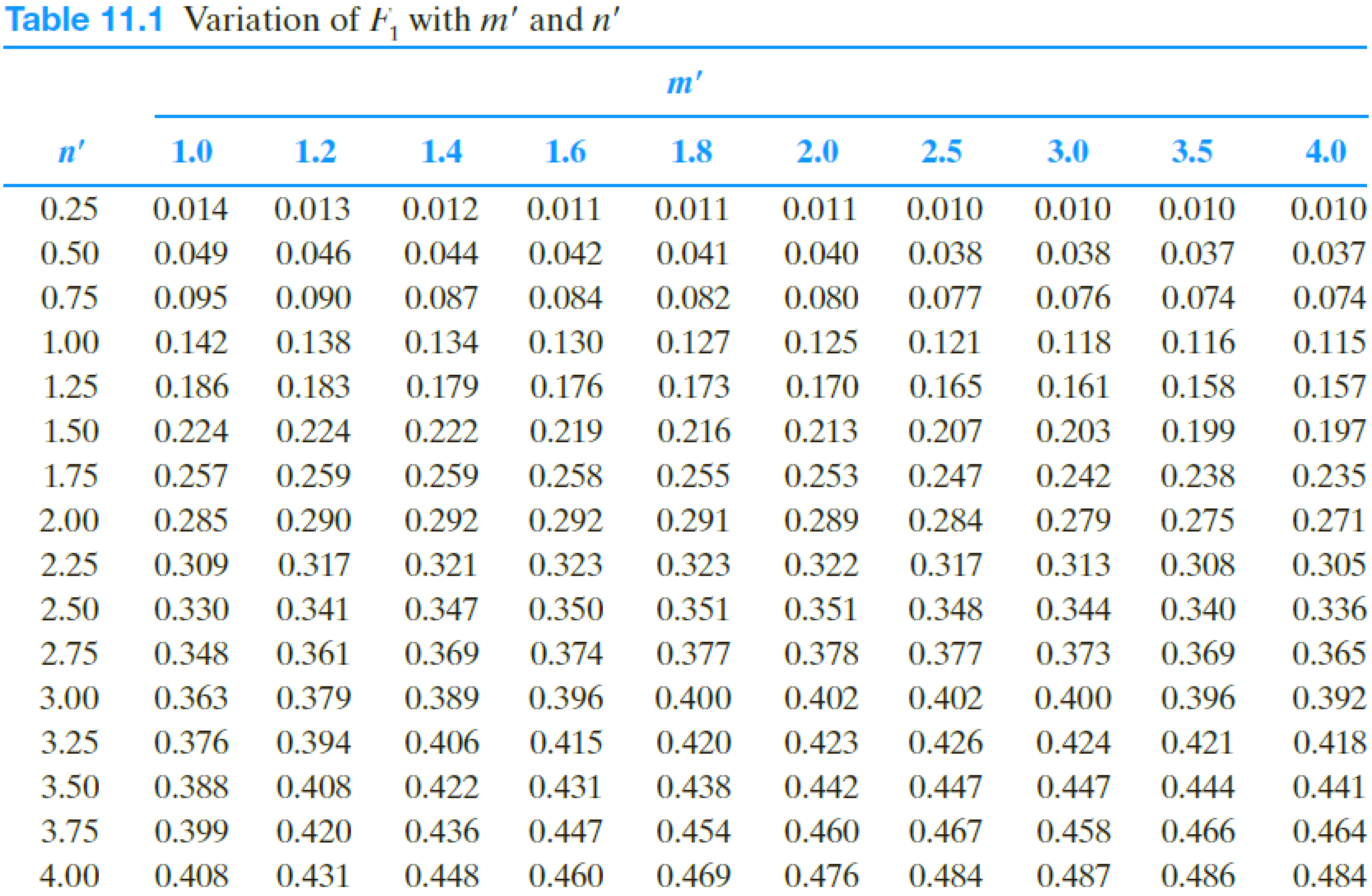


**Table 11.1** Variation of $F_1$ with $m'$ and $n'$

| | $m'$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n'$ | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| 0.25 | 0.014 | 0.013 | 0.012 | 0.011 | 0.011 | 0.011 | 0.010 | 0.010 | 0.010 | 0.010 |
| 0.50 | 0.049 | 0.046 | 0.044 | 0.042 | 0.041 | 0.040 | 0.038 | 0.038 | 0.037 | 0.037 |
| 0.75 | 0.095 | 0.090 | 0.087 | 0.084 | 0.082 | 0.080 | 0.077 | 0.076 | 0.074 | 0.074 |
| 1.00 | 0.142 | 0.138 | 0.134 | 0.130 | 0.127 | 0.125 | 0.121 | 0.118 | 0.116 | 0.115 |
| 1.25 | 0.186 | 0.183 | 0.179 | 0.176 | 0.173 | 0.170 | 0.165 | 0.161 | 0.158 | 0.157 |
| 1.50 | 0.224 | 0.224 | 0.222 | 0.219 | 0.216 | 0.213 | 0.207 | 0.203 | 0.199 | 0.197 |
| 1.75 | 0.257 | 0.259 | 0.259 | 0.258 | 0.255 | 0.253 | 0.247 | 0.242 | 0.238 | 0.235 |
| 2.00 | 0.285 | 0.290 | 0.292 | 0.292 | 0.291 | 0.289 | 0.284 | 0.279 | 0.275 | 0.271 |
| 2.25 | 0.309 | 0.317 | 0.321 | 0.323 | 0.323 | 0.322 | 0.317 | 0.313 | 0.308 | 0.305 |
| 2.50 | 0.330 | 0.341 | 0.347 | 0.350 | 0.351 | 0.351 | 0.348 | 0.344 | 0.340 | 0.336 |
| 2.75 | 0.348 | 0.361 | 0.369 | 0.374 | 0.377 | 0.378 | 0.377 | 0.373 | 0.369 | 0.365 |
| 3.00 | 0.363 | 0.379 | 0.389 | 0.396 | 0.400 | 0.402 | 0.402 | 0.400 | 0.396 | 0.392 |
| 3.25 | 0.376 | 0.394 | 0.406 | 0.415 | 0.420 | 0.423 | 0.426 | 0.424 | 0.421 | 0.418 |
| 3.50 | 0.388 | 0.408 | 0.422 | 0.431 | 0.438 | 0.442 | 0.447 | 0.447 | 0.444 | 0.441 |
| 3.75 | 0.399 | 0.420 | 0.436 | 0.447 | 0.454 | 0.460 | 0.467 | 0.458 | 0.466 | 0.464 |
| 4.00 | 0.408 | 0.431 | 0.448 | 0.460 | 0.469 | 0.476 | 0.484 | 0.487 | 0.486 | 0.484 |

# Elastic Settlement Calculation

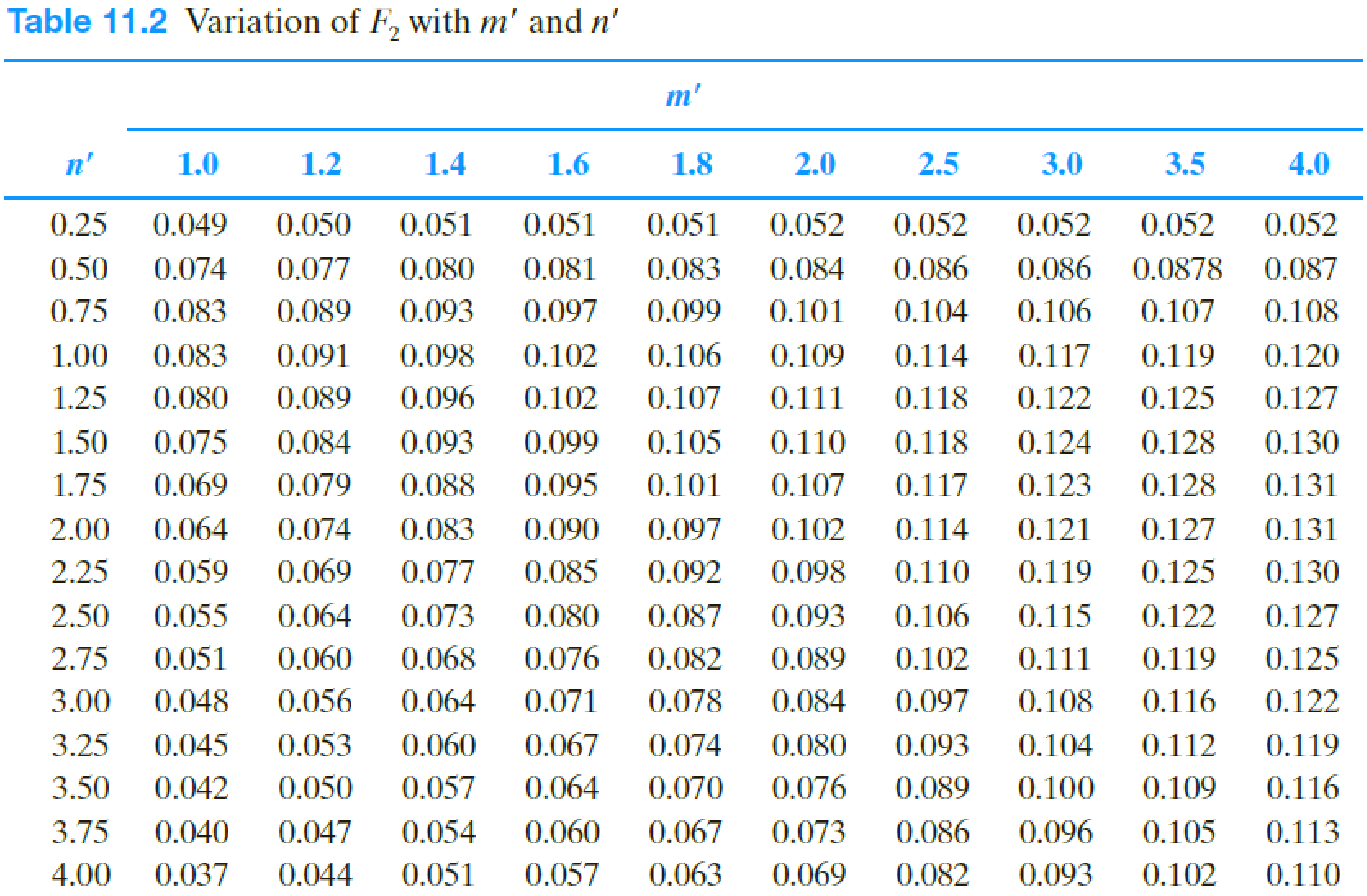


**Table 11.2** Variation of $F_2$ with $m'$ and $n'$

| | $m'$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n'$ | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| 0.25 | 0.049 | 0.050 | 0.051 | 0.051 | 0.051 | 0.052 | 0.052 | 0.052 | 0.052 | 0.052 |
| 0.50 | 0.074 | 0.077 | 0.080 | 0.081 | 0.083 | 0.084 | 0.086 | 0.086 | 0.0878 | 0.087 |
| 0.75 | 0.083 | 0.089 | 0.093 | 0.097 | 0.099 | 0.101 | 0.104 | 0.106 | 0.107 | 0.108 |
| 1.00 | 0.083 | 0.091 | 0.098 | 0.102 | 0.106 | 0.109 | 0.114 | 0.117 | 0.119 | 0.120 |
| 1.25 | 0.080 | 0.089 | 0.096 | 0.102 | 0.107 | 0.111 | 0.118 | 0.122 | 0.125 | 0.127 |
| 1.50 | 0.075 | 0.084 | 0.093 | 0.099 | 0.105 | 0.110 | 0.118 | 0.124 | 0.128 | 0.130 |
| 1.75 | 0.069 | 0.079 | 0.088 | 0.095 | 0.101 | 0.107 | 0.117 | 0.123 | 0.128 | 0.131 |
| 2.00 | 0.064 | 0.074 | 0.083 | 0.090 | 0.097 | 0.102 | 0.114 | 0.121 | 0.127 | 0.131 |
| 2.25 | 0.059 | 0.069 | 0.077 | 0.085 | 0.092 | 0.098 | 0.110 | 0.119 | 0.125 | 0.130 |
| 2.50 | 0.055 | 0.064 | 0.073 | 0.080 | 0.087 | 0.093 | 0.106 | 0.115 | 0.122 | 0.127 |
| 2.75 | 0.051 | 0.060 | 0.068 | 0.076 | 0.082 | 0.089 | 0.102 | 0.111 | 0.119 | 0.125 |
| 3.00 | 0.048 | 0.056 | 0.064 | 0.071 | 0.078 | 0.084 | 0.097 | 0.108 | 0.116 | 0.122 |
| 3.25 | 0.045 | 0.053 | 0.060 | 0.067 | 0.074 | 0.080 | 0.093 | 0.104 | 0.112 | 0.119 |
| 3.50 | 0.042 | 0.050 | 0.057 | 0.064 | 0.070 | 0.076 | 0.089 | 0.100 | 0.109 | 0.116 |
| 3.75 | 0.040 | 0.047 | 0.054 | 0.060 | 0.067 | 0.073 | 0.086 | 0.096 | 0.105 | 0.113 |
| 4.00 | 0.037 | 0.044 | 0.051 | 0.057 | 0.063 | 0.069 | 0.082 | 0.093 | 0.102 | 0.110 |

# Elastic Settlement Calculation

$$I_f = \text{depth factor (Fox, 1948)} = f\left(\frac{D_f}{B}, \mu_s, \text{and } \frac{L}{B}\right)$$

$$S_e = \Delta\sigma(\alpha B')\frac{1-\mu_s^2}{E_s} I_s I_f$$

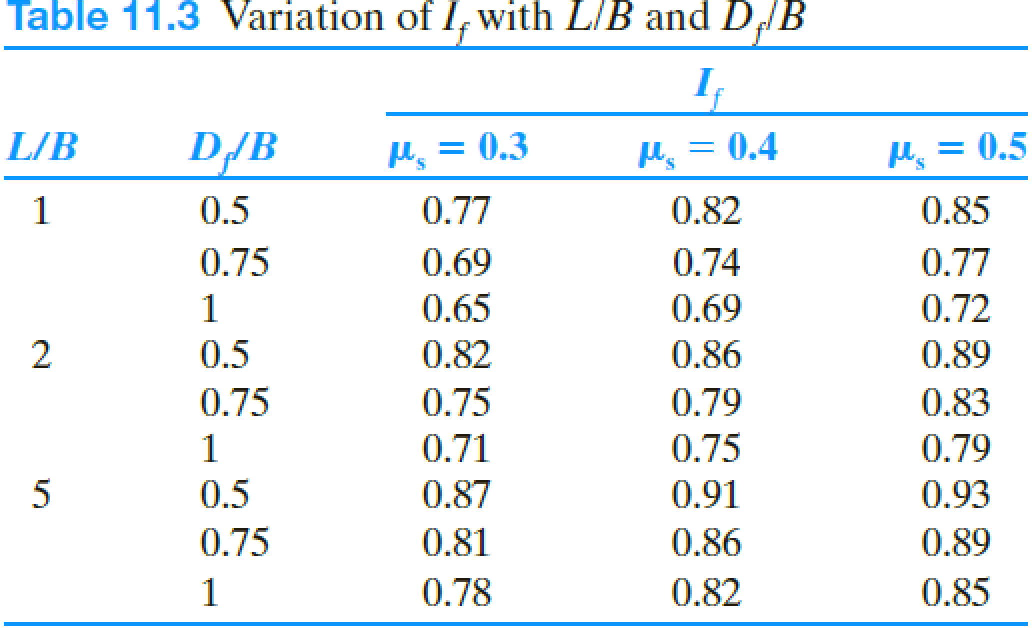


**Table 11.3** Variation of $I_f$ with $L/B$ and $D_f/B$

| | | $I_f$ | | |
|---|---|---|---|---|
| $L/B$ | $D_f/B$ | $\mu_s = 0.3$ | $\mu_s = 0.4$ | $\mu_s = 0.5$ |
| 1 | 0.5 | 0.77 | 0.82 | 0.85 |
| | 0.75 | 0.69 | 0.74 | 0.77 |
| | 1 | 0.65 | 0.69 | 0.72 |
| 2 | 0.5 | 0.82 | 0.86 | 0.89 |
| | 0.75 | 0.75 | 0.79 | 0.83 |
| | 1 | 0.71 | 0.75 | 0.79 |
| 5 | 0.5 | 0.87 | 0.91 | 0.93 |
| | 0.75 | 0.81 | 0.86 | 0.89 |
| | 1 | 0.78 | 0.82 | 0.85 |

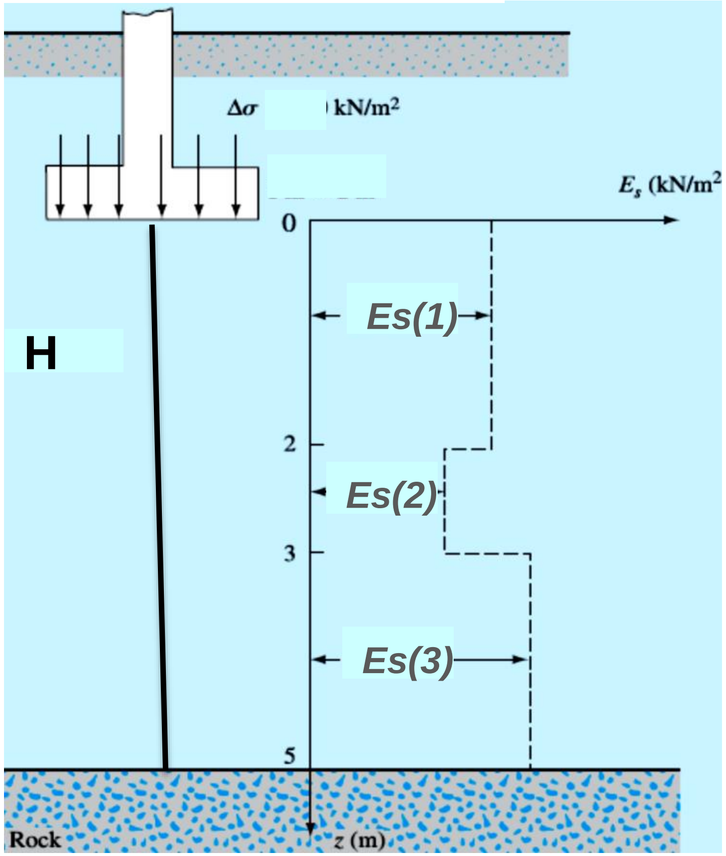
# Elastic Settlement Calculation

Due to the nonhomogeneous nature of soil deposits, the magnitude of $E_s$ may vary with depth. For that reason, Bowles (1987) recommended using a weighted average value of $E_s$

$$E_s = \frac{\sum E_{s(i)} \Delta z}{\bar{z}}$$

$E_{s(i)}$ = soil modulus of elasticity within a depth $\Delta z$

$\bar{z}$ = $H$ or $5B$, whichever is smaller



# Elastic Settlement Calculation

$$S_e = \Delta\sigma(\alpha B')\frac{1-\mu_s^2}{E_s} I_s I_f$$

$E_{s(i)}$ = soil modulus of elasticity within a depth $\Delta z$

$\bar{z} = H$ or $5B$, whichever is smaller

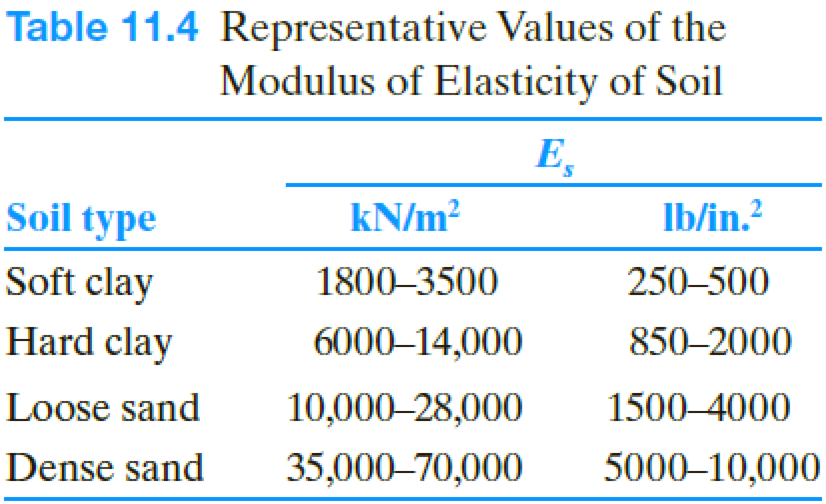


**Table 11.4** Representative Values of the Modulus of Elasticity of Soil

| Soil type | $E_s$ kN/m$^2$ | $E_s$ lb/in.$^2$ |
|---|---|---|
| Soft clay | 1800–3500 | 250–500 |
| Hard clay | 6000–14,000 | 850–2000 |
| Loose sand | 10,000–28,000 | 1500–4000 |
| Dense sand | 35,000–70,000 | 5000–10,000 |

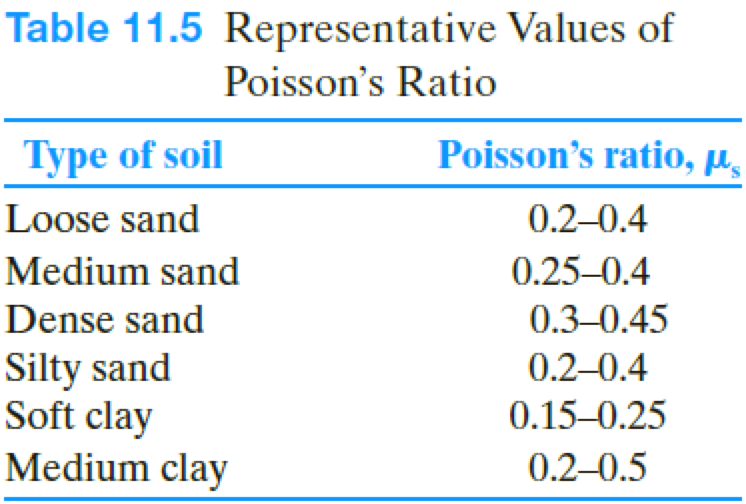
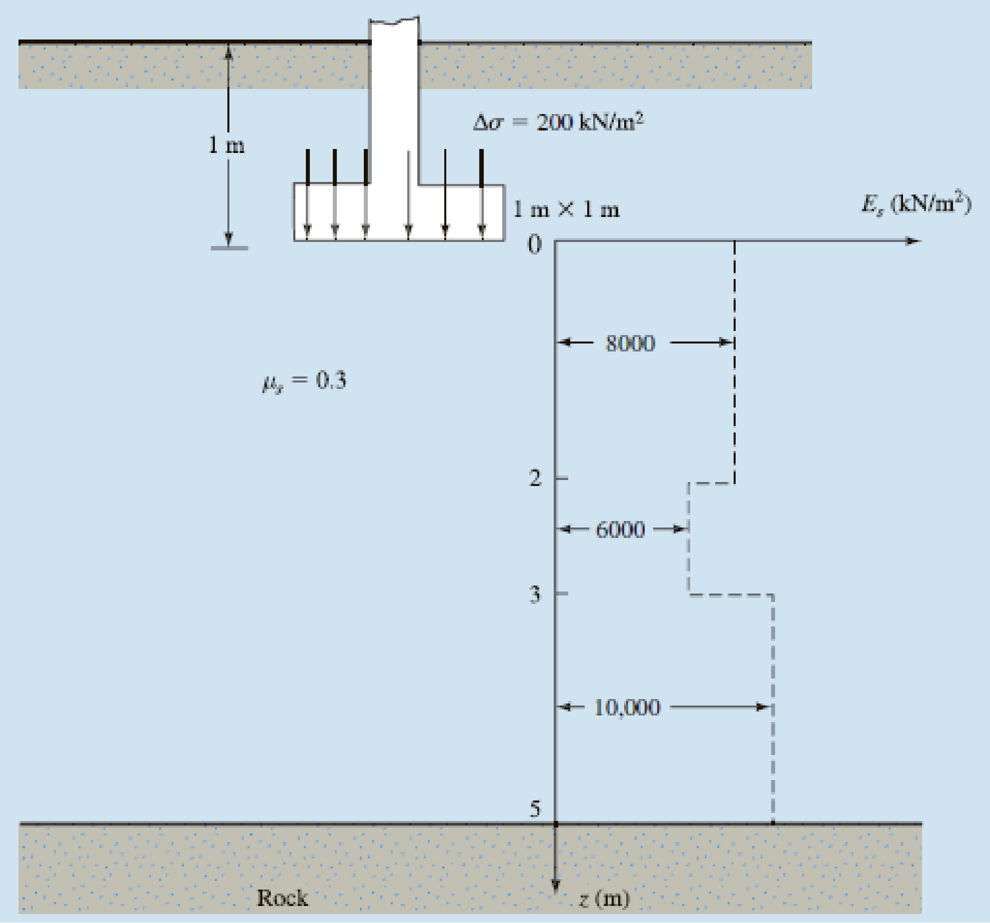


**Table 11.5** Representative Values of Poisson's Ratio

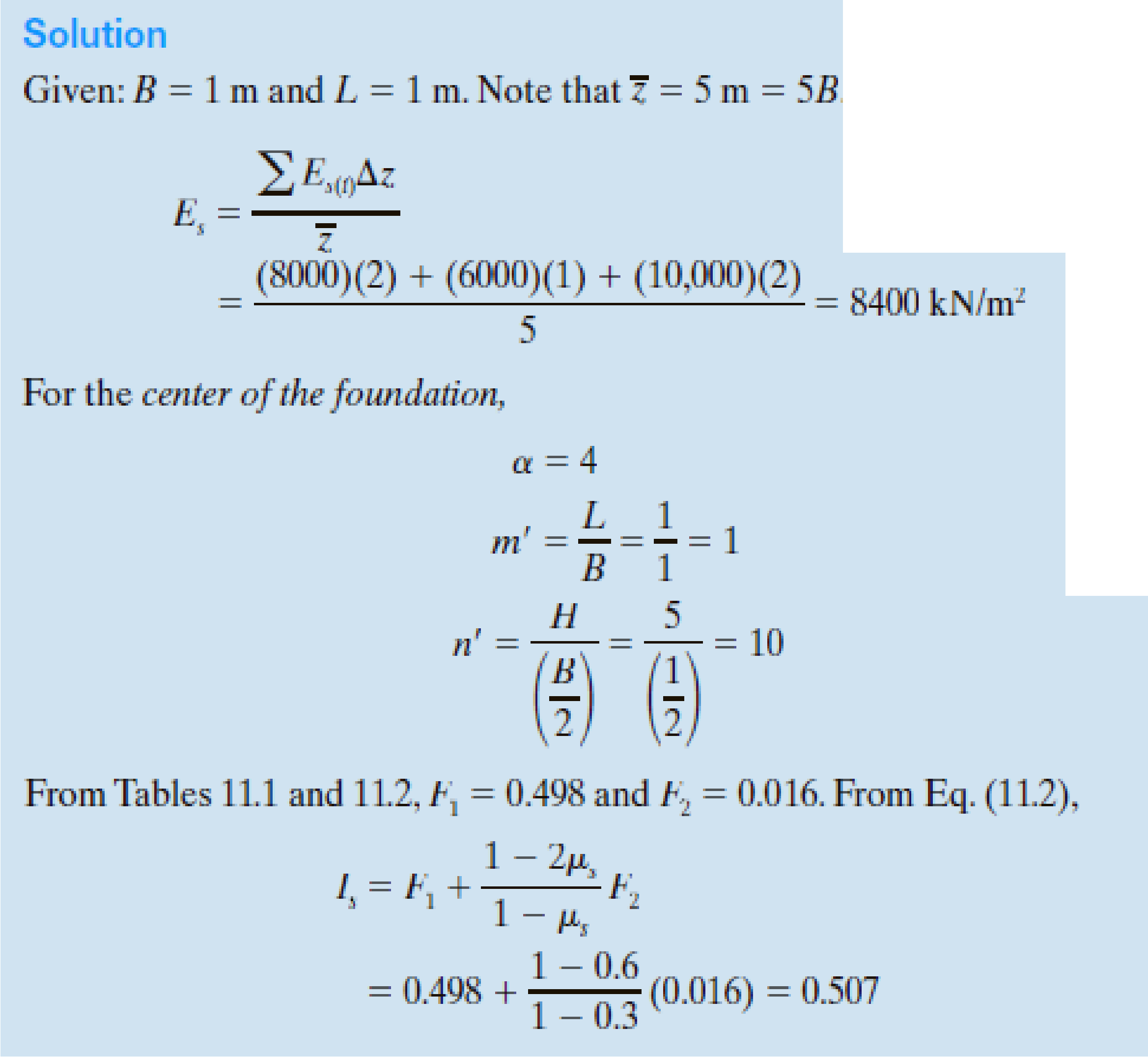
| Type of soil | Poisson's ratio, $\mu_s$ |
|---|---|
| Loose sand | 0.2–0.4 |
| Medium sand | 0.25–0.4 |
| Dense sand | 0.3–0.45 |
| Silty sand | 0.2–0.4 |
| Soft clay | 0.15–0.25 |
| Medium clay | 0.2–0.5 |

# EXAMPLE 11.1

A rigid shallow foundation 1 m × 1 m in plan is shown in Figure [redacted] Calculate the elastic settlement at the center of the foundation.



# EXAMPLE 11.1



**Solution**

Given: $B = 1$ m and $L = 1$ m. Note that $\bar{z} = 5$ m $= 5B$.

$$E_s = \frac{\sum E_{s(i)}\Delta z}{\bar{z}}$$

$$= \frac{(8000)(2) + (6000)(1) + (10{,}000)(2)}{5} = 8400 \text{ kN/m}^2$$

For the *center of the foundation,*

$$\alpha = 4$$

$$m' = \frac{L}{B} = \frac{1}{1} = 1$$

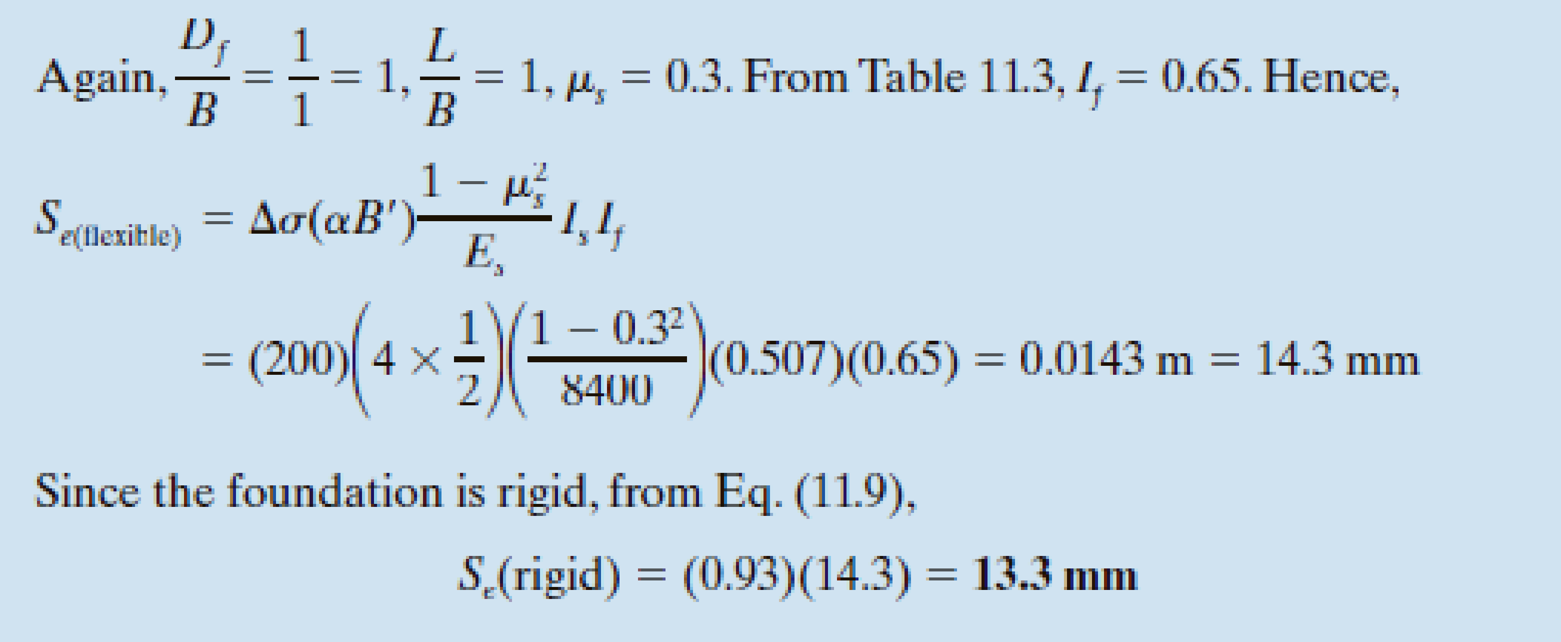
$$n' = \frac{H}{\left(\frac{B}{2}\right)} = \frac{5}{\left(\frac{1}{2}\right)} = 10$$

From Tables 11.1 and 11.2, $F_1 = 0.498$ and $F_2 = 0.016$. From Eq. (11.2),

$$I_s = F_1 + \frac{1 - 2\mu_s}{1 - \mu_s} F_2$$

$$= 0.498 + \frac{1 - 0.6}{1 - 0.3}(0.016) = 0.507$$

# EXAMPLE 11.1



Again, $\frac{D_f}{B} = \frac{1}{1} = 1$, $\frac{L}{B} = 1$, $\mu_s = 0.3$. From Table 11.3, $I_f = 0.65$. Hence,

$$S_{e(\text{flexible})} = \Delta\sigma(\alpha B')\frac{1-\mu_s^2}{E_s} I_s I_f$$

$$= (200)\left(4 \times \frac{1}{2}\right)\left(\frac{1-0.3^2}{8400}\right)(0.507)(0.65) = 0.0143 \text{ m} = 14.3 \text{ mm}$$

Since the foundation is rigid, from Eq. (11.9),

$$S_e(\text{rigid}) = (0.93)(14.3) = \mathbf{13.3\ mm}$$

# Improved Equation for Elastic Settlement

# Improved Equation for Elastic Settlement

**The improved formula takes into account**

1. The rigidity of the foundation

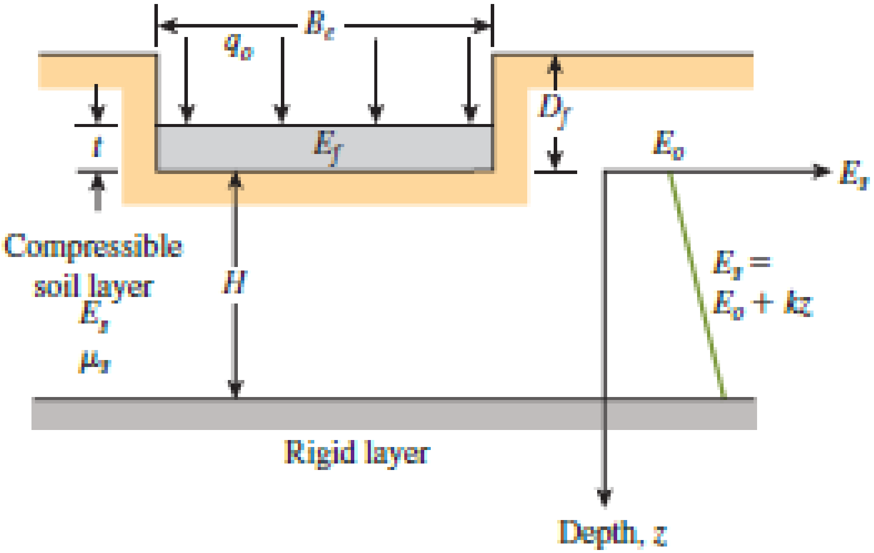
2. The depth of embedment of the foundation

3. The increase in the modulus of elasticity of the soil with depth

4. The location of rigid layers at a limited depth

# Improved Equation for Elastic Settlement

$$S_e = \frac{q_o B_e I_G I_F I_E}{E_o}(1 - \mu_s^2)$$



where

$I_G$ = influence factor for the variation of $E_s$ with depth

$$= f\left(\beta = \frac{E_o}{kB_e}, \frac{H}{B_e}\right)$$

$I_F$ = foundation rigidity correction factor

$I_E$ = foundation embedment correction factor

$B_e$ = the equivalent diameter

# Improved Equation for Elastic Settlement

$$S_e = \frac{q_o B_e I_G I_F I_E}{E_o}(1 - \mu_s^2)$$

$B_e$ of a rectangular foundation,

$$B_e = \sqrt{\frac{4BL}{\pi}}$$

$B$ = width of foundation

$L$ = length of foundation

For circular foundations,

$$B_e = B$$

$B$ = diameter of foundation.

# Improved Equation for Elastic Settlement

$$S_e = \frac{q_o B_e I_G I_F I_E}{E_o}(1 - \mu_s^2)$$

$I_G$ = influence factor for the variation of $E_s$ with depth

$$= f\left(\beta = \frac{E_o}{kB_e}, \frac{H}{B_e}\right)$$

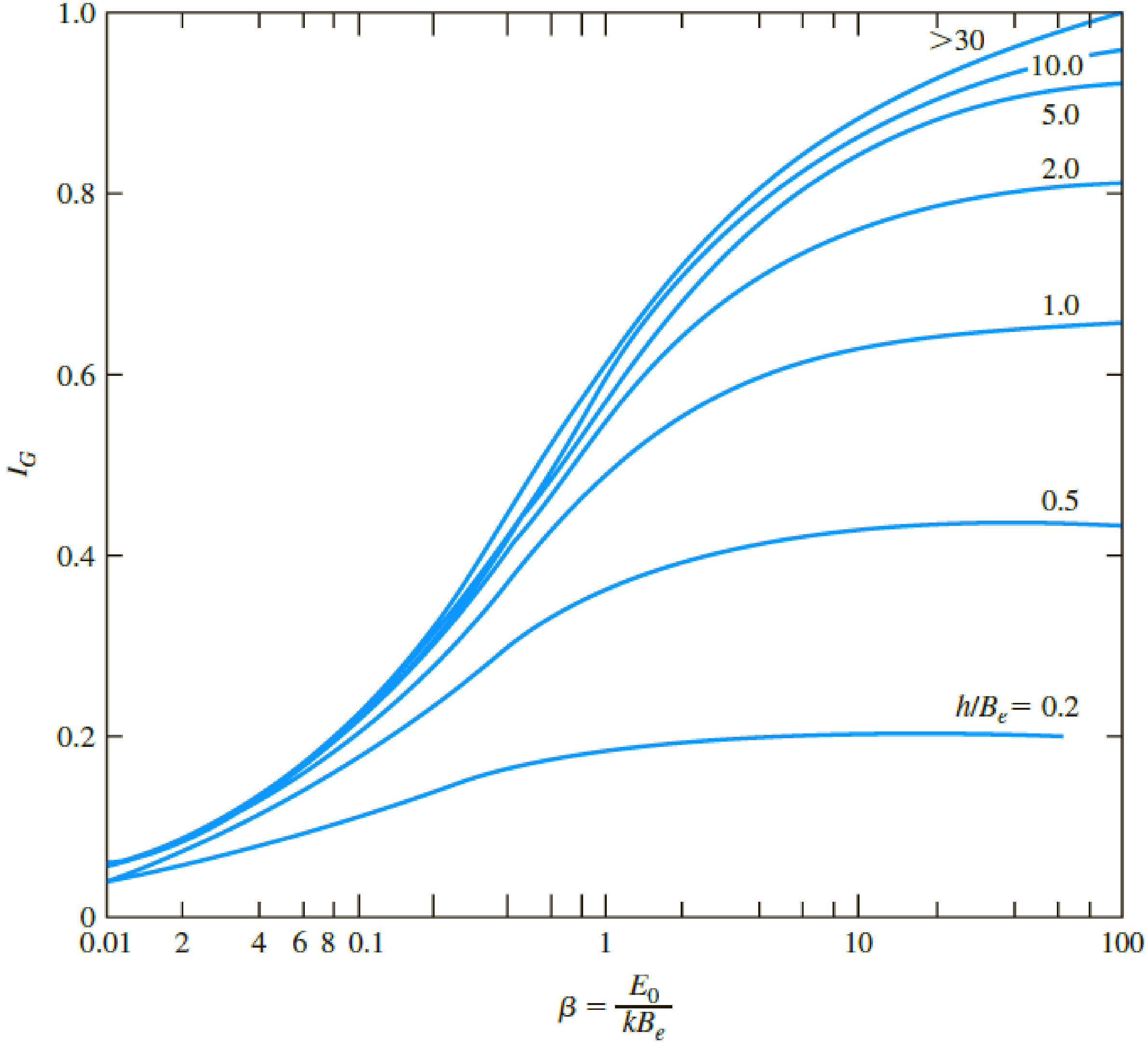
$I_F$ = foundation rigidity correction factor

$I_E$ = foundation embedment correction factor

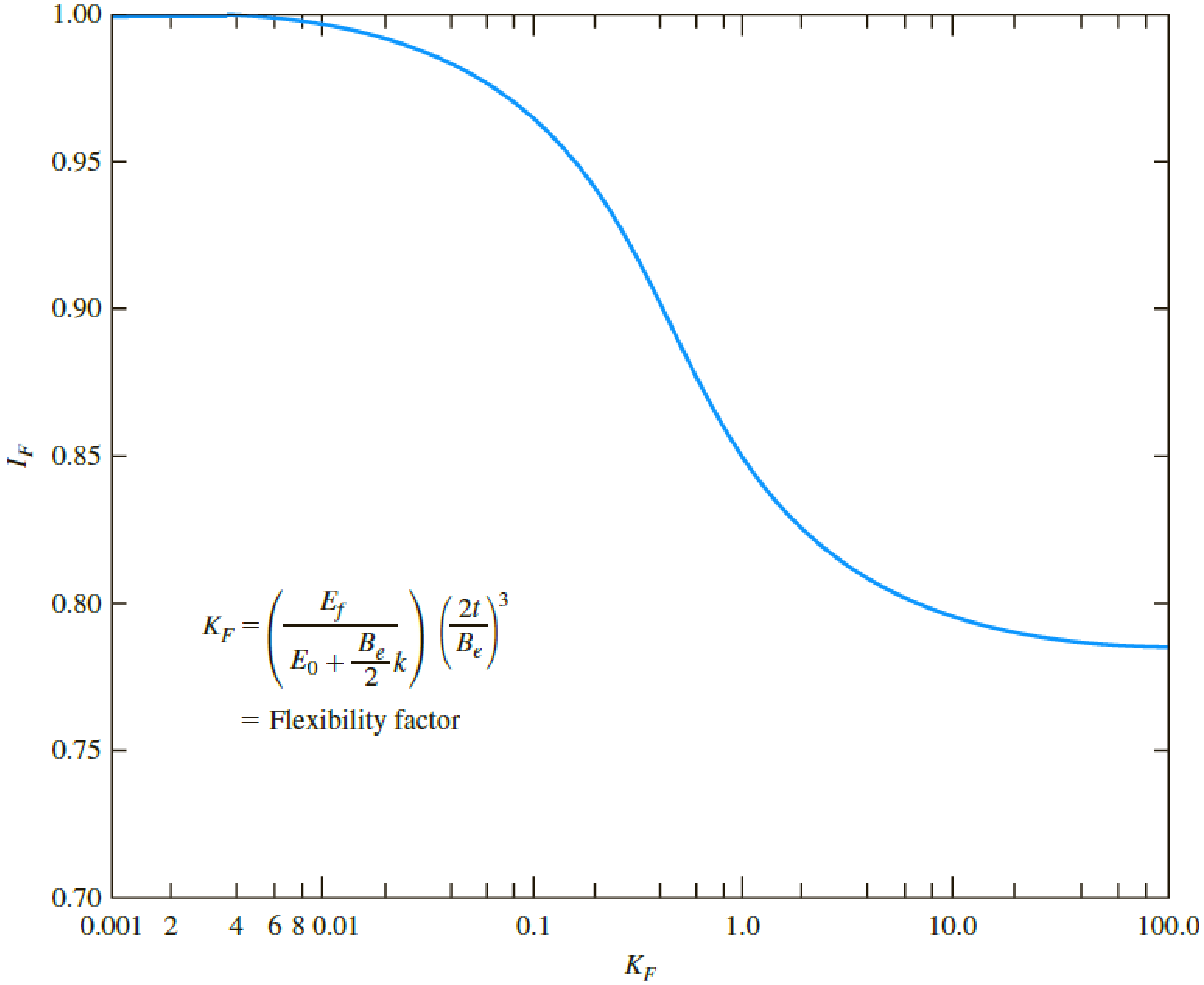
$$I_F = \frac{\pi}{4} + \frac{1}{4.6 + 10\left(\dfrac{E_f}{E_o + \dfrac{B_e}{2}k}\right)\left(\dfrac{2t}{B_e}\right)^3}$$

$$I_E = 1 - \frac{1}{3.5\exp(1.22\mu_s - 0.4)\left(\dfrac{B_e}{D_f} + 1.6\right)}$$

# $I_G$ influence factor for the variation of $E_s$ with depth



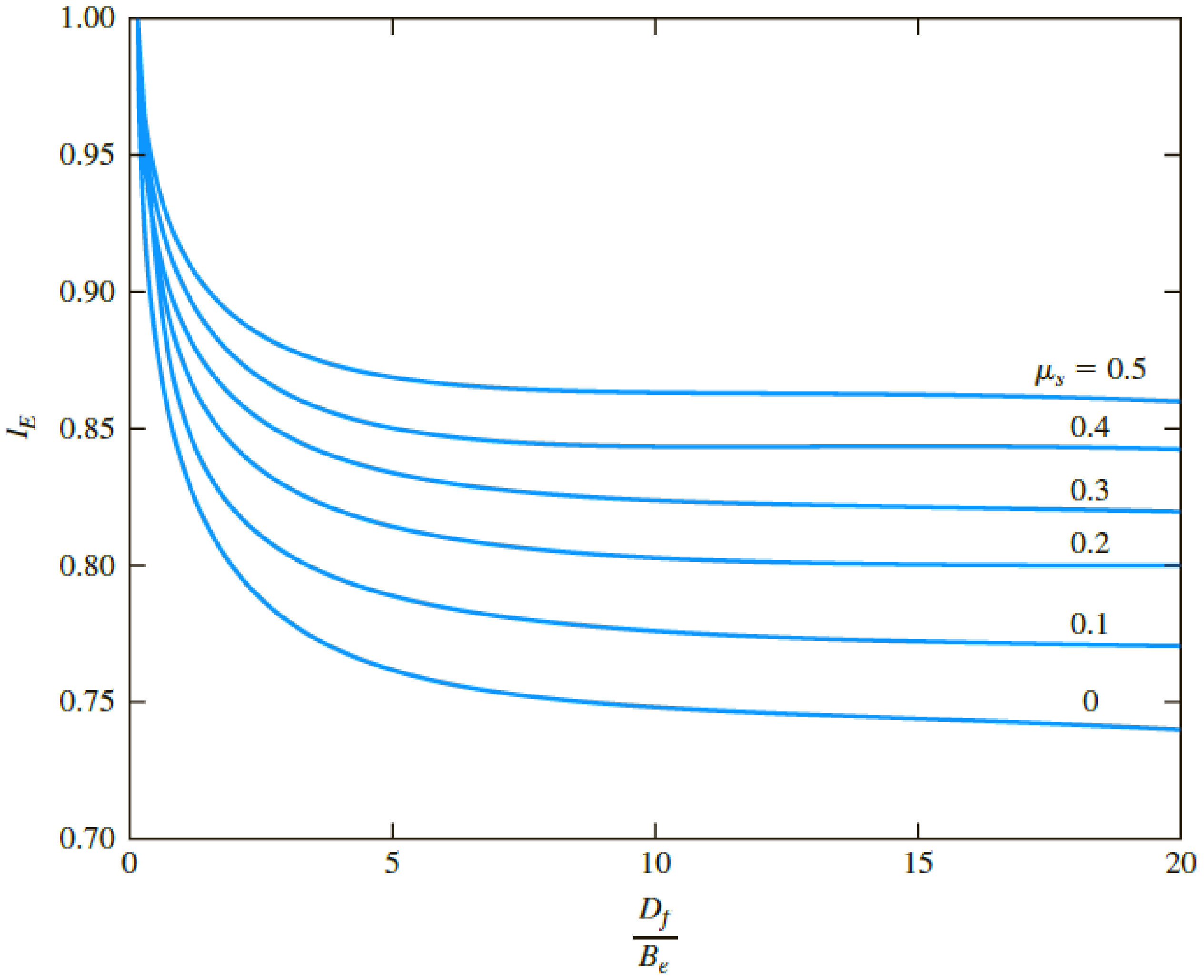
# $I_F$ foundation rigidity correction factor



$$K_F = \left(\frac{E_f}{E_0 + \frac{B_e}{2}k}\right)\left(\frac{2t}{B_e}\right)^3$$

= Flexibility factor

# $I_E$ foundation embedment correction factor



# EXAMPLE 11.2

## Example 11.2

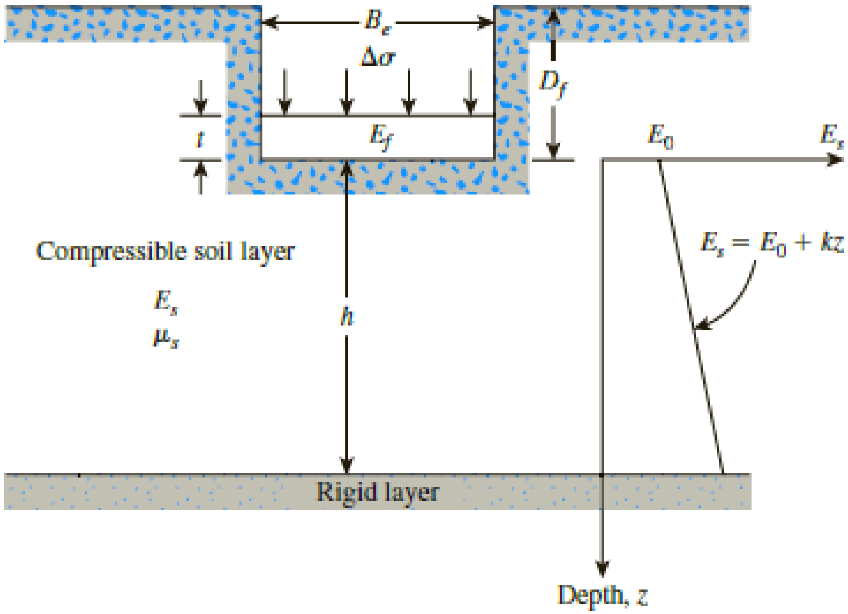
Refer to Figure 11.5. For a shallow foundation supported by a silty clay, the following are given:

Length, $L = 1.5$ m
Width, $B = 1$ m
Depth of foundation, $D_f = 1$ m
Thickness of foundation, $t$ – 0.23 m
Load per unit area, $\Delta\sigma = 190$ kN/m$^2$
$E_f = 15 \times 10^6$ kN/m$^2$

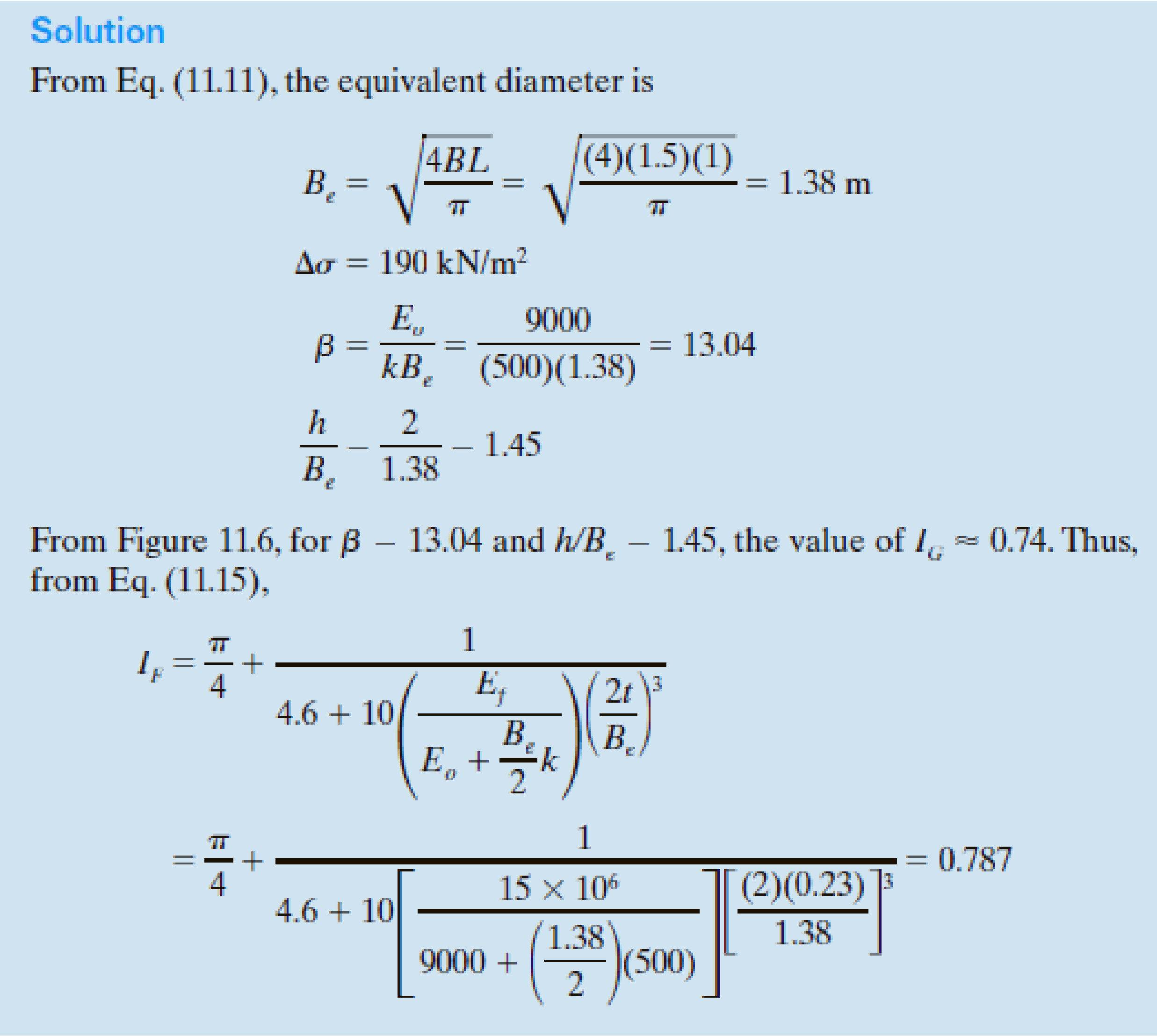
The silty clay soil had the following properties:

$h = 2$ m
$\mu_s = 0.3$
$E_o = 9000$ kN/m$^2$
$k = 500$ kN/m$^2$/m

Estimate the elastic settlement of the foundation.



# EXAMPLE 11.2



**Solution**

From Eq. (11.11), the equivalent diameter is

$$B_e = \sqrt{\frac{4BL}{\pi}} = \sqrt{\frac{(4)(1.5)(1)}{\pi}} = 1.38 \text{ m}$$

$$\Delta\sigma = 190 \text{ kN/m}^2$$

$$\beta = \frac{E_o}{kB_e} = \frac{9000}{(500)(1.38)} = 13.04$$

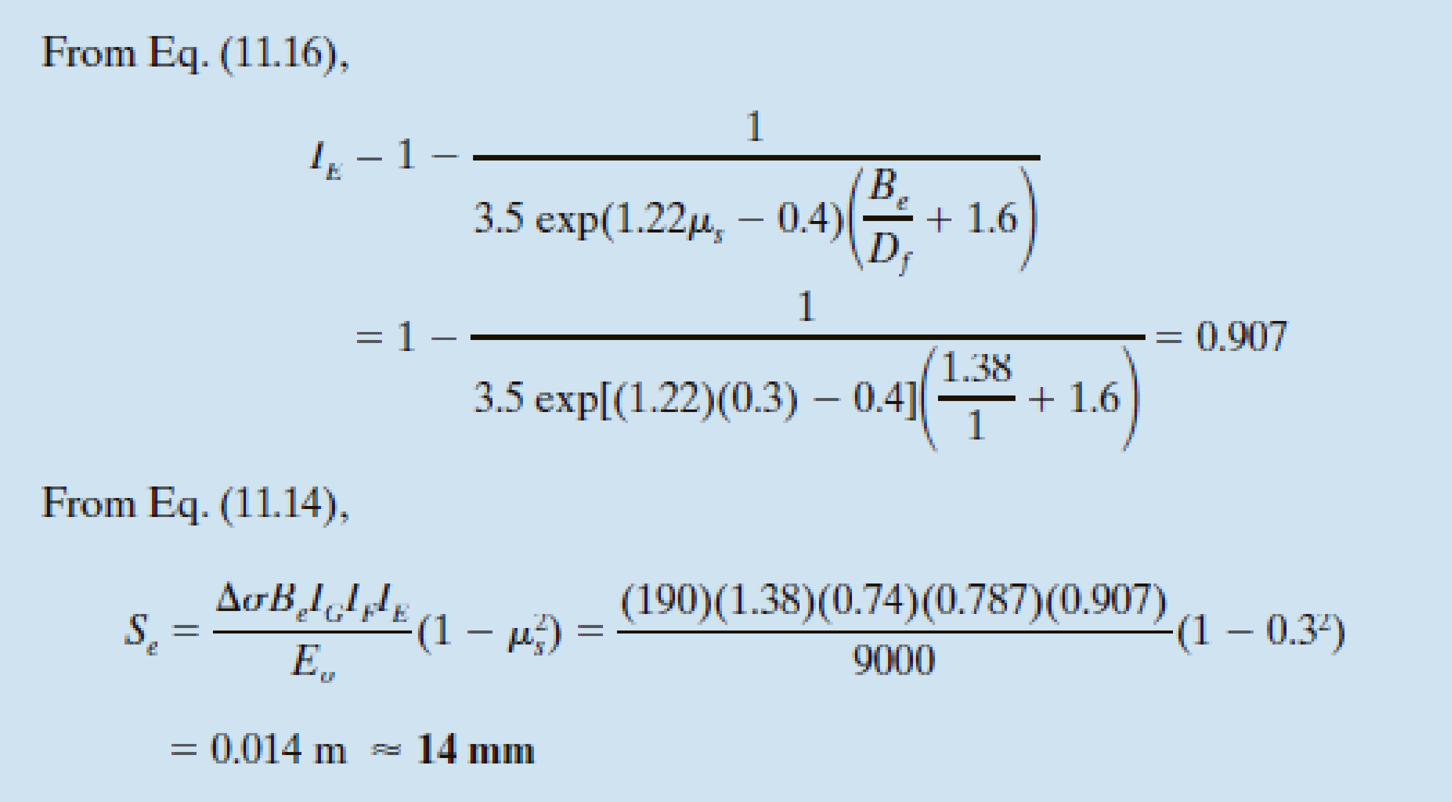
$$\frac{h}{B_e} = \frac{2}{1.38} = 1.45$$

From Figure 11.6, for $\beta = 13.04$ and $h/B_e = 1.45$, the value of $I_G \approx 0.74$. Thus, from Eq. (11.15),

$$I_F = \frac{\pi}{4} + \frac{1}{4.6 + 10\left(\dfrac{E_f}{E_o + \dfrac{B_e}{2}k}\right)\left(\dfrac{2t}{B_e}\right)^3}$$

$$= \frac{\pi}{4} + \frac{1}{4.6 + 10\left[\dfrac{15 \times 10^6}{9000 + \left(\dfrac{1.38}{2}\right)(500)}\right]\left[\dfrac{(2)(0.23)}{1.38}\right]^3} = 0.787$$

# EXAMPLE 11.2



From Eq. (11.16),

$$I_E - 1 - \frac{1}{3.5\exp(1.22\mu_s - 0.4)\left(\frac{B_e}{D_f} + 1.6\right)}$$

$$= 1 - \frac{1}{3.5\exp[(1.22)(0.3) - 0.4]\left(\frac{1.38}{1} + 1.6\right)} = 0.907$$

From Eq. (11.14),

$$S_e = \frac{\Delta\sigma B_e I_G I_F I_E}{E_o}(1 - \mu_s^2) = \frac{(190)(1.38)(0.74)(0.787)(0.907)}{9000}(1 - 0.3^2)$$

$$= 0.014 \text{ m} \approx \mathbf{14\ mm}$$

# Components of Settlement

$$S_t = S_e + S_c + S_s$$

$S_t$ = Total settlement

$S_e$ = elastic (immediate) settlement

$S_c$ = Primary consolidation settlement

$S_s$ = Secondary consolidation settlement

# CONSOLIDATION SETTLEMENT

# CONSOLIDATION SETTLEMENT

**Time-dependent settlement in saturated fine-grained soils having a low coefficient of permeability.**

**Two components:**

1. **Primary consolidation settlement**

   **(extrusion of pore water from the void space)**

2. **Secondary consolidation settlement**

   **(readjustment of soil grains @ constant effective stress)**

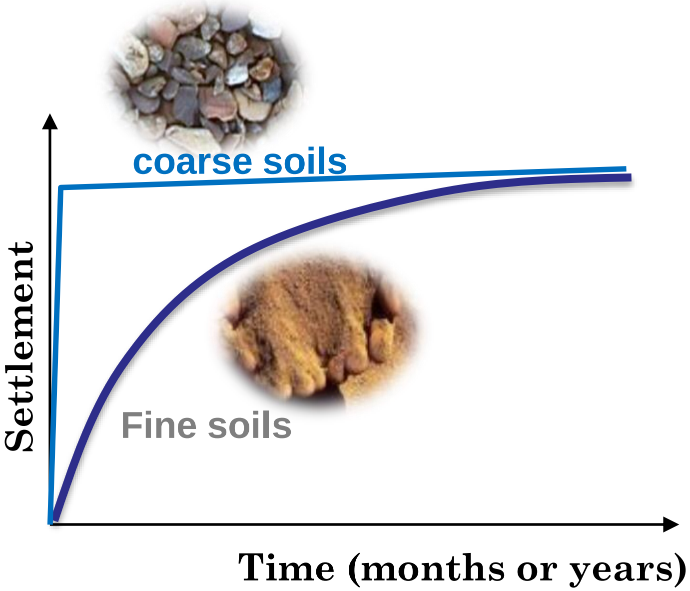
# CONSOLIDATION SETTLEMENT

- In coarse soils (sands & gravels) any volume change resulting from a change in loading occurs immediately; increases in pore pressures are dissipated rapidly due to high permeability. This is called drained loading.

- In fine soils (silts & clays) - with low permeability - the soil is undrained as the load is applied. Slow seepage occurs and the excess pore pressures dissipate slowly, and consolidation settlement occurs.

- In coarse soils (sand & gravel) the settlement takes place instantaneously.
- In fine soils (clay & silt): settlement takes far much more time to complete. Why?

So, consolidation settlement: is decrease in voids volume as pore-water is squeezed out of the soil. It is only significant in fine soil (clays & silts).



# CONSOLIDATION SETTLEMENT

**In fine-grained soil the process requires along time interval for its completion and the nature of settlement is more difficult to analyze.**

**Gradual reduction in volume == gradual reduction in void ratio, e. Therefore we have to know the change in e in order to know settlement.**

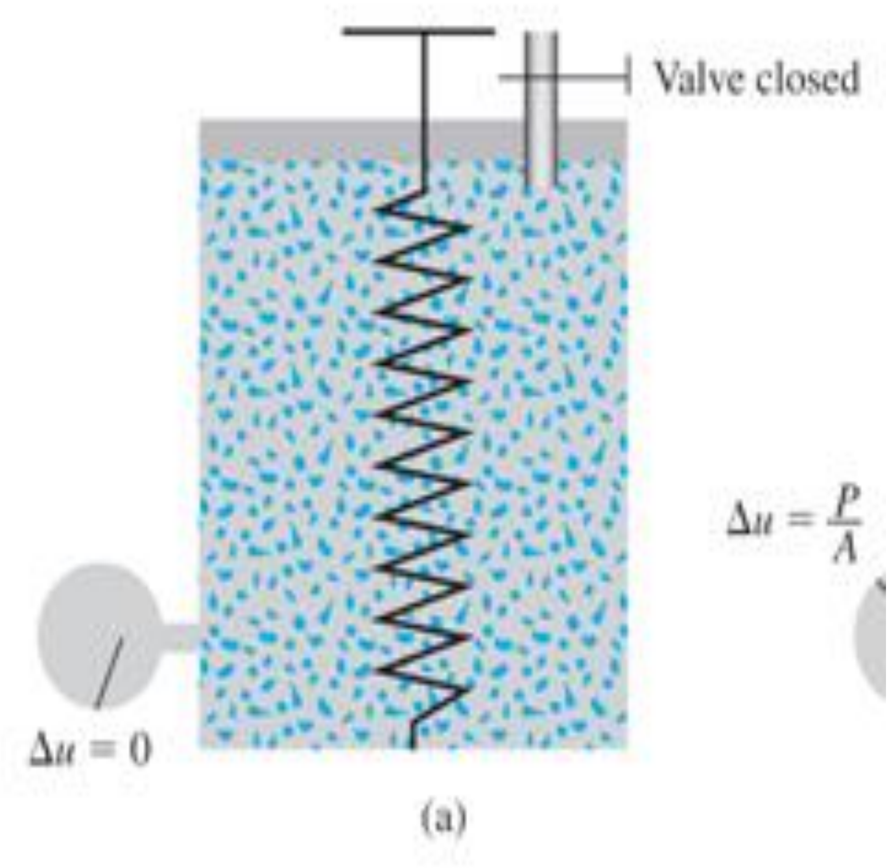
**e is our internal variable that through it we can follow the change in soil volume.**

# Description of Primary Consolidation Process

1. **When a saturated soil layer is subjected to a stress increase, the external load is initially transferred to water causing sudden increase in the pore water pressure (excess pore water pressure).**
2. **Elastic settlement occurs immediately. However, due to the low coefficient of permeability of clay, the excess pore water pressure generated by loading gradually squeezes over a long period of time.**
3. **Eventually, excess pore pressure becomes zero and the pore water pressure is the same as hydrostatic pressure prior to loading.**
4. **The associated volume change (that is, the consolidation) in the clay may continue long after the elastic settlement.**
5. **The settlement caused by consolidation in clay may be several times greater than the elastic settlement.**

# Consolidation Process – Spring analogy

i. At equilibrium under overburden stress

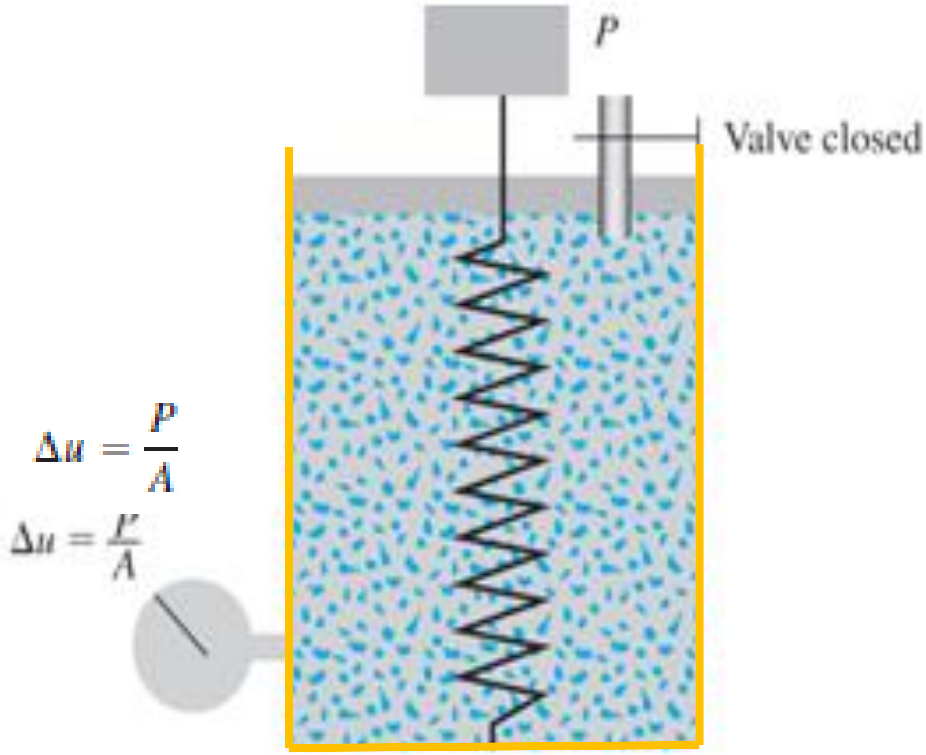


- System is analog to soil layer at equilibrium with weight of all soil layer (overburden) above it.
- In equilibrium, valve is closed.
- Piston is loaded, compresses a spring in chamber.
- Hydrostatic pressure = $u_o$

spring ≈ soil skeleton
water ≈ water in soil void
valve ≈ pore sizes in soil

# Consolidation Process- Spring analogy

ii. Under Load (t = 0)



(b)

- **Soil is loaded by stress increment $\Delta\sigma$**
- **Valve is initially closed**
- **As water is incompressible and valve is closed, no water is out, no movement of piston.**
- **Stress is ($\Delta\sigma$) is transferred to water.**
- **Pressure gauge reads an excess pore pressure ($\Delta u$) such that:**

  $\Delta u = \Delta\sigma$

  $u = u_o + \Delta u$

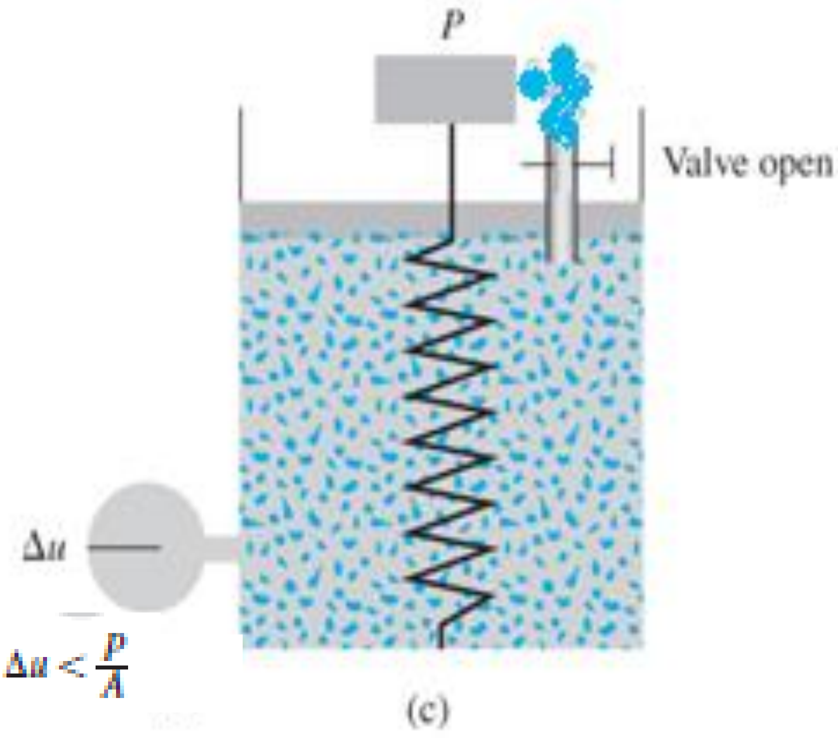
**From the principle of effective stresses:**

**$\Delta\sigma' = \Delta\sigma - \Delta u$ Then $\Delta\sigma' = 0$**

**No Settlement**

# Consolidation Process- Spring analogy

## iii. Under Load ($0 < t < \infty$)



- **To simulate fine grained cohesive soil, where permeability is slow, valve is slightly opened.**
- **Water slowly leave the chamber.**
- **As water flows out excess pore pressure ($\Delta u$) decreases, and load is transferred to the spring.**
- **Settlement is observed.**

**$\Delta u < \Delta\sigma$**

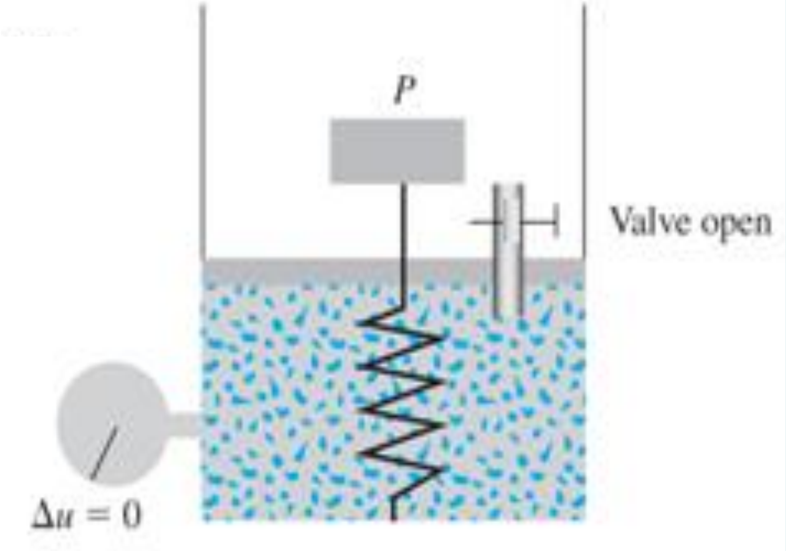
**$u = u_o + \Delta u$**

**From the principle of effective stresses:**

**$\Delta\sigma' = \Delta\sigma - \Delta u$ $\quad$ $\Delta u < \Delta\sigma$ $\quad$ Then $\Delta\sigma' > 0$**

# Consolidation Process- Spring analogy

iv. End of consolidation ($t = \infty$)



(d)

- **At the end of consolidation, no further water is squeezed out, excess pore pressure is zero.**
- **Pore water pressure is back to hydrostatic.**

$$\Delta u = 0$$
$$u = u_o$$

- **The spring (soil) is in equilibrium with applied stress.**
- **Final (ultimate) settlement is reached.**

**All stresses are transferred to soil**

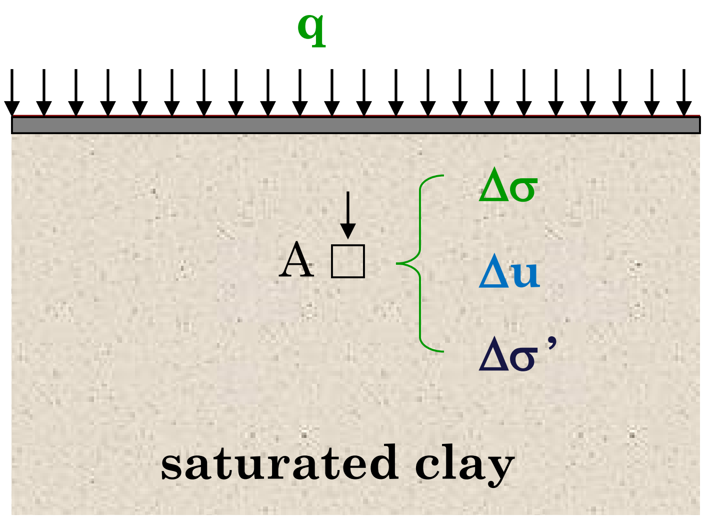
**From the principle of effective stresses:**

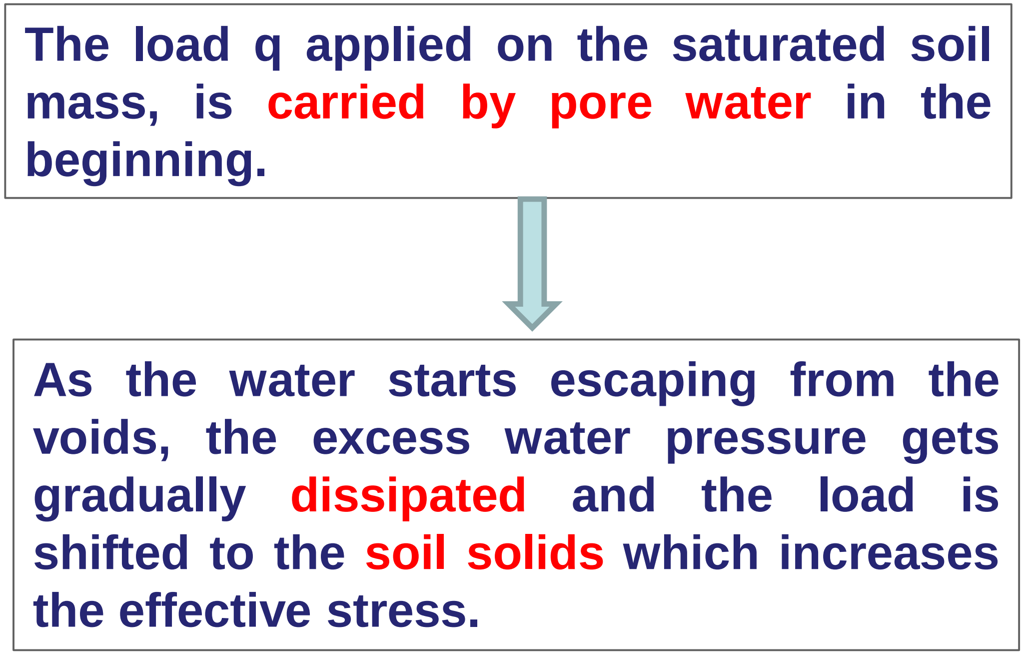
$\Delta\sigma' = \Delta\sigma - \Delta u$ $\Delta u = 0$ **Then** $\Delta\sigma' = \Delta\sigma$

# Short-term and Long-term Stresses

- ☐ With the spring analogy in mind, consider the case where a layer of saturated clay of thickness H that is confined between two layers of sand is being subjected to an instantaneous increase of total stress of $\Delta\sigma$.

- ☐ Due to a surcharge q applied at the GL, the stresses and pore pressures are increased at point A and, they vary with time.





The load q applied on the saturated soil mass, is carried by pore water in the beginning.

As the water starts escaping from the voids, the excess water pressure gets gradually dissipated and the load is shifted to the soil solids which increases the effective stress.

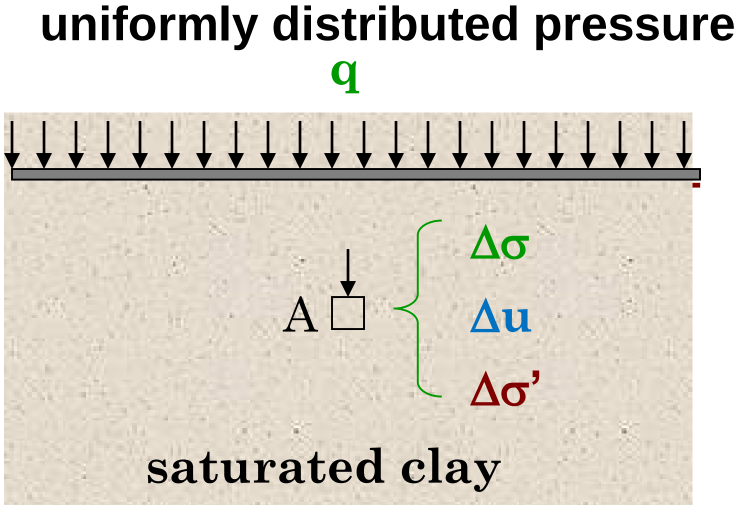
# Short-term and Long-term Stresses

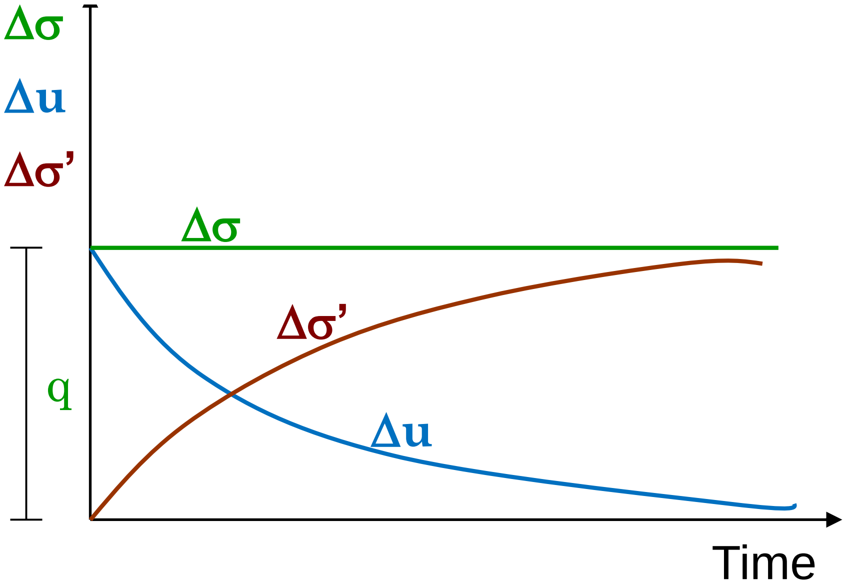
- ☐ $\Delta\sigma$, the increase in total stress remains the same during consolidation, while effective stress $\Delta\sigma'$ increases.
- ☐ $\Delta u$ the excess pore-water pressure decreases (due to drainage) transferring the load from water to the soil.

**Excess pore pressure ($\Delta u$)**

**is the difference between the current pore pressure (u) and the steady state pore pressure ($u_o$).**

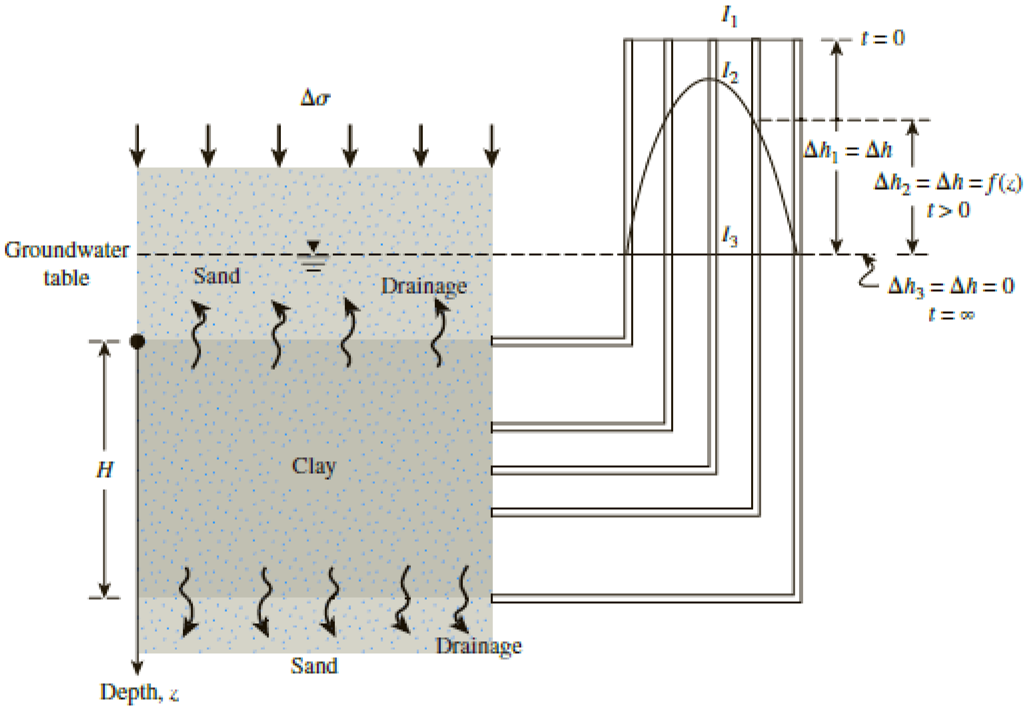
$$\Delta u = u - u_o$$



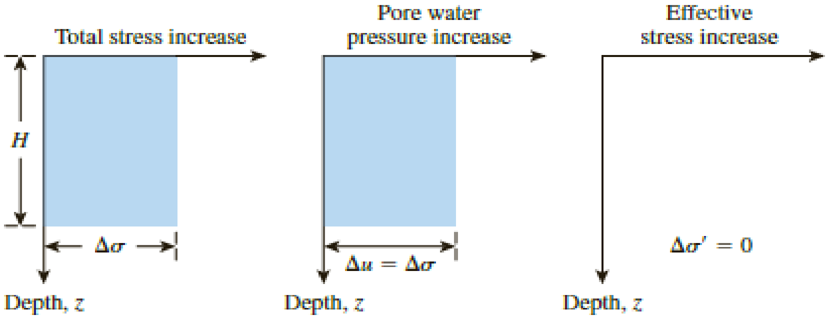


# Short-term and Long-term Stresses

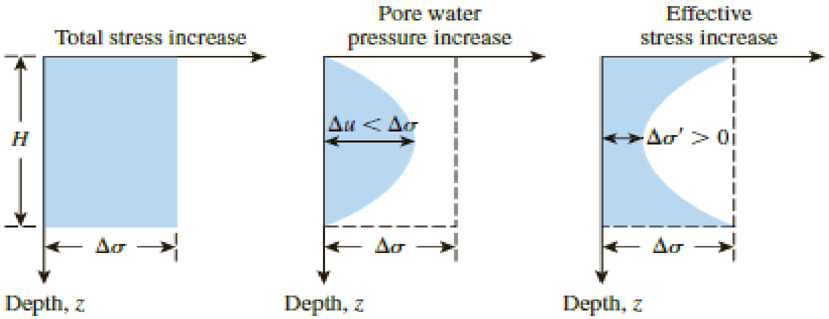
**Variation of total stress [σ], pore water pressure [*u*], and effective stress [σ′] in a clay layer drained at top and bottom as a result of an added stress, Δσ.**



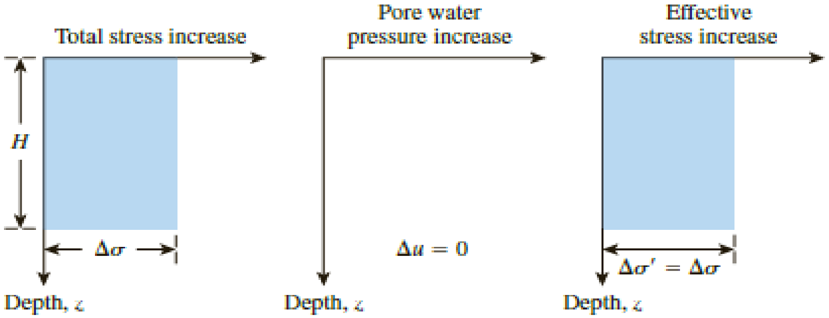
# Short-term and Long-term Stresses



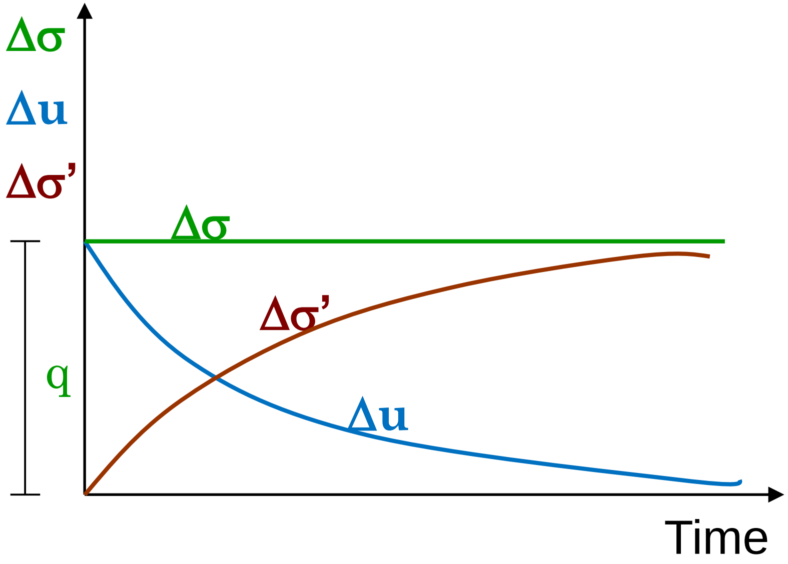
At time $t = 0$



At time $0 < t < \infty$



At time $t = \infty$



# Example

**The figure below shows how an extensive layer of fill will be placed on a certain site.**

**The unit weights are:**

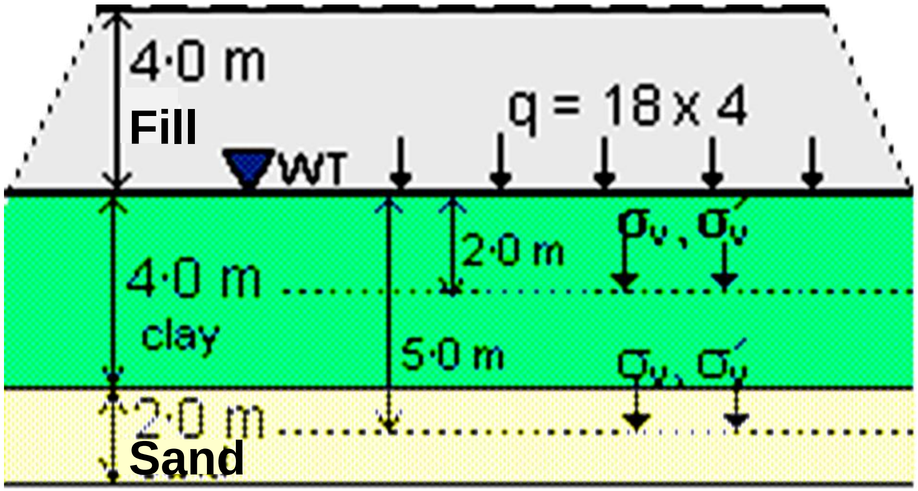
**Clay and sand = 20 kN/m³**
**Rolled fill =18 kN/m³**
**Water = 10 kN/m³**

**Calculate the total and effective stress at the mid-depth of the sand and the mid-depth of the clay for the following conditions:**

- (i) **Initially, before construction**
- (ii) **Immediately after construction**
- (iii) **Many years after construction**

Note: You know how to handle these cases from your background in CE382.

(we consider here the extreme cases with respect to loading time and the p.w.p is taken equal to the extended load).



# Solution

## (i) Initially, before construction

**Initial stresses at mid-depth of clay (z = 2.0m)**

Vertical total stress $\sigma_v = 20.0 \times 2.0 = 40.0$kPa

Pore pressure $u = 10 \times 2.0 = 20.0$ kPa

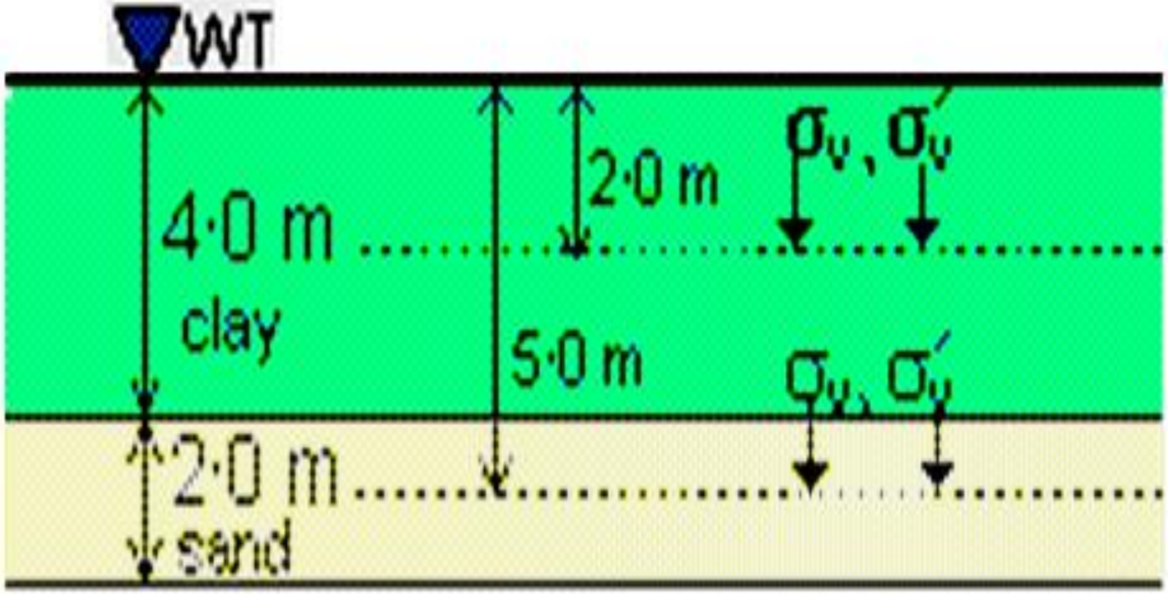
Vertical effective stress $\sigma'_v = \sigma_v - u = 20.0$kPa

**Initial stresses at mid-depth of sand (z = 5.0 m)**

Vertical total stress $\sigma_v = 20.0 \times 5.0 = 100.0$ kPa

Pore pressure $u = 10 \times 5.0 = 50.0$ kPa

Vertical effective stress $\sigma'_v = \sigma_v - u = 50.0$ kPa



# Solution

(ii) **Immediately after construction**

The construction of the embankment applies a surface surcharge:

$q = 18 \times 4 = 72.0$ kPa.

The sand is drained (either horizontally or into the rock below) and so there is no increase in pore pressure.

The clay is undrained and the pore pressure increases by 72 kPa.

Initial stresses at mid-depth of clay ($z = 2.0$m)

Vertical total stress $\sigma_v = 20.0 \times 2.0 + 72.0 = 112.0$kPa

Pore pressure $u = 10 \times 2.0 + 72.0 = 92.0$ kPa

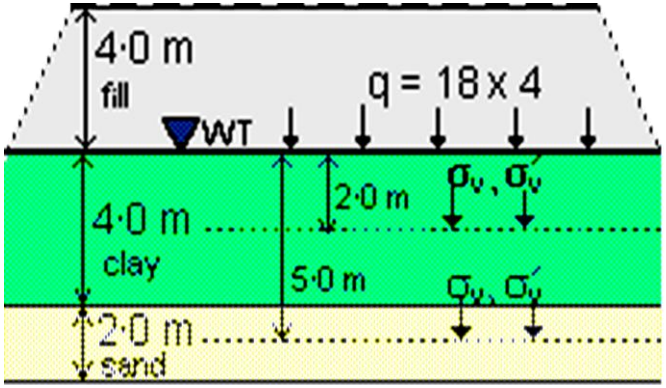
Vertical effective stress $\sigma'_v = \sigma_v - u = 20.0$kPa (i.e. no change immediately)

Initial stresses at mid-depth of sand ($z = 5.0$m)

Vertical total stress $\sigma_v = 20.0 \times 5.0 + 72.0 = 172.0$kPa

Pore pressure $u = 10 \times 5.0 = 50.0$ kPa

Vertical effective stress $\sigma'_v = \sigma_v - u = 122.0$kPa (i.e. an immediate increase)



# Solution

(iii) Many years after construction

After many years, the excess pore pressures in the clay will have dissipated. The pore pressures will now be the same as they were initially.

Initial stresses at mid-depth of clay (z = 2.0 m)

Vertical total stress $\sigma_v = 20.0 \times 2.0 + 72.0 = 112.0$ kPa

Pore pressure $u = 10 \times 2.0 = 20.0$ kPa

Vertical effective stress $\sigma'_v = \sigma_v - u = 92.0$ kPa (i.e. a long-term increase)

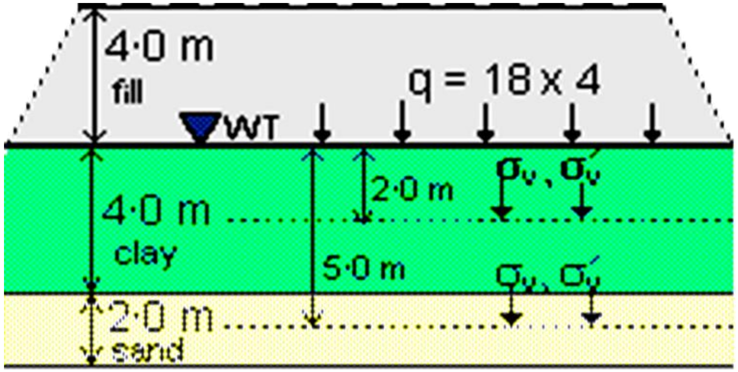
Initial stresses at mid-depth of sand (z = 5.0 m)

Vertical total stress $\sigma_v = 20.0 \times 5.0 + 72.0 = 172.0$ kPa

Pore pressure $u = 10 \times 5.0 = 50.0$ kPa

Vertical effective stress $\sigma'_v = \sigma_v - u = 122.0$ kPa (i.e. no further change)

This gradual process of drainage under an additional load application and the associated transfer of excess pore water pressure to effective stress cause the time-dependent settlement in the clay soil layer. This is called consolidation.



# The end