Analysis of Singly Reinforced Rectangular Beams:

$\varepsilon_s = \left(\frac{d-c}{c}\right)\varepsilon_{cu}$ $\rightarrow strain\ at\ tension\ steel\ centroid, d$

$\varepsilon_{min} = \left(\frac{d_{min}-c}{c}\right)\varepsilon_{cu}$ $\rightarrow strain\ at\ top\ steel\ layer, d_{min}$

$\varepsilon_t = \left(\frac{d_t-c}{c}\right)\varepsilon_{cu}$ $\rightarrow strain\ at\ bottom\ steel\ layer, d_t$

$T = A_s f_y$ $\rightarrow tension\ force, steel\ yielding$

$T = A_s E_s \left(\frac{d-c}{c}\right)\varepsilon_{cu}$ $\rightarrow tension\ force, steel\ not\ yielding$

$C_c = 085 f_c' ba$ $\rightarrow compression\ force, steel\ yielding$

$C_c = 085 f_c' b\beta_1 c$ $\rightarrow compression\ force, steel\ not\ yielding$

$\rho = \frac{A_s}{bd}$ $\rightarrow steel\ ratio$

$\rho_{min} = Max\left(\frac{\sqrt{f_c'}}{4f_y}, \frac{1.4}{f_y}\right)$ $\rightarrow minimum\ steel\ ratio, SBC304-18, 9.6.1.2$

$a = \frac{A_s f_y}{0.85 f_c' b}$ $\rightarrow depth\ of\ equivalent\ rectangular\ stress\ block, steel\ yielding$

$c = \frac{a}{\beta_1}$ $\rightarrow neutrial\ axix\ depth, steel\ yielding$

$\beta_1 = 0.85 - \frac{0.05(f_c' - 28)}{7}$ $minimum = 0.65, maximum\ 0.85$ $\rightarrow SBC304-18, 22.2.2.4.3$

$c = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\beta d}}{2\alpha}$ $\rightarrow neutrial\ axix\ depth, steel\ not\ yielding$

$\alpha = 0.85 f_c' b\beta_1$, $\beta = A_s E_s \varepsilon_{cu}$ $\rightarrow factors$

$M_n = A_s f_y\left(d - \frac{a}{2}\right)$ $\rightarrow nominal\ moment\ capacity, steel\ yielding$

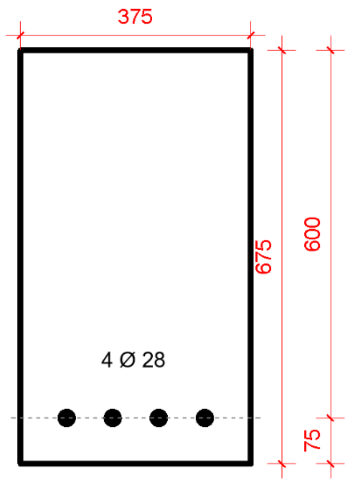
$M_n = T\left(d - \frac{a}{2}\right)$ $\rightarrow nominal\ moment\ capacity, steel\ not\ yielding$

$\emptyset = 0.9$ $\rightarrow tension\ control, \varepsilon_t \geq 0.005$

$\emptyset = 0.65$ $\rightarrow compression\ control, \varepsilon_t \leq \varepsilon_y$

$\emptyset = 0.65 + 0.25\frac{\varepsilon_t - \varepsilon_{ty}}{0.005 - \varepsilon_{ty}}$ $\rightarrow transition, \varepsilon_y < \varepsilon_t < 0.005$

$g = \frac{\sum A_{si} y_i}{A_{si}}$ $\rightarrow depth\ from\ bottom\ to\ centroid\ of\ steel$



Determine design moment for the above section. f'_c = 30 MPa, f_y = 420 MPa, β_1=0.85

1- Find, ρ and ρ_{min}:

$$\rho = \frac{A_s}{bd} = \frac{4 \times \pi \frac{28^2}{4}}{375 \times 600} = 0.0109$$

$$\rho_{min} = Max\left(\frac{\sqrt{f_c'}}{4f_y}, \frac{1.4}{f_y}\right) = Max\left(\frac{\sqrt{30}}{4 \times 420}, \frac{1.4}{420}\right) = Max(0.0033,0.0033) = 0.0033$$

$\rho \geq \rho_{min}$ ➔ ✓

3- Find Neutral axis depth (*c*) from top fibers assuming steel is yielding:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{4 \times \pi \frac{28^2}{4} \times 420}{0.85 \times 30 \times 375} = 108.12\ mm$$

β_1 given= 0.85

$$c = \frac{a}{\beta_1} = \frac{108.12}{0.85} = 127.2\ mm$$

Check assumption (steel is yielding) at most top steel layer, d_{min}

$$\varepsilon_{min} = \left(\frac{d_{min} - c}{c}\right)\varepsilon_{cu} = \left(\frac{600 - 127.2}{127.2}\right) \times 0.003 = 0.0111 \geq \varepsilon_y$$

0.0111 ≥ 0.0021 ➔ steel is yielding

4- Find Ø:

Check strain at bottom layer

$$\varepsilon_t = \left(\frac{d_t - c}{c}\right)\varepsilon_{cu} = \left(\frac{600 - 127.2}{127.2}\right) \times 0.003 = 0.0111 \geq 0.005 \geq \varepsilon_y$$

0.0111 ≥ 0.005 ➔ tension control ➔ Ø = 09

5- Find design moment capacity $\emptyset M_n$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 4 \times \pi \frac{28^2}{4} \times 420 \left(600 - \frac{108.12}{2}\right) = 564.46 \times 10^6 N.mm$$

$$\emptyset M_n = 0.9 \times 564.46 = 336.2\ kN.m$$

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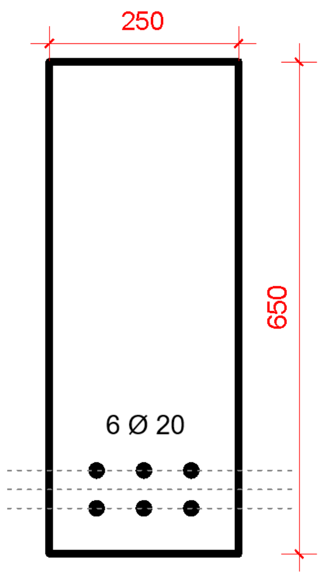

Determine design moment for the above section with six 20-mm bars in two layers. Bar / layer spacing is 30 mm. stirrups diameter is 10 mm. f'_c = 20 MPa, f_y = 420 MPa

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1- Find beam effective depth, *d*:

$$d_1 = h - cover - d_s - \frac{d_b}{2} = 650 - 40 - 10 - 10 = 590\ mm$$

$$d_2 = d_1 - S - d_b = 590 - 30 - 20 = 540\ mm$$

$$d = (d_1 + d_2)/2 = 565\ mm$$

Alternatively:

$$g = \frac{\sum A_{si} y_i}{A_{si}} = \frac{\left[\left(3 \times \pi \frac{20^2}{4}\right) \times (40 + 10 + 10)\right] + \left[\left(3 \times \pi \frac{20^2}{4}\right) \times (40 + 10 + 20 + 30 + 10)\right]}{\left(6 \times \pi \frac{20^2}{4}\right)} = 85\ mm$$

$$d = h - g = 650 - 85 = 565\ mm$$

2- Find, ρ and ρ_{min}:

$$\rho = \frac{A_s}{bd} = \frac{6 \times \pi \frac{20^2}{4}}{250 \times 565} = 0.01334$$

$$\rho_{min} = Max\left(\frac{\sqrt{f_c'}}{4f_y}, \frac{1.4}{f_y}\right) = Max\left(\frac{\sqrt{20}}{4 \times 420}, \frac{1.4}{420}\right) = Max(0.00266, 0.00333) = 0.00333$$

$\rho \geq \rho_{min}$ ➔ ✓

3- Find Neutral axis depth (*c*) from top fibers assuming steel is yielding:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{6 \times \pi \frac{20^2}{4} \times 420}{0.85 \times 20 \times 250} = 186.28\ mm$$

$f'_c = 20$ MPa < 28 ➔ $\beta_1 = 0.85$

$$c = \frac{a}{\beta_1} = \frac{186.28}{0.85} = 219.15\ mm$$

Check assumption (steel is yielding) at most top steel layer, d_{min}

$$\varepsilon_{min} = \left(\frac{d_{min} - c}{c}\right)\varepsilon_{cu} = \left(\frac{540 - 219.15}{219.15}\right) \times 0.003 = 0.0044 \geq \varepsilon_y$$

0.0044 ≥ 0.002 ➔ steel is yielding

4- Find Ø:

Check strain at bottom layer

$$\varepsilon_t = \left(\frac{d_t - c}{c}\right)\varepsilon_{cu} = \left(\frac{590 - 219.15}{219.15}\right) \times 0.003 = 0.00507 \geq 0.005 \geq \varepsilon_y$$

0.00507 ≥ 0.005 ➔ tension control ➔ Ø = 0.9

5- Find design moment capacity $ØM_n$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = 6 \times \pi \frac{20^2}{4} \times 420\left(565 - \frac{186.28}{2}\right) = 373.56 \times 10^6 N.mm$$

$$ØM_n = 0.9 \times 373.56 = 336.2\ kN.m$$

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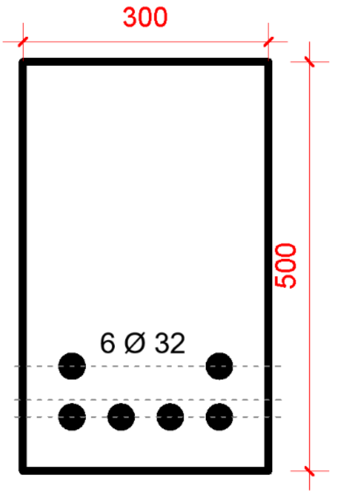

Determine design moment for the above section. Bar / layer spacing is 30 mm. stirrups diameter is 10 mm.
f'_c = 30 MPa, f_y = 420 MPa, β_1=0.85

1- Find beam effective depth, *d*:

$$g = \frac{\sum A_{si} y_i}{A_{si}} = \frac{\left[\left(4 \times \pi \frac{32^2}{4}\right) \times (40 + 10 + 16)\right] + \left[\left(2 \times \pi \frac{32^2}{4}\right) \times (40 + 10 + 32 + 30 + 16)\right]}{\left(6 \times \pi \frac{32^2}{4}\right)}$$

$$g = 86.7\ mm$$

$$d = h - g = 500 - 86.7 = 413.3\ mm$$

2- Find, ρ and ρ_{min}:

$$\rho = \frac{A_s}{bd} = \frac{6 \times \pi \frac{32^2}{4}}{300 \times 413.3} = 0.0389$$

$$\rho_{min} = Max\left(\frac{\sqrt{f_c'}}{4f_y}, \frac{1.4}{f_y}\right) = Max\left(\frac{\sqrt{30}}{4 \times 420}, \frac{1.4}{420}\right) = Max(0.0033, 0.0033) = 0.0033$$

$\rho \geq \rho_{min}$ ➔ ✓

3- Find Neutral axis depth (*c*) from top fibers assuming steel is yielding:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{6 \times \pi \frac{32^2}{4} \times 420}{0.85 \times 30 \times 300} = 264.8\ mm$$

β_1 given= 0.85

$$c = \frac{a}{\beta_1} = \frac{264.8}{0.85} = 311.5\ mm$$

Check assumption (steel is yielding) at most top steel layer, d_{min}

$$\varepsilon_{min} = \left(\frac{d_{min} - c}{c}\right)\varepsilon_{cu} = \left(\frac{372 - 311.5}{311.5}\right) \times 0.003 = 0.0006 \leq \varepsilon_y$$

0.0006 ≤ 0.002 ➔ steel is not yielding

4- Find Neutral axis depth (c) from top fibers assuming steel is not yielding

$$\alpha = 0.85 f_c' b \beta_1 = 0.85 \times 30 \times 300 \times 0.85 = 6502.5$$

$$\beta = A_s E_s \varepsilon_{cu} = 6 \times \pi \frac{32^2}{4} \times 200000 \times 0.003 = 2.89 \times 10^6$$

$$c = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\beta d}}{2\alpha} = \frac{-2.89 \times 10^6 \pm \sqrt{(2.89 \times 10^6)^2 - 4 \times 6502.5 \times 2.89 \times 10^6 \times 413.3}}{2 \times 6502.5}$$

$$c = 260.7\ mm$$

And

$$a = \beta_1 c = 0.85 \times 260.7 = 221.6\ mm$$

5- Find T and C_c assuming steel is not yielding

$$T = A_s E_s \left(\frac{d - c}{c}\right) \varepsilon_{cu} = 6 \times \pi \frac{32^2}{4} \times 200000 \left(\frac{413.3 - 260.7}{260.7}\right) \times 0.003$$

$$T = 1693891\ N = 1693.9\ kN$$

$$C_c = 085 f_c' b \beta_1 c = 0.85 \times 30 \times 300 \times 0.85 \times 260.7 = 1695201\ N = 1695.2\ kN$$

Take average

$$T = \frac{1693.9 + 1695.2}{2} = 1694.6\ kN$$

6- Find Ø:

Check strain at bottom layer

$$\varepsilon_t = \left(\frac{d_t - c}{c}\right) \varepsilon_{cu} = \left(\frac{434 - 260.7}{260.7}\right) \times 0.003 = 0.00199 \le 0.005$$

0.00199 ≤ ε_y=0.0021 ➔ compression control ➔ Ø = 0.65

7- Find design moment capacity $ØM_n$

$$M_n = T\left(d - \frac{a}{2}\right) = 1694.6 \times 10^3 \left(413.3 - \frac{221.6}{2}\right) = 512.6 \times 10^6 N.mm$$

$$ØM_n = 0.65 \times 512.6 = 333.2\ kN.m$$

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