Brouwer's conjecture

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In the mathematical field of [spectral graph theory](https://en.wikipedia.org/wiki/Spectral_graph_theory), **Brouwer's conjecture** is a conjecture by [Andries Brouwer](https://en.wikipedia.org/wiki/Andries_Brouwer%22%20%5Co%20%22Andries%20Brouwer) on upper bounds for the intermediate sums of the [eigenvalues](https://en.wikipedia.org/wiki/Eigenvalues_and_eigenvectors) of the [Laplacian](https://en.wikipedia.org/wiki/Laplacian_matrix) of a [graph](https://en.wikipedia.org/wiki/Graph_%28discrete_mathematics%29) in term of its number of edges.[[1]](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_note-:0-1)

The conjecture states that if *G* is a simple undirected graph and *L*(*G*) its Laplacian matrix, then its eigenvalues *λn*(*L*(*G*)) ≤ *λn*−1(*L*(*G*)) ≤ ... ≤ *λ*1(*L*(*G*)) satisfy

{\displaystyle \sum \_{i=1}^{t}\lambda \_{i}(L(G))\leq m(G)+\left({\begin{array}{c}t+1\\2\end{array}}\right),\quad t=1,\ldots ,n}



 where *m*(*G*) is the number of edges of *G*.

State of the art[[edit](https://en.wikipedia.org/w/index.php?title=Brouwer%27s_conjecture&action=edit&section=1)]

Brouwer has confirmed by computation that the conjecture is valid for all graphs with at most 10 vertices.[[1]](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_note-:0-1) It is also known that the conjecture is valid for any number of vertices if *t* = 1, 2, *n* − 1, and *n*.

For certain types of graphs, Brouwer's conjecture is known to be valid for all *t* and for any number of vertices. In particular, it is known that is valid for trees,[[2]](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_note-2) and for unicyclic and bicyclic graphs.[[3]](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_note-3) It was also proved that Brouwer’s conjecture holds for two large families of graphs; the first family of graphs is obtained from a clique by identifying each of its vertices to a vertex of an arbitrary c-cyclic graph, and the second family is composed of the graphs in which the removal of the edges of the maximal complete bipartite subgraph gives a graph each of whose non-trivial components is a c-cyclic graph.[[4]](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_note-4) For certain sequences of random graphs, Brouwer's conjecture holds true with probability tending to one as the number of vertices tends to infinity.[[5]](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_note-5)

References[[edit](https://en.wikipedia.org/w/index.php?title=Brouwer%27s_conjecture&action=edit&section=2)]

* 1. ^ [Jump up to:***a***](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_ref-:0_1-0) [***b***](https://en.wikipedia.org/wiki/Brouwer%27s_conjecture#cite_ref-:0_1-1) *Brouwer, Andries E.; Haemers, Willem H. (2012). Spectra of Graphs. Universitext. New York, NY: Springer New York.*[*doi*](https://en.wikipedia.org/wiki/Doi_%28identifier%29)*:*[*10.1007/978-1-4614-1939-6*](https://doi.org/10.1007/978-1-4614-1939-6)*.*[*ISBN*](https://en.wikipedia.org/wiki/ISBN_%28identifier%29)[*978-1-4614-1938-9*](https://en.wikipedia.org/wiki/Special%3ABookSources/978-1-4614-1938-9)*.*
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