CE 382

Chapter 3

Weight-Volume Relationships

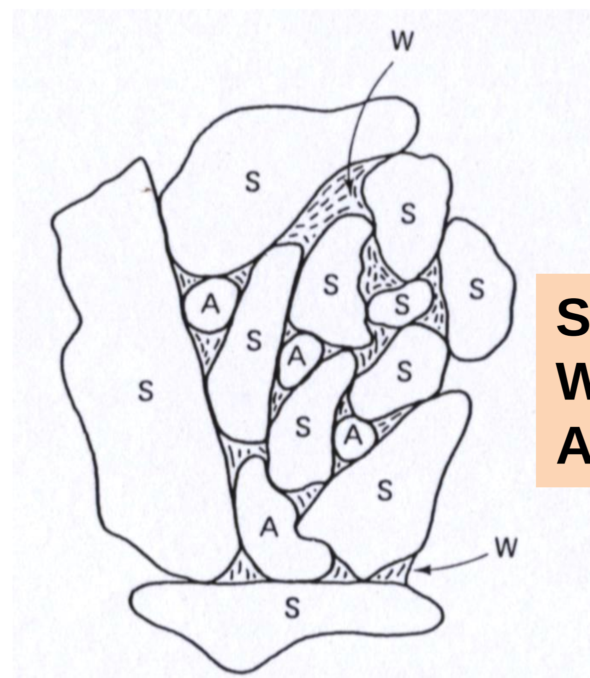
Omitted Parts:

Sections 3.6 & 3.7

GENERAL

- Soil deposits comprise the accumulated solid particles plus the void space between the particles.
- The void spaces are partially or completely filled with water or other liquid.
- Voids space not occupied by fluid are filled with air or other gas.
- Hence soil deposits are referred to as <u>three-phase system</u>,

  i.e. Solid + Liquid (water) + Gas (air)



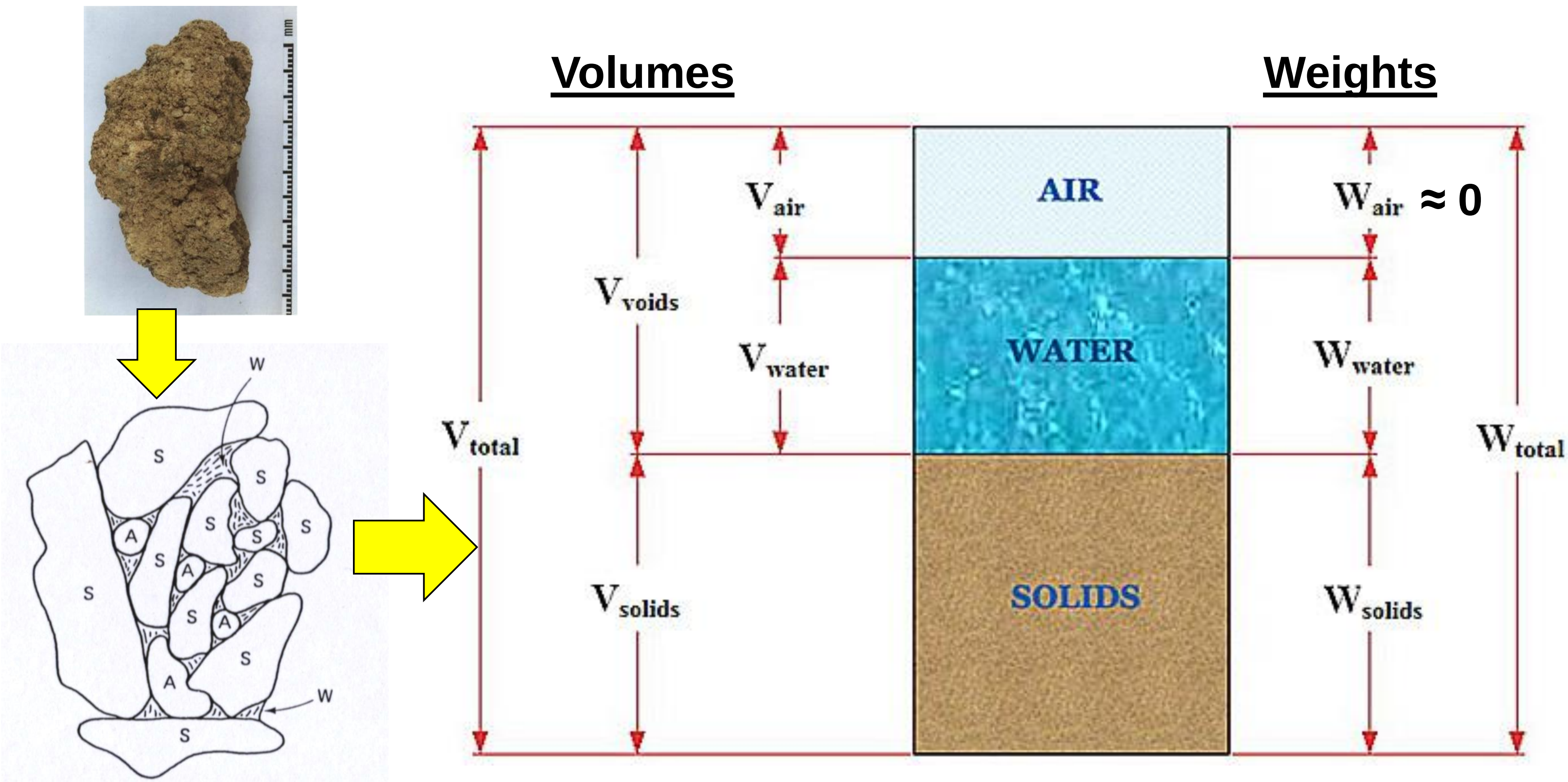
S: Solid
W: water
A: Air

GENERAL

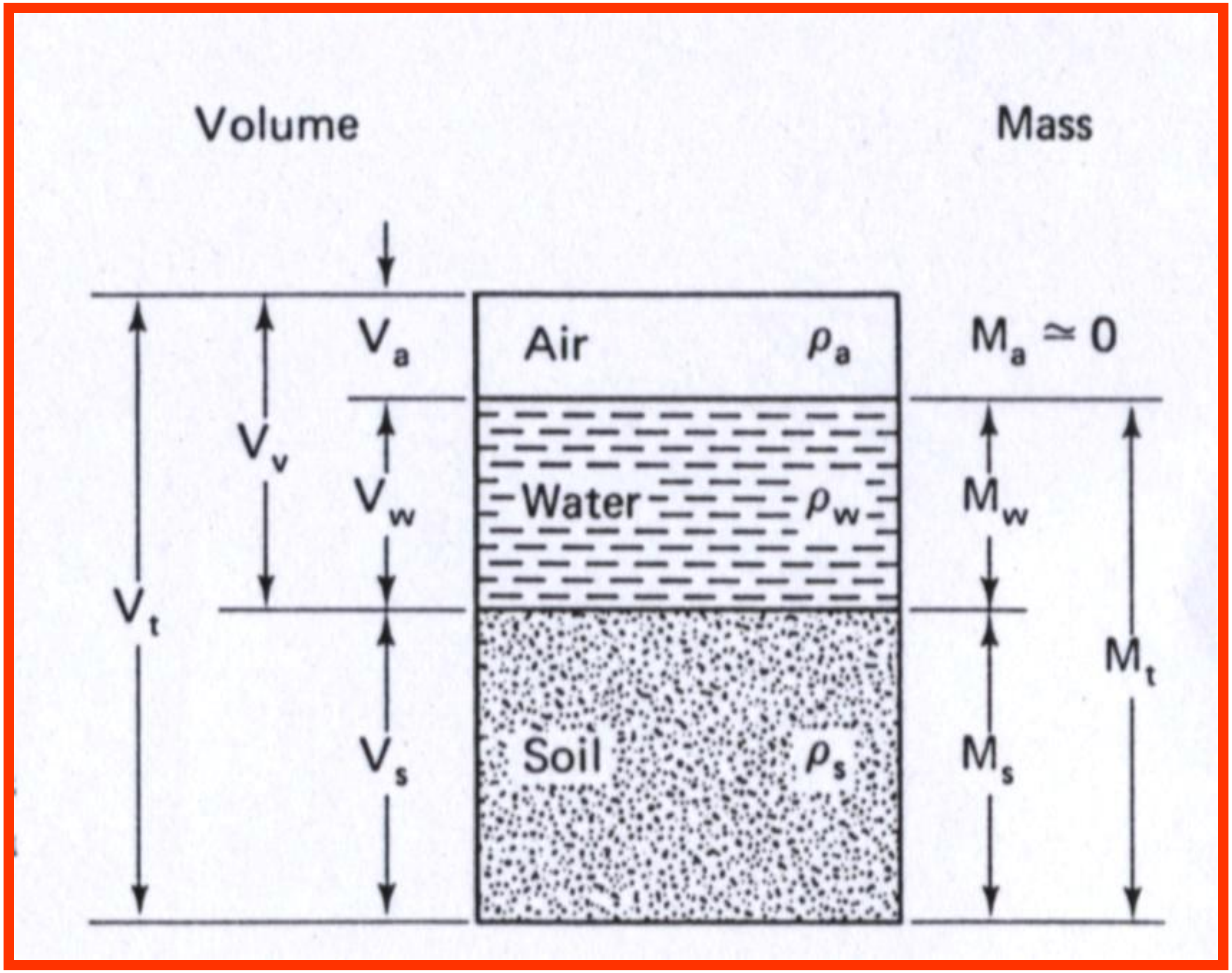
- Bulk soil as it exists in nature is a more or less random accumulation of soil particles, water, and air as shown above.
- Properties such as <u>strength, compressibility, permeability</u> are directly related to the ratio and interaction of these three phases.
- Therefore, an understanding of the terminology and definitions relating to soil composition is fundamental to the study of soil mechanics and geotechnical engineering as a whole.

PHASE DIAGRAM

For purpose of study and analysis it is convenient to represent the soil mass by a PHASE DIAGRAM, with part of the diagram representing the solid particles, part representing water or liquid, and another part air or other gas.



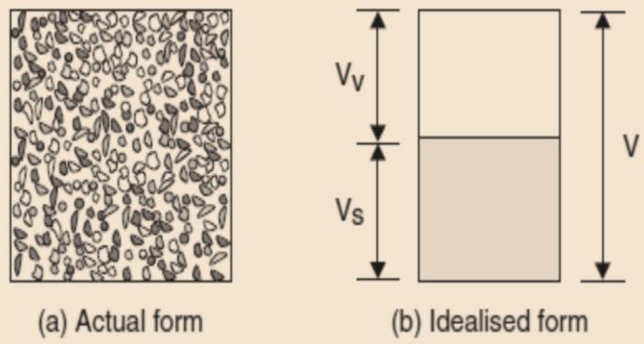
Phase diagram in terms of mass



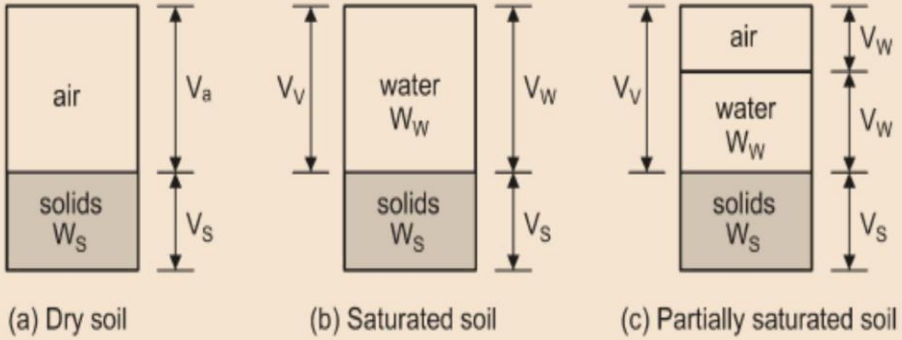
Possible Cases

Two phases:

- **Dry soil (solid + air)**
- **Fully saturated soil (solid + water)**



Cross-section through a granular soil.



Water and air contents in a soil.

Three phases:

Partially saturated soil (solid + water+ air)

PHASE DIAGRAM

- The total volume of a given soil sample can be expressed as:

$$V = V_s + V_v = V_s + V_w + V_a$$

Where

V = Total volume

V_s = Volume of soil solids

V_v = Volume of voids

V_w = Volume of water

V_a = Volume of air

- Assuming that the weight of the air is negligible, we can give the total weight of the sample as

$$W = W_s + W_w$$

Where W_s = weight of solids

W_w = weight of water

- In engineering practice we usually measure the total volume, V, the mass of water, M_w, and the mass of dry solid M_s.

Volume Relationships

There are <u>three</u> volumetric ratios that are very useful in geotechnical engineering, and these can be determined directly from the phase diagram

1. Void Ratio

$$e = \frac{V_v}{V_s}$$

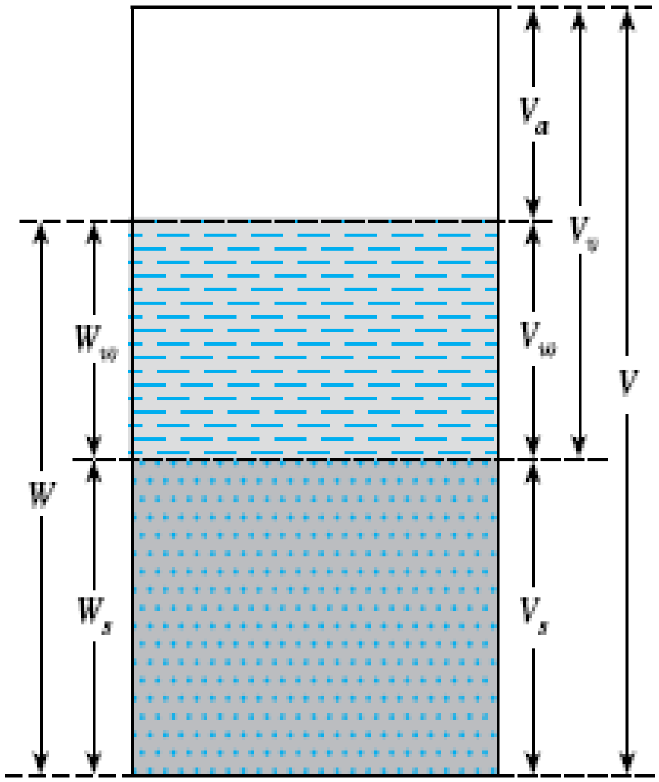
$$n = \frac{e}{1+e}$$

2. Porosity

$$n = \frac{V_v}{V}$$

3. Degree of Saturation

$$S = \frac{V_w}{V_v}$$



Porosity and degree of saturation are commonly expressed as a percentage.

Exercise

Given $V_v = V_s$, $V_w = 0.5V_v$

e = ?

n = ? %

S = ? %

Weight Relationships

There are <u>three</u> weight ratios that are very useful in geotechnical engineering, and these can be determined directly from the phase diagram

1. Moisture content (Water content)

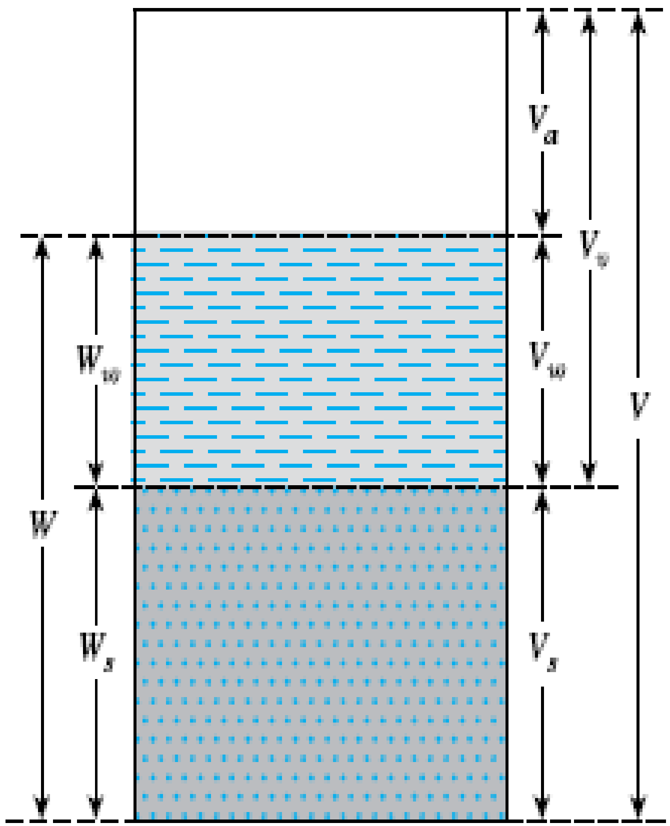
$$w = \frac{W_w}{W_s}$$

2. Unit weight (total, bulk, moist, wet)

$$\gamma = \frac{W}{V}$$ **unit : kN/m³**

3. Specific gravity

$$G_s = \frac{W_s}{W_w} = \frac{W_s}{V_s * \gamma_w}$$



Unit Weight

1. Unit weight (total, wet , bulk or moist unit weight)

$$\gamma = \frac{W}{V}$$

2. Solid unit weight

$$\gamma_s = \frac{W_s}{V_s}$$

3. Unit weight of water

$$\gamma_w = \frac{W_w}{V_w}$$

$$(\gamma_w = 9.807 \approx 10 \ \ kN/m^3)$$

4. Dry unit weight

$$\gamma_d = \frac{W_s}{V}$$

5. Saturated unit weight

$$\gamma_{sat} = \frac{W_s + W_w}{V} \qquad (S = 100\%)$$

6. Submerged unit weight

$$\gamma' = \gamma - \gamma_w$$

Density vs Unit Weight

Density

$$\rho = \frac{M}{V}$$ **unit : kg/m³**

Dry density

$$\rho_d = \frac{M_s}{V}$$

$$\rho_d = \frac{\rho}{1+w}$$

Unit Weight

$$\gamma = \rho * g$$

$$\gamma\,(\mathrm{kN/m^3}) = \frac{\mathrm{g} * \rho(\mathrm{kg/m^3})}{1000}$$

$$g = 9.81 \ \mathrm{m/sec^2}$$

$$\gamma_w = 9.81\,\mathrm{kN/m^3} = 1000 \ \mathrm{kgf/m^3}$$

$$G_s = \frac{M_s}{M_w} = \frac{M_s}{V_s * \rho_w}$$ ⟷ $$G_s = \frac{W_s}{W_w} = \frac{W_s}{V_s * \gamma_w}$$

SUMMARY

Simple Rules

- ❑Remember the basic definitions of e, n, w, s, G_s, γ, … etc.
- ❑Draw a phase diagram
- ❑Assume either V_s = 1.0 or V=1.0, if NOT given.
- ❑Often use s*e = w* G_s $G_S = \frac{W_S}{V_S * \gamma_W}$

USEFUL RELATIONSHIPS

$$\gamma = \frac{G_s(1+w)}{1+e}\gamma_w$$

$$\gamma = \frac{G_s + S*e}{1+e}\gamma_w$$

$$\gamma_d = \frac{G_s}{1+e}\gamma_w$$

$$\gamma_{sat} = \frac{G_s + e}{1+e}\gamma_w$$

$$G_s = \frac{W_s}{V_s*\gamma_w}$$

$$S*e = \mathrm{w}*G_s$$

Exercise

Prove that

$$\gamma = \frac{G_s(1+w)}{1+e}\gamma_w$$

$$\gamma = \frac{G_s + S*e}{1+e}\gamma_w$$

$$\gamma_d = \frac{G_s}{1+e}\gamma_w$$

$$\gamma_{sat} = \frac{G_s + e}{1+e}\gamma_w$$

$$S*e = \mathrm{w}*G_s$$

$$n = \frac{e}{1+e}$$

$$\gamma = \frac{W}{V} = \frac{W_w + W_s}{V_s + V_v} = \frac{\gamma_w V_w + \gamma_s V_s}{V_s + V_v} = \frac{\gamma_w V_w + \gamma_w G_s V_s}{V_s + V_v}$$

Weight-Volume Relationships

Table 3.1 Various Forms of Relationships for γ, γ_d, and γ_{sat}

Moist unit weight (γ)		Dry unit weight (γ_d)		Saturated unit weight (γ_{sat})	
Given	Relationship	Given	Relationship	Given	Relationship
w, G_s, e	$\frac{(1+w)G_s\gamma_w}{1+e}$	γ, w	$\frac{\gamma}{1+w}$	G_s, e	$\frac{(G_s+e)\gamma_w}{1+e}$
S, G_s, e	$\frac{(G_s+Se)\gamma_w}{1+e}$	G_s, e	$\frac{G_s\gamma_w}{1+e}$	G_s, n	$[(1-n)G_s+n]\gamma_w$
w, G_s, S	$\frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}}$	G_s, n	$G_s\gamma_w(1-n)$	G_s, w_{sat}	$\left(\frac{1+w_{sat}}{1+w_{sat}G_s}\right)G_s\gamma_w$
w, G_s, n	$G_s\gamma_w(1-n)(1+w)$	G_s, w, S	$\frac{G_s\gamma_w}{1+\left(\frac{wG_s}{S}\right)}$	e, w_{sat}	$\left(\frac{e}{w_{sat}}\right)\left(\frac{1+w_{sat}}{1+e}\right)\gamma_w$
S, G_s, n	$G_s\gamma_w(1-n)+nS\gamma_w$	e, w, S	$\frac{eS\gamma_w}{(1+e)w}$	n, w_{sat}	$n\left(\frac{1+w_{sat}}{w_{sat}}\right)\gamma_w$
		γ_{sat}, e	$\gamma_{sat}-\frac{e\gamma_w}{1+e}$	γ_d, e	$\gamma_d+\left(\frac{e}{1+e}\right)\gamma_w$
		γ_{sat}, n	$\gamma_{sat}-n\gamma_w$	γ_d, n	$\gamma_d+n\gamma_w$
		γ_{sat}, G_s	$\frac{(\gamma_{sat}-\gamma_w)G_s}{(G_s-1)}$	γ_d, S	$\left(1-\frac{1}{G_s}\right)\gamma_d+\gamma_w$
				γ_d, w_{sat}	$\gamma_d(1+w_{sat})$

EXAMPLE 3.2

Example 3.2

For a moist soil sample, the following are given.

Total volume: $V = 1.2\ \text{m}^3$
Total mass: $M = 2350$ kg
Moisture content: $w = 8.6\%$
Specific gravity of soil solids: $G_s = 2.71$

Determine the following.

a. Moist density
b. Dry density
c. Void ratio
d. Porosity
e. Degree of saturation
f. Volume of water in the soil sample

Solution

Part a
From Eq. (3.13),

$$\rho = \frac{M}{V} = \frac{2350}{1.2} = \mathbf{1958.3\ kg/m^3}$$

Part b
From Eq. (3.14),

$$\rho_d = \frac{M_s}{V} = \frac{M}{(1 + w)V} = \frac{2350}{\left(1 + \frac{8.6}{100}\right)(1.2)} = \mathbf{1803.3\ kg/m^3}$$

Part c
From Eq. (3.23),

$$\rho_d - \frac{G_s\rho_w}{1 + e}$$

$$e = \frac{G_s\rho_w}{\rho_d} - 1 = \frac{(2.71)(1000)}{1803.3} - 1 = \mathbf{0.503}$$

Part d
From Eq. (3.7),

$$n = \frac{e}{1 + e} = \frac{0.503}{1 + 0.503} = \mathbf{0.335}$$

Part e
From Eq. (3.19),

$s*e = w*\ G_s$

$$S = \frac{wG_s}{e} = \frac{\left(\frac{8.6}{100}\right)(2.71)}{0.503} = 0.463 = \mathbf{46.3\%}$$

Part f
The volume of water is

$$\frac{M_w}{\rho_w} = \frac{M - M_s}{\rho_w} = \frac{M - \frac{M}{1 + w}}{\rho_w} = \frac{2350 - \left(\frac{2350}{1 + \frac{8.6}{100}}\right)}{1000} = \mathbf{0.186\ m^3}$$

EXAMPLE 3.2

Alternate Solution

Refer to Figure 3.7.

Part a

$$\rho = \frac{M}{V} = \frac{2350}{1.2} = \mathbf{1958.3\ kg/m^3}$$

Part b

$$M_s = \frac{M}{1 + w} = \frac{2350}{1 + \frac{8.6}{100}} = 2163.9\ \text{kg}$$

$$\rho_d = \frac{M_s}{V} = \frac{M}{(1 + w)V} = \frac{2350}{\left(1 + \frac{8.6}{100}\right)(1.2)} = \mathbf{1803.3\ kg/m^3}$$

Part c

The volume of solids: $\frac{M_s}{G_s \rho_w} = \frac{2163.9}{(2.71)(1000)} = 0.798\ \text{m}^3$

The volume of voids: $V_v = V - V_s = 1.2 - 0.798 = 0.402\ \text{m}^3$

Void ratio: $e = \frac{V_v}{V_s} = \frac{0.402}{0.798} = \mathbf{0.503}$

Part d

$$\text{Porosity: } n = \frac{V_v}{V} = \frac{0.402}{1.2} = \mathbf{0.335}$$

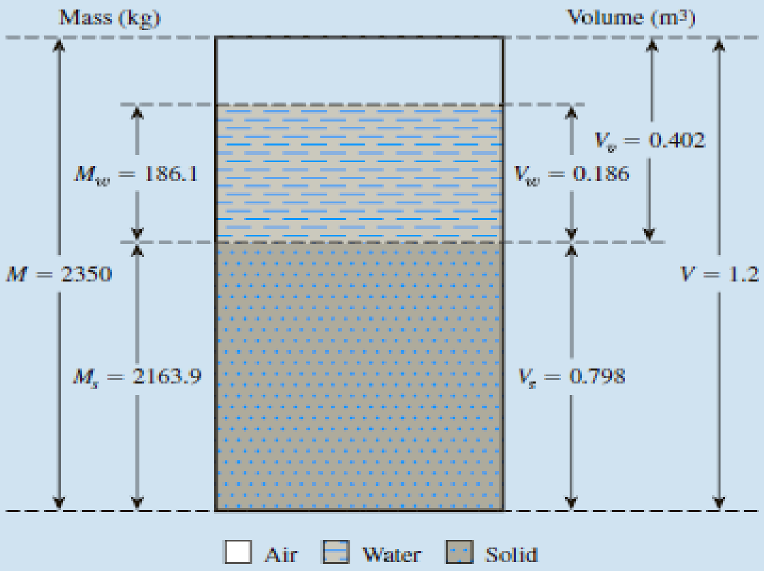


Figure 3.7

Part e

$$S = \frac{V_w}{V_v}$$

$$\text{Volume of water: } V_w = \frac{M_w}{\rho_w} = \frac{186.1}{1000} = 0.186\ \text{m}^3$$

Hence,

$$S = \frac{0.186}{0.402} = 0.463 = \mathbf{46.3\%}$$

Part f

From Part e,

$$V_w = \mathbf{0.186\ m^3}$$

EXAMPLE

The moist unit weight of a soil is 19.2 kN/m^3. Given that G_s 2.69 and $w = 9.8\%$, determine

a. Dry unit weight
b. Void ratio
c. Porosity
d. Degree of saturation

Think instead of ➡

$$\gamma = \frac{(Se + G_s)}{1+e}\gamma_w$$

$$Se = wG_s$$

a. $$\gamma_d = \frac{\gamma}{1+w} = \frac{19.2}{1+\frac{9.8}{100}} = \mathbf{17.5\ kN/m^3}$$

b. $$\gamma_d = 17.5 = \frac{G_s\gamma_w}{1+e} = \frac{(2.69)(9.81)}{1+e}; \quad e = \mathbf{0.51}$$

c. $$n = \frac{e}{1+e} = \frac{0.51}{1+0.51} = \mathbf{0.338}$$

d. $$S = \frac{wG_s}{e} = \frac{(0.098)(2.69)}{0.51} \times 100 = \mathbf{51.7\%}$$

EXAMPLE

A sample of soil has a total volume of 0.0282 m^3, a saturation rate of 56% and a water content of 18.5%. If the specific gravity of the soil is 2.529, determine the values of the wet and dry densities and void ratio of the soil.

$$From \quad wG_s = Se.........e = 0.835$$

$$From \quad \rho = \frac{(Se + G_s)}{1+e}\rho_w.........\rho = 1633Kg/m^3$$

$$From \quad \rho_d = \frac{G_s}{1+e}\rho_w.....\rho_d = 1378kg/m^3$$

GIVEN:

V= 0.0282 m^3
S = 56%
w = 18.5%
G_s = 2.529

Required:
e
ρ
ρ_d

$$\rho = \frac{M}{V} = \frac{46.043\ kg}{0.0282\ m^3} = \underline{1633\ kg/m^3}$$

$$\rho = \frac{M_s}{V} = \frac{38.855\ kg}{0.0282\ m^3} = \underline{1378\ kg/m^3}$$

$$e = \frac{V_v}{V_s} = \frac{0.012836\ m^3}{0.015364\ m^3} = \underline{0.835}$$

EXAMPLE

A saturated soil has a moisture content of 25.7% and a void ratio of 0.668. Determine the density and specific gravity of solids.

S = 1
w =25.7
e = 0.668

$$Se = wG_s$$

(Midterm Exam)

The total volume of a soil specimen is 80 cm^3 and it weighs 144g. The dry weight of the specimen is 128 g, and the density of the solids is 2.68 Mg/m^3. Find the

a)Water content

b)Wet density

c)Dry unit weight

d)Void ratio

e)Porosity

f)Degree of saturation

g)The mass of water to be added to a cubic meter of soil to reach 80% saturation.

(Midterm Exam)

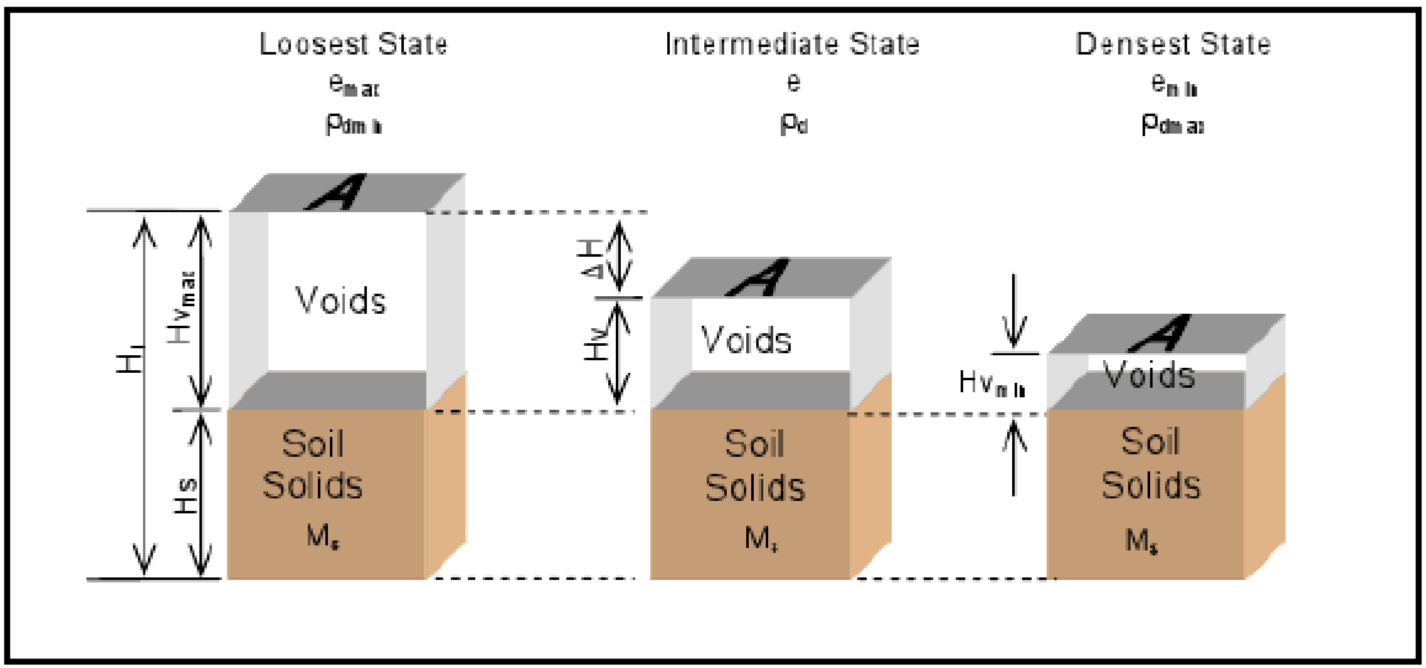
In its natural condition, a soil sample has a mass of 2290 g and a volume of 1.15 × 10^{-3} m^3. After being completely dried in an oven, the mass of the sample is 2035 g. The value of G_s for the soil solids is 2.68.

Determine the bulk density, moist unit weight, water content, void ratio, porosity, and degree of saturation.

Relative Density

- The **relative density** is the parameter that compare the volume reduction achieved from **compaction** to the maximum **possible** volume reduction.

- The relative density **Dr**, also called **density index** is commonly used to indicate the IN SITU **denseness** or **looseness** of granular soils.



Volume reduction from compaction of granular soil

Relative Density

D_r can be expressed either in terms of **void ratios** or **dry densities**.

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}}$$

where D_r = relative density, usually given as a percentage
e = *in situ* void ratio of the soil
e_{max} = void ratio of the soil in the loosest state
e_{min} = void ratio of the soil in the densest state

$$D_r = \frac{\left[\frac{1}{\gamma_{d(min)}}\right] - \left[\frac{1}{\gamma_d}\right]}{\left[\frac{1}{\gamma_{d(min)}}\right] - \left[\frac{1}{\gamma_{d(max)}}\right]} = \left[\frac{\gamma_d - \gamma_{d(min)}}{\gamma_{d(max)} - \gamma_{d(min)}}\right]\left[\frac{\gamma_{d(max)}}{\gamma_d}\right]$$

Remarks

- The relative density of a natural soil very strongly affects its engineering behavior.
- The range of values of D_r may vary from a minimum of zero for very LOOSE soil to a maximum of 100% for a very DENSE soil.
- Because of the irregular size and shape of granular particles, it is not possible to obtain a ZERO volume of voids. (Do you remember well-graded vs. poorly-graded!!)
- ASTM test designations D-4253 and D-4254 (2007) provide procedure for determining maximum and minimum dry unit weights of granular soils.

Remarks

- Granular soils are qualitatively described according to their relative densities as shown below

Relative Density (%)	Description of soil deposit
0-15	Very loose
15-50	Loose
50-70	Medium
70-85	Dense
85-100	Very dense

- The use of relative density has been restricted to granular soils because of the difficulty of determining e_{max} in clayey soils. Liquidity Index in fine-grained soils is of similar use as D_r in granular soils.

(Useful Formula)

You should know the following formulas:

$$V_{total} = V_{solid} + V_{voids} \rightarrow V_{total} = V_{solid} + V_{air} + V_{water}$$

$$W_{total} = W_{solid} + W_{water} \rightarrow (W_{air} = 0\ ,\ \ W_{solid} = W_{dry})$$

$$\gamma_{dry} = \frac{G_s \times \gamma_w}{1+e}\ ,\ \ \gamma_{dry} = \frac{\gamma_{moist}}{(1+\%w)}\ ,\ \ \gamma_{dry} = \frac{W_{dry}}{V_{total}}\ ,\ \ \gamma_{solid} = \frac{W_{dry}}{V_{solid}}$$

$$\gamma_{moist} = \frac{G_s \times \gamma_w (1+\%w)}{1+e}\ ,\ \ \gamma_{sat} = \frac{G_s \times \gamma_w \left(1+\frac{e}{G_s}\right)}{1+e} \rightarrow (S=1)$$

$$\gamma_{Z.A.V} = \frac{G_s \times \gamma_w}{1+G_s w} \rightarrow (S = 1 \rightarrow e = e_{min} = G_s w/1)$$

$$S.e = G_s.w\ ,\ \ S = \frac{V_{water}}{V_{voids}}\ ,\ \ \left(\text{at saturation} \rightarrow S = 1 \rightarrow w_{sat} = \frac{e}{G_s}\right)$$

$$w = \frac{\text{Weight of water}}{\text{Weight of solid}} = \frac{W_w}{W_s} = \frac{W_{wet} - W_{dry}}{W_{dry}} \times 100\%$$

$$e = \frac{V_{voids}}{V_{solid}} = \frac{V_T - V_s}{V_s}\ ,\ \ n = \frac{e}{1+e}\ ,\ \ n = \frac{V_{voids}}{V_{total}}$$

$$G_s = \frac{\gamma_{solid}}{\gamma_{water}}\ ,\ \ \gamma_{solid} = \frac{W_{dry}}{V_{solid}}\ ,\ \ \gamma_{water} = \frac{W_{water}}{V_{water}}$$

A proposed earth dam requires 7500 m^3 of compacted soil with relative density of 94% , maximum void ratio of 0.73, minimum void ratio of 0.4 and specific gravity =2.67.
Two borrow pits are available as described in the following table.
Choose the best borrow pit with minimum cost.

Borrow Pit	Degree of saturation %	Moisture content %	Cost ($/$m^3$)
A	82	18.43	10
B	100	24.34	5

Given:

D_r=94% , e_{max}=0.73 , e_{min}=0.4 , G_s=2.67

Solution:

First, we calculate the value of $\mathbf{V_s}$ that is required for the earth dam from **e**
We calculate **e** from $\mathbf{D_r}$
For each borrow pit, we calculate $\mathbf{V_T}$ from **e**, then the cost.

THE END