Flexural Behavior of RC Beams: Cracked Stage:

$$E_c = 4700\sqrt{f_c'} \quad \rightarrow Cocrete\ MoE, SBC304-18, 19.2.2.1.b$$

$$E_s = 200{,}000\ MPa \quad \rightarrow Rebar\ MoE, SBC304-18, 20.2.2.2$$

$$n = \frac{E_s}{E_c} \quad \rightarrow Modular\ ratio$$

$$d = h - cover - d_s - \frac{d_b}{2} \quad \rightarrow beam\ effective\ depth, one\ steel\ layer$$

$$\frac{bx^2}{2} - nA_s d + nA_s x = 0 \quad \rightarrow Neutral\ axis\ depth\ (x)\ from\ top\ fibers$$

$$I = \frac{bx^3}{3} + (nA_s)(d-x)^2 \quad \rightarrow Moment\ of\ inertia\ about\ neutrial\ axis$$

$$f = \frac{My}{I} \quad \rightarrow Bending\ Stresses$$

$$f_c = \frac{My}{I} \quad \rightarrow Bending\ Stresses\ At\ extreme\ compression\ fiber\ (y = x)$$

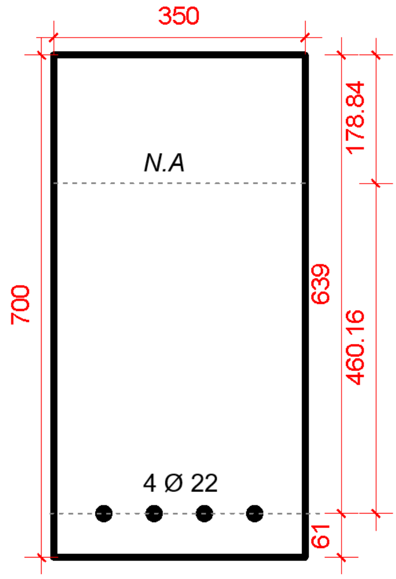
$$f_c = \frac{f_s}{n} \quad \rightarrow from\ Modular\ ratio$$

$$f_s = n\frac{My}{I} \quad \rightarrow Bending\ Stresses\ At\ steel\ level\ (y = d - x)$$

$$M_c = \frac{f_c I}{y} \rightarrow Allowable\ resisting\ moment\ based\ on\ allowable\ compression\ in\ cocrete (y = x)$$

$$M_s = \frac{f_s I}{ny} \rightarrow Allowable\ resisting\ moment\ based\ on\ to\ allowable\ tension\ in\ steel\ (y = d - x)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \rightarrow Quadratic\ formula$$



Assuming a cracked elastic section, calculate the bending stresses in the beam shown in the figure above using the transformed area method. f_c' = 25 MPa, n = 8, d_s = 10 mm, and tension steel stress is equal to 88.095 kN.m.

Since the beam is cracked, it is not homogeneous (i.e., $y \neq$ h/2) and (i.e., $I \neq I_g$).

1- Find beam effective depth, d:

$$d = h - cover - d_s - \frac{d_b}{2} = 700 - 40 - 10 - 11 = 639\ mm$$

2- Find Area of steel, A_s:

$$A_s = \# \times A_b = 4 \times \pi \frac{22^2}{4} = 1520.5\ mm^2$$

3- Find Neutral axis depth (x) from top fibers:

$$\frac{bx^2}{2} - nA_s d + nA_s x = 0$$

$$\frac{350x^2}{2} - 8 \times 1520.5 \times 639 + 8 \times 1520.5 \times x = 0$$

$$x^2 + 69.51x - 44415.98 = 0$$

Quadratic formula:

$$x = \frac{-69.51 \pm \sqrt{69.51^2 - 4 \times 1 \times -44415.95}}{2 \times 1} = (178.84)\ or (-248.35)$$

Take the positive answer, x = 178.84 *mm*

4- Find moment of inertia about neutral axis:

$$I = \frac{bx^3}{3} + (nA_s)(d - x)^2 = \frac{350 \times 178.84^3}{3} + (8 \times 1520.5)(639 - 178.84)^2 = 3.243 \times 10^9\ mm^4$$

5- Find bending Stresses at extreme compression fiber (y=x)

$$f_c = \frac{My}{I} = \frac{88.095 \times 10^6\ N.mm \times 178.84\ mm}{3.243 \times 10^9\ mm^4} = 4.86\ MPa$$

6- Find bending Stresses at steel level (y=$d-x$)

$$f_s = n\frac{My}{I} = 8 \times \frac{88.095 \times 10^6\ N.mm \times (639 - 178.84)\ mm}{3.243 \times 10^9\ mm^4} = 100\ MPa$$

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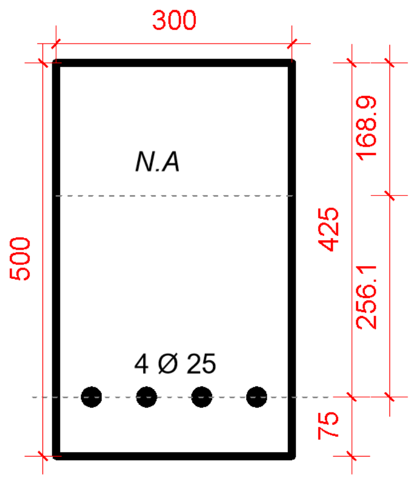

Determine the allowable resisting moments of the beam above (causing compression and tension stresses) if the allowable stress in concrete is f_c = 12 MPa and in steel is f_s = 160 MPa. f_c'= 25 MPa and beam effective depth is 425 mm.

Since the beam is cracked, it is not homogeneous (i.e., $y \neq$ h/2) and (i.e., $I \neq I_g$).

1- Find modular ratio, n:

$$n = \frac{E_s}{E_c} = \frac{200{,}000}{4700\sqrt{f_c'}} = \frac{200{,}000}{4700\sqrt{25}} = 8.51$$

2- Find Area of steel, A_s:

$$A_s = \# \times A_b = 4 \times \pi\frac{25^2}{4} = 1964\ mm^2$$

3- Find Neutral axis depth (x) from top fibers:

$$\frac{bx^2}{2} - nA_s d + nA_s x = 0$$

$$\frac{300x^2}{2} - 8.51 \times 1964 \times 425 + 8.51 \times 1964 \times x = 0$$

$$x^2 + 111.42x - 47356 = 0$$

Quadratic formula:

$$x = \frac{-111.42 \pm \sqrt{111.42^2 - 4 \times 1 \times -47356}}{2 \times 1} = (168.9)\ or (-280.34)$$

Take the positive answer, x = 168.9 mm

4- Find moment of inertia about neutral axis:

$$I = \frac{bx^3}{3} + (nA_s)(d-x)^2 = \frac{300 \times 168.9^3}{3} + (8.51 \times 1964)(425 - 168.9)^2 = 1.578 \times 10^9\ mm^4$$

5- Find allowable resisting moment based on allowable compression in concrete (y=x)

$$M_c = \frac{f_c I}{y} = \frac{12\ N/mm^2 \times 1.578 \times 10^9 mm^4}{168.9\ mm} = 112100790.3\ N.mm = 112.1\ kN.m$$

6- Find allowable resisting moment based on to allowable tension in steel (y=d – x)

$$M_s = \frac{f_s I}{ny} = \frac{160\ N/mm^2 \times 1.578 \times 10^9 mm^4}{8.51 \times (425 - 168.9)\ mm} = 115859713.1\ N.mm = 115.86\ kN.m$$

<u>Extra step:</u>

The resisting moment of the section is 112.1 kN.m (the smallest) because if that value is exceeded, concrete becomes overstressed even though the steel stress is less than its allowable stress.

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