Design of Singly Reinforced Rectangular Beams:

$$S_b = Max\left(25, d_b, \frac{4}{3}d_{agg}\right) \quad \rightarrow Minimum\ Bar\ Spacing, SBC304-18, 25.2.1$$

$$S_l = Max\left(25, \frac{4}{3}d_{agg}\right) \quad \rightarrow Minimum\ layer\ Spacing$$

$$b_{min} = nd_b + (n-1)S_b + 6d_s - d_b + 2cover \quad \rightarrow minmum\ beam\ width\ to\ accomadate\ rebars$$

$$n_{max} = \frac{b + S_b - 6d_s + d_b - 2cover}{d_b + S_b} \quad \rightarrow maximim\ number\ of\ rebars\ in\ one\ layer$$

$$R_n = \frac{M_u}{\emptyset bd^2} \quad \rightarrow factor$$

$$\rho = \frac{0.85f_c'}{f_y}\left(1 - \sqrt{1 - \frac{4R_n}{1.7f_c'}}\right) \quad \rightarrow steel\ ratio, for\ design\ purpose$$

$$\rho = \frac{A_s}{bd} \quad \rightarrow steel\ ratio$$

$$\rho_{min} = Max\left(\frac{\sqrt{f_c'}}{4f_y}, \frac{1.4}{f_y}\right) \quad \rightarrow minimum\ steel\ ratio, SBC304-18, 9.6.1.2$$

Design of a 300 × 600 mm rectangular section subjected to an ultimate bending moment M_u = 300 kN.m. given: f'_c = 25 MPa, f_y = 420 MPa, β_1=0.85, d_{agg}=20 mm, d_s=10 mm, d_b=20 mm. Assume two layers of steel.

1- Find, ρ and ρ_{min}: and A_s

Assume steel is yielding, Ø = 0.9, $d = h - 90$ = 510 mm.

$$R_n = \frac{M_u}{\emptyset b d^2} = \frac{300 \times 10^6}{0.9 \times 300 \times 510^2} = 4.272$$

$$\rho = \frac{0.85 f'_c}{f_y}\left(1 - \sqrt{1 - \frac{4R_n}{1.7 f'_c}}\right) = \frac{0.85 \times 25}{420}\left(1 - \sqrt{1 - \frac{4 \times 4.272}{1.7 \times 25}}\right) = 0.011472$$

$$A_s = \rho b d = 0.011472 \times 300 \times 510 = 1755.15\, mm^2$$

$$\rho_{min} = Max\left(\frac{\sqrt{f'_c}}{4 f_y}, \frac{1.4}{f_y}\right) = Max\left(\frac{\sqrt{25}}{4 \times 420}, \frac{1.4}{420}\right) = Max(0.00297, 0.00333) = 0.00333$$

$\rho \geq \rho_{min}$ ➔ ✓

2- Find provided area of steel:

$$\#\, rebars = \frac{A_s}{A_b} = \frac{1755.15}{\pi \frac{20^2}{4}} = 5.59 \approx 6\, rebars$$

$$A_{s\, provided} = \#\, rebars \times A_b = 6 \times \pi \frac{20^2}{4} = 1884.96\, mm^2$$

3- Check number of layers:

$$S_b = Max\left(25, d_b, \frac{4}{3} d_{agg}\right) = Max\left(25, 20, \frac{4}{3} \times 20\right) = 26.7\, mm$$

$$n_{max} = \frac{b + S_b - 6 d_s + d_b - 2 cover}{d_b + S_b} = \frac{300 + 26.7 - 6 \times 10 + 20 - 2 \times 40}{20 + 26.7} = 4.4 \approx 4$$

Six rebars need two layers as assumed. Three rebars in each layer.

4- Find effective depth, d:

$$S_l = Max\left(25, \frac{4}{3} d_{agg}\right) = Max\left(25, \frac{4}{3} \times 20\right) = 26.7\, mm$$

$$d_t = h - cover - d_s - \frac{d_b}{2} = 600 - 40 - 10 - 10 = 540\, mm$$

$$d_{min} = d_t - S - d_b = 540 - 26.7 - 20 = 493.3\, mm$$

$$d = (d_{min} + d_t)/2 = 515.65\, mm$$

5- Check assumption (steel is yielding) at most top steel layer, d_{min}

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{6 \times \pi \frac{20^2}{4} \times 420}{0.85 \times 20 \times 250} = 124.185\ mm$$

f'_c = 25 MPa < 28 ➔ $\beta_1 = 0.85$

$$c = \frac{a}{\beta_1} = \frac{124.185}{0.85} = 146.1\ mm$$

$$\varepsilon_{min} = \left(\frac{d_{min} - c}{c}\right) \varepsilon_{cu} = \left(\frac{493.3 - 146.1}{146.1}\right) \times 0.003 = 0.0071 \geq \varepsilon_y$$

$0.0071 \geq 0.0021$ ➔ steel is yielding

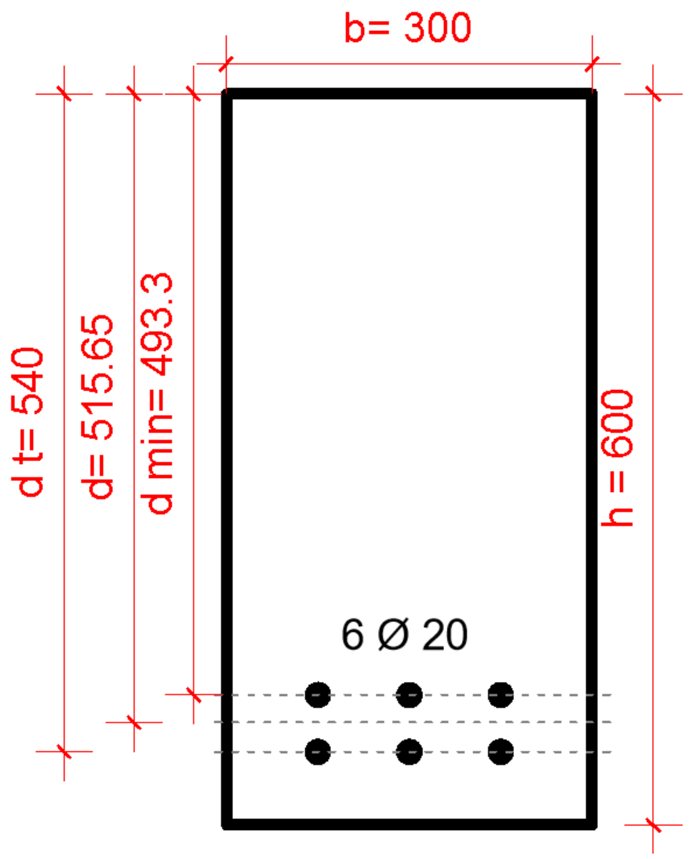
6- Check $\emptyset M_n \geq M_u$:

Since actual *d* is greater than assumed *d* ➔ this step is not necessary.

$$\emptyset M_n = \emptyset A_s f_y \left(d - \frac{a}{2}\right) = 0.9 \times 6 \times \pi \frac{20^2}{4} \times 420 \left(515.65 - \frac{124.185}{2}\right) = 323.166 \times 10^6 N.mm$$

$$\emptyset M_n = 323.166\ kN.m\ \geq M_u = 300\ kN.m$$

7- Draw beam section:



■■■■

Design of a 375 × 900 mm rectangular section subjected to an ultimate bending moment M_u = 1500 kN.m. Bar / layer spacing is 30 mm. d_s=10 mm, d_b=28 mm. f'_c = 35 MPa, f_y = 420 MPa. Assume two layers of steel.

1- Find, ρ and ρ_{min}: and A_s

Assume steel is yielding, Ø = 0.9, $d = h - 90$ = 810 mm.

$$R_n = \frac{M_u}{\emptyset b d^2} = \frac{1500 \times 10^6}{0.9 \times 375 \times 810^2} = 6.774$$

$$\rho = \frac{0.85 f_c'}{f_y}\left(1 - \sqrt{1 - \frac{4R_n}{1.7 f_c'}}\right) = \frac{0.85 \times 35}{420}\left(1 - \sqrt{1 - \frac{4 \times 6.774}{1.7 \times 35}}\right) = 0.01856$$

$$A_s = \rho b d = 0.011472 \times 375 \times 810 = 5637.7\, mm^2$$

$$\rho_{min} = Max\left(\frac{\sqrt{f_c'}}{4f_y}, \frac{1.4}{f_y}\right) = Max\left(\frac{\sqrt{35}}{4 \times 420}, \frac{1.4}{420}\right) = Max(0.00352, 0.00333) = 0.00352$$

$\rho \geq \rho_{min}$ ➔ ✓

2- Find provided area of steel:

$$\#\, rebars = \frac{A_s}{A_b} = \frac{5637.7}{\pi \frac{28^2}{4}} = 9.16 \;\approx 10\, rebars$$

$$A_{s\, provided} = \#\, rebars \times A_b = 10 \times \pi \frac{28^2}{4} = 6157.5\, mm^2$$

3- Check number of layers:

$$n_{max} = \frac{b + S_b - 6d_s + d_b - 2cover}{d_b + S_b} = \frac{375 + 30 - 6 \times 10 + 28 - 2 \times 40}{28 + 30} = 5.05 \approx 5$$

ten rebars need two layers as assumed. five rebars in each layer.

4- Find effective depth, d:

$$d_t = h - cover - d_s - \frac{d_b}{2} = 900 - 40 - 10 - 14 = 836\, mm$$

$$d_{min} = d_t - S - d_b = 836 - 30 - 28 = 778\, mm$$

$$d = (d_{min} + d_t)/2 = 807\, mm$$

5- Check assumption (steel is yielding) at most top steel layer, d_{min}

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{6 \times \pi \frac{28^2}{4} \times 420}{0.85 \times 35 \times 375} = 231.813\, mm$$

f'_c = 35 MPa > 28 ➔ β_1 should be calculated

$$\beta_1 = 0.85 - \frac{0.05(f_c' - 28)}{7} = 0.85 - \frac{0.05(35 - 28)}{7} = 0.814$$

$$c = \frac{a}{\beta_1} = \frac{231.813}{0.814} = 284.782\ mm$$

$$\varepsilon_{min} = \left(\frac{d_{min} - c}{c}\right)\varepsilon_{cu} = \left(\frac{778 - 284.782}{284.782}\right) \times 0.003 = 0.0052 \geq \varepsilon_y$$

0.0052 ≥ 0.0021 ➔ steel is yielding

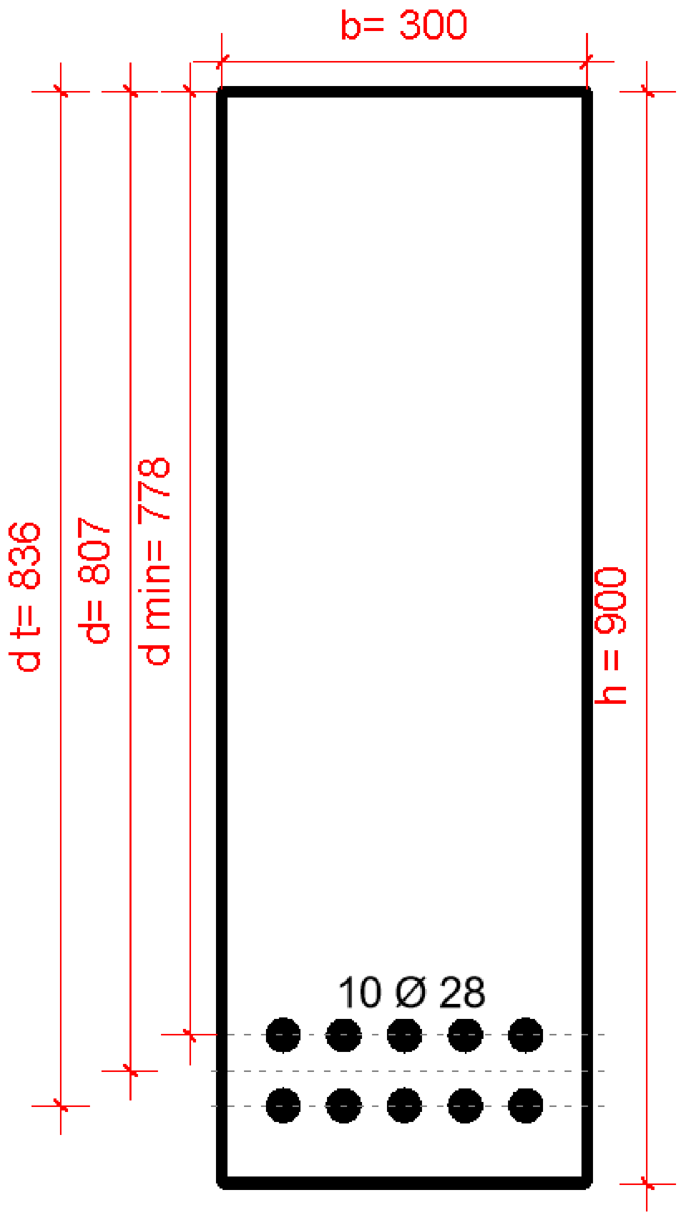
6- Check $ØM_n \geq M_u$:

Since actual d is less than assumed d ➔ this step is necessary.

$$\emptyset M_n = \emptyset A_s f_y \left(d - \frac{a}{2}\right) = 0.9 \times 10 \times \pi \frac{28^2}{4} \times 420 \left(807 - \frac{231.813}{2}\right) = 1608.55 \times 10^6 N.mm$$

$$\emptyset M_n = 1608.55\ kN.m \geq M_u = 1500\ kN.m$$

7- Draw beam section:



■■■■