Shear Analysis and Design in Beams:

$$V_n = V_c + V_s \qquad \rightarrow \textit{Nominal shear strength in beams, SBC} - 18, 22.5.1.1$$

$$V_c = \left(\frac{\sqrt{f_c'}}{6}\right) b_w d \qquad \rightarrow \textit{shear strength carried by Concrete}$$

$$V_s = \frac{A_v f_{yt} d}{s} \qquad \rightarrow \textit{shear strength carried by steel, SBC} - 18, 22.5.10.5.3$$

$$V_{s\,max} = \frac{2}{3}\sqrt{f_c'} b_w d \qquad = 4V_c \qquad \rightarrow \textit{max. shear strength rsisted by steel, SBC} - 18, 22.5.1.2$$

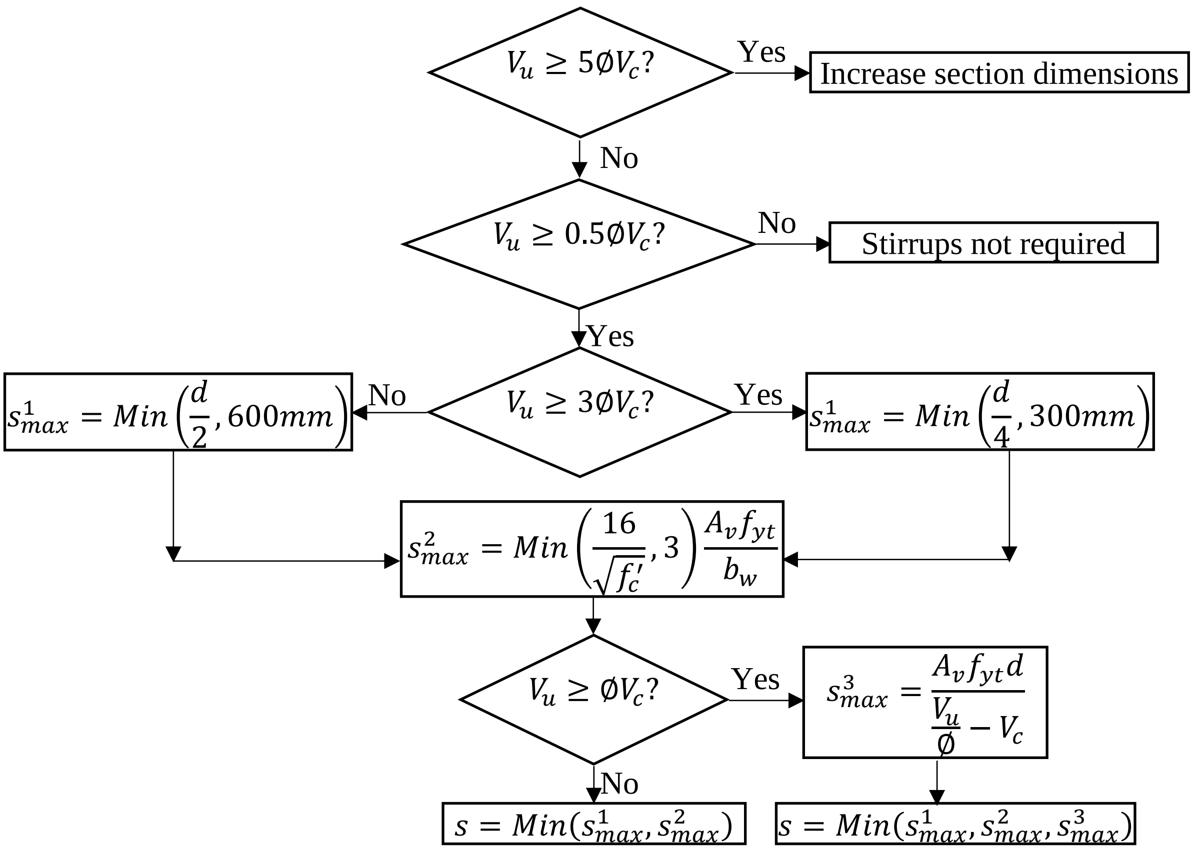
$$\emptyset = 0.75 \qquad \rightarrow \textit{shear strength reduction factor, SBC} - \textit{Table } 21.2.1$$

$$\emptyset V_e = w_u \frac{l_n}{2} \qquad \rightarrow \textit{shear force at the end of beam(simply supported beam)}$$

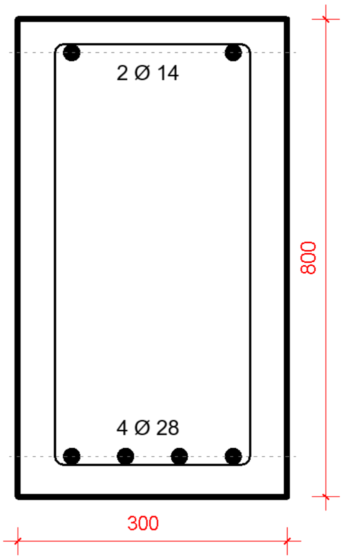
$$\emptyset V_m = w_{uL} \frac{l_n}{8} \qquad \rightarrow \textit{shear force at the middle of beam (simply supported beam)}$$

$$\left(\frac{V_u}{\emptyset}\right)_x = V_e - \frac{2x}{l_n}(V_e - V_m) \qquad \rightarrow \frac{V_u}{\emptyset} \textit{ at distance } x \textit{ from the face of supports}$$

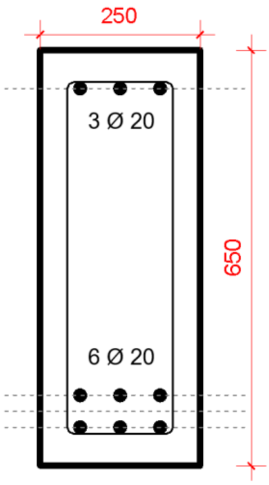
$$x = \frac{\left(V_e - \frac{V_u}{\emptyset}\right)}{V_e - V_m} \times \frac{l_n}{2} \qquad \rightarrow \textit{distance } x \textit{ from the face of supports where } \frac{V_u}{\emptyset} \textit{ occurs}$$



Beam A:
Ø 8 stirrups @ 250 mm



Beam B:
Ø 12 stirrups @ 100 mm



Which section is sufficient to resist ultimate shear stress of 300 kN? f'_c = 30 MPa, f_y = 420 MPa, S_l=30

Beam: A.

1- Find, *d*.

$$d = h - cover - d_s - \frac{d_b}{2} = 800 - 40 - 8 - 14 = 738\ mm$$

2- Find V_c:

$$V_c = \left(\frac{\sqrt{f_c'}}{6}\right) b_w d = \left(\frac{\sqrt{30}}{6}\right) 300 \times 738 = 202109.6\ N$$

3- Find V_s:

$$A_v = 2 \times \pi \frac{8^2}{4} = 100.53\ mm^2$$

$$V_s = \frac{A_v f_{yt} d}{s} = \frac{100.53\ \times 420 \times 738}{250} = 124641.1\ N$$

3- Find $ØV_n$:

$$V_n = V_c + V_s = 202109.6 + 124641.1 = 326750.7\ N$$

$$ØV_n = 0.75 \times 326.75 = 245.06\ kN$$

$$ØV_n = 245.06\ < V_u = 300$$

This beam cannot resist the existing shear stress.

Beam: B.

1- Find, *d*.

$$d_t = h - cover - d_s - \frac{d_b}{2} = 650 - 40 - 10 - 10 = 590\, mm$$

$$d_{min} = d_1 - S - d_b = 590 - 30 - 20 = 540\, mm$$

$$d = (d_t + d_{min})/2 = 565\, mm$$

2- Find V_c:

$$V_c = \left(\frac{\sqrt{f_c'}}{6}\right) b_w d = \left(\frac{\sqrt{30}}{6}\right) 250 \times 565 = 128943\, N$$

3- Find V_s:

$$A_v = 2 \times \pi \frac{12^2}{4} = 157.08\, mm^2$$

$$V_s = \frac{A_v f_{yt} d}{s} = \frac{157.08 \times 420 \times 565}{100} = 372750\, N$$

3- Find $ØV_n$:

$$V_n = V_c + V_s = 128943 + 372750 = 501693\, N$$

$$ØV_n = 0.75 \times 501.693 = 376.27\, kN$$

$$ØV_n = 376.27 > V_u = 300$$

This beam can resist the existing shear stress

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Design for shear a simply supported beam with 8-m clear span and the rectangular section of 850×400 mm, subjected to uniform loading including dead load of 45 kN/m and live load of 32 kN/m. Use two-legged 10 mm stirrups and steel depth d = 758 mm.

Determine the stirrup spacing at the critical section (as a multiple of 50 mm) and the stirrup distribution using a second spacing value 50 mm greater than the first one. f'_c = 20 MPa, f_y = 420 MPa, d_s=10 mm, S_l=28 mm.

1- Find factored shear force at end and middle of the span, V_e *and* V_m *and* $V_u/Ø$ at distance *d* from support.

$$w_u = 1.4DL + 1.7LL = 1.4 \times 45 + 1.7 \times 32 = 117.4\ kN/m$$

$$w_{uL} = 1.7LL = 1.7 \times 32 = 54.4\ kN/m$$

$$ØV_e = w_u \frac{l_n}{2} = 117.4 \times \frac{8}{2} = 469.6\ kN$$

$$V_e = \frac{469.6}{0.75} = 626.13\ kN$$

$$ØV_m = w_{uL} \frac{l_n}{8} = 54.4 \times \frac{8}{8} = 54.4\ kN$$

$$V_m = \frac{54.4}{0.75} = 72.53\ kN$$

$$\left(\frac{V_u}{Ø}\right)_x = V_e - \frac{2x}{l_n}(V_e - V_m) = 626.13 - \frac{2 \times 758}{8000}(626.13 - 72.53) = 521.226\ kN = 521226\ N$$

2- Check beam dimensions adequacy and if stirrups are required.

$$V_c = \left(\frac{\sqrt{f_c'}}{6}\right) b_w d = \left(\frac{\sqrt{20}}{6}\right) 400 \times 758 = 225992\ N$$

$$5ØV_c = 5 \times 0.75 \times 255.992 = 847.47\ kN > V_u = 521.23\ \ kN$$

$V_u < 5ØV_c$ ➔ section dimensions are satisfactory.

$$0.5ØV_c = 0.5 \times 0.75 \times 255.992 = 84.747\ kN < V_u = 521.23\ \ kN$$

$V_u > 0.5ØV_c$ ➔ stirrups are required, calculate A_v

$$A_v = 2 \times \pi \frac{{d_s}^2}{4} = 2 \times \pi \frac{10^2}{4} = 157.08\ mm^2$$

3- Find maximum spacings, S^1_{max}, S^2_{max} and S^3_{max}

$$3ØV_c = 3 \times 0.75 \times 255.992 = 508.482\ kN < V_u = 521.23\ \ kN$$

$$3ØV_c < V_u \rightarrow s^1_{max} = Min\left(\frac{d}{4}, 300mm\right) = 189.5\ mm$$

$$s^2_{max} = Min\left(\frac{16}{\sqrt{f_c'}}, 3\right)\frac{A_v f_{yt}}{b_w} = Min\left(\frac{16}{\sqrt{20}}, 3\right)\frac{157.08 \times 420}{400} = 494.8\ mm$$

$$s^3_{max} = \frac{A_v f_{yt} d}{\frac{V_u}{Ø} - V_c} = \frac{157.08 \times 420 \times 758}{\frac{521226}{0.75} - 255992} = 106.6\ mm$$

$$s = Min(s^1_{max}, s^2_{max}, s^3_{max}) = 106.6\ mm$$

Use s = 100 mm at critical section.

4- Find spacing at other sections.

First spacing s_1 = 100 mm ➔ Second spacing s_2 = 150mm ➔ Third spacing s_3 = 200 mm, … so on.

@ First spacing:

$$s_1 = 100mm\ @\ V_u = 521.226\ kN$$

@ Second spacing:

$$V_n = V_c + V_s = V_c + \frac{A_v f_{yt} d}{s} = 225992 + \frac{157.08 \times 420 \times 758}{150} = 559378\ N = 559.378\ kN$$

$$x = \frac{\left(V_e - \frac{V_u}{\emptyset}\right)}{V_e - V_m} \times \frac{l_n}{2} = \frac{(626.13 - 559.378)}{626.13 - 72.53} \times \frac{8}{2} = 0.482\ m$$

@ Third spacing:

$$V_n = V_c + V_s = V_c + \frac{A_v f_{yt} d}{s} = 225992 + \frac{157.08 \times 420 \times 758}{200} = 476031\ N = 476.031\ kN$$

$$x = \frac{\left(V_e - \frac{V_u}{\emptyset}\right)}{V_e - V_m} \times \frac{l_n}{2} = \frac{(626.13 - 476.031)}{626.13 - 72.53} \times \frac{8}{2} = 1.085\ m$$

@ Forth spacing:

$$V_n = V_c + V_s = V_c + \frac{A_v f_{yt} d}{s} = 225992 + \frac{157.08 \times 420 \times 758}{250} = 426023\ N = 426.023\ kN$$

$$x = \frac{\left(V_e - \frac{V_u}{\emptyset}\right)}{V_e - V_m} \times \frac{l_n}{2} = \frac{(626.13 - 426.023)}{626.13 - 72.53} \times \frac{8}{2} = 1.446\ m$$

@ No stirrups required:

$$\frac{V_u}{\emptyset} = V_n = 0.5\emptyset V_c = 0.5 \times 225992 = 112996\ N = 112.966\ kN$$

$$x = \frac{\left(V_e - \frac{V_u}{\emptyset}\right)}{V_e - V_m} \times \frac{l_n}{2} = \frac{(626.13 - 112.966\ )}{626.13 - 72.53} \times \frac{8}{2} = 3.708\ m$$

5- find number of stirrups.

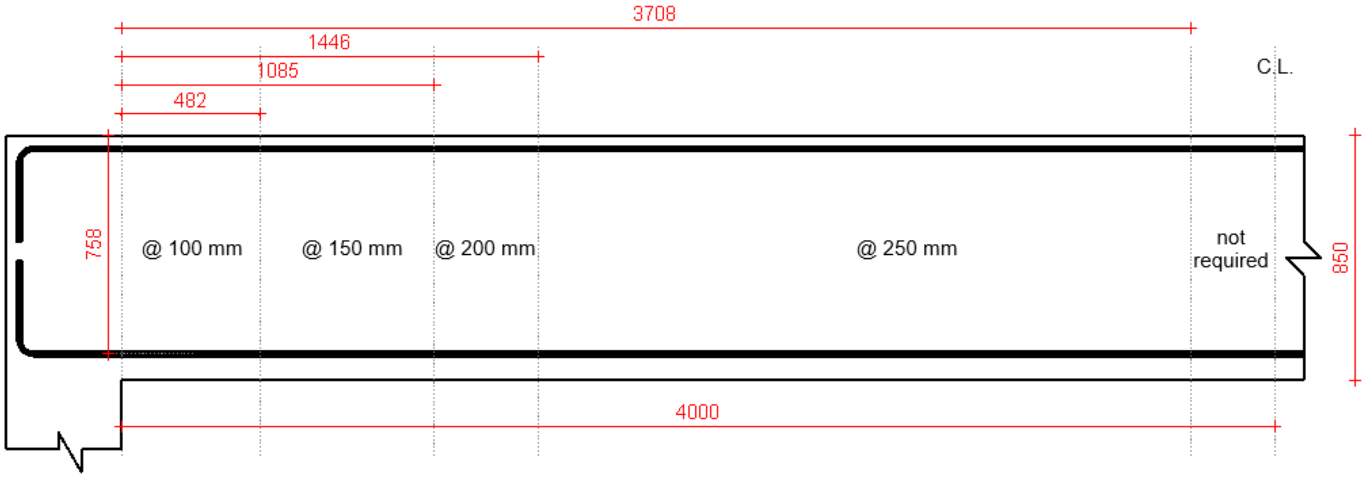
$$Number\ of\ stirrups = \frac{Distance\ in\ which\ stirrups\ of\ the\ same\ spacing\ has\ to\ be\ provided}{spacing\ between\ the\ stirrups}$$

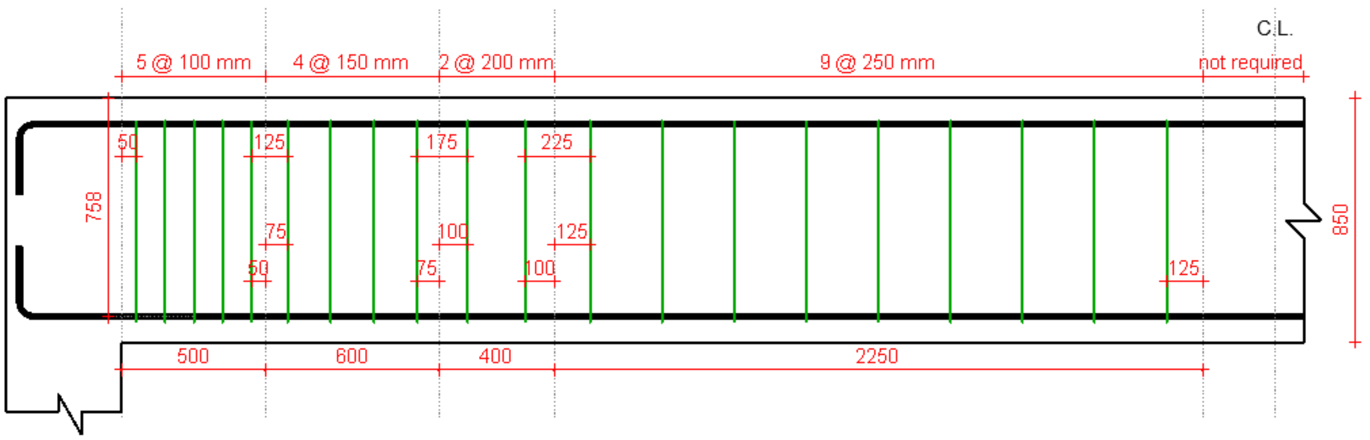
$$(Number\ of\ stirrups)_{s=100} = \frac{482 - 50}{100} = 4.32\ use\ 5@100\ mm$$

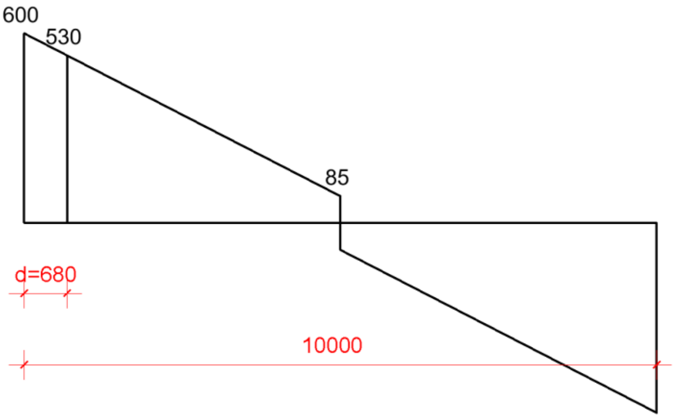
$$(Number\ of\ stirrups)_{s=150} = \frac{1085 - (50 + 5 \times 100)}{150} = 3.56\ use\ 4@150\ mm$$

$$(Number\ of\ stirrups)_{s=200} = \frac{1446 - (50 + 5 \times 100 + 4 \times 150)}{200} = 1.48\ use\ 2@200\ mm$$

$$(Number\ of\ stirrups)_{s=250} = \frac{3708 - (50 + 5 \times 100 + 4 \times 150 + 2 \times 200)}{250} = 8.63\ use\ 9@250\ mm$$







Using the shear force envelope given (V_u/Ø), (a) check that the section is large enough (800×500 mm) for shear design and (b) determine the required spacing of stirrups to resist maximum ultimate shear at the shear critical section of the girder (use Ø12 stirrups). f'_c = 28 MPa, f_y = 420.

(a)

$$V_c = \left(\frac{\sqrt{f_c'}}{6}\right) b_w d = \left(\frac{\sqrt{28}}{6}\right) 500 \times 680 = 299852\ N = 299.852\ kN$$

$$5V_c = 5 \times 299.852 = 1499.26\ kN > V_u = 530\ kN$$

$V_u < 5ØV_c$ ➔ section dimensions are satisfactory.

(b)

Find maximum spacings, S^1_{max}, S^2_{max} and S^3_{max}

$$A_v = 2 \times \pi \frac{{d_s}^2}{4} = 2 \times \pi \frac{12^2}{4} = 226.1\ mm^2$$

$$3ØV_c = 3 \times 0.75 \times 299.852 = 674.667\ kN > V_u = 530\ kN$$

$$3ØV_c > V_u \rightarrow s^1_{max} = Min\left(\frac{d}{2}, 600mm\right) = 340\ mm$$

$$s^2_{max} = Min\left(\frac{16}{\sqrt{f_c'}}, 3\right)\frac{A_v f_{yt}}{b_w} = Min\left(\frac{16}{\sqrt{28}}, 3\right)\frac{226.1 \times 420}{500} = 569.7\ mm$$

$$s^3_{max} = \frac{A_v f_{yt} d}{\frac{V_u}{Ø} - V_c} = \frac{157.08 \times 420 \times 758}{\frac{530}{0.75} - 299852} = 158.7\ mm$$

$$s = Min(s^1_{max}, s^2_{max}, s^3_{max}) = 158.7\ mm$$

Use s = 150 mm at critical section.

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