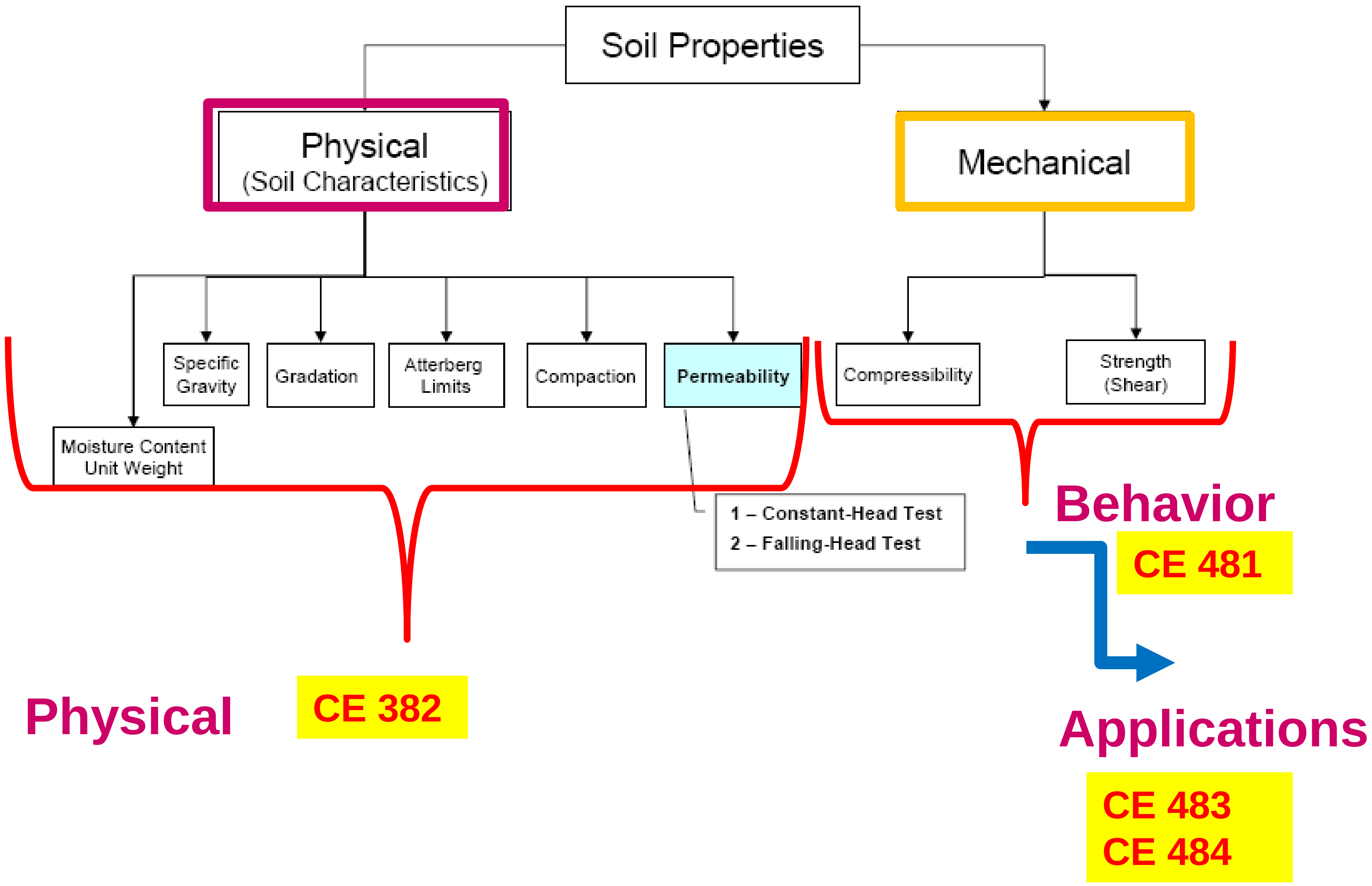
Chapter 7
PERMEABILITY

Omitted Topic

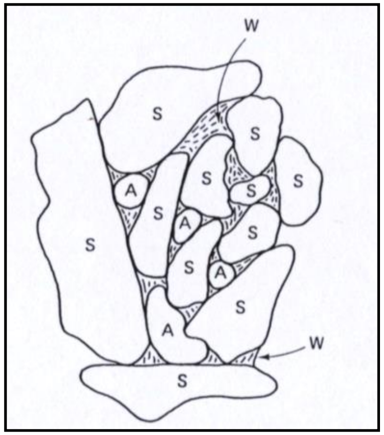
Section 7.6,7.7,7.11,7.12,7.13

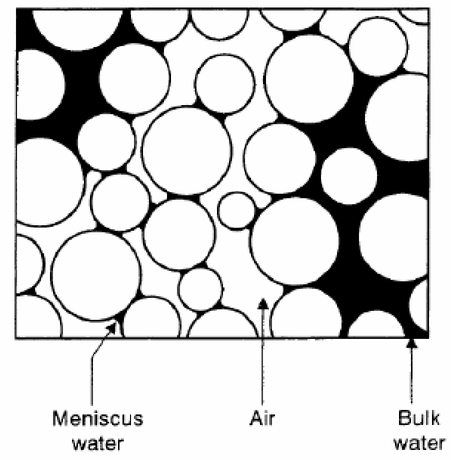
PERMEABILITY



PERMEABILITY

- Soils are permeable due to the existence of interconnected voids.





- One of the most important consideration in soil mechanics is the effects of water in the soil on its engineering properties, and hence behavior.

- Most of geotechnical engineering problems somehow have water associated with them in various ways.

PERMEABILITY

- Permeability is one of the most important soil properties of interest to geotechnical engineering.

- The following applications illustrate the importance of permeability in geotechnical design:

  - Permeability influences the rate of settlement of a saturated soil under load.

  - The design of earth dams is very much based upon the permeability of the soils used. Filters made of soils are designed based upon their permeability.

  - The stability of slopes and retaining walls can be greatly affected by the permeability of the soils

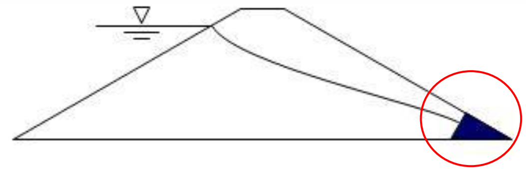
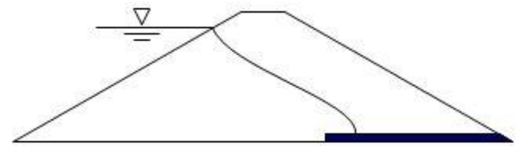
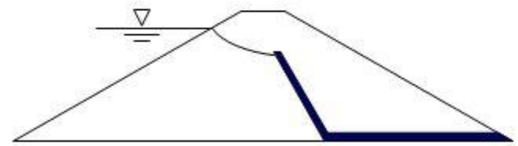
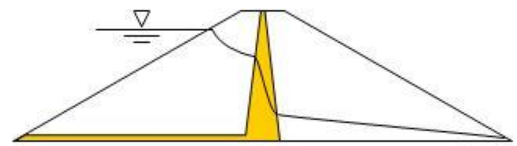
  - Permeability of soils is required in solving pumping seepage water from construction excavations.

PERMEABILITY

PERMEABILITY

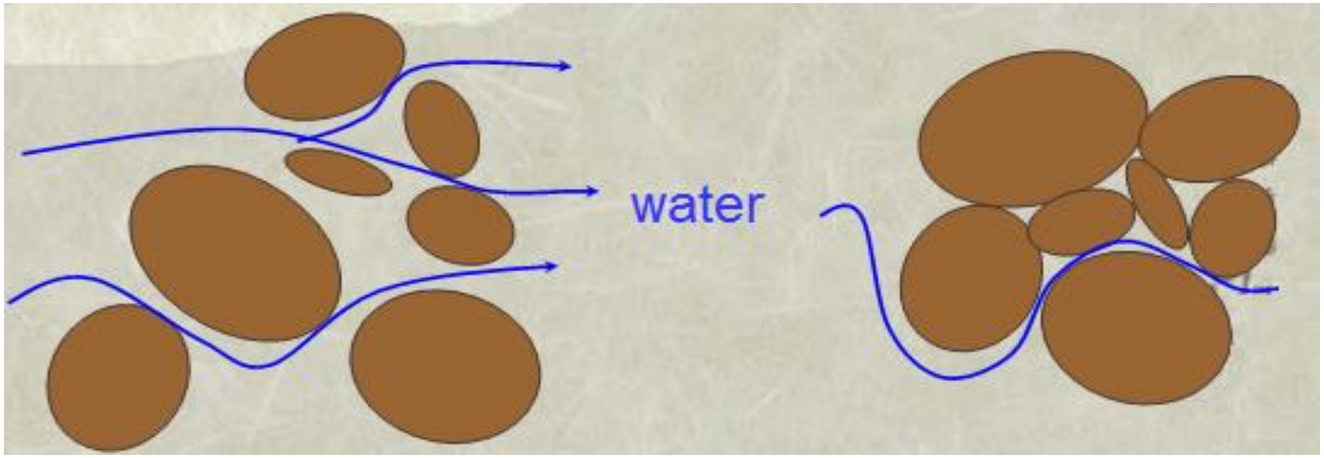
Reservoirs

Dam design: Seepage: Earth dams

Toe drain	
Blanket drain	
Chimney and blanket drain	
Impermeable core and blanket	

DEFINITION OF PERMEABILITY

- **Permeability is a measure of a given porous medium ability to permit fluid flow through its voids.**
- **Any material with voids is Porous and if the voids are interconnected, possesses permeability.**
- **Therefore, rock, concrete, soil, and many engineering materials are both POROUS and PERMEABLE. However, among them soils, even in their densest state, are more permeable.**



FLOW THROUGH POROUS MEDIA

- Fluid flow can be described or classified in different ways like:

  - Steady vs. Unsteady
  - Laminar vs. Turbulent
  - 1-D vs. 3D

- In our discussion in this chapter we will assume that the flow is laminar. This really is the case in most soils.

- Whether the flow is steady or not and the number of dimensions we consider, this will be decided when we present SEEPAGE in the following chapter.

FLOW THROUGH POROUS MEDIA

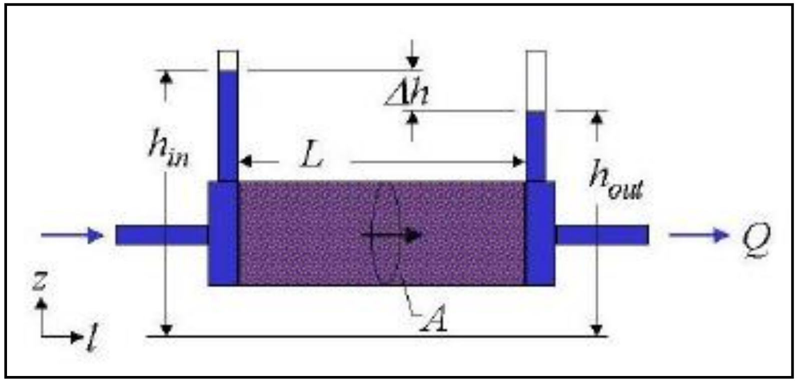
- **The soil is regarded as <u>RIGID</u> and stationary with a steady flow of water through the pore spaces.**

<u>REQUIREMENTS FOR STEADY STATE FLOW</u>

- **The soil is fully saturated**
- **The pressure (and hence total) gradient is unchanging**
- **Soil mass is constant**
- **Flow rate is constant**

DARCY'S LAW

- A French engineer named Darcy (1856) noticed that the velocity of the drainage drinking water flowing to a village named Dijon was:
  - Proportional to the difference in elevation between the water's entry point and its discharge point.
  - Inversely proportional to the distance over which the change in elevation occurred.



$$v \alpha \frac{\Delta h}{L} \quad \text{.........(1)}$$

Where:

v = discharge velocity, which is the quantity of water flowing in unit time through a unit GROUS cross-sectional area of soil at right angles to the direction of flows.

Δh = loss of head between two points.

L = Distance over which the change or loss of head occurs.

DARCY'S LAW

- The ratio $\frac{\Delta h}{L}$ is termed the HYDRAULIC GRADIENT, and is denoted by the symbol i. Therefore, Eq. 1 becomes:

$$v \,\alpha\, i$$

- Darcy' introduced a constant of proportionality called the Darcy Coefficient of Permeability, k and Eq. 2 becomes:

$$v = k\, i.$$

- Commonly in civil engineering k is called simply hydraulic conductivity or the coefficient of permeability or, even more simply, the Permeability.

- Eq. 3 is called Darcy's law. It was primarily based on the observations made by Darcy for flow of water through clean sands.

DARCY'S LAW

Assumptions in Deriving Darcy's Law

I. Soils

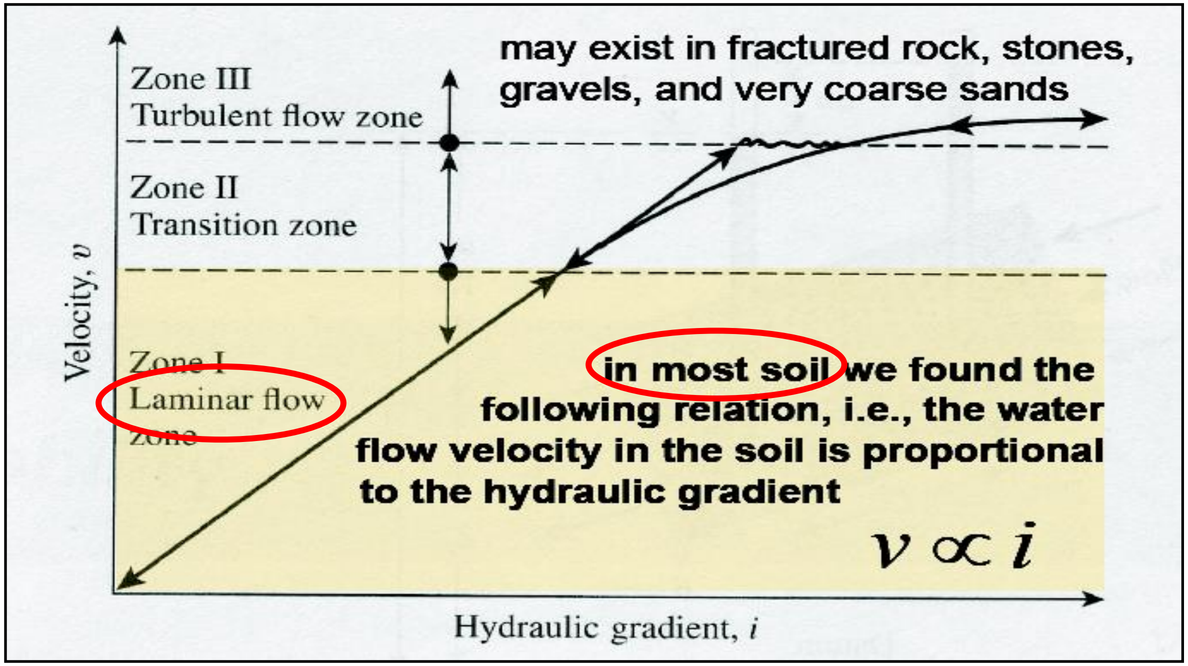
- ❑ Homogeneous & isotropic
- ❑ Fully saturated

II. Flow

- ❑ The flow is laminar, no turbulent flows
- ❑ The flow is in steady state, no temporal variation

DARCY'S LAW

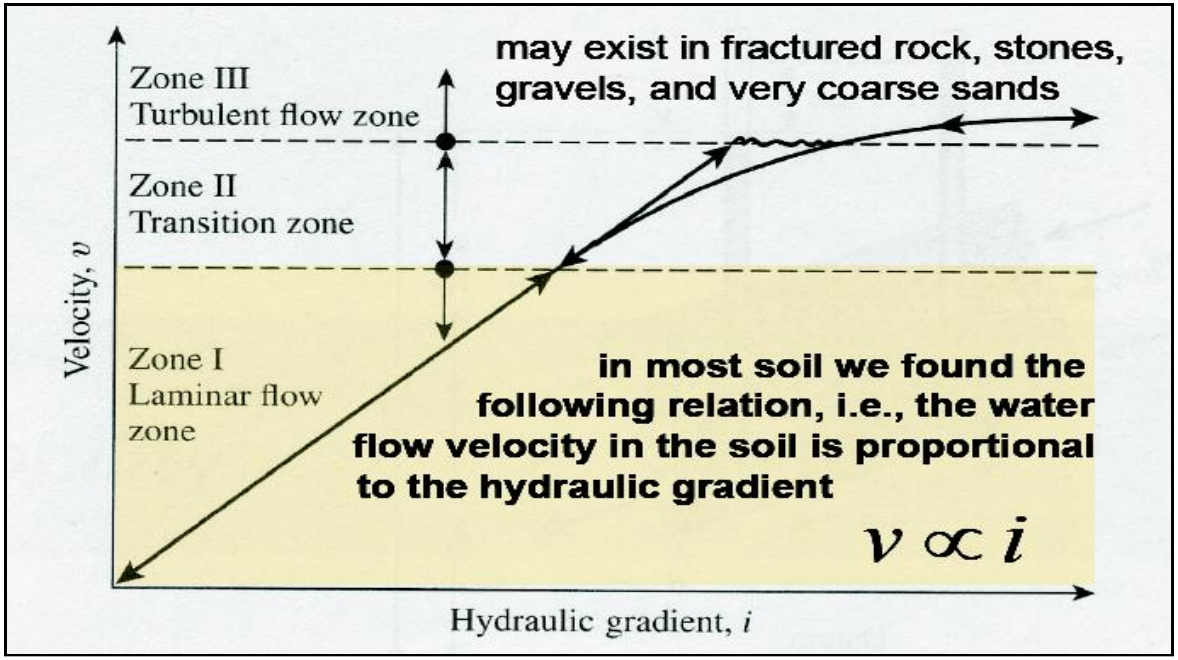
Variation of Velocity with Hydraulic Gradient



- When the hydraulic gradient is increased gradually, the flow remains **laminar** in **Zones I** and **II**, and the velocity, **v**, bears a linear relationship to the hydraulic gradient **i**.
- At a higher hydraulic gradient, the flow becomes **turbulent** (Zone III). When the hydraulic gradient is decreased, laminar flow conditions exist only in Zone I.

DARCY'S LAW

Variation of Velocity with Hydraulic Gradient

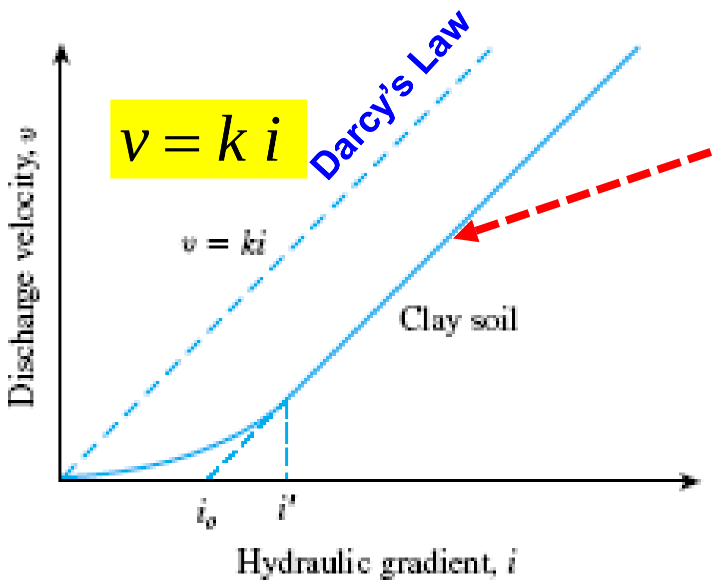


- At high gradient the flow will be turbulent and the relationship between v and I will not be linear. Hence Eq. 2 may not be valid.

- This is the case in gravels and very coarse sands. However, in most soil (mixture) the flow of water through the voids can be considered LAMINAR and Eq. 2 is valid.

DARCY'S LAW

Is Darcy's law valid for very low hydraulic gradients?



$$v = k(i - i_0) \qquad (\text{for } i \geq i')$$

$$v = ki^m \qquad (\text{for } i < i')$$

Figure 7.4
Variation of discharge velocity with hydraulic gradient in clay

These equations imply that for very low hydraulic gradients, the relationship between v and *i* is nonlinear (only study by Hansbo, 1960).

Several other studies refute the preceding findings. Mitchell (1976) discussed these studies in detail. Taking all points into consideration, he concluded that Darcy's law is valid.

Conclusion:

Darcy's law is valid even for very low hydraulic gradients

DARCY'S LAW

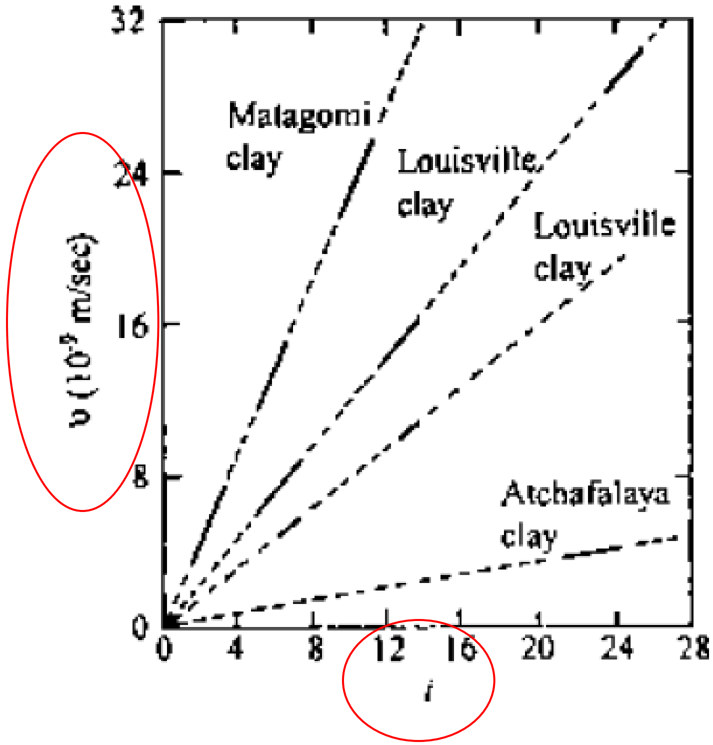


Figure 5.3 Discharge velocity-gradient relationship for four clays (after Tavenas *et al.*, 1983b).

Conclusion:

- **In most soil (mixture) the flow of water through the voids can be considered LAMINAR Darcy's law is valid.**
- **Darcy's law is valid even for very low hydraulic gradients**

FLOW RATE

FLOW RATE (Flux)

- If A is the cross-sectional area through which flow is occurring, the flow rate, q, is determined by:

$$q = vA$$

or

$$q = kiA \quad \text{........(4)}$$

QUANTITY OF FLOW

- If flow occurs over a period of time t, the total quantity of water Q flowing during this period can be found as:

$$Q = qt$$

or

$$Q = kiAt \quad \text{..................(5)}$$

All terms in Eq. (5) are easily measured or determined except **k**. Next we will discuss techniques in evaluating **k**. But first we will address some important concepts regarding **hydraulic gradient.**

FLOW RATE

To determine the quantity of flow, two parameters are needed:

* ***k*** **=hydraulic conductivity (how permeable the soil medium?)**
*** *i* = hydraulic gradient (how large is the driving head?)**

i **can be determined**

1. **From the head loss and geometry….. (1-D case)**
2. **Flow net (next chapter)……….(2D case)**

k can be determined using:

1. **Laboratory Testing**
2. **Field Testing**
3. **Empirical Equations**

Heads & Hydraulic Gradient

Bernoulli's Equation

- **According to Bernoulli's equation, the total energy available, described as a measurable distance (called head) above a reference datum is:**

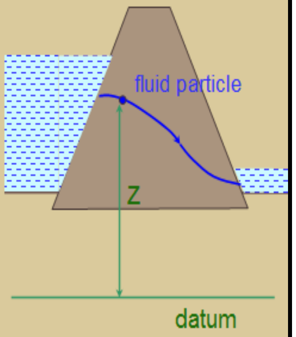
$$h = \frac{u}{\gamma_w} + \frac{v^2}{2g} + Z$$

↑ Pressure head ↑ Velocity head ↑ Elevation head

where h = total head
u = pressure
v = velocity
g = acceleration due to gravity
γ_w = unit weight of water

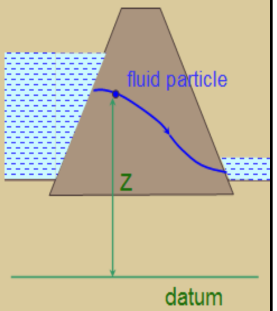
The energy of a fluid particle is made of:

1. Kinetic energy
   - due to **velocity**
2. Strain energy
   - due to **pressure**
3. Potential energy
   - due to **elevation** (z) with respect to a datum



Expressing energy in unit of length:

Total head = Velocity head + Pressure head + Elevation head

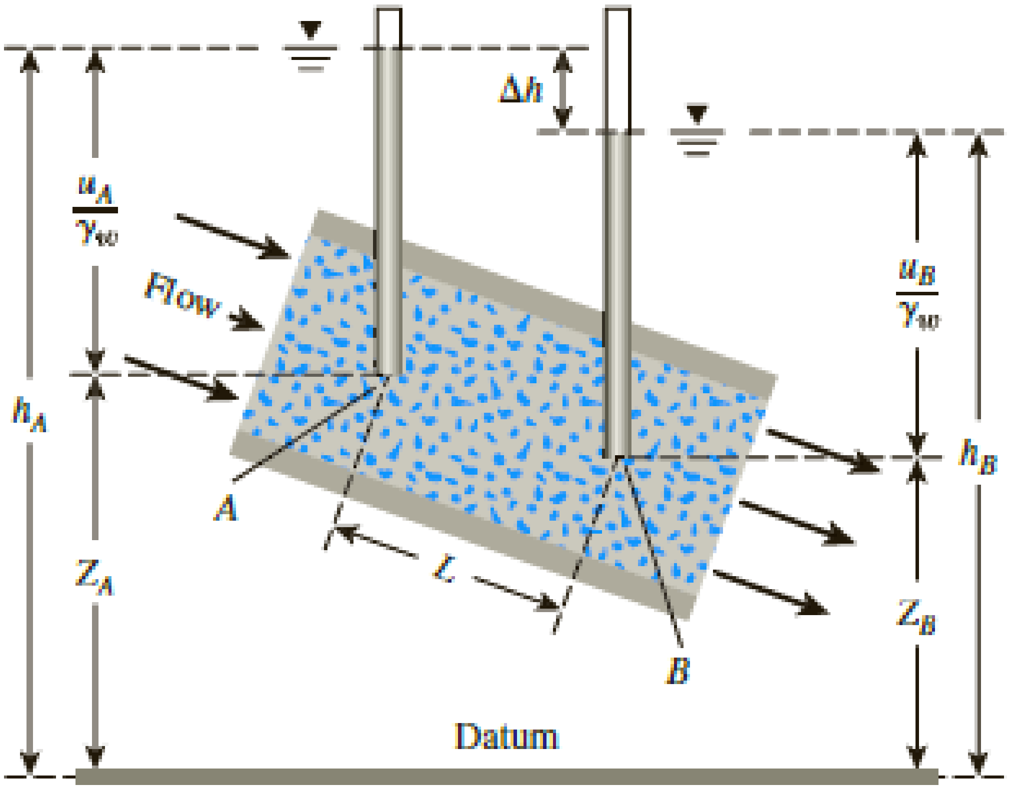


Heads

- In soils seepage velocity is normally very small (further if it is squared) that VELOCITY HEAD can be neglected. Therefore, the total head at any point is given as:

$$h = \frac{u}{\gamma_w} + Z$$

The pressure head at a point can be measured by inserting a PIEZOMETER TUBE into the pipe. The level will rise to a level representing the current pressure head at the that point. (If the flow is steady and the velocity head =0)



Piezometric Levels or Heads

- **The levels to which water rises in the piezometer tubes situated at points A and B are know as the piezometric levels of points A and B, respectively.**

- **The loss of head between point A and B is given by:**

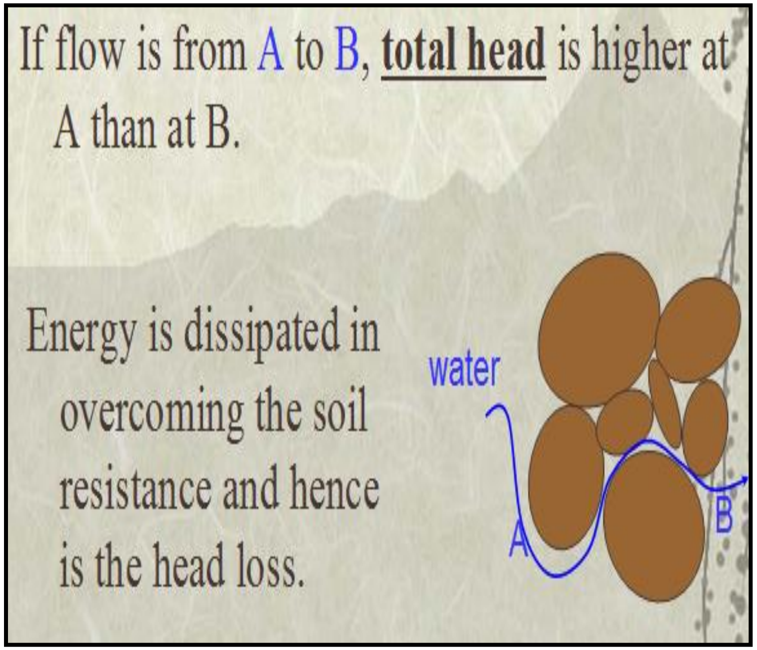
$$\Delta h = h_A - h_B = \left(\frac{u_A}{\gamma_w} + Z_A\right) - \left(\frac{u_B}{\gamma_w} + Z_B\right)$$

$$\Delta h = Z_A - Z_B + \left(\frac{u_A - u_B}{\gamma_w}\right)$$

- **The hydraulic gradient (or head loss) is as defined before expressed as:**

$$i = \frac{\Delta h}{L}$$

- **It is the SLOPE of the ENERGY LINE defined by the free surface of flowing water in OPEN CHANNELS or the slope of the PIEZOMETRIC HEADS (PIEZOMETRIC LEVELS) between two points in CONFINED FLOW.**



REMARKS

- A standpipe referred to as a PIEZOMETER is used for measuring pressure. It operates by converting pressure head to the more readily measurable elevation head.

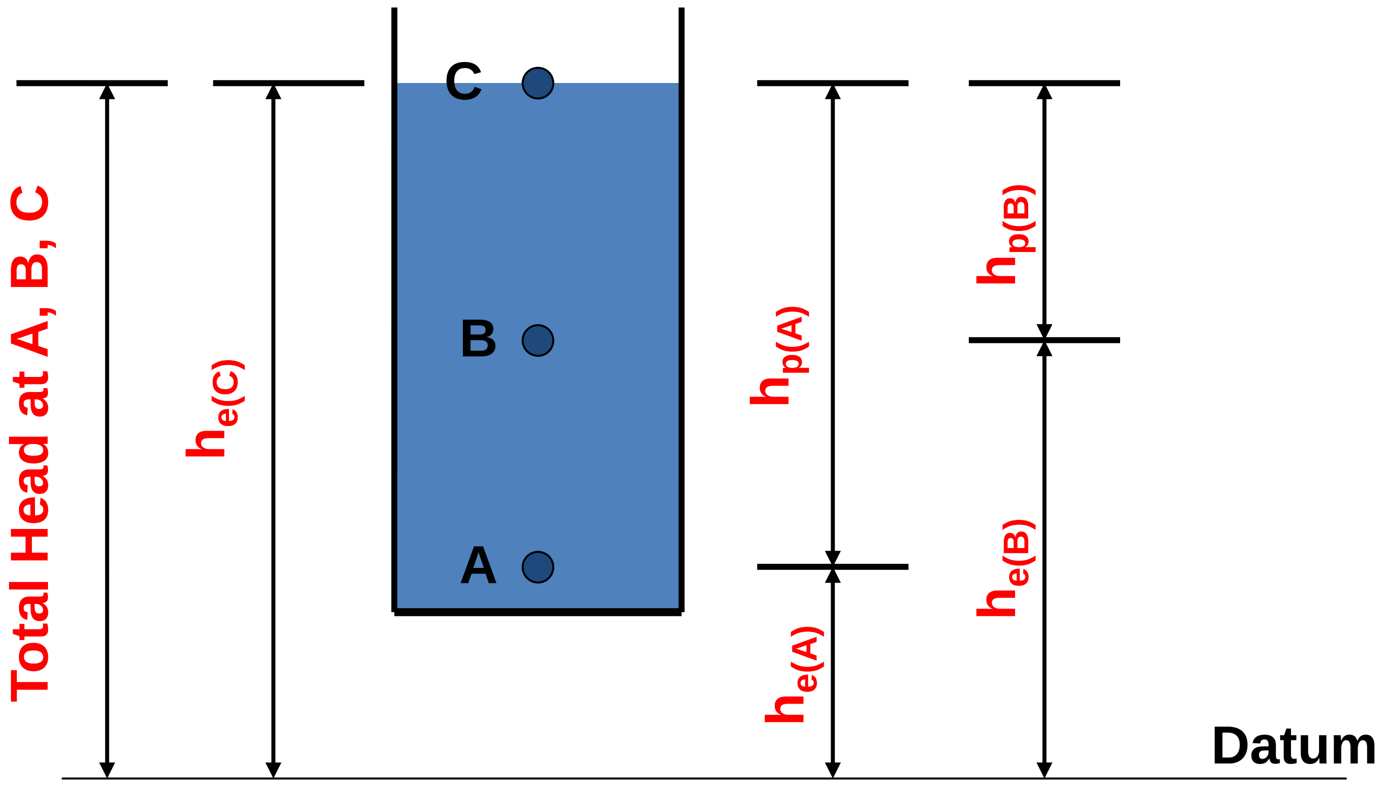
- The quantity (h_p+h_e) is called the piezometric head , piezometric level, or total head since it is the head that would be measured by a piezometer referenced to some datum plane.

- The elevation of the water column in the standpipe is the TOTAL HEAD (h_p +h_e), whereas the actual height of rise of the water column in the standpipe is the PRESSURE HEAD, h_p.

- Elevation head at a point = Extent of that point from the datum.

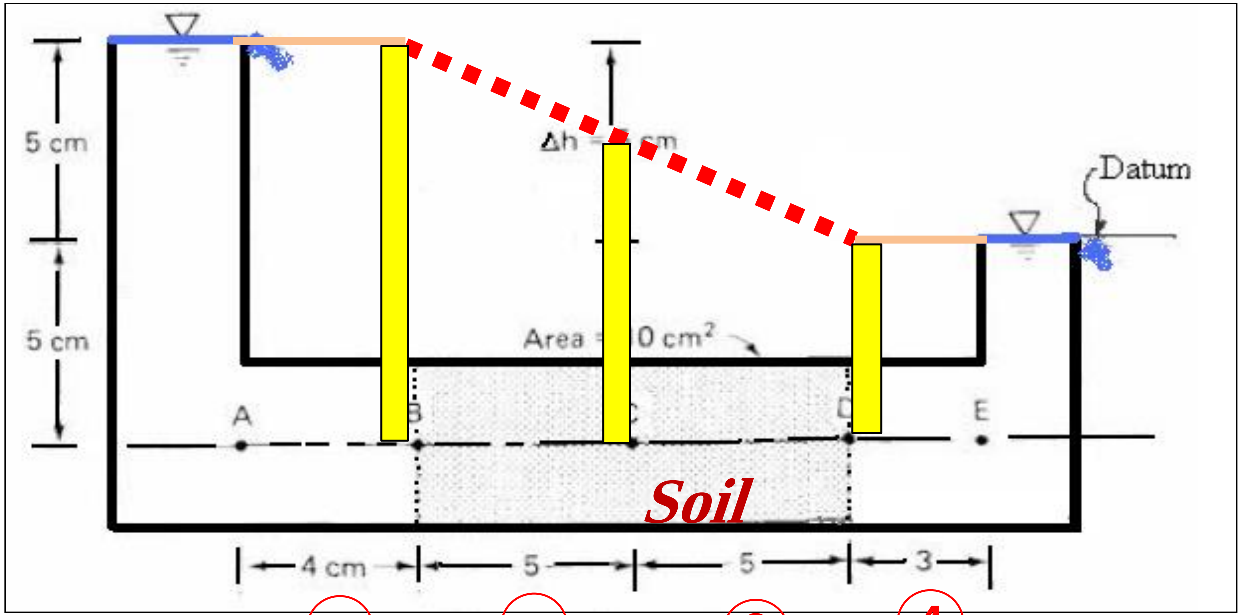
- It is most often convenient to establish the datum plane at the tail water elevation.

Example 1



Remember always to look at <u>total head</u>

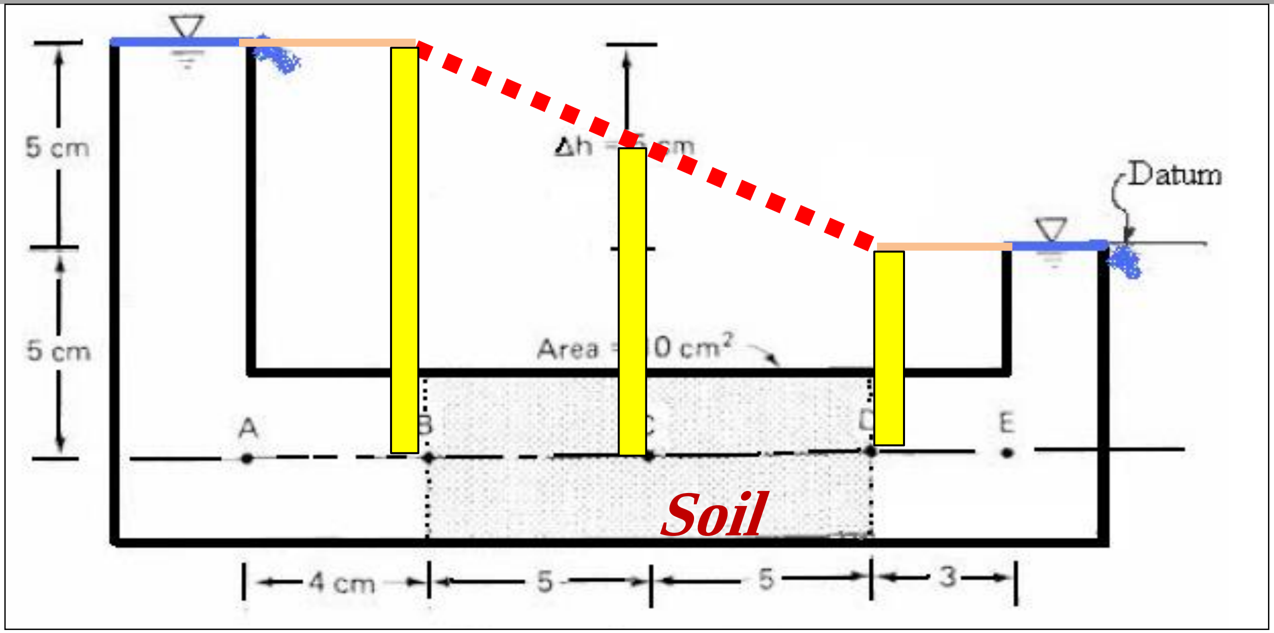
Example 2



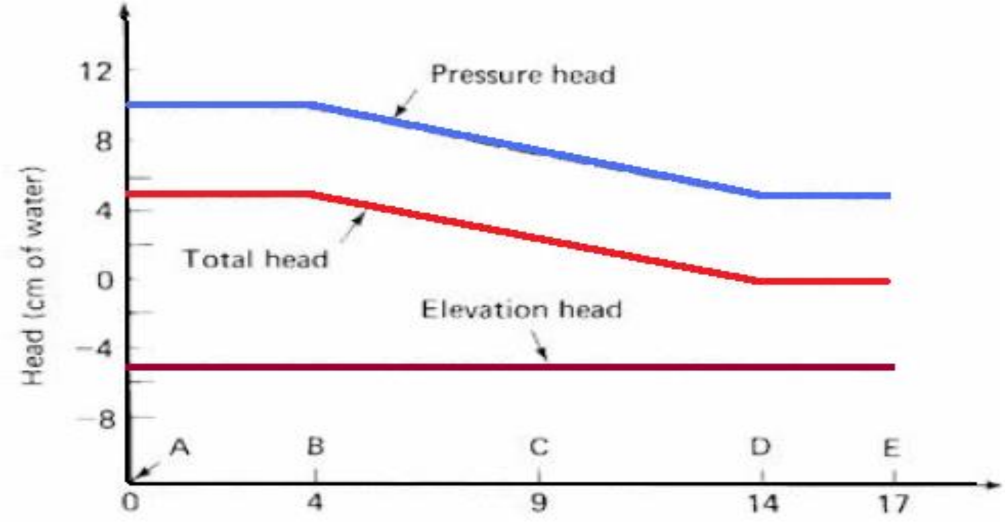
$h = h_p + h_e$

Point	Pressure Head ③	Elevation Head ①	Total Head ②	Head Loss ④
A	10	−5	5	0
B	10	−5	5	0
C	$7\frac{1}{2}$	−5	$2\frac{1}{2}$	$2\frac{1}{2}$
D	5	−5	0	5
E	5	−5	0	5

Example 2



5 cm
5 cm
Δh = 5 cm
Datum
Area = 10 cm²
A
B
C
D
E
Soil
4 cm
5
5
3



Pressure head
Total head
Elevation head
Head (cm of water)
12
8
4
0
−4
−8
A
B
C
D
E
0
4
9
14
17

Example (2nd midterm exam Fall 38-39)

For the case shown in Fig.1 below,

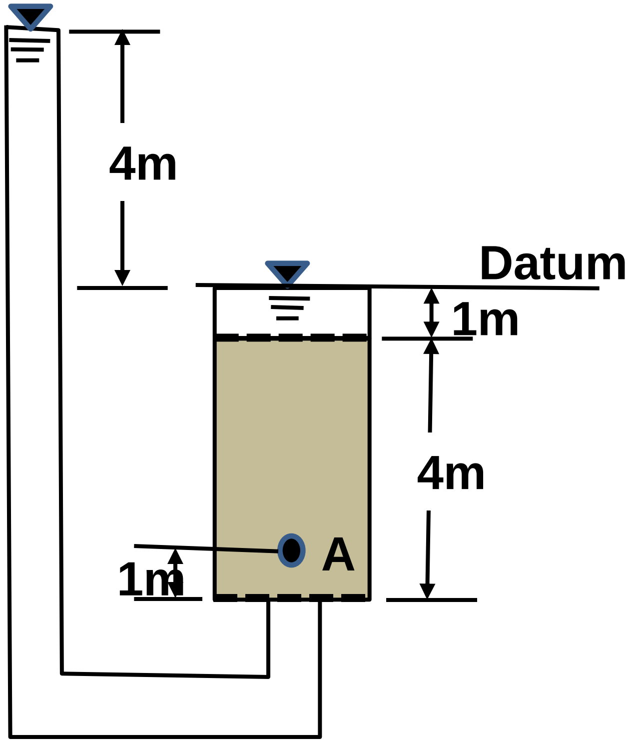
i. Determine the pressure, elevation, and total head at the entering end, exit end, and point A of the sample. Include steps of your solution.
ii. Discharge and seepage velocity for a permeability of 0.1 cm/s and a porosity of 50%.

Start with elevation
Find total head and then pressure head

Or
Find pressure head and then total head

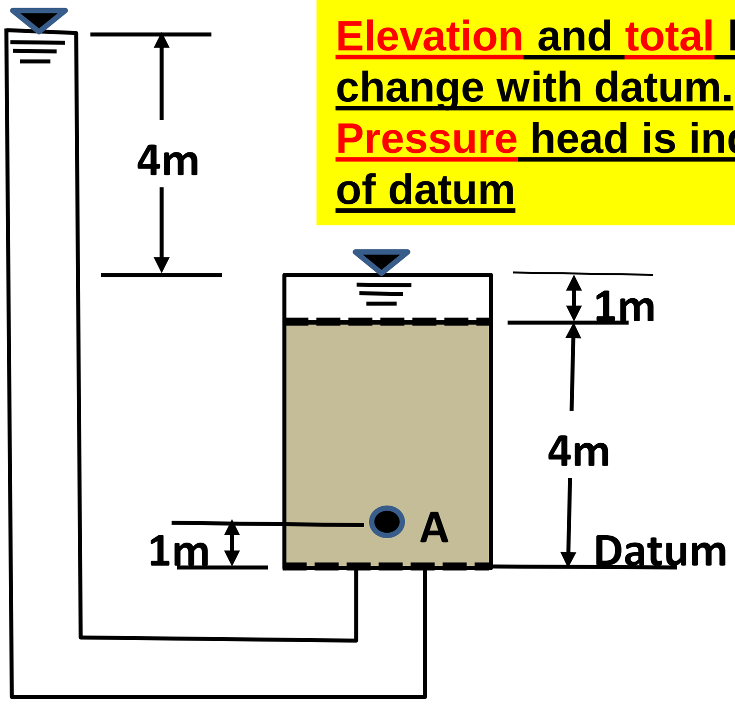
	Elevation Head	Pressure Head	Total Head
Entering Point	-5	9	4
Point A	-4	7	3
Exit Point	-1	1	0

By interpolation

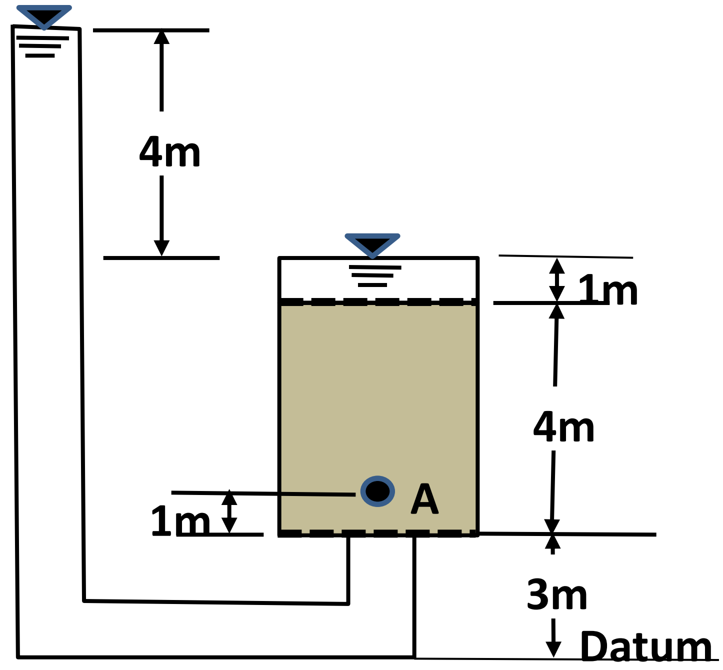


Heads

Elevation and total heads change with datum.
Pressure head is independent of datum

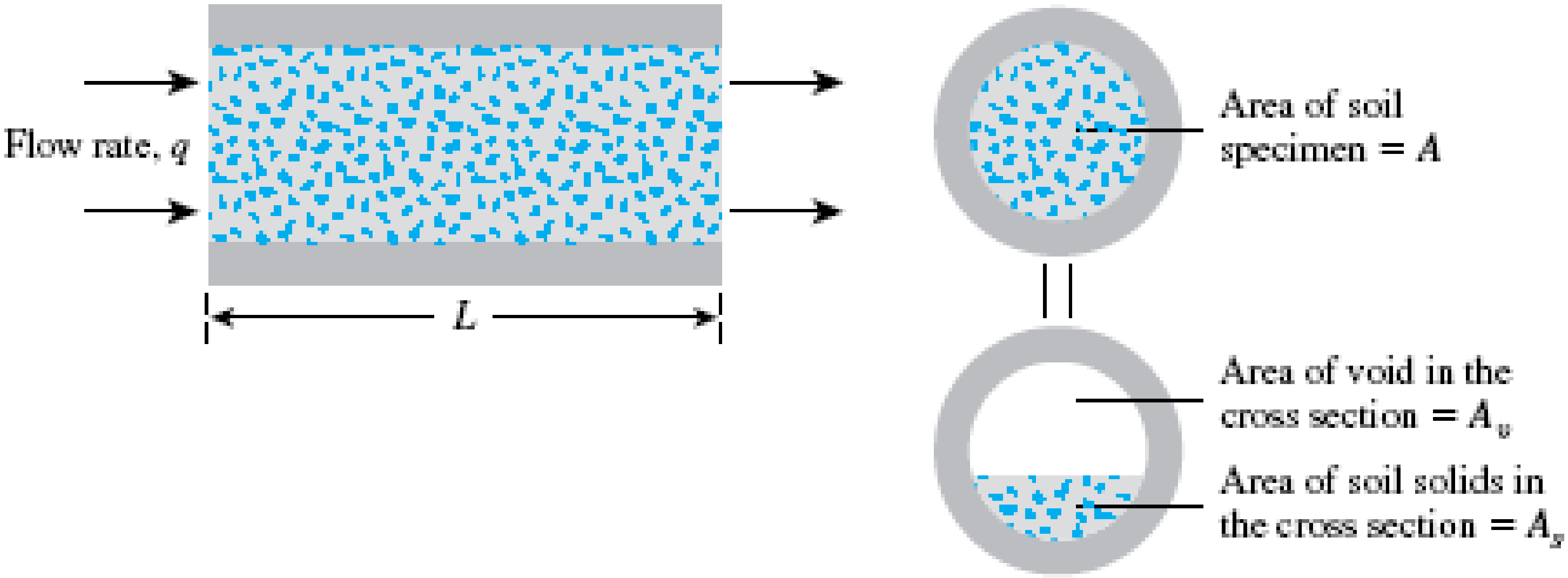


	Elevation Head	Pressure Head	Total Head
Entering Point	0	9	9
Point A	1	7	8
Exit Point	4	1	5



	Elevation Head	Pressure Head	Total Head
Entering Point	3	9	12
Point A	4	7	11
Exit Point	7	1	8

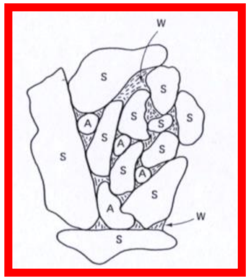
DISCHARGE AND SEEPAGE VELOCITIES



DISCHARGE AND SEEPAGE VELOCITIES

- The discharge **(apparent)** seepage velocity of water based on the gross cross-sectional area of the soil, or

$$v = \frac{q}{A}$$



- However, water cannot be flowing through solid particles but only through the voids or pores between the grains.

- The **average** velocity at which the water flows through the soil pores is obtained by:

$$v_s = \frac{q}{A_v}$$

v_s is called the **SEEPAGE VELOCITY**

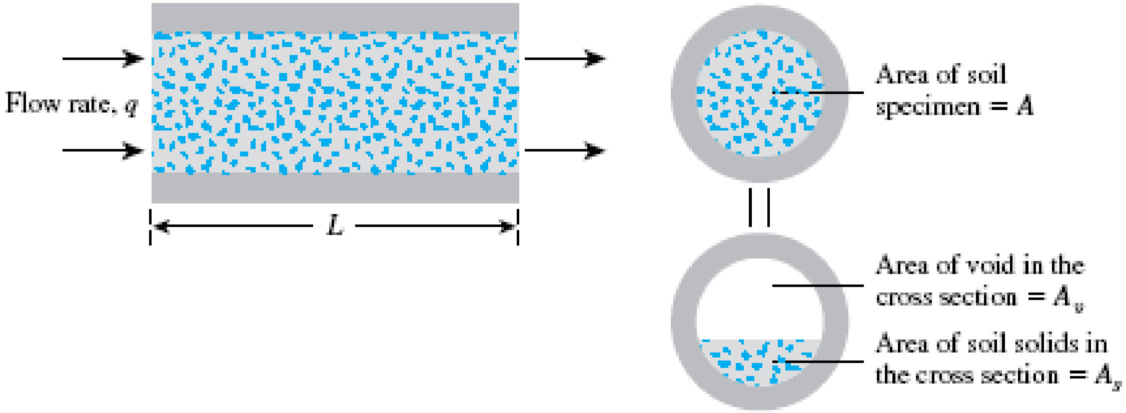
DISCHARGE AND SEEPAGE VELOCITIES

- **From the law of conservation of mass (for incompressive steady state flow, this low reduces to the EQUATION OF CONTINUITY), we get :**

$$q = Av = A_v v_s$$

Therefore

$$v_s = \frac{A}{A_v} v$$



But with the 3rd dimension

$$\frac{A}{A_v} = \frac{V}{V_v} = \frac{1}{n}$$

Hence

$$v_s = \frac{v}{n}$$

$$v_s = \left(\frac{1+e}{e}\right) v$$

REMARKS

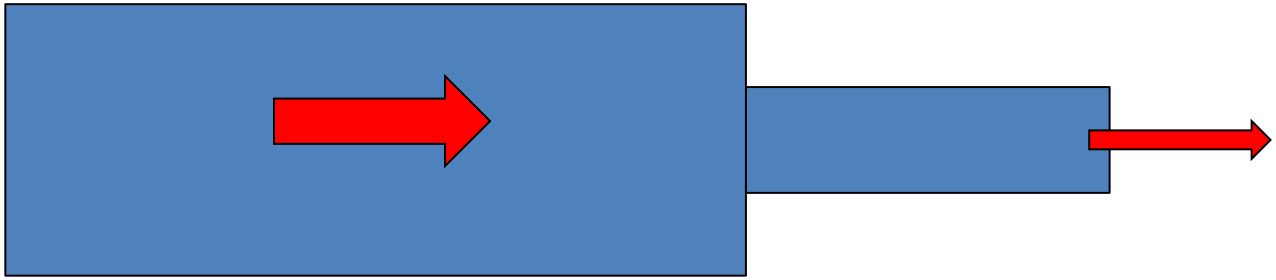
$$v_s = \frac{v}{n}$$

- Since 0%< n < 100%, it follows that seepage velocity is always **greater** than the discharge (**superficial**) velocity.

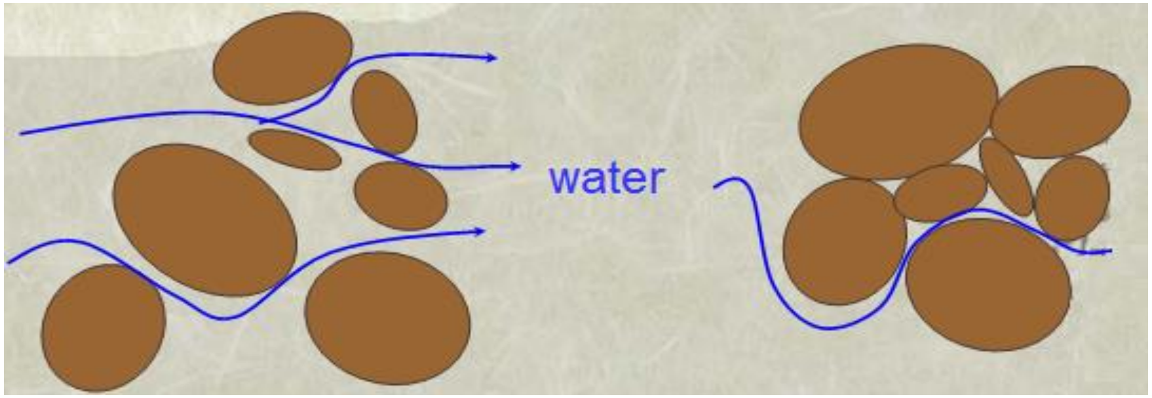
- ν is also called the **apparent seepage velocity**. It is a **superficial, fiction** but convenient engineering velocity. (Also called **velocity of approach**).

- $\nu_s > \nu$ not only because $A_v < A$ but also because the flow is not straight – line, and must follow **TORTUOUS** paths around the grains. (**Recall the continuity equation**).

REMARKS



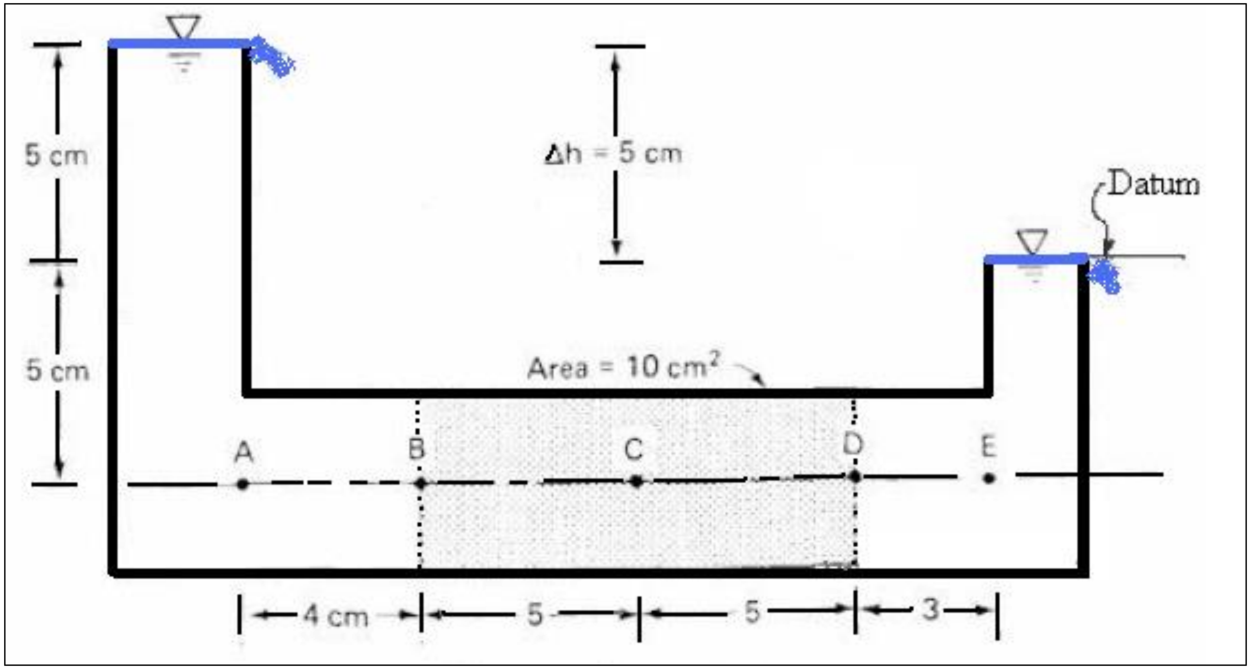
- $\nu_s > \nu$ not only because $A_v < A$ but also because the flow is not straight –line, and must follow TORTUOUS paths around the grains. (Recall the continuity equation).



Example 3

If the horizontal cylinder of soil shown below has a coefficient of permeability of 0.01 cm/sec and a void ratio of 0.70. Determine: -

i. **The amount of flow through the soil per hour**
ii. **The pore water pressure in kN/m^2 at points B, C, and D**
iii. **The discharge velocity**
iv. **The seepage velocity**



Example 3

I. $q = vA = kiA = 0.01X(5/10)X10 = 0.05$ cm³/s = **180 cm³/hr**

II.

$$h = \frac{u}{\gamma_w} + Z$$

$0.05 = u_{(B)}/9.81 - 0.05$

$u_{(B)} = 0.1 \text{ X } 9.81 =$ **0.981 kN/m².**

$0.025 = u_{(C)}/9.81 - 0.05$

$u_{(C)} = 0.075 \text{ X } 9.81 =$ **0.736 kN/m²**

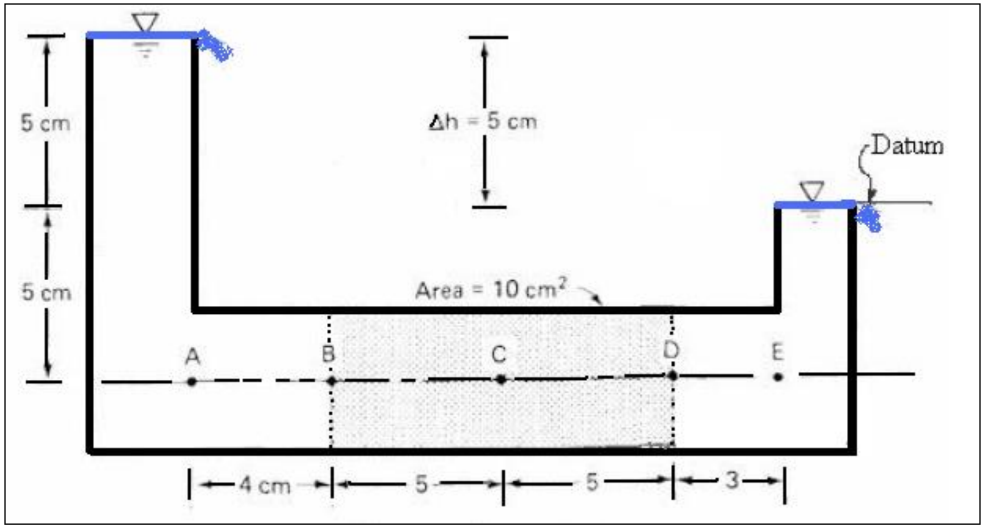
$0 = u_{(D)}/9.81 - 0.05$

$u_{(D)} = 0.05 \text{ X } 9.81 =$ **0.491 kN/m²**

III. $v = k\,I = 0.01 \text{ X } 5/10 =$ **0.005 cm/sec**

IV. $v_s = v\,(1+e)/e = 0.005\,(1+0.7)/0.7$

= **0.012 cm/sec**



Point	Pressure Head	Elevation Head	Total Head	Head Loss
A	10	−5	5	0
B	10	−5	5	0
C	7½	−5	2½	2½
D	5	−5	0	5
E	5	−5	0	5

Example 7.5

Example 7.5

A permeable soil layer is underlain by an impervious layer, as shown in Figure 7.8a. With $k = 5.3 \times 10^{-5}$ m/sec for the permeable layer, calculate the rate of seepage through it in m³/hr/m width if $H = 3$ m and $\alpha = 8°$.

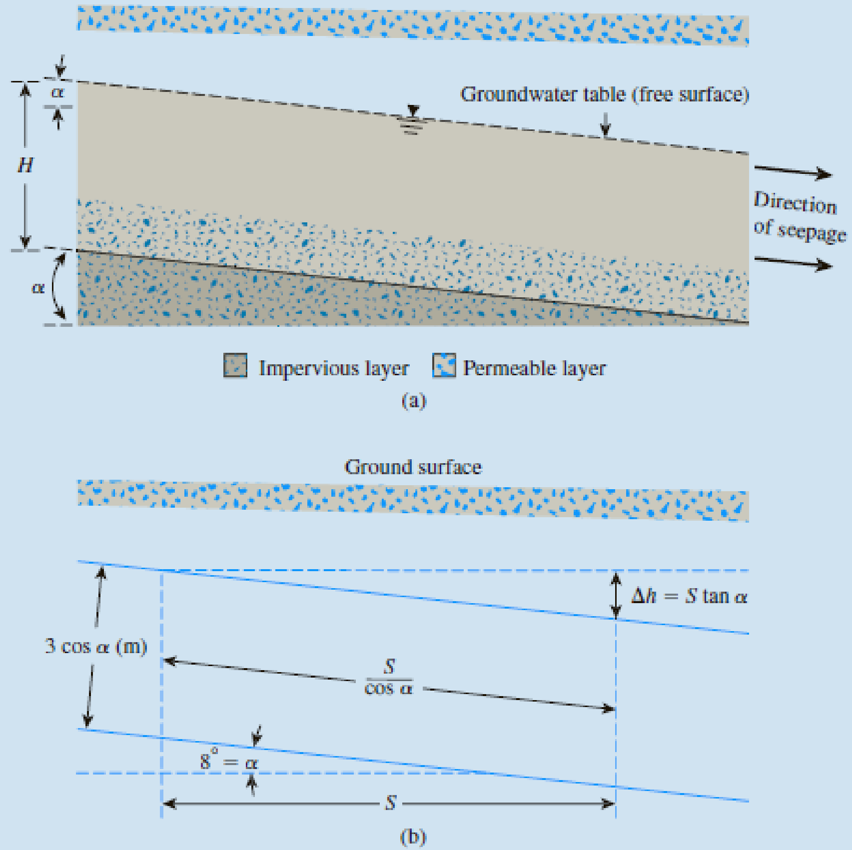


Figure 7.8

Example 7.5

Solution

From Figure 7.8b,

$$i = \frac{\text{head loss}}{\text{length}} = \frac{S \tan \alpha}{\left(\frac{S}{\cos \alpha}\right)} = \sin \alpha$$

$$q = kiA = (k)(\sin \alpha)(3 \cos \alpha)(1)$$

$$k = 5.3 \times 10^{-5} \text{ m/sec}$$

$$q = (5.3 \times 10^{-5})(\sin 8°)(3 \cos 8°)(3600) = \mathbf{0.0789\ m^3/hr/m}$$

↑ To change to m/hr

Example 7.6

Example 7.6

Find the flow rate in m^3/sec/m length (at right angles to the cross section shown) through the permeable soil layer shown in Figure 7.9 given $H = 8$ m, $H_1 = 3$ m, $h = 4$ m, $S = 50$ m, $\alpha = 8°$, and $k = 0.08$ cm/sec.

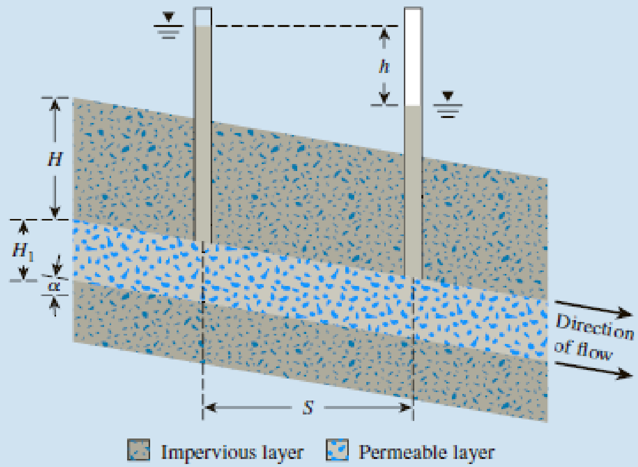


Figure 7.9 Flow through permeable layer

Solution

$$\text{Hydraulic gradient } (i) = \frac{h}{\frac{S}{\cos\alpha}}$$

From Eqs. (7.6) and (7.7),

$$q = kiA = k\left(\frac{h\cos\alpha}{S}\right)(H_1\cos\alpha \times 1)$$

$$= (0.08 \times 10^{-2}\text{ m/sec})\left(\frac{4\cos 8°}{50}\right)(3\cos 8° \times 1)$$

$$= \mathbf{0.19 \times 10^{-3}\ m^3/sec/m}$$

Hydraulic Conductivity

The Hydraulic conductivity depends on several factors, most of which are listed below:

Solids

- **Grain size distribution**
- **Pore size distribution**
- **Roughness of mineral particles**

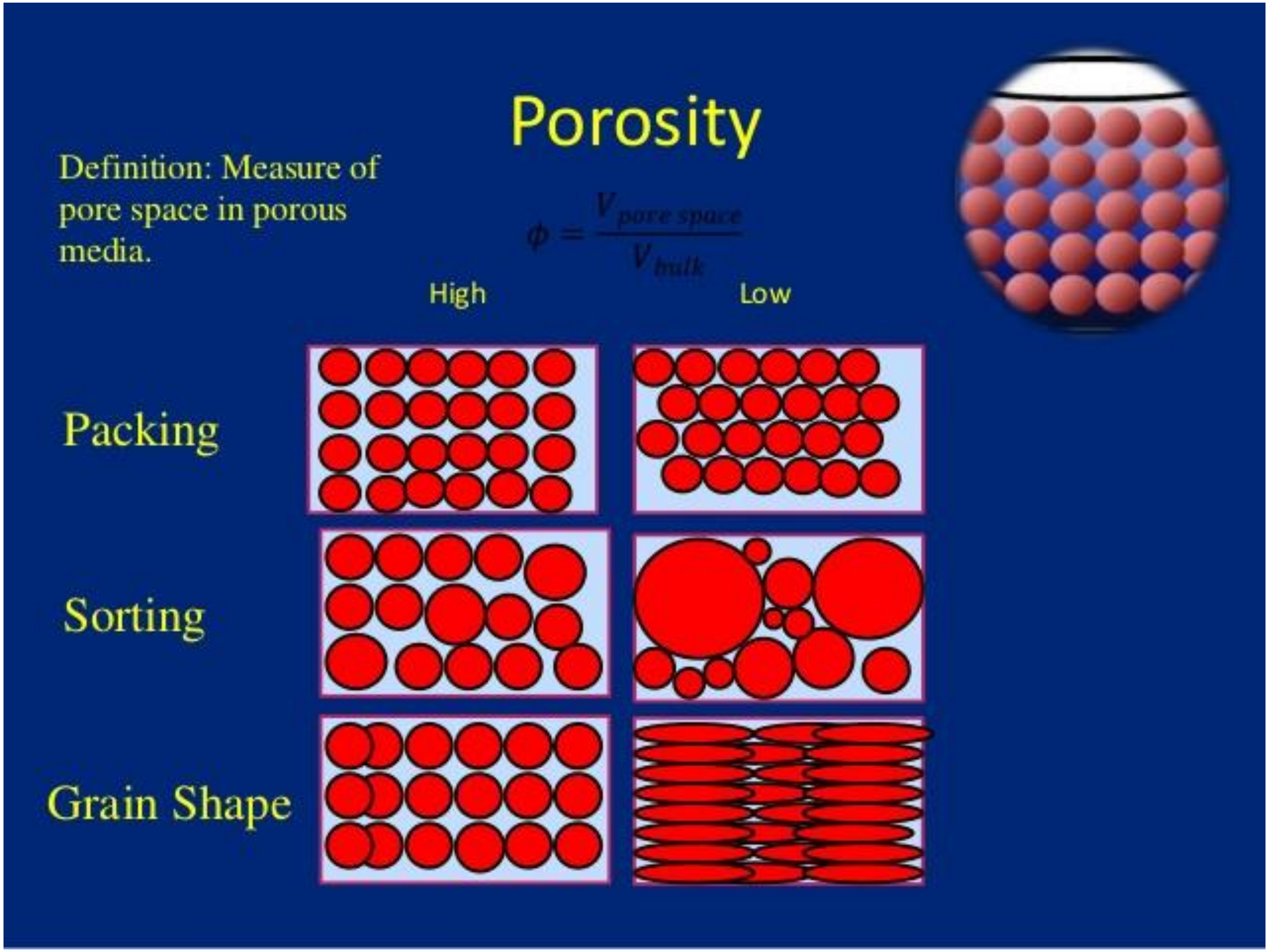
Voids

- **Void ratio** $(k \alpha e)$
- **Degree of saturation** $(k \alpha s)$
- **Double layer thickness** $(k\alpha \frac{1}{d})$
- **Soil structure (flocculated structure has higher k than dispersed structure).**

Permeant

- **Ionic concentration**
- **Viscosity of the permeant**
- **Density and concentration of the permeant**

POROSITY

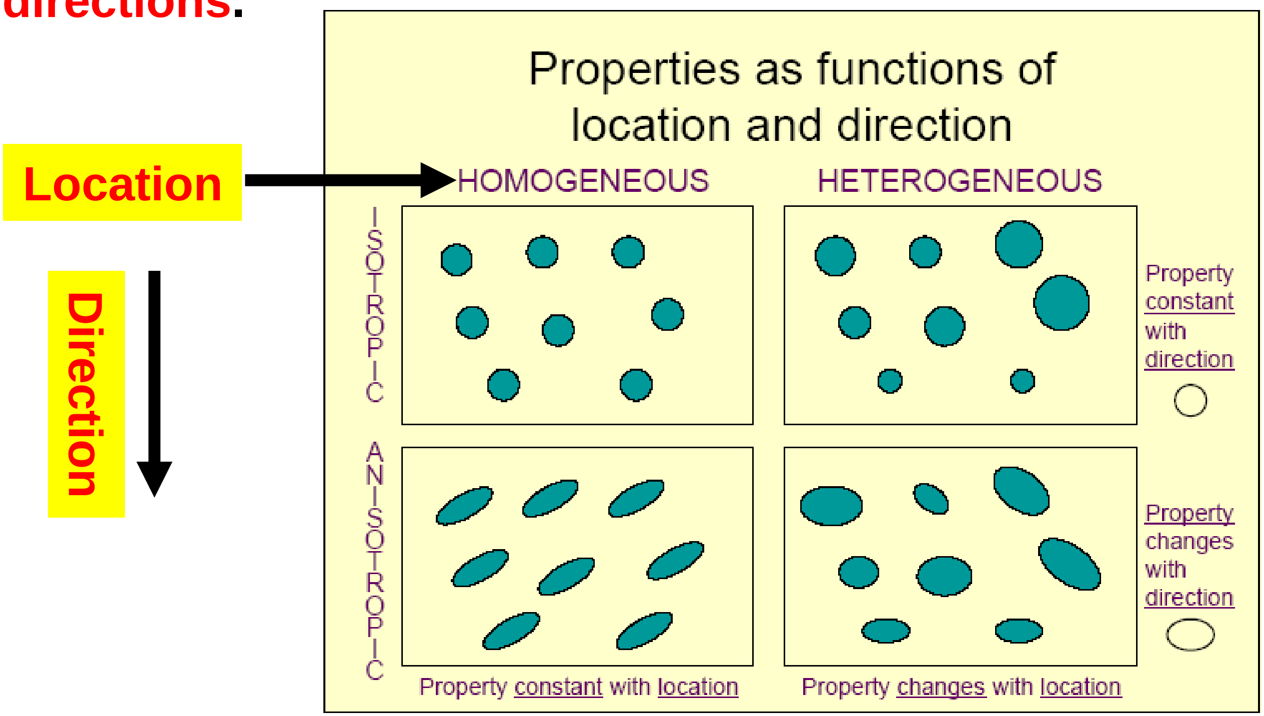

Porosity
Definition: Measure of pore space in porous media.
$\phi = \frac{V_{pore\ space}}{V_{bulk}}$
High
Low
Packing
Sorting
Grain Shape

REMARKS

- **k typically cannot be correlated with porosity. For example, clay has a very high porosity but very low permeability.**

- **However, within a single lithologic type (such as sandstone) k increases with increasing porosity.**

- **For most natural formations, k changes with locations and directions.**



Typical values of k

Soil Type	k (cm/sec)	Range
Clean Gravel	$>10^{0}$	Very high
Clean coarse sand	10^{-1}-10^{0}	High
Fine sand	10^{-3}- 10^{-1}	Medium
Silty sand, silt	10^{-5}-10^{-3}	Low
Dense silt, clayey silt	10^{-7}-10^{-5}	Very low
Clay, silty clay	$<10^{-7}$	Practically impermeable

Intrinsic Permeability vs Hydraulic Conductivity

- The coefficient of proportionality k ,in Eq. (3) is called the hydraulic conductivity. The term coefficient of permeability is also sometimes used as a synonym for hydraulic conductivity .

- Permeability is a portion of hydraulic conductivity, and is a property of the porous media only, not the fluid.

- The hydraulic conductivity of a soil is related to the properties of the fluid flown through it by:

$$k = \frac{\gamma_w}{\eta}\overline{K}$$

where γ_w = unit weight of water

η = viscosity of water

$\overline{K}$ = absolute permeability

(intrinsic permeability or absolute depends only on properties of the solid matrix)

Unit of η is N.s/m^2

The *absolute permeability* is expressed in units of L^2 (That is cm^2)

Variation with Temperature

It is conventional to express the value of k at a temperature of 20°C. Within the range of test temperatures, we can assume that $\gamma_{w(T_1)} \simeq \gamma_{w(T_2)}$.

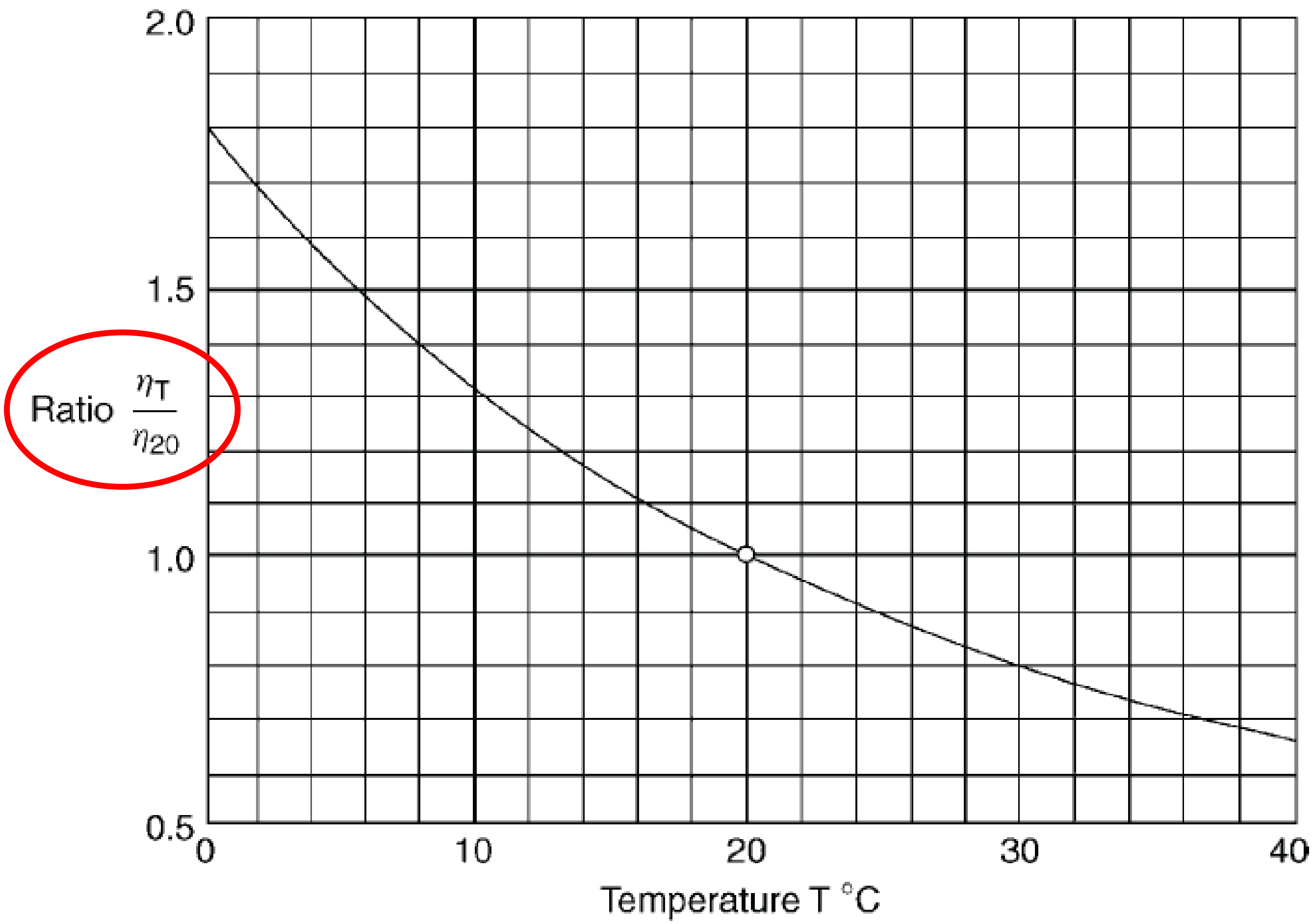
$$k_{20°C} = \left(\frac{\eta_{T°C}}{\eta_{20°C}}\right) k_{T°C}$$

Table 7.2 Variation of $\eta_{T°C}/\eta_{20°C}$

Temperature, T (°C)	$\eta_{T°C}/\eta_{20°C}$	Temperature, T (°C)	$\eta_{T°C}/\eta_{20°C}$
15	1.135	23	0.931
16	1.106	24	0.910
17	1.077	25	0.889
18	1.051	26	0.869
19	1.025	27	0.850
20	1.000	28	0.832
21	0.976	29	0.814
22	0.953	30	0.797

If we measure permeability at a cold temperature the viscosity will be high and k at 20c should be higher, and vise versa.

Variation with Temperature



REMARKS

- **Intrinsic Permeability is a measure of how well a porous media transmits a fluid. It has nothing to do with the fluid itself. It is measured in (length)^2.**

- **The Hydraulic Conductivity is a measure of how easily water moves through the porous media. It depends not only on the permeability of the matrix, but also is a function of the fluid. It is a measure of (length)/(time).**

- **The unit of k is the same as velocity i.e. distance/time. Hence hydraulic conductivity is sometimes defined as the "Superficial velocity of water flowing through soil under unit hydraulic gradient".**

Determination of the Coefficients of Permeability

The coefficient of permeability can be determined:

1. In the laboratory
2. In the field
3. From empirical relations
4. From Consolidation test (CE 481)

I. Laboratory

A device called a permeameter is used in the laboratory. There are two standard types of laboratory test procedures:

1. The constant-head test
2. The falling-head test

CONSTANT-HEAD TEST (ASTM D2434)

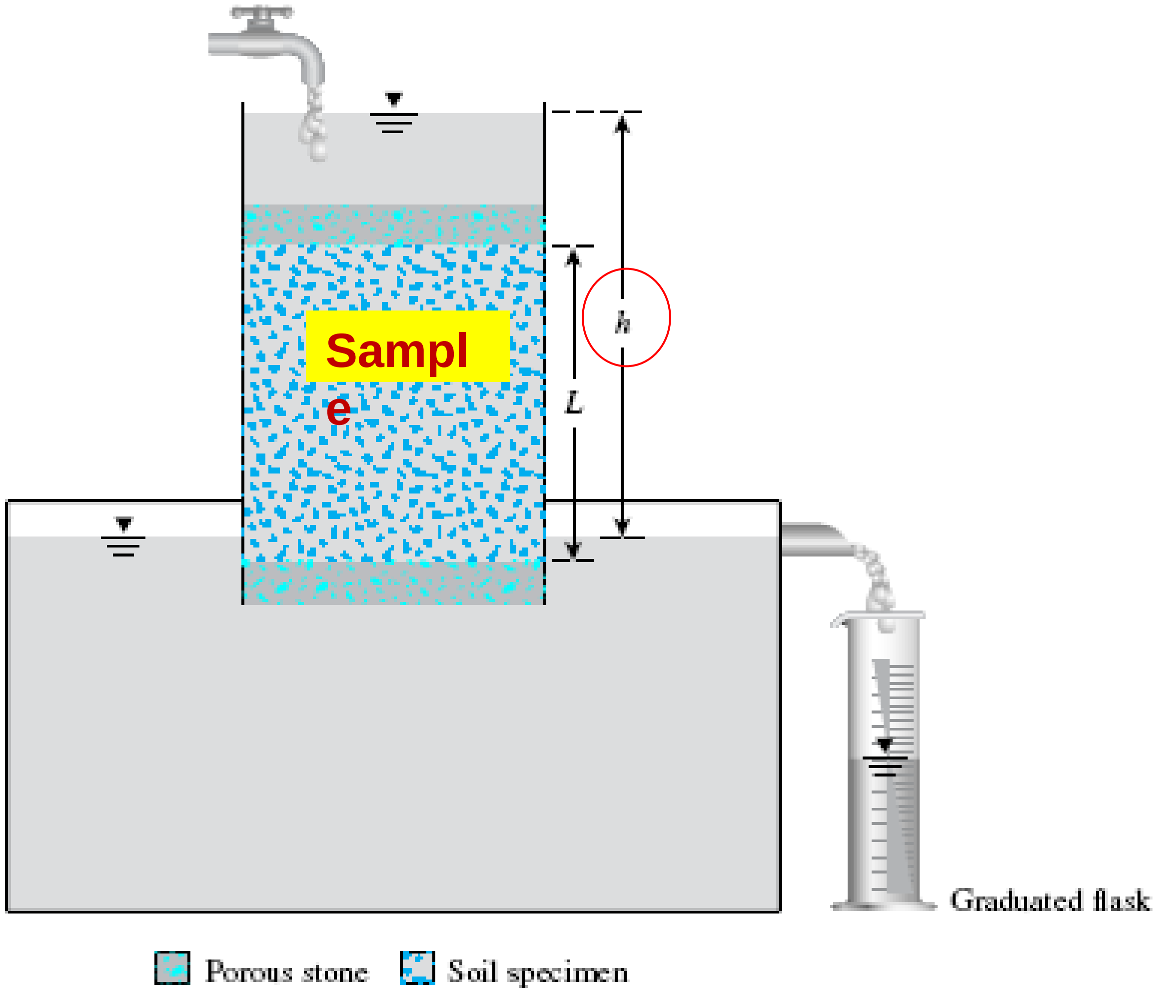


Figure 7.5 Constant-head permeability test

CONSTANT-HEAD TEST (ASTM D2434)

The total volume of water collected can be expressed as:

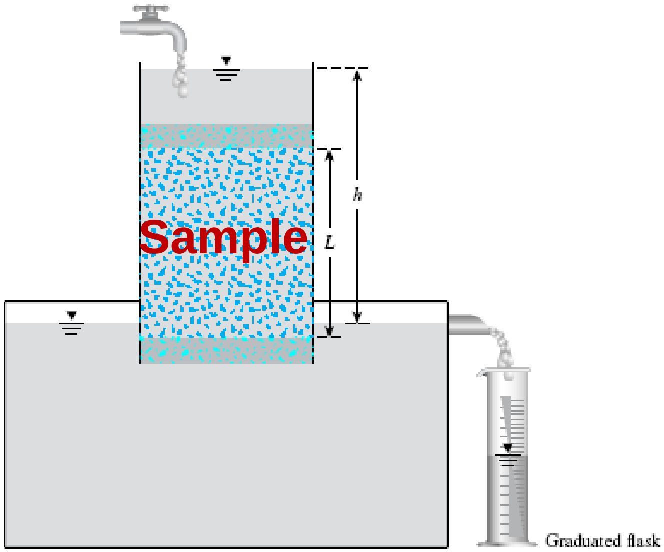
$$Q = Avt = A(ki)t \longrightarrow k = \frac{QL}{Aht}$$

where

Q = volume of water collected
A = area of cross section of the soil specimen
t = Duration of water collection

Notes:

- **The water used in the test should be de-aired.**
- **This test is more suitable for soils with high k (i,e. gravels, sand, coarse silts). Why?**
- **The test applies a constant head of water to each end of a soil in a "permeameter".**



Porous stone Soil specimen

Figure 7.5 Constant-head permeability test

Example 7.1

Example 7.1

The results of a constant-head permeability test for a fine sand sample having a diameter of 150 mm and a length of 300 mm are as follows:

- Constant head difference = 500 mm
- Time of collection of water = 5 min
- Volume of water collected = 350 cm^3
- Temperature of water = 24°C

Determine the hydraulic conductivity for the soil at 20°C.

Solution

For a constant-head permeability test,

$$k = \frac{QL}{Aht}$$

Given that $Q = 350$ cm^3, $L = 300$ mm, $A = (\pi/4)(150)^2 = 17671.46$ mm^2, $h = 500$ mm, and $t = 5 \times 60 = 300$ sec, we have

change to mm^3
↓

$$k = \frac{(350 \times 10^3) \times 300}{17{,}671.46 \times 500 \times 300} = 3.96 \times 10^{-2} \text{ mm/sec}$$

$$= 3.96 \times 10^{-3} \text{ cm/sec}$$

$$k_{20} = k_{24}\frac{\eta_{24}}{\eta_{20}}$$

From Table 7.2,

$$\frac{\eta_{24}}{\eta_{20}} = 0.91$$

So, $k_{20} = (3.96 \times 10^{-3}) \times 0.91 =$ **3.6×10^{-3} cm/sec.**

Example

EXAMPLE (Midterm Exam)

In a constant-head permeability test, the length of the specimen is 150 mm and the diameter of the cross section is 32 mm. If k = 0.085 cm/sec and a rate of flow of 160 cm3/min has to be maintained during the test, what should be the head difference across the specimen? Also, determine the discharge velocity under the test conditions.

FALLING-HEAD TEST

Note: The test applies a constant head of water only at the discharge point.

The velocity of fall in the standpipe is

$$v = -\frac{dh}{dt}$$

the –ve means a decreasing value of h as t increases (a "falling" head)

Flow into the sample $$q_{in} = -a\frac{dh}{dt}$$

Where a is the cross-sectional area of the standpipe

Flow out of the sample $$q_{out} = vA = k\frac{h}{L}A$$

From continuity equation $q_{out} = q_{in}$

$$-a\frac{dh}{dt} = k\frac{h}{L}A$$

Integration with limits of time from 0 to t and with limits of head difference from h_1 to h_2 gives

Integrating yields

$$k = \frac{aL}{At}\ln\left(\frac{h_1}{h_2}\right)$$

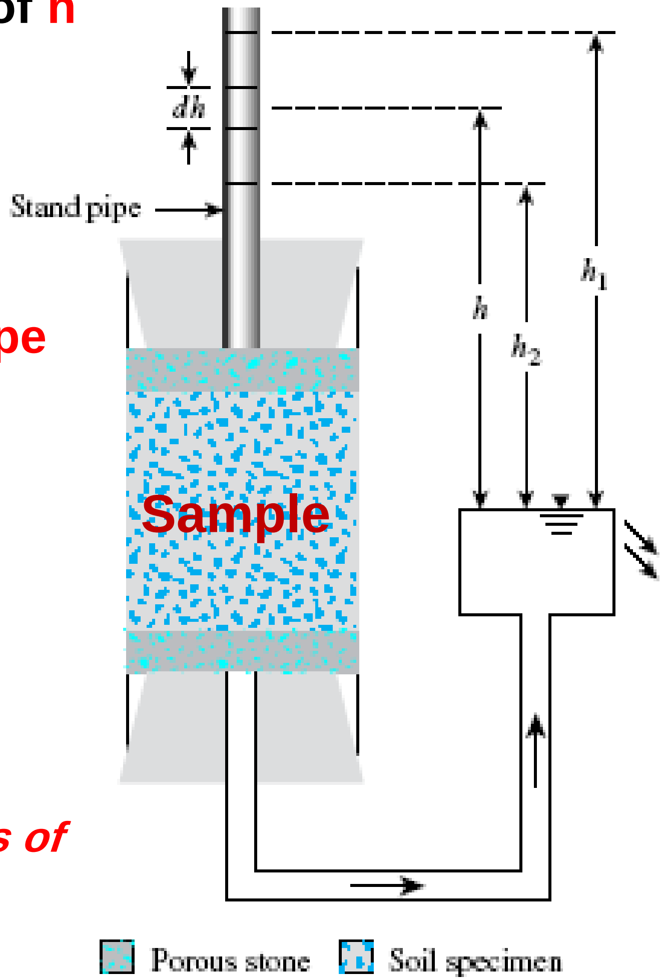


Figure 7.6 Falling-head permeability test

Example 7.2

Example 7.2

For a falling-head permeability test, the following values are given:

- Length of specimen = 200 mm
- Area of soil specimen = 1000 mm^2
- Area of standpipe = 40 mm^2
- At time $t = 0$, the head difference is 500 mm
- At time $t = 180$ sec, the head difference is 300 mm

Determine the hydraulic conductivity of the soil in cm/sec.

Solution

From Eq. (7.22),

$$k = 2.303 \frac{aL}{At} \log_{10}\left(\frac{h_1}{h_2}\right)$$

We are given $a = 40$ mm^2, $L = 200$ mm, $A = 1000$ mm^2, $t = 180$ sec, $h_1 = 500$ mm, and $h_2 = 300$ mm,

$$k = 2.303 \frac{(40)(200)}{(1000)(180)} \log_{10}\left(\frac{500}{300}\right) = 2.27 \times 10^{-2} \text{ mm/sec}$$

$$= \mathbf{2.27 \times 10^{-3} \text{ cm/sec}}$$

Example 7.3

Example 7.3

For a falling-head permeability test, the following are given: length of specimen = 15 in., area of specimen = 3 in.^{2}, and k = 0.0688 in./min. What should be the area of the standpipe for the head to drop from 25 to 12 in. in 8 min.?

Solution

From Eq. (7.22),

$$k = 2.303 \frac{aL}{At} \log_{10} \frac{h_1}{h_2}$$

$$0.0688 = 2.303 \left(\frac{a \times 15}{3 \times 8} \right) \log_{10} \left(\frac{25}{12} \right)$$

$$a = \mathbf{0.15\ in.^2}$$

Limitations of Laboratory Tests

- Soil specimen is not representative of the natural deposit.
- Effect of the boundary conditions due to the small size of the specimen.
- Air bubbles may be trapped in the test specimen, or air may come out of solution of the water.
- When k is very small, say $10^{-5} – 10^{-9}$ cm/sec, evaporation may affect the measurements.
- Temperature variation, especially in test of long duration, may affect the measurements.
- Migration of fines in testing sands and silts.
- To expedite the test, the laboratory hydraulic gradient $\Delta h/L$ is often made 5 or more, whereas in the field more realistic values may be on the order of 0.1 to 2.0

Consolidation Test – CE481

- One way to find **k** for fine-grained soils is to conduct consolidation test and from its results **k** can be found as:

$$k = \gamma_w m_v c_v$$

From Terzaghi 1-D Theory of consolidation

where

m_v = coefficient of volume compressibility
C_v = coefficient of consolidation

- Consolidation of soils is addressed in the course CE 481

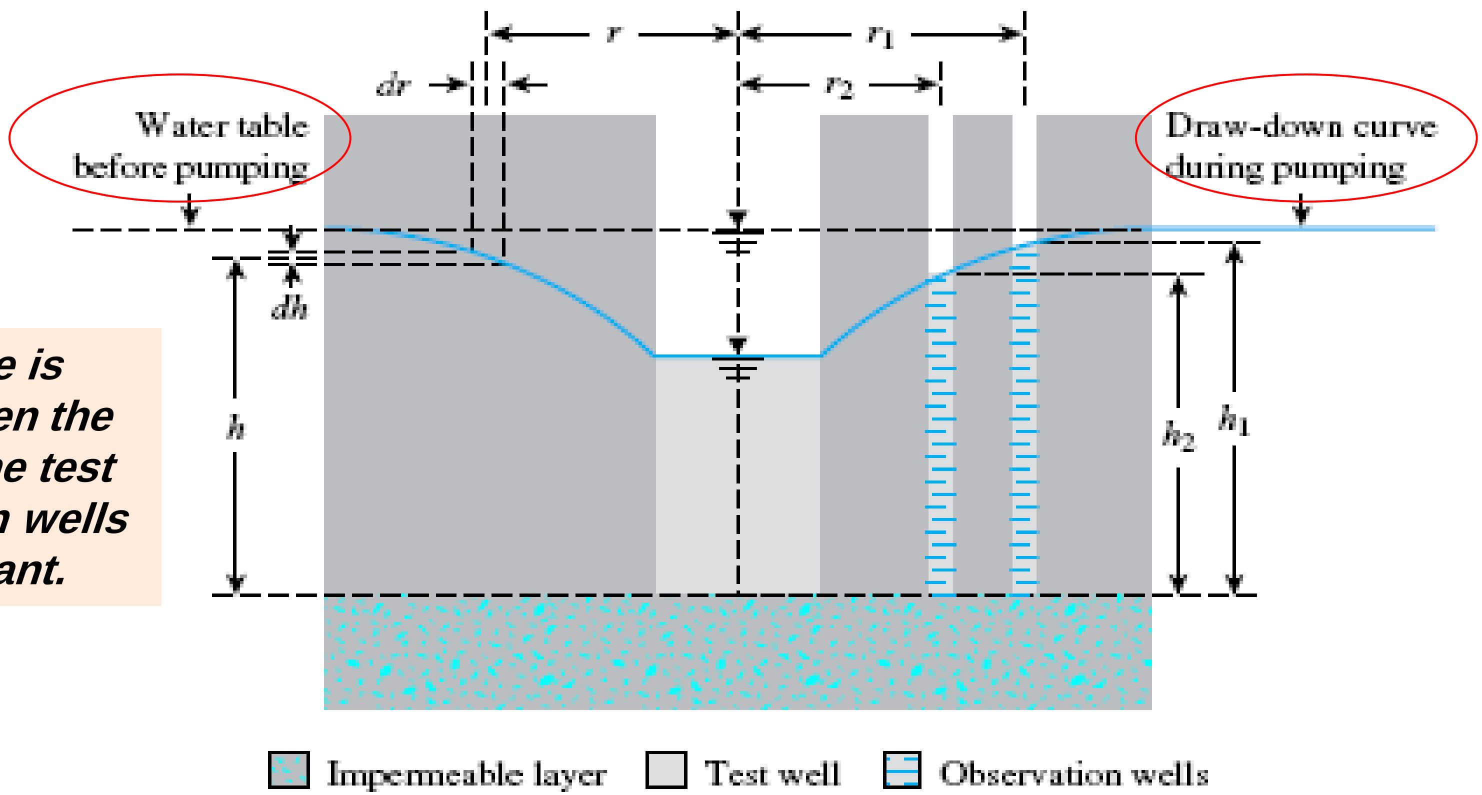
- This is very practical especially for very-fine-grained soil where permeability test would take long period of time.

In Situ Methods

For important projects the in situ determination of permeability may be justified.

A. Unconfined aquifer

- **Required to determine the permeability of the top layer**



The steady state is established when the water level in the test and observation wells becomes constant.

The hydraulic gradient at any point in the water-bearing stratum is constant and is equal to the slope of groundwater surface (Dupuit's assumption).

In Situ Methods

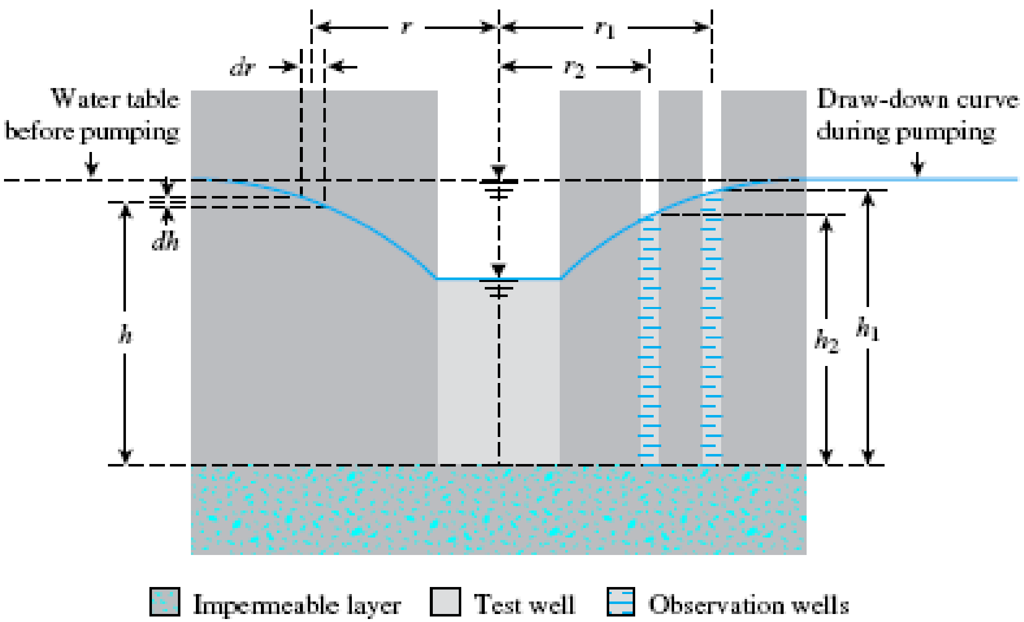
- The layer is permeable, unconfined, and underlain by impermeable layer
- The rate of flow of groundwater into the well, which is equal to the rate of discharge from pumping can be expressed as:

$$q = vA$$

$$q = k\left(\frac{dh}{dr}\right)2\pi r h$$

$$\int_{r_2}^{r_1} \frac{dr}{r} = \left(\frac{2\pi k}{q}\right) \int_{h_2}^{h_1} h\, dh$$

$$k = \left(\frac{q}{\pi(h_1^2 - h_2^2)}\right)\ln\left(\frac{r_1}{r_2}\right)$$



$$k = \frac{2.303 q \log_{10}\left(\frac{r_1}{r_2}\right)}{\pi(h_1^2 - h_2^2)}$$

In Situ Methods

B. Confined aquifer

The discharge is equal to

$$q = k\left(\frac{dh}{dr}\right)2\pi r H$$

$$\int_{r_2}^{r_1} \frac{dr}{r} = \int_{h_2}^{h_1} \frac{2\pi k H}{q}\, dh$$

$$k = \frac{q \log_{10}\left(\frac{r_1}{r_2}\right)}{2.727H(h_1 - h_2)}$$

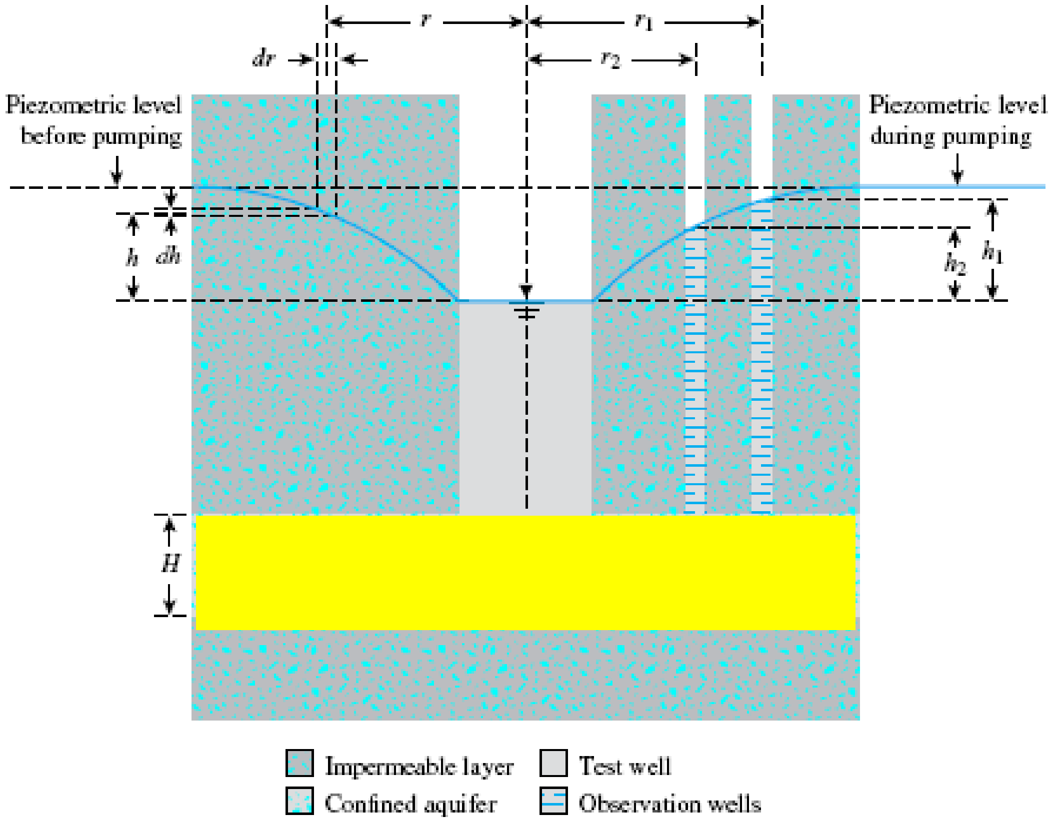


Figure 7.22 Pumping test from a well penetrating the full depth in a confined aquifer

Example 7.17

EXAMPLE 7.17

A pumping test from a confined aquifer yielded the following results: $q = 0.303\ \text{m}^3/\text{min}$, $h_1 = 2.44$ m, $h_2 = 1.52$ m, $r_1 = 18.3$ m, $r_2 = 9.15$ m, and $H = 3.05$ m. Refer to Figure 7.24 and determine the magnitude of k of the permeable layer.

Solution

From Eq. (7.49),

$$k = \frac{q \log_{10}\left(\frac{r_1}{r_2}\right)}{2.727H(h_1 - h_2)} = \frac{(0.303) \log_{10}\left(\frac{18.3}{9.15}\right)}{(2.727)(3.05)(2.44 - 1.52)}$$

$$= 0.01192\ \text{m/min} \approx \mathbf{0.0199\ cm/sec}$$

EXAMPLE (2nd Midterm Exam Fall 40-41)

A pumping well test was made in a sand layer extending to a depth of 15 m where an impermeable stratum was encountered. The initial ground-water level was at the ground surface. Observation wells were sited at distances of 3 m and 7.5 m from the pumping well. A steady state was established at about 20 hours when the discharge was 3.8 L/s. The drawdowns at the two observation well were 1.5 m and 0.35 m. Calculate the coefficient of permeability for the sand layer.

$$k = \frac{2.303q \log_{10}\left(\frac{r_1}{r_2}\right)}{\pi(h_1^2 - h_2^2)}$$

$$k = \frac{q \log_{10}\left(\frac{r_1}{r_2}\right)}{2.727H(h_1 - h_2)}$$

EQUIVALENT HYDRAULIC CONDUCTIVITY

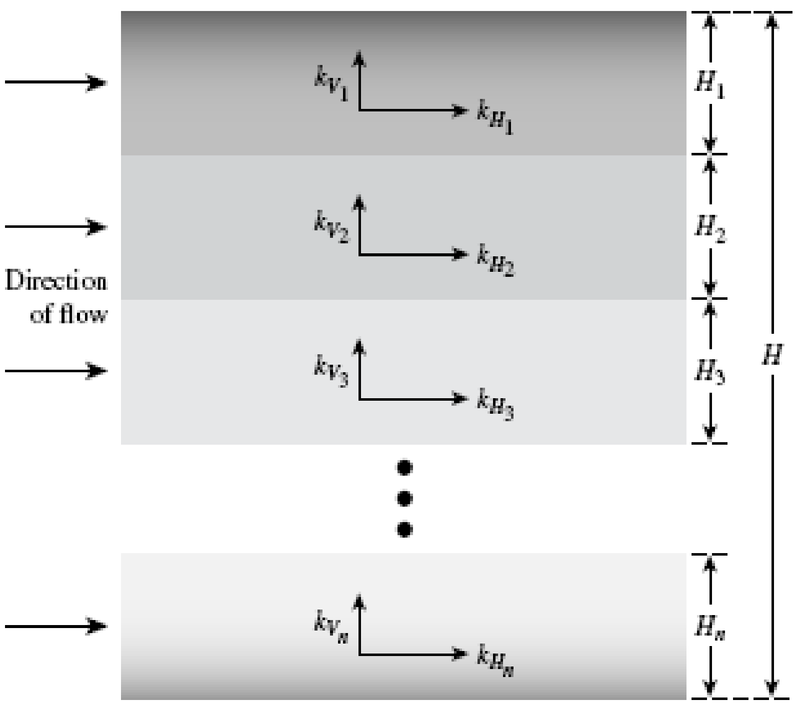
Flow in Horizontal Direction

Head is the SAME

Q is sum

$$q = q_1 + q_2 + q_3 + \dots\dots\dots\dots q_n$$

- **We sum flow rates**
- **We have same gradient**



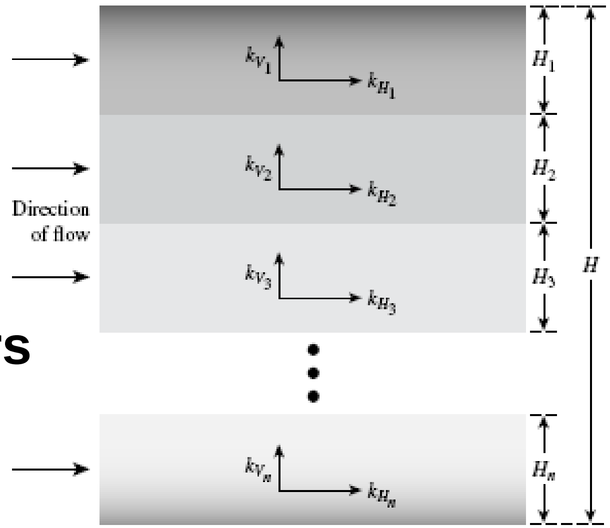
EQUIVALENT HYDRAULIC CONDUCTIVITY

Flow in Horizontal Direction

The total flow through the cross section in unit time is given as:

$$q = v \cdot 1 \cdot H$$

Flow is equal to the sum of flow in individual layers



$$= v_1 \cdot 1 \cdot H_1 + v_2 \cdot 1 \cdot H_2 + v_3 \cdot 1 \cdot H_3 + \cdots + v_n \cdot 1 \cdot H_n$$

$$v = k_{H(\mathrm{eq})} i_{\mathrm{eq}}; \quad v_1 = k_{H_1} i_1; \quad v_2 = k_{H_2} i_2; \quad v_3 = k_{H_3} i_3; \quad \ldots \; v_n = k_{H_n} i_n;$$

$$k_{H(\mathrm{eq})} = \frac{1}{H}\left(k_{H_1} H_1 + k_{H_2} H_2 + k_{H_3} H_3 + \cdots + k_{H_n} H_n\right)$$

$$q = k_{H(eq)} . i . 1 . H$$

$$k_{H(eq)} = \sum_{m=1}^{n} \frac{k_m H_m}{H}$$

EQUIVALENT HYDRAULIC CONDUCTIVITY

Flow in Vertical Direction

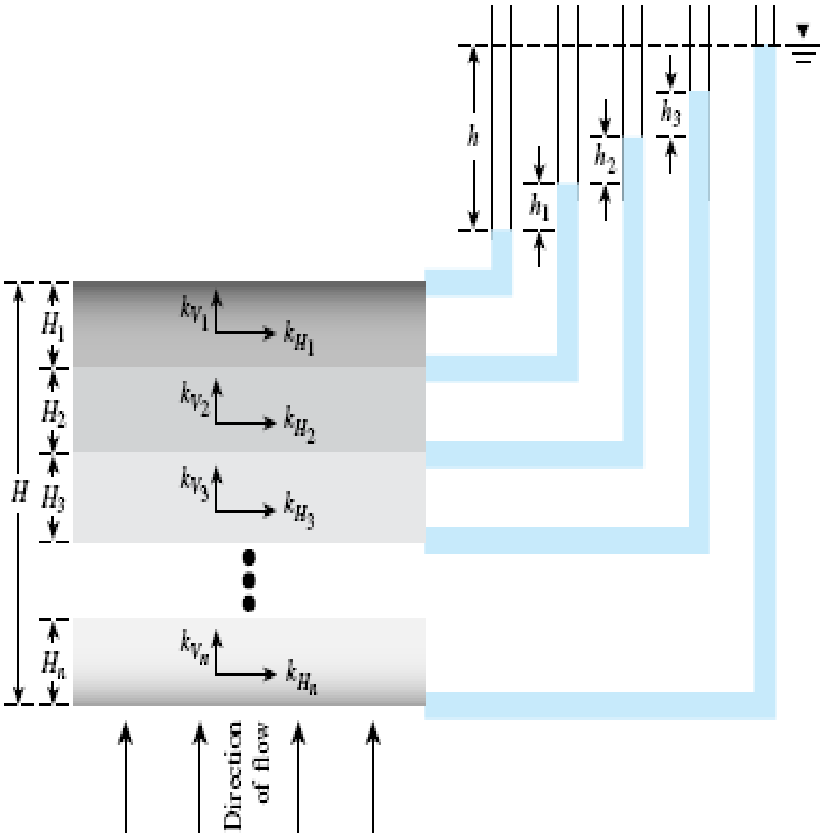
Q is the SAME

Head is sum

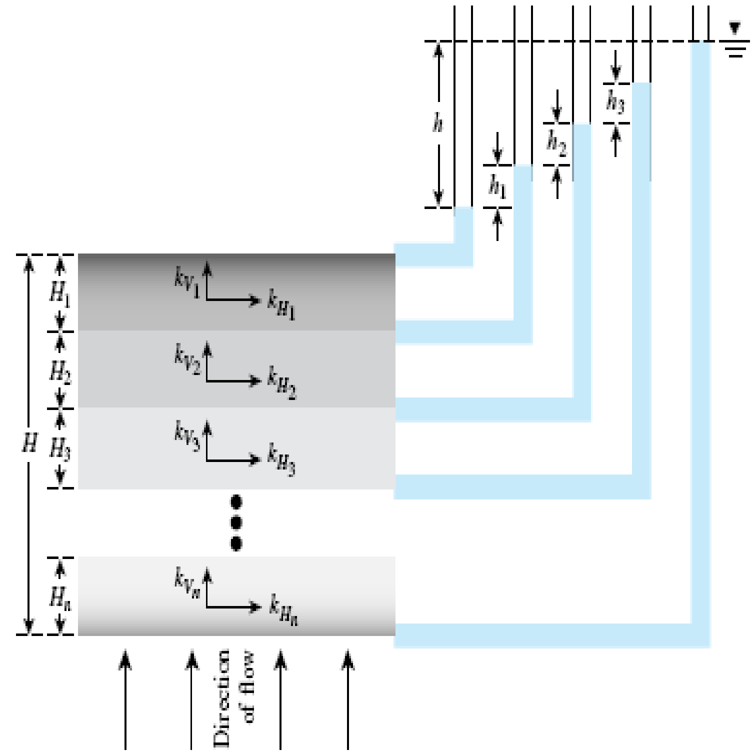
$$h = h_1 + h_2 + h_3 + \ldots\ldots + h_n$$

- **We sum heads**
- **We have same velocity**

(because same q and since A is same v must be the same)



Flow in Vertical Direction



From Darcy's law

$$v_1 = k_{V_1} i_1 = k_{V_1} \frac{h_1}{H_1} \ldots\ldots\ldots v_n = k_{V_n} \frac{h_n}{H_n}$$

$$h = \frac{H_1}{k_{v_1}} v_1 + \frac{H_2}{k_{v_2}} v_2 + \ldots\ldots + \frac{H_n}{k_{v_n}} v_n \qquad (**)$$

$$v = k_{V(eq)} \frac{\mathrm{h}}{\mathrm{H}} \rightarrow h = \frac{H}{k_{v(eq)}} v \qquad (***)$$

$$v = v_1 = v_2 = \ldots\ldots\ldots\ldots v_n \qquad (****)$$

Equating the R.H.S of Eqs. (**) & (***), considering (****) yields

$$k_{V(eq)} = \frac{H}{\left(\frac{H_1}{k_{V_1}}\right) + \left(\frac{H_2}{k_{V_2}}\right) + \left(\frac{H_3}{k_{V_3}}\right) + \cdots + \left(\frac{H_n}{k_{V_n}}\right)}$$

$$k_{V(eq)} = \sum_{m=1}^{n} \frac{H}{H_m / k_m}$$

Example 7.14

Example 7.14

A layered soil is shown in Figure 7.20. Given:

- $H_1 = 1$ m $k_1 = 10^{-4}$ cm/sec
- $H_2 = 1.5$ m $k_2 = 3.2 \times 10^{-2}$ cm/sec
- $H_3 = 2$ m $k_3 = 4.1 \times 10^{-5}$ cm/sec

Estimate the ratio of equivalent hydraulic conductivity,

$$\frac{k_{H(eq)}}{k_{V(eq)}}$$

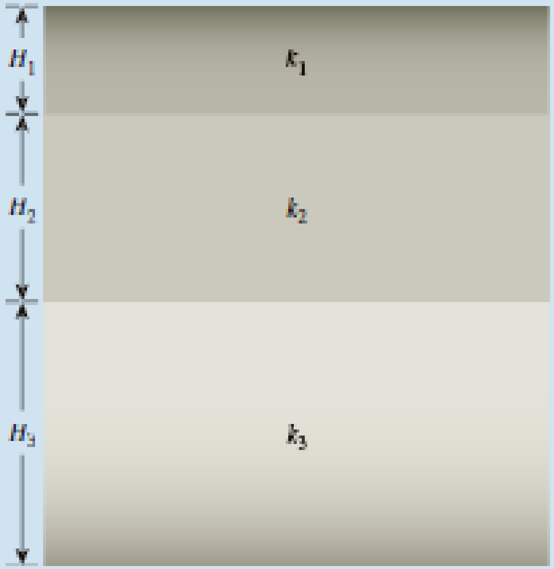


Figure 7.20 A layered soil profile

Example 7.14

Solution

From Eq. (7.40),

$$k_{H(\text{eq})} = \frac{1}{H}(k_{H_1}H_1 + k_{H_2}H_2 + k_{H_3}H_3)$$

$$= \frac{1}{(1 + 1.5 + 2)}[(10^{-4})(1) + (3.2 \times 10^{-2})(1.5) + (4.1 \times 10^{-5})(2)]$$

$$= 107.07 \times 10^{-4} \text{ cm/sec}$$

Again, from Eq. (7.45),

$$k_{V(\text{eq})} = \frac{H}{\left(\frac{H_1}{k_{V_1}}\right) + \left(\frac{H_2}{k_{V_2}}\right) + \left(\frac{H_3}{k_{V_3}}\right)}$$

$$= \frac{1 + 1.5 + 2}{\left(\frac{1}{10^{-4}}\right) + \left(\frac{1.5}{3.2 \times 10^{-2}}\right) + \left(\frac{2}{4.1 \times 10^{-5}}\right)}$$

$$= 0.765 \times 10^{-4} \text{ cm/sec}$$

Hence,

$$\frac{k_{H(\text{eq})}}{k_{V(\text{eq})}} = \frac{107.07 \times 10^{-4}}{0.765 \times 10^{-4}} \approx \mathbf{140}$$

Example 7.15

Example 7.15

Figure 7.21 shows three layers of soil in a tube that is 100 mm × 100 mm in cross section. Water is supplied to maintain a constant-head difference of 300 mm across the sample. The hydraulic conductivities of the soils in the direction of flow through them are as follows:

Soil	k (cm/sec)
A	10^{-2}
B	3×10^{-3}
C	4.9×10^{-4}

Find the rate of water supply in cm³/hr.

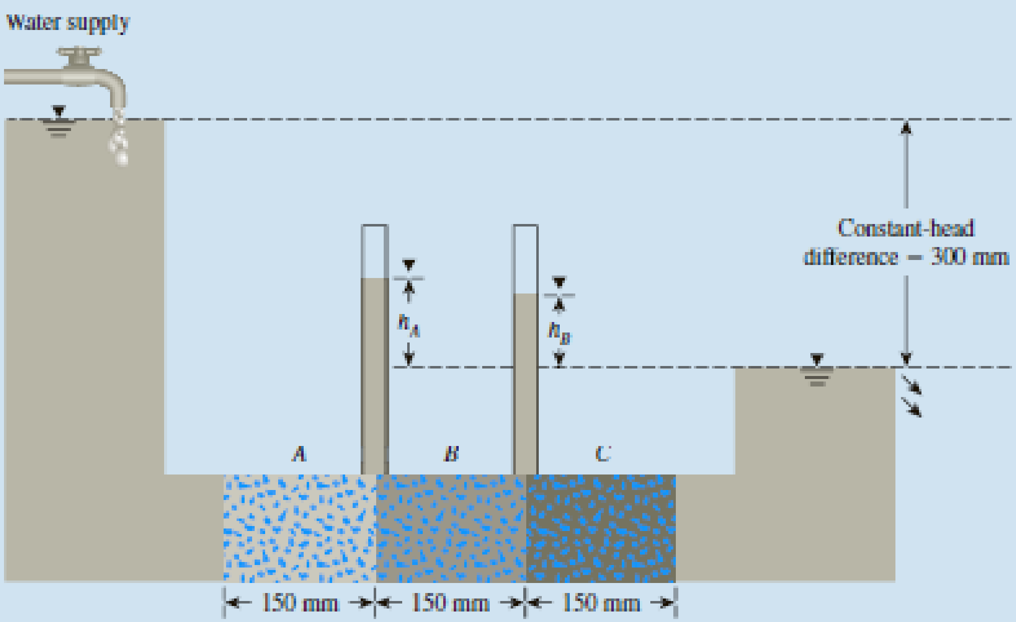


Figure 7.21 Three layers of soil in a tube 100 mm × 100 mm in cross section

Example 7.15

Solution

From Eq. (7.45),

$$k_{V(eq)} = \frac{H}{\left(\frac{H_1}{k_1}\right) + \left(\frac{H_2}{k_2}\right) + \left(\frac{H_3}{k_3}\right)} = \frac{450}{\left(\frac{150}{10^{-2}}\right) + \left(\frac{150}{3 \times 10^{-3}}\right) + \left(\frac{150}{4.9 \times 10^{-4}}\right)}$$

$$= 0.001213 \text{ cm/sec}$$

$$q = k_{V(eq)} i A = (0.001213)\left(\frac{300}{450}\right)\left(\frac{100}{10} \times \frac{100}{10}\right)$$

$$= 0.0809 \text{ cm}^3\text{/sec} = \mathbf{291.24\ cm^3/hr}$$

Example 7.16

EXAMPLE 7.16

Refer to Example 7.15 and Figure 7.21. Determine the magnitudes of h_A and h_B.

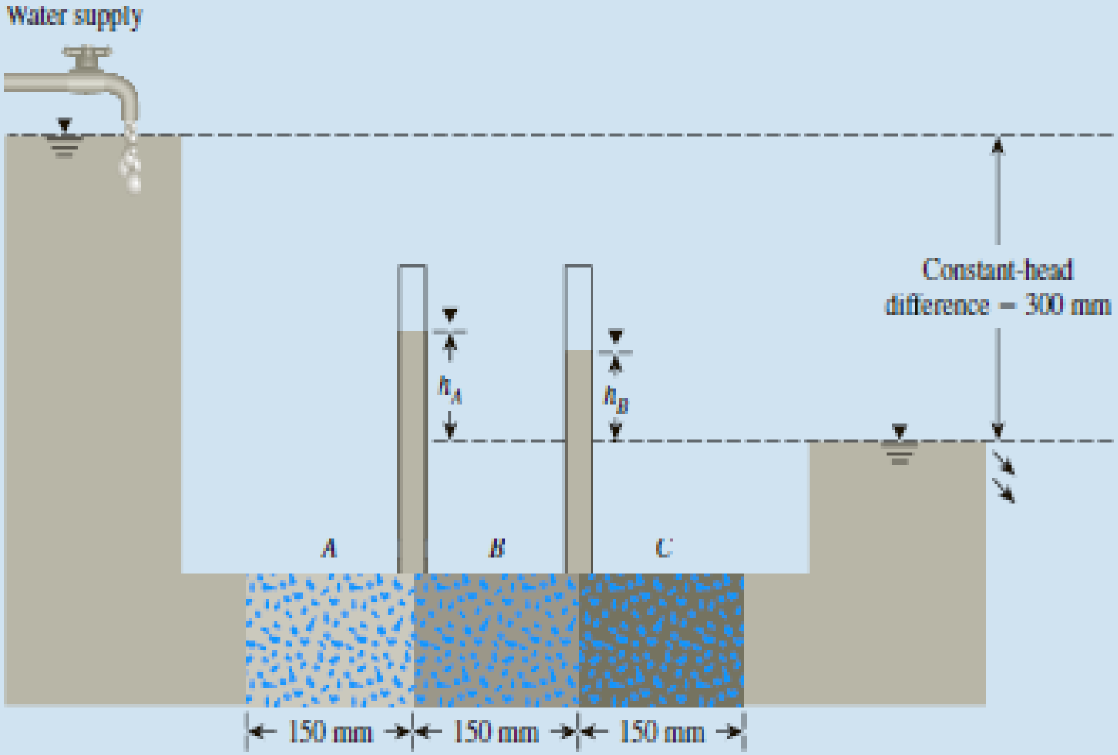


Figure 7.21 Three layers of soil in a tube 100 mm × 100 mm in cross section

Example 7.16

Solution

The loss of head during flow through Soil *A* can be calculated as

$$q = k_A i_A A = k_A \frac{\Delta h_A A}{L_A}$$

where Δh_A and L_A are, respectively, the head loss in Soil *A* and the length of Soil *A*. Hence,

$$\Delta h_A = \frac{q L_A}{k_A A}$$

From Example 7.15, $q = 0.0809\ \text{cm}^3/\text{sec}$, $L_A = 15\ \text{cm}$, and $k_A = 10^{-2}\ \text{cm/sec}$. Thus,

$$\Delta h_A = \frac{(0.0809)(15)}{(0.01)(10 \times 10\ \text{cm}^2)} = 1.2135\ \text{cm} \approx 12.14\ \text{mm}$$

Hence,

$$h_A = 300 - 12.14 = \mathbf{287.86\ mm}$$

Similarly, for Soil *B*,

$$\Delta h_B = \frac{q L_B}{k_B A} = \frac{(0.0809)(15)}{(0.003)(10 \times 10)} = 4.045\ \text{cm} = 40.45\ \text{mm}$$

Hence,

$$h_B = 300 - \Delta h_A - \Delta h_B = 300 - 12.14 - 40.45 = \mathbf{247.41\ mm}$$

EXAMPLE (2nd Midterm Exam Fall 40-41)

As shown in Figure 2, assume that the coefficients of permeability for the soils are $3K_1 = K_2 = 1.5K_3 = 2K_4$. If $K_1 = 3.5 \times 10^{-2}$ cm/sec, calculate the flow rate.

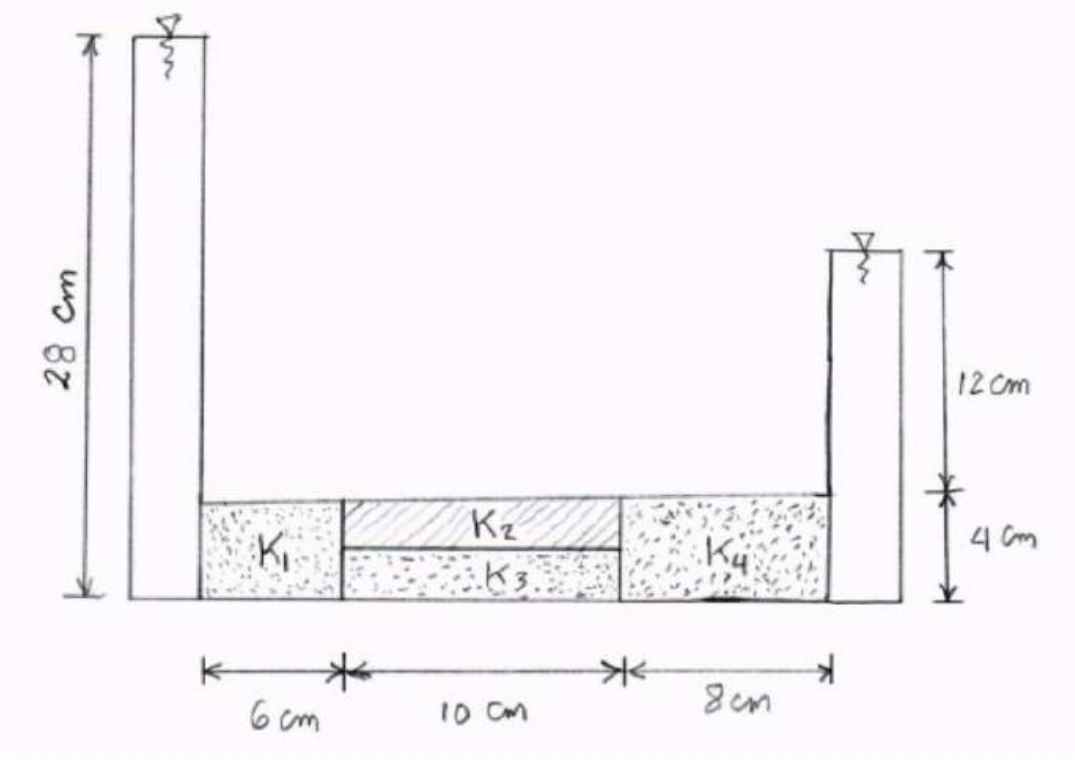


Figure 2

THE END