Analysis of Doubly Reinforced Rectangular Beams:

$$\varepsilon_S' = \left(\frac{c - d'}{c}\right)\varepsilon_{cu} \qquad \rightarrow strain\ at\ compression\ steel\ centroid, d'$$

$$a = \frac{A_s f_y - A_s'(f_y - 0.85 f_c')}{0.85 f_c' b} \qquad \rightarrow depth\ of\ equivalent\ stress\ block, both\ steel\ yielding$$

$$c = \frac{a}{\beta_1} \qquad \rightarrow neutrial\ axix\ depth, both\ steel\ yielding$$

$$c = \frac{-B \pm \sqrt{B^2 - 4AC_0}}{2A} \qquad \rightarrow neutrial\ axix\ depth, compression\ steel\ not\ yielding$$

$$A = 0.85 f_c' b \beta_1 \quad , \quad B = 600A_s' - 0.85A_s' f_c' - A_s f_y \quad , C_0 = 600A_s' d' \qquad \rightarrow factors$$

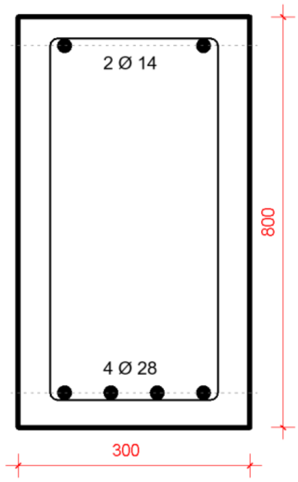
$$C_c = 085 f_c' ba \qquad \rightarrow compression\ force\ in\ concrete$$

$$C_s = A_s'(f_y - 0.85 f_c') \qquad \rightarrow compression\ force\ in\ steel, compression\ steel\ yielding$$

$$C_s = A_s'(f_s' - 0.85 f_c') \qquad \rightarrow compression\ force\ in\ steel, compression\ steel\ not\ yielding$$

$$f_s' = E_s \varepsilon_S' \qquad \rightarrow compression\ steel\ stress, not\ yielding$$

$$M_n = C_c\left(d - \frac{a}{2}\right) - C_s(d - d') \qquad \rightarrow nominal\ moment\ capacity$$



Determine design moment for the above section. f'_c = 20 MPa, f_y = 420 MPa, d_s=10

1- Find, d, d', d_t, d_{min}, A_s, A'_s and β_1

$$d = d_t = d_{min} = h - cover - d_s - \frac{d_b}{2} = 800 - 40 - 10 - 14 = 736\ mm$$

$$d' = cover + d_s + \frac{d_b}{2} = 40 + 10 + 7 = 57\ mm$$

$$A_s = 4 \times \pi \frac{28^2}{4} = 3463\ mm^2$$

$$A'_s = 2 \times \pi \frac{14^2}{4} = 307.9\ mm^2$$

f'_c = 20 MPa < 28 ➔ β_1 = 0.85

2- Find Neutral axis depth (c) from top fibers assuming both steel is yielding:

$$a = \frac{A_s f_y - A'_s(f_y - 0.85 f'_c)}{0.85 f'_c b} = \frac{3463 \times 420 - 307.9 \times (420 - 0.85 \times 20)}{0.85 \times 20 \times 300} = 178.51\ mm$$

$$c = \frac{a}{\beta_1} = \frac{178.51}{0.85} = 210\ mm$$

Check assumption (steel is yielding) for both tension and compression steel, d_{min} and d'

$$\varepsilon_{min} = \left(\frac{d_{min} - c}{c}\right)\varepsilon_{cu} = \left(\frac{736 - 210}{210}\right) \times 0.003 = 0.0075 \geq \varepsilon_y$$

0.0075 ≥ 0.0021 ➔ tension steel is yielding

$$\varepsilon'_S = \left(\frac{c - d'}{c}\right)\varepsilon_{cu} = \left(\frac{210 - 57}{210}\right) \times 0.003 = 0.0022 \geq \varepsilon_y$$

0.0022 ≥ 0.0021 ➔ compression steel is yielding

3- Find C_c and C_s

$$C_c = 085 f_c' ba = 0.85 \times 20 \times 300 \times 178.51 = 910389.6\ N$$

$$C_s = A_s'(f_y - 0.85 f_c') = 307.9 \times (420 - 0.85 \times 20) = 124074\ N$$

4- Find Ø:

Check strain at bottom layer

$$\varepsilon_t = \left(\frac{d_t - c}{c}\right)\varepsilon_{cu} = \left(\frac{736 - 210}{210}\right) \times 0.003 = 0.0075 \geq 0.005 \geq \varepsilon_y$$

0.0075 ≥ 0.005 ➔ tension control ➔ Ø = 0.9

5- Find design moment capacity $ØM_n$

$$M_n = C_c\left(d - \frac{a}{2}\right) - C_s(d - d') = 910389.6 \times \left(736 - \frac{178.51}{2}\right) - 124074 \times (736 - 57)$$

$$M_n = 673.04 \times 10^6\ N.mm$$

$$\emptyset M_n = 0.9 \times 673.04 = 605.7\ kN.m$$

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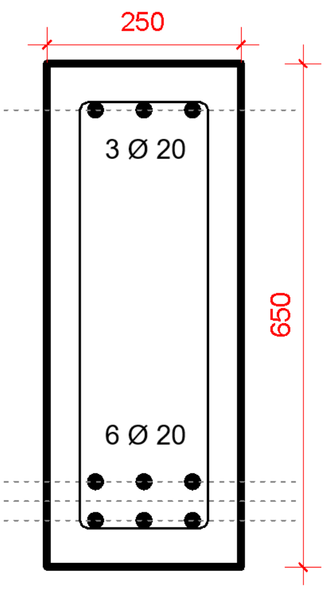

Determine design moment for the above section. f'c = 30 MPa, fy = 420 MPa, ds=10, Sl=30

1- Find, *d, d', dt, dmin, As, A's and β1*

$$d_t = h - cover - d_s - \frac{d_b}{2} = 650 - 40 - 10 - 10 = 590\ mm$$

$$d_{min} = d_1 - S - d_b = 590 - 30 - 20 = 540\ mm$$

$$d = (d_t + d_{min})/2 = 565\ mm$$

$$d' = cover + d_s + \frac{d_b}{2} = 40 + 10 + 10 = 60\ mm$$

$$A_s = 6 \times \pi \frac{20^2}{4} = 1885\ mm^2$$

$$A_s' = 3 \times \pi \frac{20^2}{4} = 942.5\ mm^2$$

f'_c = 30 MPa > 28 ➔ calculate β_1

$$\beta_1 = 0.85 - \frac{0.05(f_c' - 28)}{7} = 0.85 - \frac{0.05(30 - 28)}{7} = 0.84$$

2- Find Neutral axis depth (*c*) from top fibers assuming both steel is yielding:

$$a = \frac{A_s f_y - A_s'(f_y - 0.85 f_c')}{0.85 f_c' b} = \frac{1885\ \times 420 - 942.5 \times (420 - 0.85 \times 30)}{0.85 \times 30 \times 250} = 65.86\ mm$$

$$c = \frac{a}{\beta_1} = \frac{65.86}{0.84} = 78.41\ mm$$

Check assumption (steel is yielding) for both tension and compression steel, d_{min} and *d'*

$$\varepsilon_{min} = \left(\frac{d_{min} - c}{c}\right)\varepsilon_{cu} = \left(\frac{540 - 78.41}{78.41}\right) \times 0.003 = 0.0177 \geq \varepsilon_y$$

0.0177 ≥ 0.0021 ➔ tension steel is yielding

$$\varepsilon_S' = \left(\frac{c-d'}{c}\right)\varepsilon_{cu} = \left(\frac{78.41-60}{78.41}\right) \times 0.003 = 0.0007 < \varepsilon_y$$

0.0007 < 0.0021 ➔ compression steel is NOT yielding

3- Find revised neutral axis depth (*c*) from top fibers assuming compression steel is not yielding

$$A = 0.85 f_c' b \beta_1 = 0.85 \times 30 \times 250 \times 0.84 = 5355$$

$$B = 600A_s' - 0.85A_s' f_c' - A_s f_y = 600 \times 942.5 - 0.85 \times 942.5 \times 30 - 1885 \ \times 420 = -250227.855$$

$$C_0 = 600 A_s' d' = 600 \times 942.5 \times 60 = 3929200.66$$

$$c = \frac{-B \pm \sqrt{B^2 - 4AC_0}}{2A} = \frac{-(-250227.855) \pm \sqrt{-250227.855^2 - 4 \times 5355 \times 3929200.66}}{2 \times 5355}$$

$$c = (106.32\ )\text{or}(-50.06)$$

Take positive answer c= 106.32 mm

$$a = c\beta_1 = 106.32 \times 0.84 = 89.31$$

4- Find revised ε_s' and f_s'

$$\varepsilon_S' = \left(\frac{c-d'}{c}\right)\varepsilon_{cu} = \left(\frac{106.32-60}{106.32}\right) \times 0.003 = 0.00131$$

$$f_s' = E_s \varepsilon_S' = 200000 \times 0.00131 = 261.4\ MPa$$

5- Find C_c and C_s

$$C_c = 085 f_c' ba = 0.85 \times 30 \times 250 \times 89.31 = 569348.51\ N$$

$$C_s = A_s'(f_s' - 0.85 f_c') = 942.5 \times (261.4 - 0.85 \times 30) = 222332.84\ N$$

6- Find Ø:

Check strain at bottom layer

$$\varepsilon_t = \left(\frac{d_t - c}{c}\right)\varepsilon_{cu} = \left(\frac{590-106.32}{106.32}\right) \times 0.003 = 0.0136 \geq 0.005 \geq \varepsilon_y$$

0.0136 ≥ 0.005 ➔ tension control ➔ Ø = 0.9

7- Find design moment capacity $ØM_n$

$$M_n = C_c\left(d - \frac{a}{2}\right) - C_s(d-d') = 569348.51\ \times \left(565 - \frac{89.31}{2}\right) - 222332.84 \times (565-60)$$

$$M_n = 408.54 \times 10^6\ N.mm$$

$$ØM_n = 0.9 \times 408.04 = 367.7\ kN.m$$

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