2023 International Conference on Topology and its Applications,

July 3-7, 2023, Nafpaktos, Greece

ABSTRACTS

Department of Mathematics,
University of Patras, Greece

2023 International Conference on Topology and its Applications,

July 3-7, 2023, Nafpaktos, Greece

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Contents

Abstracts

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A transversality condition of codimension one submanifolds

Abstract. In this work, we wish to derive a condition of transversality of two hypersurfaces M^n and P^n of $\overline{M}^{n+1}$ along the boundary ∂M^n, provided that $\partial M^n \subset P^n$. This condition is given by the transversality of the classical Newton transformation T_r. In particular we proof that at a point p of the boundary ∂M^n and for every $1 \leq r \leq n-1$ we have:

$$\langle T_r \nu, \nu \rangle = \rho^r \sigma_r \tag{1}$$

Where $\rho = \langle \xi, \nu \rangle$, σ_r is the r-th symmetric function of the principal curvatures of the inclusion $\partial M^n \subset P^n$ with respect to the outward unit normal vector field ν normal to ∂M^n, and ξ is the vector field normal to P^n in $\overline{M}^{n+1}$.

Relation (1) shows that the ellipticity of the Newton transformation T_r, for some $1 \leq r \leq n-1$ on M^n, implies the transversality of the hypersurfaces M^n and P^n along ∂M^n. A similar formula of (1) was also obtained in [1] by the author and M.Benalili in context of pseudo-Riemannian spaces. It is to emphasize the importance of the application of Newton transformations in intrinsic Riemannian geometry (see [3], [4], [7] and [8]).

The formula for the Newton transformations implies the relation between transversality of M^n and P^n and ellipticity of T_r provided that P^n is totally geodesic in $\overline{M}^{n+r}$.

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Surgery by William Paul Thurston in the glued space of star geometric bodies in three or more dimensions

Mathematics Subject Classification (MSC): 51F, 51H, 51H20, 52A23

Abstract. In this article, gluing convex (nonconvex) star-hedral polyhedral along their sides using proper pairing, we will construct Euclidean, spherical and hyperbolic manifolds, and with present the author's method of **'transforming of manifolds'** that allows gluing the fundamental domain of the fundamental group of manifolds of a star geometric body based on structural models of metric geometry and metric topology, and we obtain a partition of a star geometric body with the symmetry of an arbitrary simpletial group by cutting and re-gluing in three or more dimensions.

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The *B*-completion of fuzzy quasi-metric spaces

Mathematics Subject Classification (MSC): 54A40

Abstract. We describe an approach for completing any fuzzy quasi-metric space. The completion is constructed as an addition to the bicompletion of the original fuzzy quasi-metric space. For balanced fuzzy quasi-metric spaces, the completion produced coincides, up to isometry, with the Doitchinov completion. We present and explore a new class of maps, which we call balanced maps, in response to the question of whether uniformly continuous maps between fuzzy quasi-metric spaces can be extended to the constructed completion.

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Quantale-valued generalizations of approach vector spaces over quantale valued approach division rings

Mathematics Subject Classification (MSC): 54H13, 34A05, 46A99

Abstract. The motivation of this work is to study the concept of quantale-valued approach vector spaces over the quantale-valued approach division rings. The reason behind this generalization is crept into the notion of approach vector spaces attributed to R. Lowen and S. Verwulgen. They considered approach vector spaces over the reals, whereas our approach is more general, as we consider arbitrary division rings instead of real vector spaces. In so doing, we first introduce the concepts of approach rings, approach division rings, and explore some of their basic facts. We determine a suitable class of approach division rings that can be used as a scalar domain for approach vector spaces, here we talk about the action of approach division ring on approach vector spaces. Furthermore, we generalize approach rings and approach division rings into quantale-valued approach rings and quantale-valued approach division rings exploring various results from quantale-valued approach vector spaces and some of their categorical aspects.

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Ekeland variational principle and some of its equivalents on a weighted graph, completeness and OSC property

Mathematics Subject Classification (MSC): 47H10, 05C22, 06F30, 54E50

Abstract. We prove a version of Ekeland Variational Principle (EkVP) in a weighted graph G and its equivalence to Caristi fixed point theorem and to Takahashi minimization principle. The usual completeness and topological notions are replaced with some weaker versions expressed in terms of the graph G. The main tool used in the proof is the OSC property for sequences in a graph. Converse results, meaning the completeness of graphs for which one of these principles holds is also considered.

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A generalization of Doitchinov's quasi-uniform completion

Mathematics Subject Classification (MSC): 54D35, 54E15, 54E50, 54E55

Abstract. In this talk, we develop a completion theory for all quasi-uniform spaces, which we call Λ-completion. One of the most important features of this completion theory is the adoption of requirements posed by Doitchinov for a natural generalization of the classical theory of uniform completeness. The core concept of this approach is the notion of *cut of nets*, which results from the combination of two notions: (1) The notion of *Dedekind-MacNeille cut* in order theory, and (2) the notion of D-*Cauchy net* of Doitchinov. In the uniform case, the notion of cut of nets coincides with the well known notion of equivalence class of nets.

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A topological characterization of the existence of some notions of generalized stable sets

Mathematics Subject Classification (MSC): 91A30, 91A35, 91B06, 91B14, 91B15

Abstract. The theory of optimal choice sets is a solution theory that has a long and well-established tradition in social choice and game theories. Some of important general solution concepts of choice problems when the set of best alternatives does not exist (this problem occurs when the preferences yielded by an economic process are cyclic) are the Stable Set (Von

Neumann-Morgenstern set), Generalized Stable set (Van Deemen set), Extended Stable set, m-Stable set and w-Stable set. In this talk, we present a topological characterization of the Generalized Stable set (Van Deemen set), the Extended Stable set, the m-Stable set and the w-Stable set. This is done in a general framework for which dominance relation refers to an arbitrary binary relation defined on a set of alternatives that is not necessarily finite.

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Based-free involutions

Mathematics Subject Classification (MSC): Primary 57N20; Secondary 57S99

Abstract. By a *space* we mean a separable metrizable topological space. An involution on a space is called *based-free*, if it has a unique fixed-point. For Hilbert space ℓ_2 we denote by σ the standard based-free involution given by the formula $\sigma(x) = -x$. It was proved in [1] that (ℓ_2, σ) is universal for all spaces with based-free involutions. This means that for every space X with a based-free involution $\tau : X \to X$, there exists an equivariant topological embedding $(X, \tau) \hookrightarrow (\ell_2, \sigma)$.

Let B denote the unit ball in ℓ_2, that is, $B := \{x \in \ell_2 : \|x\| \leq 1\}$. By abuse of notation, we denote the restriction $\sigma \restriction B$ by σ as well. One of the goals of this talk is to provide a more transparent and direct proof of this fact. Our argument is based on the following result which is also of an independent interest: for every space (X, τ) with a based-free involution, the equivariant maps $(X, \tau) \to (B, \sigma)$ separate points and closed sets. We also establish two new characterizations of equivariant based-free compactifications.

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Sierpiński carpet and rigidity of locally symmetric rank one manifolds

Abstract. We create some analogue of the Sierpiński carpet for nilpotent geometry on horospheres in symmetric rank one negatively curved spaces $H^n_{\mathbb{F}}$ over division algebras $\mathbb{F} \neq \mathbb{R}$, i.e over complex $\mathbb{C}$, quaternionic $\mathbb{H}$, or octonionic/Cayley numbers $\mathbb{O}$. The original Sierpiński carpet in the plane was described by Wacław Sierpiński in 1916 as a fractal generalizing the Cantor set.

Making such a Sierpiński carpet with a positive Lebesgue measure at the sphere at infinity $\partial H^n_{\mathbb{F}}$ and defining its "stretching", we construct a non-rigid discrete $\mathbb{F}$-hyperbolic groups $G \subset \mathrm{Isom}\, H^n_{\mathbb{F}}$ whose limit set $\mathit{\Lambda}(G)$ is the whole sphere at infinity $\partial H^n_{\mathbb{F}}$. This answers questions by G.D.Mostow [6], L.Bers [4] and S.L.Krushkal [5] about uniqueness of a conformal or CR structure on the sphere at infinity $\partial H^n_{\mathbb{F}}$ compatible with the action of a discrete group $G \subset \mathrm{Isom}\, H^n_{\mathbb{F}}$.

Previously, for the real hyperbolic spaces, this problem was solved by Apanasov [1, 2]. Due to D.Sullivan [7] rigidity theorem generalized by Apanasov [2] and [3], Theorem 5.19, the complement of the constructed class of discrete groups $G \subset \mathrm{Isom}\, H^n_{\mathbb{F}}$ (having a positive Lebesgue measure of the set of vertices of its fundamental polyhedra at infinity) whose limit set $\Lambda(G)$ is the whole sphere at infinity $\partial H^n_{\mathbb{F}}$ consists of groups rigid in the sense of Mostow.

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Differential calculuses in group algebras

Abstract. The task of studying derivations in different algebras is well known. The method proposed in the joint work of the author with A.S. Mishchenko and A.I. Stern will be discussed. The method consists in identifying derivations and characters on the groupoid of an adjoint action.

It turns out that if we consider groupoids of another action as a groupoid, we can generate other families of operators, not only those obeying the Leibniz rule, with the help of characters. In particular, one can obtain in this way the well-known differential calculus of Fox.

It is noteworthy that in describing such operator families, among other things, other interesting geometrical constructions arise, in particular the ends of groups.

The results which will be discussed during the talk also have an interpretation in terms of coarse geometry, which will also be discussed.

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Outer automorphisms of the mapping class groups of some sporadic nonorientable surfaces

Mathematics Subject Classification (MSC): 20F38, 57K20

Abstract. In this talk, we will first mention the definitions of some basic concepts such as curves on a nonorientable surface N of genus g, the mapping class group $\mathrm{Mod}(N)$ of N, puncture slide, and $Y-$homeomorphisms. Then, we will give an idea of the proof of the fact that the automorphism group of $\mathrm{Mod}(N)$ is isomorphic to $\mathrm{Mod}(N)$ for the genus $g > 4$ (joint with B. Szepietowski, [1]). Later, we consider the cases not covered by this result. In particular, we will give an outline of the proof of the fact that the case $g = 1$. If time permits, I will also discuss the necessary tools used in proofs.

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Liftable homeomorphisms of real projective plane

Mathematics Subject Classification (MSC): 57M12, 57M60

Abstract. Let $p : \tilde{S} \to S$ be a (branched) covering. A homeomorphism of S, say f, lifts to a homeomorphism of $\tilde{S}$, if there exists a homeomorphism $\tilde{f}$ of $\tilde{S}$ such that $p\tilde{f} = fp$. In this talk, we give conditions for regular branched finite abelian covers of the real projective plane, where each homeomorphism of the base (preserving the branch locus) lifts to a homeomorphism of the covering surface.

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Inner automorphisms of an abelian extension of a quandle

Mathematics Subject Classification (MSC): 57K12, 57K10

Abstract. Let $(Q, *)$ be a quandle and A an abelian group. A quandle 2-cocycle is a function $\phi : Q \times Q \to A$ satisfying (i) $\phi(x, x) = 0$ for all $x \in Q$ and (ii) $\phi(x, y) + \phi(x * y, z) = \phi(x, z) + \phi(x * z, y * z)$ for $x, y, z \in Q$. By a quandle 2-cocycle, one can define a so-called quandle cocycle link invariant, which are not only classical knot invariants but higher dimensional knot invariants, or define a new quandle structure on $Q \times A$ by defining an operation $(x, a)\tilde{*}(y, b) = (x * y, a + \phi(x, y))$ for $(x, a), (y, b) \in Q \times A$. We call such a quandle as an abelian extension of Q by A and denote it by $Q \times_\phi A$.

In this talk, we will discuss about inner automorphisms of ableian extension $Q \times_\phi A$ in terms of inner automorphisms of the underlying quandle Q.

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$\mathbb{R}$-rigid countably compact spaces

Mathematics Subject Classification (MSC): Primary 54C30; Secondary 54D35, 03E17

Abstract. A space X is called Y-rigid if any continuous map $f : X \to Y$ is constant. For each cardinal κ we construct an infinite regular countably compact space X_κ such that X_κ is Y-rigid for any T_1 space Y of pseudocharacter $\leq \kappa$. This result resolves two problems posed by Tzannes in [4]. A regular separable first-countable countably compact space is called a *Nyikos* space. The existence in ZFC of a noncompact Nyikos space is still an open problem. We construct a consistent example of Nyikos $\mathbb{R}$-rigid space. To construct the aforementioned example we developed a technique of embedding into first-countable countably compact spaces. In particular, we show that assuming $\min\{\mathfrak{s}, \mathfrak{b}\} = \mathfrak{c}$ each regular first-countable space of weight $< \mathfrak{c}$ embeds into a regular first-countable countably compact space. Finally, assuming $\mathfrak{b} = \mathfrak{c}$ and the existence of a $P_\mathfrak{c}$-point we prove that each Tychonoff first-countable space embeds into a Tychonoff first-countable countably compact space. The results can be found in [1, 2, 3].

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Topology and dynamics of W-sets in 2-dimensional manifolds

Mathematics Subject Classification (MSC): 37B30, 57K99

Abstract. In this talk we introduce the notion of W-set for a flow defined on a compact 2-manifold. These sets encapsulate the asymptotic dynamics of the flow and are a natural generalization of Morse decompositions. We present some results that relate the topology and Conley index of an W-set with the topology of the phase space. In addition, we present topological characterizations of the sphere and the torus in terms of the existence of flows having particular W-sets.

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Semi-equivelar gems of PL d-manifolds

Mathematics Subject Classification (MSC): Primary 57Q15; Secondary 52B70, 05C10, 05C15, 52C20

Abstract. We define a notion of $(f_0, f_1, \ldots, f_d)$-type semi-equivelar gems for closed connected d-manifolds, related to regular embedding of gems Γ representing M on a surface S such that the face-cycles at all the vertices of Γ on S are of the same type. The term is inspired by semi-equivelar maps of surfaces. Given a surface S having non-negative Euler characteristic, we find all regular embeddings on S and then construct a genus-minimal semi-equivelar gem (if exists) of each such type embedded on S. Further, we construct some semi-equivelar gems as follows:

(1) For each connected surface K, we construct a genus-minimal semi-equivelar gem that represents K. In particular, for $K = \#_n(\mathbb{S}^1 \times \mathbb{S}^1)$ (resp., $\#_n(\mathbb{RP}^2)$), the semi-equivelar gem of type $((4n+2)^3)$ (resp., $((2n+2)^3)$) is constructed.

(2) Given a closed connected orientable PL d-manifold M of regular genus at most 1, we show that M admits a genus-minimal semi-equivelar gem.

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Smooth structures on Connected Sum of Projective spaces

Mathematics Subject Classification (MSC): Primary 57N70, 57R55; Secondary 55P10, 57Q60

Abstract. The study of the inertia group of manifolds leads to the study of the classification of smooth structures on it. We will discuss the same in the talk for the connected sum of manifolds. We show that the concordance inertia group of individual manifolds completely determines the concordance inertia group of the connected sum of manifolds. Further, we discuss the classification of smooth structure on M, up to taking connected sum with projective spaces. In particular, we classify all smooth structures on $\#_k\mathbb{C}P^n$, up to isotopy, in lower dimensions.

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A Generalization for the Jones Polynomial for N-Polar Knots

Mathematics Subject Classification (MSC): 57M27

Abstract. We introduce some invariants of 2-polar knots focusing on the X_p polynomial that has been introduced recently. The main observation in the paper is that we introduce a methodology to obtain the general X_p polynomial for n-polar knots and links, by defining what we call the finishing states, and then we use rooted labeled trees to classify n-polar finishing states and hence general polynomial invariants for n-polar knots. We illustrate the case of planar X_p polynomial for 3-polar knots as an example.

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Interval-valued topology on soft sets

Mathematics Subject Classification (MSC): 54A40, 54A05, 06D72

Abstract. In this paper, we firstly introduce the interval-valued fuzzy topology on the family of soft sets by using an interval-valued fuzzy mapping. Also, with the help of examples, we elucidate the relationships among these concepts. In fact, the interval-valued fuzzy topology $\tau = [\tau^-, \tau^+]$ produces two fuzzy topologies τ^-, τ^+ on the family of soft sets given in [1]. Later, we obtain that each interval-valued fuzzy topology is a descending family of soft topologies. Finally, we give some topological structures such as

interval-valued fuzzy neighborhood system of a soft point, base and subbase of τ. Using these concepts, we define the concepts of interval-valued fuzzy subspace, direct sum, product, continuous mapping and investigate relationships among them.

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On the structure of the level set

Mathematics Subject Classification (MSC): 53A04, 53A05, 53A31

Abstract. In this paper, we consider the question of the structure of the set of submersions for which all level surfaces are linearly connected sets [1].

Let $C^1(R^n,\, R^1)$ the set of all differentiable functions of the class C^1. On the set $C^1(R^n,\, R^1)$, we introduce the weak (C^1-compact-open) topology [3].

The set of all C^r-smooth mappings $f : M \to N$ is denoted by $C^r(M, N)$, where M, N are smooth manifolds of the class C^r. Suppose that $r = 1, 2, ...$ [2].

The weak topology (C^1-compact-open topology) in $C^r(M, N)$ is generated by sets defined as follows.

Let $f \in C^r(M, N)$ and let $(\varphi,\, U)$, $(\psi,\, V)$ be maps of manifolds M, N. Let, further, $K \subset U$ be a compact set such that $f(K) \subset V$; let, $0 < \varepsilon < +\infty$.

Prebasic neighborhood

$$\aleph^r(f, (\varphi,\, U), (\psi,\, V), K, \varepsilon) \tag{1}$$

weak topology is defined as the set of such C^r-maps $g : M \to N$ as $g(K) \subset V$ and for any $x \in \varphi(K)$, $k = 0, ..., r$,

$$\|D^k \left(\psi f \varphi^{-1}\right)(x) - D^k \left(\psi g \varphi^{-1}\right)(x)\| < \varepsilon.$$

This means that local representations of maps f, g, together with their first r derivatives, differ by no more than at ε each point of the compact set K.

The weak topology in $C^r(M, N)$ is generated by the sets (1); this defines the topological space $C^r_W(M, N)$. The neighborhood of a point f with respect to this topology is therefore any set containing the intersection of a finite number of sets of type (1) [3].

Here, as a manifold N, we consider a one-dimensional manifold R^1 and assume that the $r = 1$. Space $C^1(R^n,\ R^1)$ is considered with a weak topology (C^r-compact-open topology). It is known that a space $C^r(M, N)$ with a weak topology has a countable base.

Denote by the $LS(R^n,\ R^1)$ set of submersions for which all level surfaces are linear connected. The following theorem gives information about the structure of the set $LS(R^n,\ R^1)$.

Theorem. The set $LS(R^n,\ R^1)$ is a closed subset of the space $C^1(R^n,\ R^1)$ of all differentiable functions of the class C^1.

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On axiomatic homology theory of general topological spaces

Mathematics Subject Classification (MSC): 55N05, 55N07, 55U35

Abstract. On the category $\mathcal{K}^2_{CM}$ of pairs of compact metric spaces the exact homology theory was defined by N. Steenrod, that is known as the classical Steenrod homology theory. J. Milnor constructed the exact homology theory on the category $\mathcal{K}^2_C$ of pairs of compact Hausdorff spaces, which is isomorphic to the Steenrod homology theory on the subcategory $\mathcal{K}^2_{CM}$ and which satisfies the so-called "modified continuity" property: if $X_1 \leftarrow X_2 \leftarrow X_3 \leftarrow \dots$ is an inverse sequence of compact metric spaces with inverse limit X, then for each integer n there is an exact sequence:

$$0 \to \varprojlim{}^1 H_{n+1}(X_i) \xrightarrow{\beta} H_n(X) \xrightarrow{\gamma} \varprojlim H_n(X_i) \to 0, \tag{1}$$

where H_* is the Steenrod (Milnor) homology theory. There are exact homology theories defined by other authors (A. N. Kolmogoroff, G. Chogoshvili, K. A. Sitnikov, A. Borel and J. C. Moore, H. N. Inasaridze, D. A. Edwards and H. M. Hastings, W. S. Massey, E. G. Sklyarenko) that are isomorphic to the Steenrod homology theory on the category $\mathcal{K}^2_{CM}$ and so, satisfy the modified continuity axiom.

On the category $\mathcal{K}^2_C$ the axiomatic characterization is obtained by N. Berikashvili, L. Mdzinarishvili and Kh. Inasaridze, L. Mdzinarishvili, Kh. Inasaridze. The connection between these axiomatic systems is studied in the paper [1].

In the paper [2] we have generalized the result for general topological spaces. In particular, we have defined the Alexander-Spanier normal cohomology theory based on all normal coverings and have shown that it is isomorphic to the Alexandroff-Čech normal cohomology [2]. Using this fact and methods developed in [3], we constructed an exact, the so-called

Alexander-Spanier normal homology theory $\bar{H}_*^N(-,-;G)$ on the category $\mathcal{K}^2_{Top}$, which is isomorphic to the Steenrod homology theory on the subcategory of compact pairs $\mathcal{K}^2_C$. Moreover, we gave an axiomatic characterization of the constructed homology theory [2]. In this paper we will use the method of construction of the strong homology theory to show that the homology theory $\bar{H}_*^N(-,-;G)$ is strong shape invariant.

The talk partially is based on joint works with co-authors Vladimer Baladze (BSU) and Leonard Mdzinarishvili (GTU).

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On a space of τ-continuous functions

Abstract. Throughout the paper all spaces are assumed to be Hausdorff. Denote by $F(X,Y)$ the set of all functions from X to Y and $C_\tau(X,Y)$ the set of all τ-continuous functions from X to Y with the topology of pointwise convergence.

Let X and Y be topological spaces. A function $f : X \to Y$ is said to be τ-continuous [1] if for every subspace A of X such that $|A| = \tau$, the restriction $f|_A$ is continuous. It is clear that every continuous function is τ-continuous.

The following example shows that a τ-continuous function need not to be continuous, in general.

Example. Let $\mathbb{R}$ be the real line and θ_Z the family consisting of all sets whose cardinality of the complement is at most countable. It is easy to verify that $(\mathbb{R}, \theta_Z)$ is a topological space. This topological space is called Zariski space. For $X = \mathbb{R}$ with the Zariski topology and $Y = \mathbb{R}$ with the usual topology, the identity map from X to Y is ω-continuous but not continuous.

Corollary 1. If X is Zariski space, then any mapping of X to a topological space Y is ω-continuous.

A subset F of a space X is said to be τ-closed [2] if for every $B \subset F$ with $|B| = \tau$,the closure B in X of the set B is contained in F. The τ-closure of a set A is defined as $[A]_\tau = \bigcup\{[B] : B \subset A, |B| = \tau\}$ and a set A is said to be τ-dense in X if $[A]_\tau = X$.

Note that every closed set is τ-closed. Conversely, the set of all rational numbers is ω-closed but not closed in $(\mathbb{R}, \theta_Z)$.

In a paper [3] obtained the following result:

Proposition 1. For a mapping $f : X \to Y$ of arbitrary topological spaces X and Y the following conditions are equivalent:

(1) $f : X \to Y$ is τ-continuous;

(2) for every closed set F in Y, the preimage $f^{-1}(F)$ is τ-closed in X;

(3) for every τ-closed set F in Y, the preimage $f^{-1}(F)$ is τ-closed in X;

(4) $f([A]_\tau) \subset [f(A)]_\tau$ for an arbitrary subset $A \subset X$;

(5) $[f^{-1}(B)]_\tau \subset f^{-1}([B]_\tau)$ for an arbitrary subset $B \subset Y$.

Proposition 2. If $Y \subset Z$, then $C_\tau(X, Y)$ is a subspace of the space $C_\tau(X, Z)$; it is closed if Y is closed in Z.

Theorem. Let X be a topological space. Then $C_\tau(X, Y)$ is dense in $F(X, Y)$.

Corollary 2. Let X be a topological space. Then $c(C_\tau(X, \mathbb{R})) = \aleph_0$.

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On functional tightness of the space of semiadditive functionals of finite support

Abstract. In this talk, we study the behavior of the functional tightness of topological spaces under the influence of the functor of semiadditive functionals of finite support. It is proved that the functor OS_n preserves the functional tightness of compact spaces.

Let X be a compactum (compact and Hausdorff). By $C(X)$ denote the space of all continuous functions $f\colon X \to R$ with usual pointwise operations and the sup-norm, i.e. with the norm $\|f\| = \sup\{|\mathrm{f(x)}| : \mathrm{x} \in \mathrm{X}\}$. For each $c \in R$ by c_X denote constant function defining by the formula $c_X(x) = c$, $x \in X$.

A functional $\nu\colon C(X) \to R$ is called:

1) weakly additive if $\nu(\varphi + c_X) = \nu(\varphi) + c \cdot \nu(1_X)$ for all $c \in R$ and $\varphi \in C(X)$;

2) order-preserving, if for functions $\varphi, \psi \in C(X)$ from $\varphi \leq \psi$ it follows $\nu(\varphi) \leq \nu(\psi)$;

3) normed if $\nu(1_X) = 1$;

4) positively-homogeneous, if $\nu(\lambda\varphi) = \lambda\nu(\varphi)$ for all $\varphi \in C(X)$, $\lambda \in R_+$, where $R_+ = [0, +\infty)$;

5) semiadditive, if $\nu(f + g) \leq \nu(f) + \nu(g)$ for all $f, g \in C(X)$.

For a compactum X by $OS(X)$ [1] denote the set of all functionals satisfying above conditions 1) - 5). Elements of the set $OS(X)$, are shortly called semiaadditive functionals. Note that each functional $\nu \in OS(X)$ is a continuous mapping of $C(X)$ to R, i.e. the set $OS(X)$ is a subset of $C_p(C(X))$. This set is equipped with the pointwise topology. Sets in the form

$$O(\mu;\varphi_1,...,\varphi_k;\varepsilon) = \{\nu \in OS(X) : |\mu(\varphi_i) - \nu(\varphi_i)| < \varepsilon,\ i = 1\,,...,k\}$$

where $\varphi_i \in C\,(X)$, $i = 1,...,k$, $\varepsilon > 0$, generates a neighborhood base of a functional μ in $OS(X)$.

Recall that a functional μ is said to be supported on a set F if $\phi(F) = 0$ implies $\mu(\phi) = 0$ for every $\phi \in C(X)$. The minimal closed set F on which the functional μ is supported, is called the support of μ. The support of a functional μ is denoted by $supp(\mu)$.

For a space X and the functor OS put

$$OS_n(X) = \{\mu \in OS(X) \colon |\mathrm{supp}(\mu)| \leq n\},$$

where n is a natural number.

Let X and Y be topological spaces. A function $f\colon X \to Y$ is said to be τ-continuous [2] if for every subspace A of X such that $|A| \leq \tau$, the restriction $f|_A$ is continuous.

Definition. [2] The functional tightness $t_0(X)$ of a space X is minimum cardinal number τ such that every τ-continuous real-valued function on X is continuous.

Theorem. For an arbitrary infinite compactum X we have $t_0(X) = t_0(OS_n(X))$.

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Uniformly linked space and its hyperspace

Abstract.
Definition 1 [1]. Let X be a nonempty set. A family $\mathcal{U}$ of covers of a set X is called uniformity on X if the following conditions are satisfied:

(P1) If $\alpha \in \mathcal{U}$ and α is inscribed in some cover β of the set X, then $\beta \in \mathcal{U}$.

(P2) For any $\alpha_1 \in \mathcal{U}$, $\alpha_2 \in \mathcal{U}$ there exists $\alpha \in \mathcal{U}$, which is inscribed in α_1 and α_2.

(P3) For any $\alpha \in \mathcal{U}$, there exists $\beta \in \mathcal{U}$ strongly star inscribed in α.

(P4) For any x, y of a pair of different points of X, there exists $\alpha \in \mathcal{U}$ such that no element of α contains both x and y.

A family $\mathcal{U}$ consisting of a set X, satisfying conditions (P1) - (P3) is called pseudo-uniformity on X and a pair $(X, \mathcal{U})$ is called a pseudo-uniform space.

A family $\mathcal{U}$ consisting of a set X, satisfying conditions (P1) - (P4) is called uniformity on X and a pair $(X, \mathcal{U})$ is called a uniform space.

Proposition 1 [1]. If $\mathcal{B}$ is the base of a uniform space $(X, \mathcal{U})$, then $P(\mathcal{B}) = \{P(\alpha) : \alpha \in \mathcal{B}\}$ forms a base of some uniformity $\exp\mathcal{U}$ on $\exp X$.

A uniform space $(\exp X, \ \exp\mathcal{U})$ is called a hyperspace of closed subsets of a uniform space $(X, \mathcal{U})$, and uniformity $\exp\mathcal{U}$ is called Hausdorff uniformity on $\exp X$.

Remark 1 [1]. Let $\exp_c X$ be the set of all nonempty compact subsets of the uniform space $(X, \mathcal{U})$. For each $\alpha \in \mathcal{U}$, put

$$K(\alpha) = \{\langle\alpha'\rangle : \alpha' \subseteq \alpha \,\text{and}\, \alpha' \text{ is finite}\}.$$

Note that $K(\alpha)$ is the cover of the set $\exp_c X$.

Definition 2 [1]. A uniform space $(X, \mathcal{U})$ is called uniformly linked if for any cover $\alpha \in \mathcal{U}$ there exists a natural number n, such that to any points

$x, y \in X$ one can choose a linked sequence $\{A_1,\, A_2,\, \dots\,,\, A_k\} \subset \alpha$, such that $k \leq n$, $x \in A_1,\ y \in A_k$.

Proposition 2 [1]. For a uniform space $(X, \mathcal{U})$, the following conditions are equivalent:

(1) The uniform space $(X, \mathcal{U})$ is uniformly connected.

(2) The uniformity of $\mathcal{U}$ does not contain disjoint covers consisting of at most one element.

(3) For any $\alpha \in \mathcal{U}$ and for any point $x \in X$, $\bigcup_{n=1}^{\infty} \alpha_n(x) = X$.

(4) For any $\alpha \in \mathcal{U}$ and for any points of $x, y \in X$ there exists a finite linked sequence $\{A_1,\, A_2,\, \dots\,,\, A_k\} \subset \alpha$ such that $x \in A_1,\ y \in A_k$.

Theorem 1. A uniform space $(X, \mathcal{U})$ is uniformly linked if and only if the uniform space $(\exp_c X, \exp_c \mathcal{U})$ is uniformly linked.

It follows from Proposion 2 that every uniformly linked uniform space $(X, \mathcal{U})$ is uniformly connected.

Corollary 1. A uniform space $(X, \mathcal{U})$ is uniformly connected if and only if the uniform space $(\exp_c X,\, \exp_c \mathcal{U})$ is uniformly connected.

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Order convergence in the spaces of continuous functions

Mathematics Subject Classification (MSC): 46A40, 54A20, 54C35

Abstract. A net in a poset converges in order if it is "squeezed" between an increasing and a decreasing nets with the same extrema. This mode of convergence is induced by a topology only in rare cases, however it is often induced by a filter convergence structure. Order convergence has always been an important tool in studying vector lattices, along with some other modes of convergence induced by the algebraic and order structure of a vector lattice. In this talk I will present an explicit characterization of order convergence in various spaces of real-valued continuous functions including $C(X)$, $C_b(X)$ and others.

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On questions of PS Alexandrov, A weakly infinite-dimensional Compactum not having Property *C*

Mathematics Subject Classification (MSC): 54F54

Abstract. We construct a compact subspace X of the Hilbert space I^ω such that X is not a C-space and X is weakly infinite-dimensional. In fact, $\dim_{\mathrm{w}} X = \omega_0$ where $\dim_{\mathrm{w}}$ is the transfinite extension of the covering dimension for weakly infinite-dimensional spaces in the sense of Smirnov, [B1]. We answer the question stated in [A+G], whether the notions weakly infinite dimensionality and property C coincide, in the negative.

The example also has implications for the open question on the product theorem for weakly infinite-dimensional compact spaces. The space $X \times X$

is strongly infinite-dimensional and so the product theorem is not valid for weakly-infinite dimensional spaces even in the compact case.

In [F], V.V. Fedorchuk defined intermediate classes of weakly infinite-dimensional spaces. The space X constructed is not 3-C. Therefore the question of equivalence of m-C spaces is also answered sharp in the negative.

P.S. Alexandroff noted that for finite dimensional spaces homological dimension and covering dimension coincide. However A.N. Dranishnikov [D1] showed an infinite dimensional space have finite homological dimension. S. Nowak proposed an alternative and conjectures the coincidence of stable cohomological dimension and covering dimension. A.N. Dranishnikov [D2] showed that if there is a counterexample of an infinite-dimensional space having finite stable cohomological dimension it must be a weakly-infinite dimensional compactum not having property C. Can we provide an example of an infinite dimensional space with finite stable cohomological dimension using these new techniques?

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On τ-metrizable, projectively τ-metrizable and almost τ-metrizable topological groups

Mathematics Subject Classification (MSC): 54C05, 54C10, 54C20

Abstract. Let $\mathbb{R}_+ = [0,\infty), \mathbb{R} = (-\infty,\infty)$ and τ an infinite cardinal number. Let $\mathbb{R}_+^\tau$ and $\mathbb{R}^\tau$ denote the Tichonoff product τ-pieces of copies of the spaces $\mathbb{R}_+$ and $\mathbb{R}$ corresponding to the natural topology.

Let X be a non-empty set. The mapping $\rho_\tau : X \times X \to \mathbb{R}_+^\tau$ is called the τ-metric on X and the metric R_τ to R_+^τ.

Theorem 1. A topological group is G -metrizable if and only if the inequalities $\chi(G) \leq \tau$ hold.

Theorem 2. τ - metrizable groups and only they are limits of the projective length τ, consisting of metrizable topological groups and continuous homeomorphisms.

Theorem 3. For a topological group G the following conditions are equivalent:

(1) The topological group G is projectively τ-metrizable;

(2) The topological group G is the limit of the projective spectrum $S = \{G_a, f_\alpha^\beta, M\}$ composed of τ-metrizable topological groups G and open and perfect homomorphisms $f_\alpha^\beta : \alpha, \beta \in M$.

Theorem 4. The following conditions are equivalent:

(1) The topological group G is almost τ-metrizable;

(2) The topological group G has a compact subgroup H with character $\chi(H,G) \leq \tau$. G/H is factor space and the natural mapping $f : G \to G/H$ is open and perfect.

Theorem 5. The factor-space G/H of an almost τ-metrizable group G with respect to the closed subgroup H is an almost τ-metrizable space.

Theorem 6. If K is a compact group on the space X, that the orbit space X/K is τ-metrizable, then in any neighborhood of the unit of the group K there is a normal divisor N, the orbit space X/N is also τ-metrizable.

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Construction of the set of all (μ)-paracompact and close to them extensions by uniform structures

Mathematics Subject Classification (MSC): 54E15

Abstract. It is well known that every (μ)-paracompact space is (μ)-complete with respect to its universal uniformity U_X, and the system of all open covers of the space X forms the base of universal uniformity U_X[1], [2]. If Y is a dense subspace of the space X, and V is the uniformity on Y induced by the uniformity U, then the space (X, U) is the (μ)-completion of the uniform space (Y, V). Thus, generally speaking, V is not a universal uniformity, but has a special feature which we call as (μ)-preparacompactness. It turns out that by (μ)-preparacompact uniform structures of space Y one can construct all his (μ)-paracompact extensions namely, to obtain these extensions as (μ)-completions of space Y on (μ)-preparacompact uniform

structures. Therefore, the construction of paracompact and related extensions of the Tychonoff space by uniform structures is, in our opinion, the most convenient and natural.

In this work the set of all compact, superparacompact, strongly paracompact, paracompact and μ-paracompact extensions of a Tychonoff space with the help of his uniform structures has been constructed.

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Polyhedral joins and graph complexes

Mathematics Subject Classification (MSC): 05E45, 55P10, 55P15, 05C76

Abstract. Given two graphs G and H one can define their lexicographic product $G \circ H$ as the graph obtained by taking a copy of H for each vertex of G and adding all the possible edges between two copies if the corresponding vertices are adjacent in G. This construction seems natural and one can ask for analogous constructions for simplicial complexes.

Given a simplicial complex K with vertex set $\{1, \ldots, n\}$ and a family of pairs of CW-complexes $\{(X_i, A_i) : 1 \leqslant i \leqslant n\}$, we define the polyhedral join of the family as the union over the simplices σ of $\overset{*}{D}(\sigma) = *_{i=1}^{n} Y_i$, where

$$Y_i = \begin{cases} X_i & \text{if } i \in \sigma \\ A_i & \text{if } i \notin \sigma \end{cases}$$

This space not only generalizes the lexicographic product, but its homotopy type is needed for calulating the honmotopy type of a family of graph complexes associated to $G \circ H$.

Given a graph G and any $d \in \mathbb{N} \cup \{\infty\}$, we associate a simplicial complex $\mathcal{F}_d(G)$ whose vertices are the same as those of G and where a subset S is a simplex if the induced subgraph on S is a forrest with maximal degree at most d (for $d = \infty$ all degrees are allowed).

In this talk I will give some general results about polyhedral joins and explain how this is used to calculate the homotopy type of the complexes $\mathcal{F}_d(G \circ H)$ for some graph families.

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On topological properties of partial metric spaces and applications to fixed point theory

Mathematics Subject Classification (MSC): 54E35, 54D30, 54E50, 54H25

Abstract. The notion of a partial metric space was introduced by Matthews [4] who showed, roughly speaking, how metric–like tools can be extended to non–Hausdorff topologies. He also indicated that this class of spaces plays an important role in the study of denotational semantics of a programming language.

In this talk we are going to present some necessary and sufficient conditions under which the topology generated by a partial metric is equivalent to the topology generated by a suitably defined metric. Next, we are going to focus on two basic topological properties of partial metric spaces, namely completeness and compactness. In particular, it appears that in these spaces compactness is equivalent to sequential compactness. Moreover, Hausdorff compact partial metric spaces are metrizable.

Finally, we will focus on some new extensions of the Generalized Banach Contraction Principle (see e.g. [1]) in this class of spaces. We will also discuss the significance of bottom sets of partial metric spaces in fixed point theorems for mappings acting in these spaces.

The results presented in this talk come from the papers [2] and [3].

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Some notes on induced functions on hyperspaces

Mathematics Subject Classification (MSC): 54B20, 54C05, 54C35

Abstract. Let X be a topological space and $\mathrm{CL}(X)$ be the hyperspace of all nonempty closed subsets of X. A continuous function $f : X \to Y$ between topological spaces induces a function between the hyperspaces $\mathrm{CL}(X)$ and $\mathrm{CL}(Y)$, defined by

$$A \mapsto \overline{f(A)}.$$

The problem of wether the induced function is also continuous has been studied and solved when $\mathrm{CL}(X)$ and $\mathrm{CL}(Y)$ are endowed with the Vietoris topology or the Hausdorff metric topology.

In this talk we will discuss the continuity of the induced function with respect to the Attouch-Wets metric topology and the Fell topology. We will give a characterization of its continuity when $\mathrm{CL}(X)$ and $\mathrm{CL}(Y)$ are endowed with the Attouch-Wets metric topology. Regarding the Fell topology, we will characterize its continuity in the case when $f : X \to Y$ is closed.

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Plat closure for links in Dunwoody and Takahashi manifolds

Mathematics Subject Classification (MSC): 57K10, 57K30

Abstract. Dunwoody and Takahashi manifolds are two intensively studied families of closed connected orientable 3-manifolds. The fist one was introduced by Dunwoody in [2] using Heegaard diagrams and generalized in [1]; the second one arose in [4] by means of Dehn surgery and was generalized in [3]. In this talk, after recalling some properties of Dunwoody and Takahashi manifolds, I'll present some new result, obtained in collaboration with Paolo Cavicchioli, on the equivalence moves for links, represented via plat closure, lying in these two families.

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An algorithmic method to compute plat-like Markov moves in genus two 3-manifolds

Mathematics Subject Classification (MSC): 57K10, 57M15, 57-04, 05C99

Abstract. This dissertation will deal with the equivalence of links in 3-manifolds of Heegaard genus two. We construct an algorithm (implemented in `C++`) which, starting from a description of such a manifold introduced by Casali and Grasselli that uses 6-tuples of integers and determines a Heegaard decomposition of the manifold, allows to find the words in $B_{2,2n}$, the braid group on $2n$ strands of a surface of genus two, that realize the plat-equivalence for links in that manifold. In this way we extend the result obtained by Cattabriga and Gabrovšek for 3-manifolds of Heegaard genus one to the case of genus two. We describe explicitly the words for a notable family of 3-manifolds.

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On π-compatible topologies

Mathematics Subject Classification (MSC): Primary 54A10, 54D10, 54D15

Abstract. Let X be a non-empty set, $\mathcal{P}(X)$ the family of all subsets of X, τ a topology on X and $\mathcal{I}_m(X,\tau)$ the family of all meager sets of the topological space (X,τ).

An interesting collection of subsets of X extending τ as well as $\mathcal{I}_m(X,\tau)$, is the family $\mathcal{B}_p(X,\tau)$ of all subsets of X possessing the Baire property in (X,τ). A subset A of X has the Baire property in the space (X,τ) if $A = (O \setminus M) \cup N$, where $O \in \tau$ and $M, N \in \mathcal{I}_m(X,\tau)$. It is well known that the family $\mathcal{B}_p(X,\tau)$ is a σ-algebra of sets.

Let us recall that for the real numbers $\mathbb{R}$ with the Euclidean topology τ_E we have $\mathcal{B}_p(\mathbb{R},\tau_E) \neq \mathcal{P}(\mathbb{R})$, and there is a lot of information about elements of the family $\mathcal{P}(\mathbb{R}) \setminus \mathcal{B}_p(\mathbb{R},\tau_E)$. It would be interesting to know for what topologies τ on $\mathbb{R}$ the equality $\mathcal{B}_p(\mathbb{R},\tau) = \mathcal{B}_p(\mathbb{R},\tau_E)$ is valid.

One can even pose a general question.

Question. Let X be a set and τ be a topology on X. For what topologies σ on the set X does the equality $\mathcal{B}_p(X,\tau) = \mathcal{B}_p(X,\sigma)$ hold?

The π-compatibility of topologies σ and τ on a set X introduced in [CN1] implies $\mathcal{I}_m(X,\tau) = \mathcal{I}_m(X,\sigma)$ and $\mathcal{B}_p(X,\tau) = \mathcal{B}_p(X,\sigma)$ but it is not equivalent to the equalities. It turns out that a stronger version of π-compatibility between two topologies, the notion of the admissible extension (see [CN2]), was implicitly occurred in literature several times. I discuss some new facts

about these relations between topologies which will be valid for the mentioned cases.

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Self-homotopy invariants with spheres

Mathematics Subject Classification (MSC): Primary 55P10, 55Q05, 55Q20

Abstract. We introduce the self-homotopy invariants, such as the self-closeness number and the self-length. We focus on computing the self-closeness numbers and self-lengths of the product and wedge of two spheres. The self-closeness number is the minimum number $N\mathcal{E}(X)$ such that $\mathcal{A}^n_\sharp(X) = \mathcal{E}(X)$, where $\mathcal{A}^n_\sharp(X)$ consists of homotopy classes of self-map of X that induce an automorphism from $\pi_i(X)$ to $\pi_i(X)$ for $i = 0, 1, \cdots, n$. The self-length $\mathcal{L}_{\mathcal{E}(X)}$ is the number of strict inclusions of the monoid chain on $\mathcal{A}^n_\sharp(X)$ for all $k \geq 0$.

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Gem-induced trisections of PL 4-manifolds

Mathematics Subject Classification (MSC): 57Q15, 57K40, 57M15

Abstract. According to Gay and Kirby ([1]) a *trisection* of a smooth, oriented, closed 4-manifold M is a decomposition of M into three 4-dimensional handlebodies, with disjoint interiors, mutually intersecting in 3-dimensional handlebodies, so that the intersection of the three “pieces” is a closed orientable surface. The minimum genus of the intersecting surface is called the *trisection genus* of M.

In [2] Spreer and Tillmann computed the trisection genus of standard simply-connected PL 4-manifolds, via their representation by *simple cry-*

stallizations, i.e. edge-colored graphs dual to triangulations whose 1-skeleton coincides with that of a 4-simplex.

In this talk we propose generalizations of Spreer and Tillmann's work by presenting and analyzing *gem-induced trisections*, i.e. trisections that are induced by any edge-colored graph representing a compact PL 4-manifold with empty or connected boundary. In particular, we prove that the graph-defined PL invariant *regular genus* provides an upper bound for the value of the trisection genus.

We also present some results about the possible minimality of the genus of a gem-induced trisection. As a consequence, we are able to generalize Spreer and Tillmann's result by determining a wider class of closed simply-connected PL 4-manifolds whose trisection genus is realized by a gem-induced trisection and coincides with the second Betti number and also with half the value of the regular genus.

Moreover, in case of a compact PL 4-manifold admitting a handle decomposition with no 3-handles, we give an estimation of the trisection genus in terms of a surgery description ([3],[4]).

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Higher topological complexity of Seifert fibered manifolds

Mathematics Subject Classification (MSC): 55M30, 57N65, 55S99, 55P99

Abstract. We improve the cohomological lower bound for higher topological complexity by describing higher topological complexity weights for cohomology classes. As an application, we show that in many cases the higher n^{th} topological complexity of the Seifert fibered manifold is either $3n$ or $3n+1$.

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An algorithm focused on determining the best parameterization tool for uncertain environments based on decision making

Mathematics Subject Classification (MSC): 03E75, 90B50, 91B06

Abstract. Decision-making problem in an uncertain environment is found prime importance in current periods of time. Innovative methods based on soft set theory with applications in many fields of the daily life of uncertain environments have already been developed and proposed. In this paper, we find out the best possible parameter from the given fixed set of parameters in any soft sets over the universe U, which is given for the solution of an uncertainty problem. Moreover, we construct an algorithm, which proceeds toward an application for a type of uncertainty problem. The paragon outcome is achieved in the scope of group work to compute the success value of the group. We hope that this research for the selection of the best possible parameter of a universal set in any soft set will be worthwhile to researchers working on problems having uncertainty in many features for further studies in this field.

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On the fixed-point property of p-convex compacta

Abstract. Recently an analytic proof of the so-called Schauder Conjecture has been claimed. The argument involves p-convexity, $0 < p < 1$. The claimed proof will be scrutinized.

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Macroscopic dimension conjecture for RAAG

Abstract. The Gromov's macroscopic dimension conjecture states that the macroscopic dimension of the universal cover X of a closed n-manifold M with a positive scalar curvature metric does not exceed $n - 2$. We prove a strong version of Gromov's conjecture for spin manifolds whose fundamental group is a right-angled Artin group (RAAG).

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On classes of frames containing J-frames

Mathematics Subject Classification (MSC): 06D22

Abstract. J-frames are a pointfree enunciation of Michael's J-spaces ([3]). In this talk, we present a study of classes of frames satisfying conditions which are weaker (and some which are stronger) than those defining J-frames and establish conditions under which these frames are J-frames. Mimicking the defining conditions of a J-frame, modulo replacing "compact" with "connected", we define a class of CJ-frames. This class contains all connected frames. After showing that a J-frame has no points if and only if it is connected, we shall infer that all non-spatial J-frames are CJ-frames. A characterisation of J-frames via their pointfree remainders in some well-known extensions will be discussed. All metrizable regular continuous J-frames are separable (à la [1, 2]); a sketch of a proof for this will be discussed.

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Infinite Horizon Random FBSDEs and Stationary Stochastic Viscosity Solutions

Mathematics Subject Classification (MSC): Primary 60H15; Secondary 35R60, 60H25

Abstract. Stimulated by various continuous time future expectations models with random coefficients from economic theory, we study a class of infinite horizon fully coupled forward-backward stochastic differential equations (FBSDEs). Under standard Lipschitz and monotonicity conditions, and by means of the contraction mapping principle, we establish existence, uniqueness, a comparison property and dependence on a parameter of adapted solutions. Making further the connection with infinite horizon quasilinear backward stochastic partial differential equations (BSPDEs) via a generalization of the well known four-step-scheme, we are led to the notion of stochastic viscosity solutions. Given additional stationary conditions on the coefficients of the FBSDEs system, this stochastic viscosity solution becomes stationary as well.

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There is No Standard Model of ZFC and ZFC_2 with Henkin semantics. Generalized Lob's Theorem. Strong Reflection Principles and Large Cardinal Axioms. Consistency Results in Topology

Mathematics Subject Classification (MSC): 03C25; 03C30

Abstract. In this article we proved so-called strong reflection principles corresponding to formal theories Th which has omega-models. A possible generalization of the Lob's theorem is considered. Main results is: (1) let k be an inaccessible cardinal, then $\neg Con(ZFC + \exists k)$, (2) there is a Lindelöf T_3 indestructible space of pseudocharacter $\aleph_1$ and size $\aleph_2$ in L.
https://arxiv.org/abs/1301.5340

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Contemporary Phase Space Methods applied on Magnetohydrodynamic flow data

Abstract. In this study we apply the modern method of Recurrence Plots (RP) and the recently proposed Visual Boundary Recurrence Plots (VBRP) method on magnetohydrodynamic flow data. Recurrence Plots is a graphical tool based on phase space topology. Recurrence Plots are constructed by calculating Euclidian distances between vectors. Visual Boundary Recurrence Plot visualizes each point of the recurrence plot with colors depending on whether the neighboring points are recurrent or not, concentrating on

vertical and horizontal directions studying the stability of the textures of the recurrence plots. RPs and VBRPs applied on magnetohydrodynamic (MHD) turbulent channel flow data (velocity timeseries). From the RP results we extracted valuable information about the dynamics of the two different systems. However, the application of the VBRP method gave us the advantage to identify additional correlations of two systems inside. Finally, we introduced a metric distance among two VBRPs that allow us to discuss their topological closeness.

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On typical behaviour of projections of a compact set in R^N

Mathematics Subject Classification (MSC): 54E52, 54F45, 57N35

Abstract. We apply ideas of geometric measure theory and Baire category theory to topological problems, namely, to topological embeddings of compact sets into Euclidean space.

In 1947, Borsuk constructed a Cantor set in R^N, $N \geq 3$, such that its projection onto any $(N-1)$-plane contains an $(N-1)$-dimensional ball. This can be strengthened: a desired Cantor set can be obtained from an arbitrary Cantor set by an arbitrarily small isotopy of the ambient space R^N.

As a corollary, Borsuk obtained a simple arc in R^N, $N \geq 3$, whose orthogonal projection onto any $(N-1)$-plane contains an $(N-1)$-dimensional ball. This can also be done for knots, i.e. for embeddings of the circle S^1 into R^N. Borsuk's work combined with known results on Cantor sets implies: a compactum with $(N-1)$-dimensional projections can be obtained from an arbitrary uncountable compactum $X \subset R^N$ by an arbitrarily small isotopy of the space R^N.

The question arises: how do the dimensions of the projections of a compact set $X \subset R^N$ behave under a typical ambient isotopy or under a typical ambient homeomorphism? (Typical in the sense of the Baire category.)

We solve this problem. Our main result strengthens Väisälä's theorem (1979) connecting Hausdorff dimension and Shtan'ko embedding dimension (denoted by "dem"). In its turn, Väisälä's theorem extends results of Nöbeling (1931) and Szpilrajn (1937) on relationship between Hausdorff dimension and topological dimension.

As a consequence, we find out how the projections of a knot in R^3 "typically" behave. Recall that a typical knot in R^3 is wild (J. Milnor, 1964) and even wild in every point (H.-G. Bothe, 1966).

As another consequence, we get new criteria of tameness and wildness of a Cantor set in R^N in terms of its projections.

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Local completeness of $C_k(X)$

Mathematics Subject Classification (MSC): 46A03, 54C35

Abstract. Locally complete spaces form a significant and widely studied class of locally convex spaces, and the problem to find a characterization of locally complete spaces among important classes of locally convex spaces arises naturally.

For a Tychonoff space X, we denote by $C_k(X)$ the space $C(X)$ of all real-valued continuous functions on X endowed with the compact-open topology. Then the problem can be formulated as follows:

Problem 1.1. *Characterize Tychonoff spaces X for which the space $C_k(X)$ is locally complete.*

For the important partial cases when X is a pseudocompact space, the answer to Problem 1.1 was obtained by Warner [2]:

Theorem 1.2 ([2]). *For a pseudocompact space X the following assertions are equivalent:* (i) *$C_k(X)$ is locally complete,* (ii) *$C_k(X)$ is sequentially complete,* (iii) *X is Warner bounded.*

Since 1958 only a simplification of the original proof of Warner was obtained, see Theorem 2.13 in [1]. We propose a complete solution of Problem 1.1 by proving the following theorem:

Theorem 1.3. *For a Tychonoff space X the following assertions are equivalent:*

(i) *every strongly functionally compact-finite sequence of functionally closed subsets of X is locally finite;*

(ii) *X is a sequentially Ascoli space;*

(iii) *$C_k(X)$ is locally complete.*

If the space X is pseudocompact we also extend the classical Warner's Theorem 1.2.

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A characterization of proper morphisms by the lifting property

Mathematics Subject Classification (MSC): 18B30, 54B30, 54C10, 54C20, 54D30, 54D10

Abstract. It was probably in the 1950's when factorization systems appeared for first time. Isbell [2] already used the concept of lifting property, but it was not until 1967 that Quillen [1] gave the formal definition of this property to introduce model categories. Since then the lifting property has had a prominent role in category theory, but also in different areas of topology such as homotopy theory, covering spaces, extensions of continuous functions and others.

Recently Gavrilovich [3] characterized some classical properties and concepts in terms of lifting properties; for example, compactness, connecteness, dense image, discrete spaces, induced topology, separation axioms and proper maps. In particular, proper maps with domain and codomain T_4 were charaterized as those maps in $((\{a\} \longrightarrow \{a \searrow b\})^r_{<5})^{lr}$ (that is, the map has the right lifting property with respect to the class of morphism that have the left lifting property with respect to those that have the right lifting property with respect to the inclusion of the point that is open in Sierpinski's space, and have domain and codomain of cardinality less than 5). In the same article the author proposed the following conjecture:

Conjecture 1. *In the category of topological spaces* $((\{a\} \longrightarrow \{a \searrow b\})^r_{<5})^{lr}$ *is the class of proper maps.*

We give a characterization of proper maps with regular domain by means of a lifting property [4]. This partially answers the conjecture above.

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Beyond the Volume Conjecture

Abstract. Quantum Topology is a recent area of research that originated with V. Jones's discoverty of the Jones polynomial of a knot. Witten reformulated this topological invariant in terms of quantum field theory, and Kashaev in 1995 made a startling conjecture relating the Jones polynomial of a knot and its parallels to Thurston's 3-dimensional Hyperbolic Geometry. This Volume Conjecture is currently proven only for a handful of knots (less than 10), and tested experimentally for many more. The talk will survery the volume conjecture and its exponentially small refinements. Joint work with Don Zagier.

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Zero-Dimensional Extensions of Topologies

Mathematics Subject Classification (MSC): 54F45, 54A05, 54A10

Abstract. Undoubtedly, Topological Dimension Theory contains many important chapters, including the studies of different notions of dimensions such as the small inductive dimension and the covering dimension. In this paper, we extend such studies. Given a topological space (X, τ), we introduce the notions of zero-dimensional extensions of τ with respect to the small inductive dimension ind and the covering dimension dim. Based on these meanings, new cardinal invariants are inserted, the so-called zero-dimensional structural numbers with respect to ind and dim, succeeding to present the topology τ as an intersection of zero-dimensional extensions of it. We study properties of these invariants and their "behavior" in different classes of topological spaces.

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The notion of convergence on τ-metric spaces

Mathematics Subject Classification (MSC): 54A20, 40A35, 40A05

Abstract. In this paper we study the notion of convergence on τ-metric spaces, investigating new results and properties. Moreover, we study the meanings of statistical and ideal convergences on this class of spaces, investigating also their behavior under the view of the classical notions of convergence, statistical convergence and ideal convergence on the usual environment of metric spaces.

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An extension of covering dimension for continuous mappings

Mathematics Subject Classification (MSC): 54F45

Abstract. In Dimension Theory, there are "mapping theorems" establishing relationships between the dimensions of the domain and range of a continuous mapping ([AN], [C], [E], [P]). Most of the theorems deal with mappings that satisfy additional conditions such as being closed mappings. For a given continuous mapping $f : X \to Y$, the *dimension* of f is usually defined as follows:

$$\dim f = \sup\{\dim(f^{-1}(y)) : y \in f(X)\}.$$

In this talk we introduce and investigate a different notion of covering dimension for continuous mappings between topological spaces, which is closer to the classical definition of the Lebesgue covering dimension of a space. This notion leads to the definition of new interesting classes of continuous mappings. We also discuss results concerning continuous mappings between metric spaces.

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On the notion of convergence of a function along an ideal

Mathematics Subject Classification (MSC): 54A20

Abstract. The concept of convergence of a function along an ideal, alias $\mathcal{I}$-convergence, is the dual concept of convergence of a function along a filter, introduced by H. Cartan. In this paper, we seek to treat this notion as a primitive notion of convergence and examine the conditions that make it possible to recognize whether this type of convergence is topological. Specifically, we consider a non-empty set X and a class $\mathcal{C}$ consisting of triples of the form $(f, x, \mathcal{I})$, where f is a function with domain D and values in X, $\mathcal{I}$ is a proper ideal on D and $x \in X$, and provide a set of axioms of convergence, on the class $\mathcal{C}$, which prove to be necessary and sufficient to ensure the existence of a unique topology τ on X subject to the following condition: $(f, x, \mathcal{I}) \in \mathcal{C}$ if and only if f $\mathcal{I}$-converges to x, relative to the topology τ.

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Bi-equivariant extension of continuous maps

Mathematics Subject Classification (MSC): 54H15, 20M20

Abstract. Let X be a topological space and let G be an arbitrary topological group. A *binary action* of group G on X is a continuous map $\alpha : G \times X^2 \to X$ such that

$$\alpha(gh, x_1, x_2) = \alpha(g, x_1, \alpha(h, x_1, x_2)), \quad \alpha(e, x_1, x_2) = x_2,$$

or

$$gh(x_1, x_2) = g(x_1, h(x_1, x_2)), \quad e(x_1, x_2) = x_2$$

for all $g, h \in G$ and $x_1, x_2 \in X$, where e is the identity of G.

By a *topological binary transformation group* or *binary G-space* we mean a triple (G, X, α), where α is a binary action of group G on X.

A continuous map $f : X \to Y$ between binary G-spaces (G, X, α) and (G, Y, β) is called a *bi-equivariant map* provided

$$f(\alpha(g, x_1, x_2)) = \beta(g, f(x_1), f(x_2))$$

or

$$f(g(x_1, x_2)) = g(f(x_1), f(x_2))$$

for all $g \in G$ and $x_1, x_2 \in X$.

A subset B of a binary G-space X is called bi-invariant if $G(B, B) = B$. A minimal bi-invariant subset $\widetilde{A} \subset X$ which contains a set $A \subset X$ is called the *bi-invariant extension* of A.

For any subset A of X, let us recursively define the sets A^n, $n = 1, 2, \dots$, as follows:

$$A^1 = G(A, A), \quad A^2 = G(A^1, A^1), \quad \dots, \quad A^n = G(A^{n-1}, A^{n-1}), \quad \dots$$

We denote an element $x = g_1(a_1, a_2) \in A^1$ by $[g_1; a_1, a_2]$. An element $x \in A^2$ which has a form $x = g_1(g_2(a_1, a_2), g_3(a_3, a_4))$ is denoted by

$$[g_1, g_2, g_3; a_1, a_2, a_3, a_4].$$

Similarly, any element $x \in A^n$ is defined by a collection of some elements $g_1, \dots g_{2^n-1} \in G$ and $a_1, \dots, a_{2^n} \in A$: $x = [g_1, \dots, g_{2^n-1}; a_1, \dots, a_{2^n}]$.

Now let X and Y be binary G-spaces and let A be a subset of X. We say that a continuous map $f : A \to Y$ is a *structural map* if

$$[g_1, \dots, g_{2^n-1}; a_1, \dots, a_{2^n}] = [g'_1, \dots, g'_{2^m-1}; a'_1, \dots, a'_{2^m}],$$

implies

$$[g_1, \dots, g_{2^n-1}; f(a_1), \dots, f(a_{2^n})] = [g'_1, \dots, g'_{2^m-1}; f(a'_1), \dots, f(a'_{2^m})].$$

where $g_1, \dots g_{2^n-1}, g'_1, \dots g'_{2^m-1} \in G$, $a_1, \dots, a_{2^n}, a'_1, \dots, a'_{2^m} \in A$, $n, m \in N$.

A binary G-space X is called *distributive* if

$$g(h(x, x_1), h(x, x_2)) = h(x, g(x_1, x_2))$$

for any $x, x_1, x_2 \in X$ and $g, h \in G$.

Theorem 1. Let X and Y be distributive binary G-spaces and let A be a closed subset of X. Then every continuous structural map $f : A \to Y$ can be extended uniquely to a continuous bi-equivariant map $\widetilde{f} : \widetilde{A} \to Y$ where $\widetilde{A} \subset X$ is the bi-invariant extension of A.

For a distributive binary G-space X we can define the orbit space $X|G$ and a continuous section $\sigma : X|G \to X$ of the orbit projection $\pi : X \to X|G$. The image of a section $\sigma : X|G \to X$ is closed in X. Every closed subset of X touching each orbit in exactly one point defines a continuous section of $\pi : X \to X|G$. Because of this, we will use the term "section" for the closed set which is the image of a section $\sigma : X|G \to X$.

Theorem 2. Let G be a compact topological group, X and Y distributive binary G-spaces, and A a section of the orbit projection $\pi : X \to X|G$. Suppose $f : A \to Y$ is a continuous map such that

$$g(a, a) = h(k(a', a'), s(a'', a''))$$

implies

$$g(f(a), f(a)) = h(k(f(a'), f(a')), s(f(a''), f(a'')))$$

where $a, a', a'' \in A$ and $g, h, k, s \in G$. Then f has a unique continuous bi-equivariant extension $\widetilde{f} : X \to Y$.

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Entropy and Cardinality of the Rational Numbers

Mathematics Subject Classification (MSC): 53Z05, 82B10, 58A05, 26A03, 03E25, 03E10

Abstract. In this article, we will discuss the cardinality of rational numbers. We will elaborate a comparison between two approaches used to find the cardinalities of finite dimensional Cartezian products of the set of positive integers. In the first approach, used in continuum hypothesis, such products are considered to be countable. In the second approach, used in statistical mechanics, such products have a greater cardinality than the set of positive integers. The later agrees with experiments and is consistent with the foundations of topology and differential geometry. We will demonstrate that the set of rational numbers need not to be countable. This article implies that the axiom of choice can be a better technique to prove theorems that use second-countability. This is important for the mathematical foundation of quantum statistical mechanics, metrization theorems and physics of spacetime.

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Affine Connections in Quantum Gravity and New Fields

Mathematics Subject Classification (MSC): 53B50, 83C45, 83F05, 57R30, 83D05

Abstract. In this article, we will discuss the possibility of using affine connections to explain inflation and dark energy. We have previously introduced two massless scalar fields using connections more general than the Levi-Civita connections in the Einstein-Palatini action. In this article, we will develop a scheme to add suitable potential terms for these fields. We will construct a Lagrangian formalism to include these scalar fields in a theory of gravity coupled with ordinary matter and radiation. These fields need not to be present in the Lagrangians of gauge theories with conserved fermionic currents. The same remains valid for scalar fields. We will discuss a generalization of this aspect. A careful application of the Stokes's theorem in curved spacetime reveals that we need to introduce the right-handed neutrinos in the electroweak theory in curved spacetime even with the Levi-Civita connections. This is required to have conserved vector currents for the neutrinos and can be important for dark matter research. The above mentioned scalars contribute positive and negative stress tensors to Einstein's equation and can be useful to explain inflation and dark energy. We will discuss a model that can have an additional massless scalar field. We will discuss the possibility of introducing higher spin fields using second rank symmetric traceless tensors. We will also discuss the corresponding little group analysis in flat spacetime. We will show that we can use massless (A,A) type fields in Minkowski space to introduce massless integer spin particles.

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On some weakenings of the Hausdorff and the Urysohn separation axioms and new related cardinal functions

Mathematics Subject Classification (MSC): 54A25, 54D10, 54D20, 03E99

Abstract. The *Hausdorff number* $H(X)$ of a topological space X (see [1]) is the least cardinal number κ such that for every subset $A \subseteq X$ with $|A| \geq \kappa$ there exists an open neighbourhood U_a for every $a \in A$ such that $\bigcap_{a \in A} U_a = \emptyset$. A space X is said to be *n-Hausdorff* if $H(X) \leq n$. In an analogous way is possible to define also the *Urysohn number* and the class of *n-Urysohn* spaces (see [2],[3]). In [4] we present two new cardinal functions defined in the class of n-Hausdorff and n-Urysohn spaces that extend pseudocharacter and closed pseudocharacter respectively and some bounds on the cardinality of n-Hausdorff and n-Urysohn spaces that represent variations of known results. Moreover, some properties in the class of n-Urysohn n-H-closed spaces are given.

A joint work with M. Bonanzinga, N. Carlson

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On dense sets of products of separable spaces

Mathematics Subject Classification (MSC): 54B10

Abstract. The classical Hewitt - Marczewski - Pondiczery theorem states that if $d(X_s) \leqslant \tau$ $(\omega \leqslant \tau)$ for every $s \in S$ and $|S| \leqslant 2^\tau$ then

$$d(\prod_{s\in S} X_s) \leqslant \tau.$$

We consider the problem of the existence of a dense set of a cardinality ω in the product $\prod\limits_{s\in S} X_s$ for $\tau = \omega$ which contains no convergent nontrivial sequences.

For $\tau = \omega$ the existence of such set were proved for I^c (W.H. Priestly, 1970), for D^c, where D is the two point discrete space (P. Simon, 1978), for Z^c, where Z is separable not single point T_1-space (A. Gryzlov, 2018), for a product of 2^c separable decomposable spaces, i.e. spaces, which contain two not empty closed disjoint sets (A. Gryzlov, 2020).

We prove the following.

Such set exist in the product of 2^ω many separable spaces, which contain a simple sequence, that has no limit.

We say that a sequence λ is simple, if for every $x_n \in \lambda$ the set $\{n' \in \omega : x_n = x'_n\}$ if finite.

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Topological and combinatorial characterizations of normal 3-pseudomanifolds with $g_2 \leq 5$

Mathematics Subject Classification (MSC): Primary 05E45; Secondary 05C30, 57Q15, 57Q25

Abstract. In recent years, characterizing normal pseudomanifolds with respect to small g_2 has become a very popular topic. For normal 3-pseudomanifolds with $g_2 \leq 3$ and 3-manifolds with $g_2 \leq 9$, the topological and combinatorial characterizations are known. In this talk, we characterize normal 3-pseudomanifolds with $g_2 \leq 5$. First, we show that a normal 3-pseudomanifold with $g_2 \leq 5$ has no more than two singular vertices. Then, we show that a normal 3-pseudomanifold K with $g_2 \leq 5$ is obtained from some boundary complex of 4-simplices by a sequence of possible operations of types connected sum, bistellar 1-move, edge contraction, edge expansion, and an edge folding. As a result, K is a triangulation of either a sphere or a suspension of $\mathbb{RP}^2$.

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Functional countability in LOTS

Mathematics Subject Classification (MSC): Primary 54F05; Secondary 06A05, 54A35, 54C30

Abstract. A topological space X is called functionally countable if $f[X]$ is countable for any continuous function $f : X \to \mathbb{R}$. The diagonal of a space X is the subset $\Delta_X = \{\langle x, x\rangle : x \in X\}$ of $X \times X$. In [2] a 2021 paper, Vladimir Tkachuk studied spaces X such that $(X \times X) \setminus \Delta_X$ is functionally countable, in this paper Tkachuk asked the following:

Suppose that X is a linearly ordered space and $(X \times X) \setminus \Delta_X$ is functionally countable. Must be X separable?

In this talk we focus our attention in this question and we show that if X is an uncountable linearly ordered space such that $(X \times X) \setminus \Delta_X$ is functionally countable, then X must be an Aronszajn line. This result was proved in [1].

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Several types of Jordan curve theorems in an applied topological setting

Mathematics Subject Classification (MSC): 54A05, 54D10, 54F05, 54C08, 54C10, 54F65

Abstract. This talk refers to several types of Jordan curve theorems in an applied topological setting. With an Alexandroff topological structure, a Marcus-Wyse topological structure, an H-topological structure, a Khalimsky topological structure, and so on, we can establish the corresponding Jordan curve theorem. Since each of them has its own feature, depending on the situation, the usuage of it can be considered. In particular, the present talk mainly deals with the semi-Jordan curve theorem on the MW-topological plane and refers to some applications into the field of applied topology such as digital topology, mathematical morphology as well as computer science.

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Quasicontinuity, measurability and the topology of uniform convergence on compacta

Mathematics Subject Classification (MSC): 54C08, 26A21, 54A25

Abstract. The notion of quasicontinuity is a classical one. It has found many applications in the study of topological groups, in the study of continuity points of separately continuous mappings and in characterizations of minimal usco and minimal cusco maps [1].

The aim of the talk is to present some recent results on quasi-continuous mappings. Among other results we will mention the following ones. There are 2^c real quasicontinuous non-Lebesgue measurable functions defined on the interval $[0, 1]$ [4]. Let X be an uncountable Polish space. Then there are 2^c real quasi-continuous non Borel measurable functions on X [5]. The density of the space of quasicontinuous mappings from $\mathbb{R}$ to $\mathbb{R}$ equipped with the topology of uniform convergence on compacta is 2^c [2].

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Factorization of maps, large cardinals and G_δ-covers

Mathematics Subject Classification (MSC): 54D80, 03E55

Abstract.
Theorem *Let $\mathcal{C}$ be an epireflective class in $\mathtt{Top}_T$ (or in $\mathtt{Unif}_H$) generated by a space A of Ulam non-measurable cardinality. Then every continuous (or uniformly continuous, resp.) map into A from a limit of an inverse system in $\mathcal{C}$ strongly factorizes via a subsystem of cardinality less than $\mathfrak{s}_1$.*

If $\mathcal{C}$ contains a countable discrete space (or a countable finest precompact discrete uniform space, resp.) then the bound $\mathfrak{s}_1$ cannot be decreased.

By $\mathfrak{s}_1$ the first ω_1-strongly compact cardinal is denoted. It is shown that existence of the factorization is in a close connection with G_δ-covers and that the needed covers exist for cardinals less than $\mathfrak{s}_1$ and do not exist under the condition mentioned in Theorem. The result is proved for a general case of any measurable cardinal and μ-strongly compact cardinal numbers.

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Product of spaces, dimension and universality

Abstract. There are many papers, concerning the logarithmic law for the dimension of the product of spaces. In particular, examples of spaces, for which the logarithmic law is not true for its products, were constructed. We mension here some of them. First such spaces were given by L. Pontryagin. He construct two metric compact spases X and Y for which $dim(X) = dim(Y) = 2$ and $dim(X \times Y) = 3$. On the other hand V. Filippov (1971) construct two (non-metrizable) compact spaces X and Y for which $Ind(X) = ind(X) = 1$, $Ind(Y) = ind(Y) = 2$ and $Ind(X \times Y) \geq ind(X \times Y) > 3$. Later, A. Karassev and K. Kozlov (2015) (using a result of B. Pasynkov (1988)) proved that for these spaces we have $Ind(X \times Y) = ind(X \times Y) = 4$. Many examples of compact metric spaces, for which the logarithmic law is not true, are given by A. Dranishnikov (1988). He proved that for each natural numbers $n \leq m$ and each $r : n < r \leq m + n$, there are compact metric spaces X_n and X_m such that $dim(X_n) = n$, $dim(X_m) = m$ and $dim(X_n \times X_m) = r$.

The following propositions are corollaries of the given below main theorem.

Proposition 1. *For each separable metrizable space Y and each countable ordinals α and β in the non-empty class of all separable metrizable spaces X, for which $ind(X) = \alpha$ and $ind(Y \times X) = \beta$, there exists a universal element.*

Proposition 2. *For each (completely) regular space Y of weight $\leq \tau$ and each ordinals α and β in the non-empty class of all (completely) regular spaces X of weight $\leq \tau$, for which $ind(X) = \alpha$ and $ind(Y \times X) = \beta$, there exists a universal element.*

Proposition 3. *For each* T_0*-space* Y *of weight* $\leq \tau$ *and each ordinals* α *and* β *in the non-empty class of all* T_0*-spaces* X *of weight* $\leq \tau$*, for which* $ind(X) = \alpha$ *and* $ind(Y \times X) = \beta$*, there exists a universal element.*

(For many Y, α and β, the considered in these propositions classes of spaces, may be empty.)

We note that although the Propositions $1-3$ have the similar formulations, they are independent each other.

The main result is the following theorem.

Theorem. *For each space* Y *of weight* $\leq \tau$*, each ordinals* α *and* β *and each saturated class* $\mathbb{S}$ *of spaces of weight* $\leq \tau$ *in the non-empty class of all elements* $X \in \mathbb{S}$*, for which* $ind(X) = \alpha$ *and* $ind(Y \times X) = \beta$*, there exists a universal element.*

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Thurston's mapping class group-equivariant deformation retraction of Teichmüller space

Abstract. Thurston constructed a mapping class group-equivariant deformation retraction of the Teichmüller space of a closed, orientable surface. This talk will survey his construction, as well as a dual construction due to Schmutz, and discuss some consequences for the structure of moduli space.

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Descriptive complexity in number theory and dynamics

Mathematics Subject Classification (MSC): Primary 03E15; Secondary 11K16, 37B10, 37E99

Abstract. Informally, a real number is normal in base b if in its b-ary expansion, all digits and blocks of digits occur as often as one would expect them to, uniformly at random. We will denote the set of numbers normal in base b by $\mathcal{N}(b)$. Kechris asked several questions involving descriptive complexity of sets of normal numbers. The first of these was resolved in 1994 when Ki and Linton proved that $\mathcal{N}(b)$ is $\mathbf{\Pi}^0_3$-complete. Further questions were resolved by Becher, Heiber, and Slaman who showed that $\bigcap_{b=2}^{\infty} \mathcal{N}(b)$ is $\mathbf{\Pi}^0_3$-complete and that $\bigcup_{b=2}^{\infty} \mathcal{N}(b)$ is $\mathbf{\Sigma}^0_4$-complete. Many of the techniques used in these proofs can be used elsewhere. We will discuss recent results where similar techniques were applied to solve a problem of Sharkovsky and Sivak and a question of Kolyada, Misiurewicz, and Snoha. Furthermore, we will discuss a recent result where the set of numbers that are continued fraction normal, but not normal in any base b, was shown to be complete at the expected level of $D_2(\mathbf{\Pi}^0_3)$. An immediate corollary is that this set is uncountable, a result (due to Vandehey) only known previously assuming the generalized Riemann hypothesis.

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Quasi-Cauchy spaces and completion

Mathematics Subject Classification (MSC): 54A20, 54A40, 54E15, 54E35, 54E70

Abstract. Based on Cauchy pair filters, introduced by Lindgren and Fletcher [3] for studying completeness in quasi-uniform spaces [2], we develop an axiomatic theory of non-symmetric Cauchy spaces that we call quasi-Cauchy spaces. A quasi-Cauchy structure is a set of pair filters satisfying three natural axioms. We study the categorical properties of the category of quasi-Cauchy spaces and completions of non-complete quasi-Cauchy spaces. We further show that symmetric quasi-Cauchy spaces can be identified with classical Cauchy spaces introduced by Keller [1].

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A topological insight into the polar involution of convex sets

Mathematics Subject Classification (MSC): Primary 52A20, 52A21, 54B20, 54C10, 54C55; Secondary 54H15, 57S25

Abstract. Let us denote by $\mathcal{K}_0^n$ the family of all closed convex sets $A \subset \mathbb{R}^n$ containing the origin $0 \in \mathbb{R}^n$. For $A \in \mathcal{K}_0^n$, its polar set is denoted by A°. Namely,

$$A^\circ := \left\{ x \in \mathbb{R}^n : \sup_{a \in A} \langle a, x \rangle \leq 1 \right\},$$

where $\langle \cdot, \cdot \rangle$ stands for the usual inner product on $\mathbb{R}^n$.

In this talk, we will discuss the topological nature of the polar map $A \to A^\circ$ on $(\mathcal{K}_0^n, d_{AW})$, where d_{AW} denotes the Attouch-Wets metric. We will show that $(\mathcal{K}_0^n, d_{AW})$ is homeomorphic to the Hilbert cube $Q = \prod_{i=1}^{\infty}[-1, 1]$ and that the polar map is topologically conjugate with the standard based-free involution $\sigma : Q \to Q$, defined by $\sigma(x) = -x$ for all $x \in Q$. We will also characterize all the inclusion-reversing involutions on $\mathcal{K}_0^n$ which are conjugate σ. In this sense, we will show that the polar map is essentially the only possible decreasing involution with a unique fixed point on $\mathcal{K}_0^n$. More precisely, we will prove that every based-free decreasing involution $f : \mathcal{K}_0^n \to \mathcal{K}_0^n$ is conjugate with the standard involution on Q and f is of the form $f(A) = T(A^\circ)$ for some positive-definite linear isomorphism $T : \mathbb{R}^n \to \mathbb{R}^n$.

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Jones polynomials of 3-strand weaving knots

Mathematics Subject Classification (MSC): 57K10, 57K14

Abstract. In this talk, we discuss a formula for the Jones polynomial of 3-strand weaving knots, and its evaluations at certain roots of unity. This formula is derived by diagonalization of the Burau matrices associated with weaving 3-braids over an appropriate function field. Afterwards, we substitute specific values for the variable t of this formula to recover information about the knot determinant and the unknotting number of the corresponding knots.

This is joint work with my research advisor M. Prabhakar.

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On Asplund spaces $C_k(X)$ with the compact-open topology

Mathematics Subject Classification (MSC): 54C35, 54G12, 54H05, 46A03

Abstract. A famous theorem of Namioka and Phelps says that for a compact space X, the Banach space $C(X)$ is Asplund iff X is scattered. We extend this result to the space of continuous real-valued functions endowed with the compact-open topology $C_k(X)$ for several natural classes of non-compact Tychonoff spaces X. The concept of Δ_1-spaces recently introduced has been shown to be applicable for this research.

This is a joint work with Ondrej Kurka and Arkady Leiderman.

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Preservation of inverse limits and G-fibrations under the twisted product functor

Mathematics Subject Classification (MSC): 54H11, 54H15, 55P91

Abstract. Given a continuous homomorphism of topological groups $\alpha : G' \to G$, every G-space and every G-map can be regarded as G'-space and G'-map respectively, so we get the restriction functor $res : G\text{-}Top \to G'\text{-}Top$. That functor preserves many properties from objects and morphisms in G-Top and is right adjoint of the functor of twisted product $G \times_\alpha - : G'\text{-}Top \to G\text{-}Top$. A natural question is whether the properties of the objects and morphisms in G'-Top are also preserved under the twisted product functor. In this talk we will give sufficient conditions in order to have the

preservation of inverse limits and equivariant fibrations under the twisted product functor, that is to say, the twisted product of the inverse limit of an inverse sequence in G'-Top is the inverse limit of the corresponding inverse sequence in G-Top; also if $p : E \to B$ is a G'-fibration, then the induced G-map $\tilde{p} : G \times_\alpha E \to G \times_\alpha B$ is a G-fibration.

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Investigation of topology effect on the mixing process between the nanoparticles and the biological fluid inside T shaped micromixers

Abstract. During metastasis of cancer cells, circulating tumor cells (CTCs) are released from the primary tumor, reach the bloodstream, and colonize new organs. A potential reduction of metastasis may be accomplished through the use of nanoparticles in micromixers in order to capture the CTCs that circulates in blood. In the present study, the effective mixing of nanoparticles and the blood that carries the CTCs are investigated. The mixing procedure was studied under various inlet velocity ratios (Vp/Vc) and several T-shaped micromixer geometries with different topologies of the rectangular cavities by using computational fluid dynamics techniques. Two streams are mixed in T-shaped microfluidic reactors with various small rectangular cavities under various inlet conditions between the two streams.

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A note on vector spaces with an order unit

Mathematics Subject Classification (MSC): 54C10, 46B40

Abstract. An element 1_X of an ordered real vector space X is said to be an *order unit* in X if for each $x \in X$ there is a real number $\alpha > 0$ such that $\alpha 1_X \geq x$. We present a few results on such spaces equipped with the topology whose base is the collection of balls $B(x, \varepsilon) := \{y \in X : \|y - x\| < \varepsilon\}$ $(x \in X, \varepsilon > 0)$, where for $x \in X$, $\|x\| := \inf\{\lambda > 0 : -\lambda 1_X \leq x \leq \lambda 1_X\}$. The main result is an open mapping theorem for weakly additive, order-preserving mappings between vector spaces with order units.

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A criterion of compact sets in L^p-spaces and its application

Mathematics Subject Classification (MSC): 54C35, 46B50, 46E30, 57N20

Abstract. Compactness of subsets in function spaces have been studied in analysis. In this talk, we shall give a criterion for subsets in L^p-spaces on metric measure spaces to be compact, which is a generalization of the results by A.N. Kolmogorov [1] and M. Riesz [2]. Using this criterion, we investigate the topological types of subspaces consisting of Lipschitz functions with bounded supports in L^p-spaces.

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Applications of joint ergodicity to topological dynamical systems

Mathematics Subject Classification (MSC): 37-XX, 37A05

Abstract. Whenever we have a multiple ergodic average that converges to the expected limit (i.e., the product of the integrals of the functions appearing in the average), we say that we have joint ergodicity for the sequences of iterates. In such cases we can, almost immediately, get applications to topological dynamical systems. In this talk, I will discuss recent developments in the topic.

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Remarks on hyperspaces of knots

Mathematics Subject Classification (MSC): 54F16, 54H05, 57K10

Abstract. The local contractibility and Borel complexity of the hyperspaces of polygonal or tame knots are discussed.

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On Some Contractive Mappings and a New Version of Implicit Function Theorem in Topological Spaces

Mathematics Subject Classification (MSC): 47H10, 54H25

Abstract. The main goal of this article is to study about the existence of fixed points of some contractive mappings in topological spaces. At first, we define two new contractive mappings, viz., h-$\mathcal{A}$-contractive and h-$\mathcal{A}_1$-contractive mappings on a topological space X, where $h : X \times X \to \mathbb{R}^+$ is a function and $\mathcal{A}, \mathcal{A}_1$ are two implicit collections of functions. Then we obtain some fixed point results concerning these two contractive mappings. Finally, we obtain a new version of implicit function theorem on topological spaces in the light of one of our obtained fixed point result.

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ℓ-dominance for some classes of scattered spaces

Mathematics Subject Classification (MSC): 54C35, 54G12

Abstract. We say that a Tychonoff space Y is ℓ-dominated by a Tychonoff space X if there exists a linear continuous operator onto $T : C_p(X) \to C_p(Y)$.

There are many topological properties which are invariant under ℓ-dominance, and there are many which are not. For instance, it is known that if Y is ℓ-dominated by a metrizable compact space X, then Y also is a metrizable compact space, while Y does not have to be compact if X is (a non-metrizable) compact space.

We report on new results obtained in several recent joint papers [1], [2], [3]. All undefined notions can be found in the papers.

Theorem 1. [1] *Let Y be ℓ-dominated by X.*

(1) *If X is σ-scattered (σ-discrete), then Y is σ-scattered (σ-discrete, respectively).*

(2) *If X is a scattered Eberlein compact space, then Y also is a scattered Eberlein compact space.*

(3) *If X is a Δ-space, then Y also is a Δ-space.*

(4) *Let X and Y be metrizable spaces (for instance, let both be subsets of the real line $\mathbb{R}$). If X is a Q-set, then Y also is a Q-set.*

(5) *Let X and Y be metrizable spaces. If X is scattered, then Y also is scattered.*

Theorem 2. [2] *Let Y be ℓ-dominated by X.*

(1) *If X is a Δ_1-space, then Y also is a Δ_1-space.*

(2) *If X is pseudocompact and every countable set in X is scattered, then Y has the same properties.*

(3) *If X is a compact scattered space, then Y is a pseudocompact space such that its Stone–Čech compactification βY is scattered.*

Theorem 3. [3] *Let $X = [1, \alpha]$, where α is a fixed infinite countable ordinal. Then Y is ℓ-dominated by X if and only if Y is homeomorphic to $[1, \beta]$, where β is a countable ordinal such that either $\beta < \alpha$, or $\alpha \leq \beta < \alpha^{\omega}$.*

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C*-Algebra, Linear Operators and Quantum Mechanics

Mathematics Subject Classification (MSC): 46L60, 47L90, 46N50, 81Q10

Abstract. This paper describes in a short presentation the connection of C*-Algebra, Linear operators and Quantum Mechanics. Starting from C*-Algebra brief introduction we proceed on the Linear Adjoint Operators and we prove some basic theorems and useful outcomes. The linear operators are one of the basic ground-place for Quantum Mechanics mathematical representations emphasizing the different representations and Dirac observables. We finally conclude on the subject by discussing more on observables and how Quantum Mechanics mathematical construction is different from the Classical mechanics.

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Equivariant simultaneous extension operators for continuous maps

Mathematics Subject Classification (MSC): 54C55, 54H15

Abstract. Let us suppose that A is a closed invariant subset of a metrizable G-space Z, and V is a locally convex linear G-space, where G is a compact Lie group. Let $C(Z,V)$ denote the vector space of continuous maps from Z into V, and similarly for $C(A,V)$. We equip these mapping spaces with the compact-open topology and the action defined by $(gf)(x) = gf(g^{-1}x)$. In this talk, we will discuss the existence of an equivariant linear homeomorphic embedding $\varLambda : C(A,V) \to C(Z,V)$ and an invariant neighborhood X of A such that for every $f \in C(A,V)$, we have that $\varLambda(f)|_A = f$, $Im\varLambda(f) \subset conv(Imf \cup \{0\})$ and $Im(\varLambda(f)|_X) \subset conv(Imf)$ (here, $Im(f)$ denotes the image of the map f and $conv$ denotes the convex hull).

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Topologizing the space of minimal primes of an M-frame

Mathematics Subject Classification (MSC): 06D22

Abstract. An M-frame is an algebraic frame satisfying the Finite Intersection Property. Given an M-frame, call it L, we can topologize the set of minimal prime elements of L, which we will denote by $\mathrm{Min}(L)$. One such way we could topologize $\mathrm{Min}(L)$ is with the Zariski topology as is done with the prime ideals of a commutative ring. The other is the inverse topology which has a similar construction to that of the Zariski topology. Our aim in this talk to is to study these topological spaces and the interplay that exists between the topological properties of $\mathrm{Min}(L)$ and the frame-theoretic properties of L.

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Non-Hausdorff and non-Urysohn homogeneous spaces

Mathematics Subject Classification (MSC): 54A25, 54D10, 54D20, 54D35, 54D80

Abstract. Let $n \geq 2$ be an integer. For a topological space X the *Hausdroff number* $H(X)$ (resp. the *Urysohn number* $U(X)$) is the least cardinal number κ such that for every subset $A \subseteq X$ with $|A| \geq \kappa$ there exist open neighbourhoods U_a, $a \in A$, such that $\bigcap_{a \in A} U_a = \emptyset$ (resp. $\bigcap_{a \in A} \overline{U_a} = \emptyset$); a space X is said *n-Hausdorff* (resp. *n-Urysohn*), if $H(X) \leq n$ (resp. $U(X) \leq n$) [1,2]. We present results on n-Hausdorff homogeneous and n-Urysohn homogeneous spaces. In particular, new cardinal bounds for these spaces and the construction of an n-H-closed homogeneous extension for n-Hausdorff spaces are given.

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Maximal equivariant compactifications

Mathematics Subject Classification (MSC): 54H15, 54D35, 54F05

Abstract. I am going to present results from [2]. Let G be a locally compact group. Then for every G-space X the maximal G-proximity β_G can be characterized by the maximal topological proximity β as follows:

$$A \ \overline{\beta_G} \ B \Leftrightarrow \exists V \in N_e \quad VA \ \overline{\beta} \ VB.$$

Here, $\beta_G \colon X \to \beta_G X$ is the maximal G-compactification of X (which is an embedding for locally compact G by a classical result of J. de Vries [3]), V is a neighbourhood of e and $A \ \overline{\beta_G} \ B$ means that the closures of A and B do not meet in $\beta_G X$.

Note that the local compactness of G is essential. This theorem comes as a corollary of a general result about maximal $\mathcal{U}$-uniform G-compactifications for a useful wide class of uniform structures $\mathcal{U}$ on G-spaces for not necessarily locally compact groups G. It helps, in particular, to derive the following result. Let $(\mathbb{U}_1, d)$ be the Urysohn sphere and $G = \mathrm{Iso}(\mathbb{U}_1, d)$ is its isometry group with the pointwise topology. Then for every pair of subsets A, B in $\mathbb{U}_1$, we have

$$A \ \overline{\beta_G} \ B \Leftrightarrow \exists V \in N_e \quad d(VA, VB) > 0.$$

Note also that, by [1], $\beta_G \mathbb{U}_1$ is metrizable and can be identified with the Gromov compactification of the metric space $(\mathbb{U}_1, d)$.

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Mayer-Vietoris sequence for generating families in diffeological spaces

Abstract. We prove a version of the Mayer-Vietoris sequence for De Rham differential forms in diffeological spaces. It is based on the notion of a generating family instead of that of a covering by open subsets.

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A topological dynamical view of transposition hypergroups

Mathematics Subject Classification (MSC): 22-XX, 54Hxx

Abstract. The notion of "hypergroup", which generalizes the one of "group", has proven to be a central notion for various areas of mathematics. In particular, join spaces (i.e., commutative transposition hypergroups) play a unifying role in the study of classical geometries; each of the desciptive, spherical, and projective geometry can be formulated in terms of join spaces. We will present examples of transposition and rev-transposition hypergroups, and topological results on these settings.

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Homotopy contexts of Howie towers

Mathematics Subject Classification (MSC): 57M20, 57M07

Abstract. The concept of (A, B)-tower lifting was invented by James Howie and has been successfully applied in asphericity and cohomological finiteness problems in the early 80s.

We will consider this concept in a broader homotopy context to better understand the reasons for its effectiveness.

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Closed copies of $\boldsymbol{N}$ in $\mathbb{R}^{\omega_1}$

Mathematics Subject Classification (MSC): Primary 54C45; Secondary 03E17, 03E50, 03E55, 54D35, 54D40, 54D60, 54G20

Abstract. We investigate closed copies of N in powers of $\mathbb{R}$ with respect to C^*- and C-embedding. We show that $\mathbb{R}^{\omega_1}$ contains closed copies of N that are not C^*-embedded.

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Characterization of maximal ideals by $\mathcal{F}-\lim$

Abstract. Let $\{R_i\}_{i\in I}$ be an infinite family of rings and $R=\prod_{i\in I} R_i$ their product. In this work, we investigate the prime ideals of R by $\mathcal{F}-\lim$. Special attention is paid to relationship between the prime ideals of R_i and the elements of $Spec(\prod_{i\in I} R_i)$ use $\mathcal{F}-\lim$.

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The Hewitt-Nachbin number of space of the compact complete linked systems

Mathematics Subject Classification (MSC): 18A22, 18F60, 54A25

Abstract. A system $\xi = \{F_\alpha : \alpha \in A\}$ of closed subsets of a space X is called *linked* if any two elements from ξ intersect [1].

A.V. Ivanov defined the space NX of complete linked systems (CLS) of a space X in a following way:

Definition 1 [2]. A linked system M of closed subsets of a compact X is called *a complete linked system* (a CLS) if for any closed set of X, the condition

"Any neighborhood OF of the set F consists of a set $\Phi \in M$"

implies $F \in M$.

A set NX of all complete linked systems of a compact X is called *the space NX of CLS of X*. This space is equipped with the topology, the open basis of which is formed by sets in the form of

$E = O(U_1, U_2, \ldots, U_n)\langle V_1, V_2, \ldots, V_s\rangle = \{M \in NX :$ for any $i = 1, 2, \ldots, n$ there exists $F_i \in M$ such that $F_i \subset U_i$, and for any $j = 1, 2, \ldots, s$, $F \cap V_j \neq \varnothing$ for any $F \in M\}$, where $U_1, U_2, \ldots, U_n$, $V_1, V_2, \ldots, V_s$ are nonempty open in X sets [2].

Definition 2 [3]. Let M be a complete linked systems of a space X. The CLS M will be said a *compact complete linked system* if M contains at least one compact element.

We denote a compact complete linked system M by a CCLS.

Definition 3 [3]. We call an *N-compact kernel* of a topological space X the space

$$N_cX = \{M \in NX : M \ is \ a \ CCLS\}.$$

Put $q(X) = min\{\tau \geq \aleph_0 : X$ is τ-placed in $\beta X\}$; is called the *Hewitt-Nachbin number* of X. We say that X is a *$\mathcal{Q}_\tau$-space* if $q(X) \leq \tau$ [4].

Theorem 1. *Let X be an infinite T_1-space. Then $q(N_cX) \leq d(X)$.*

A space X is called an *m_τ-space*, where τ is given cardinal, if for each canonical closed set F in X and each point $x \in F$ there is a set P of type G_τ in X such that $x \in P \subset F$. Clearly, X is an m_τ-space for $|X| = \tau$. This allows us to give the following definition: put $m(X) = min\{\tau \geq \aleph_0 : X$ is an m_τ-space$\}$ [4].

Theorem 2. *If $q(N_cX) \leq \tau$ and $m(NX) \leq \tau$, then N_cX is τ-placed in NX.*

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Discussion on some fixed point models and their applications

Mathematics Subject Classification (MSC): Primary 47H10; Secondary 54H25

Abstract. Few historical fixed point theorems became as a model like algorithms in computer science. Few of them are ([1, 2, 4, 5]). Also, the purpose of this discussion on this forum based on a paper ([3]) by speaker of this talk. A careful reading of the above article will evident that the author intended to discuss various supportive tools to obtain common fixed points in metric and various other related spaces. It was also observed that how the concepts of continuity, commutativity, containment of ranges, etc. are important for establishing fixed and common fixed points. Lots of applications also obtain in the area of Mathematical Sciences, Biological Sciences, Medical Sciences, Social Sciences particularly Economics, etc.

Apart from the above highlighted paper we referred to ([6]) and ([7]) by B.E.Rhoades that motivated large number of researchers not only in 20th century but also in 21st century. Here we are very much focussed on the results which generalizes Banach([1]), Kannan([5]), Boyd and Wong([2]) either in the lines of Banach([1]) or Jungck([4]). Also, we shall discuss some applications of those results.

Acknowledgements: Dedicated to Late P V Lakshmaiah on his 32nd Death Anniversary.

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On Pure Twisted Virtual Braids

Mathematics Subject Classification (MSC): 57K10, 57K14

Abstract. Twisted virtual braids are a combinatorial generalization of virtual braids. Recently, S. Kamada et al. [1] proved theorems for twisted links corresponding to the Alexander theorem and the Markov theorem in knot theory. They have also provided a group presentation and a reduced group presentation of the twisted virtual braid group.

In this talk, we discuss the pure twisted virtual braid group. We discuss the idea of the proof that the twisted virtual braid group is a semi-direct product of the pure twisted virtual braid group and the symmetric group.

Also, provide a structure for the pure twisted virtual braid group in terms of a semi-direct product of groups.

If time permits, we might present the notion of abstract twisted virtual braids as equivalence classes of braid diagrams on a surface (possibly non-orientable), joining two distinguished boundary components.

This is joint work with V. G. Bardakov, T. A. Kozlovskaya, M. Prabhakar.

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Fréchet subspaces of minimal usco/cusco maps

Mathematics Subject Classification (MSC): 54C60, 46A04

Abstract. Minimal usco and minimal cusco maps play a crucial role in various branches of mathematics, including functional analysis, optimization, and the study of the differentiability of Lipschitz functions, among others. Consequently, understanding the topological properties of the spaces containing these maps is of great importance.

We explore the topological properties of the spaces of minimal usco and minimal cusco maps, specifically with respect to the topologies of uniform convergence on bornologies. Our investigation focuses on metrizability and complete metrizability. Furthermore, we investigate Fréchet locally convex subspaces of these spaces.

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First Countable Locally Compact Pseudocompactifications and Countably-Compactifications

Mathematics Subject Classification (MSC): Primary 54C30; Secondary 54D35, 03E17

Abstract. A *pseudocompactification* of a space X is a pseudocompact space Y in which X is densely embedded. A *countably-compactification* is defined similarly.

Theorem 1. Each locally compact, first countable Hausdorff space has a locally compact, first countable, Hausdorff pseudocompactification.

Theorem 2. $\mathfrak{b} = \mathfrak{c}$ is equivalent to the statement that each locally compact, first countable Hausdorff space has a locally compact, first countable, Hausdorff countably-compactification.

Here $\mathfrak{b}$ is the least cardinality of an unbounded family of functions f_α : $\omega \to \omega$ that is unbounded in the eventual domination order $f <^* g \iff$

$\exists k(f(n) < g(n))\forall n > k$. One direction in Theorem 2 is provided by a well-known example using an unbounded set of graphs of functions well-ordered wrt $<^*$.

Example. There is a locally compact, locally countable [hence first countable] Hausdorff space X of weight (= cardinality) $\mathfrak{b}$ for which no first countable countably-compactification exists.

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First Countable 0-dimensional Pseudocompactifications and Countably-Compactifications

Mathematics Subject Classification (MSC): Primary 54C30; Secondary 54D35, 03E17

Abstract. A *0-dimensional space* is a Hausdorff space with a base of clopen (= closed-and-open) sets.

Theorem 1. If X is a 0-dimensional, first countable space with a base $\mathcal{B}$ such that every point of X is in fewer than $\mathfrak{s}$ members of $\mathcal{B}$, then X has a 0-dimensional, first countable pseudocompactification.

Here $\mathfrak{s}$ is the least cardiality of a splitting family on the power set of ω, meaning a family $\mathcal{S}$ of subsets of ω such that for every subset A of ω, there exists $S \in \mathcal{S}$ such that $A \cap S$ and $A \setminus S$ are both infinite.

Corollary. Every Hausdorff space with a point-countable base of clopen sets has a 0-dimensional, first countable pseudocompactification.

Problem. If a 0-dimensional space has a point-countable base, must it have a point-countable base of clopen sets?

Theorem 2. $[\mathfrak{b} = \mathfrak{c}]$ If X is a first countable space with a base $\mathcal{B}$ of clopen sets, such that $|\mathcal{B}| < \mathfrak{c}$, and such that each point is in $< \mathfrak{s}$ members of

$\mathcal{B}$, then X can be (densely) embedded into a 0-dimensional, first countable, countably compact space.

Corollary. $[\mathfrak{b} = \mathfrak{c}]$ Every Hausdorff space of weight $< \mathfrak{c}$ with a point-countable base of clopen sets can be densely embedded into a 0-dimensional, first countable, countably compact space.

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Joint continuity in semitopological monoids

Mathematics Subject Classification (MSC): 54H15, 20M20, 54H11

Abstract. In this study, using Reznichenko's results [3, 4], we generalize Lawson's theorems on joint continuity in compact (locally compact) semi-topological semigroups [1, 2].

Theorem 1. *Let S be a pseudocompact Tychonoff right topological semigroup with right identity $\mathbf{e}$, X be a Tychonoff pseudocompact space, $\pi : S \times X \to X$ be a separately continuous action such that $\pi(\mathbf{e}, x) = x$ for all $x \in X$, and (S, X) be a Grothendieck pair. Then π is continuous at $(\mathbf{e}, x)$ for all $x \in X$.*

Corollary 2. *Let S be a pseudocompact Tychonoff semitopological monoid, (S, S) be a Grothendieck pair and G be a subgroup of S. Then G is a paratopological group.*

Corollary 3. *Let S be a pseudocompact Tychonoff semitopological semigroup which is a weak q_D-space, G be a semi-open subsemigroup of S with identity. Then multiplication restricted to $G \times S$ is continuous at points $G^- \times S$ where G^- is a set of unit elements of G.*

Corollary 4. *Let S be a pseudocompact Tychonoff semitopological semigroup which is a weak q_D-space, G be a semi-open subgroup of S. Then G is a paratopological group.*

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Finitely open or closed functions

Mathematics Subject Classification (MSC): Primary 54C08, 26A15, 26A21, 54H05, 54E40

Abstract. Recall, that a subset of X is an LC_n-set if it can be written as a union of n locally closed in X sets ($n \in \mathrm{N}$). A set is locally closed if it is the intersection of a closed and an open set.

We say that a function $f : X \to Y$ is *finitely open or closed* if X admits a finite cover γ such that, for each $C \in \gamma$, the restriction $f|C$ is open or closed.

We will consider the following question (X, Y can be Polish spaces or their subspaces):

Let $f : X \to Y$ be a continuous function such that, for every open set O and some $n \in \mathrm{N}$, the image $f(O)$ is an LC_n-set. Is f finitely open or closed?

In other words: Is every open-LC_n continuous function decomposable into finitely many of open or closed functions?

A natural generalization of this question is to replace continuous functions f with LC_n-measurable functions.

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On *W*-convexity

Mathematics Subject Classification (MSC): 54E35, 52A01

Abstract. Künzi and Yilzid introduced the concept of convexity structures in the sense of Takahashi in quasi-pseudometric spaces in 2016. In this talk, we continue the study of this theory, introducing the concept of W-convexity for real-valued pair of functions defined on an asymmetrically normed real vector space. Moreover, we show that all minimal pairs of functions defined on an asymmetrically normed real vector space equipped with a convex structure which is W-convex whenever W is translation-invariant.

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New best proximity point and pair results via MNC and their application to q-calculus

Mathematics Subject Classification (MSC): Primary 47H10; Secondry 34A08, 47H08, 47H09

Abstract. In this paper primary motive is to establish new best proximity point (pair) theorems with the utilization of techniques such as measure of noncompactness and several auxiliary functions. The obtained results are then applied to demonstrate existence of optimum solutions of a system of fractional order q-differential equations.

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On topology expansion using ideals

Mathematics Subject Classification (MSC): 54A10, 54A05, 54B99, 54E99

Abstract. In ideal topological space $\langle X, \tau, \mathcal{I} \rangle$ using a local function defined by ideal $\mathcal{I}$ we obtain a new, finer, topology τ^*. The aim of this paper is to find an ideal which creates a new topology with a specific set A in it, or which preserves a specific property, like preserving the family of regular open sets, or connectivity.

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One-local retract in modular metric spaces

Mathematics Subject Classification (MSC): 55

Abstract. In this talk, we illustrate the extension of the well-known results on one-local retract from metrics to the framework of modular metrics. We show that any self map $\psi : X_w \to X_w$ has at least one fixed point whenever the collection of all q_w-admissible subsets of X_w is both compact and normal.

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Relations on ultrafilters between the Rudin–Keisler and Comfort preorders

Abstract. As usual, βX denotes the standard Čech–Stone compactification of the discrete space X, which we identify with the set of ultrafilters over X (see [1, 2, 7]). We consider here ultrafilters over ω although most of our results remain true for ultrafilters over any infinite set X. The *Rudin–Keisler* preorder $\leq_{\mathrm{RK}}$ on $\beta\omega$ is defined by letting $\mathfrak{u} \leq_{\mathrm{RK}} \mathfrak{v}$ iff there exists $f : \omega \to \omega$ such that $\widetilde{f}(\mathfrak{v}) = \mathfrak{u}$, where $\widetilde{f} : \beta\omega \to \beta\omega$ is the continuous extension of f. The *Comfort* preorder $\leq_{\mathrm{C}}$ on $\beta\omega$ is defined by letting $\mathfrak{u} \leq_{\mathrm{C}} \mathfrak{v}$ iff any $\mathfrak{v}$-compact space is $\mathfrak{u}$-compact, where a space X is $\mathfrak{u}$-*compact* iff $\widetilde{f}(\mathfrak{u}) \in X$ for any $f : \omega \to X$. (See [1, 7] for more on ultrafilters and $\leq_{\mathrm{RK}}$, and [3, 4] for $\leq_{\mathrm{C}}$.)

For $\mathfrak{u}, \mathfrak{v} \in \beta\omega$ and any ordinal α, define: $\mathfrak{u}\, R_0\, \mathfrak{v}$ iff $\mathfrak{u}$ is principal, $R_{<\alpha} = \bigcup_{\beta<\alpha} R_\beta$, and $\mathfrak{u}\, R_\alpha\, \mathfrak{v}$ iff there exists a continuous map $f : \beta\omega \to \beta\omega$ such that $f(\mathfrak{v}) = \mathfrak{u}$ and $f(n)\, R_{<\alpha}\, \mathfrak{v}$ for all $n < \omega$. The hierarchy is non-degenerate and lies between $\leq_{\mathrm{RK}}$ and $\leq_{\mathrm{C}}$ as stated in the following theorem.

Theorem 1. *$R_1 = \leq_{\mathrm{RK}}$; $R_{<\alpha} \subset R_\alpha$ for all $\alpha < \omega_1$; $R_{<\omega_1} = R_{\omega_1} = \leq_{\mathrm{C}}$.*

If X, Y are spaces and α is an ordinal, $f : X^\alpha \to Y$ is *right-continuous w.r.t.* $A \subseteq X$ iff for all $\beta < \alpha$ the shift $x \mapsto f(a_0, a_1, \ldots, x, b_{\beta+1}, b_{\beta+2}, \ldots)$ is continuous whenever $a_0, a_1, \ldots \in A$ and $b_{\beta+1}, b_{\beta+2}, \ldots \in X$. As shown in [9, 10], if $n < \omega$, X is discrete, and Y is compact Hausdorff, then every $f : X^n \to Y$ uniquely extends to $\widetilde{f} : (\beta X)^n \to Y$ that is right-continuous w.r.t. X. This fact provides a canonical way to obtain, for an arbitrary first-order model $\mathfrak{A}$, its *ultrafilter extension* $\beta\mathfrak{A}$ ([9, 10], cf. also [6]), generalizing the well-known construction of ultrafilter extensions of semigroups comprehensively treated in [7]. (Some historical remarks can be found in [8].)

If $n < \omega$, the relations R_n can be redefined in terms of ultrafilter extensions of n-ary operations on ω as follows: $\mathfrak{u}\, R_n\, \mathfrak{v}$ iff there exists $f : \omega^n \to \omega$

such that $\widetilde{f}(\mathfrak{v}, \ldots, \mathfrak{v}) = \mathfrak{u}$. Moreover, $R_m \circ R_n = R_{nm}$ (so R_n are not preorders for $2 \le n < \omega$). These observations can be expanded to all R_α by using ω-ary operations on ω. Such an operation is identified with a continuous map of the Baire space ω^ω into the discrete space ω; these maps admit a natural hierarchy ranked by countable ordinals.

Proposition 1. *Any continuous $f : \omega^\omega \to \omega$ uniquely extends to $\widetilde{f} : (\beta\omega)^\omega \to \beta\omega$ that is right-continuous w.r.t. ω (in other words, ω-ary operations on ω extend to such operations on $\beta\omega$).*

Proposition 2. *Let $\alpha < \omega_1$ and $\mathfrak{u}, \mathfrak{v} \in \beta\omega$. Then $\mathfrak{u} \, R_\alpha \, \mathfrak{v}$ iff there exists a continuous $f : \omega^\omega \to \omega$ of rank α such that $\widetilde{f}(\mathfrak{v}, \mathfrak{v}, \ldots) = \mathfrak{u}$.*

The composition of arbitrary $R_{<\alpha}$ is expressed via a multiplication-like operation on ordinals. To simplify notation, denote $\sup_{\gamma<\alpha}(\gamma \cdot \beta)$ by $({<}\alpha) \cdot \beta$; the explicit calculation of these ordinals, used in getting the following result, is rather cumbersome.

Theorem 2. *Let $\alpha, \beta < \omega_1$.*

- (i) *$R_\alpha \circ R_\beta = R_\gamma$ where $\gamma = \beta \cdot \alpha$ if $\beta = 0$ or $\alpha < \omega$, $\gamma = \beta \cdot (\alpha + 1) - 1$ if $0 < \beta < \omega$ and $\alpha \ge \omega$, and $\gamma = \beta \cdot (\alpha + 1)$ if $\alpha, \beta \ge \omega$;*
- (ii) *If $\alpha > 0$ is limit, then $R_{<\alpha} \circ R_\beta = R_{<\gamma}$ where $\gamma = \beta \cdot \alpha$;*
- (iii) *If $\beta > 0$ is limit, then $R_\alpha \circ R_{<\beta} = R_{<\gamma}$ where $\gamma = ({<}\beta) \cdot \alpha$ if $\alpha < \omega$, and $\gamma = ({<}\beta) \cdot (\alpha + 1)$ otherwise;*
- (iv) *If $\alpha, \beta > 0$ are limit, then $R_{<\alpha} \circ R_{<\beta} = R_{<\gamma}$ where $\gamma = ({<}\beta) \cdot \alpha$.*

Corollary 1. *Let $2 \le \alpha \le \omega_1$. Then $R_{<\alpha}$ is a preorder iff α is multiplicatively indecomposable.*

Define preorders between $\le_{\mathrm{RK}}$ and $\le_{\mathrm{C}}$ by letting $\le_0 = \le_{\mathrm{RK}}$ and $\le_{1+\alpha} = R_{<\omega^{\omega^\alpha}}$ for all $\alpha \le \omega_1$. So, if α is infinite, $R_{<\alpha} = \le_\alpha$ iff α is an epsilon number. Also $\le_\alpha \circ \le_\beta = \le_\gamma$ where $\gamma = \max(\alpha, \beta)$.

As shown in [5], for any ultrafilter $\mathfrak{v}$ and semigroup S, the set $\{\mathfrak{u} : \mathfrak{u} \le_{\mathrm{C}} \mathfrak{v}\}$ forms a subsemigroup of βS. This can be expanded to arbitrary first-order models and relations $R_{<\alpha}$ as follows.

Corollary 2. *For every $\alpha > 1$, ultrafilter $\mathfrak{v}$, and model $\mathfrak{A}$ of any signature, $\{\mathfrak{u} : \mathfrak{u}\, R_{<\alpha}\, \mathfrak{v}\}$ forms a submodel of the model $\beta\mathfrak{A}$ iff α is additively indecomposable. Consequently, for all $\alpha > 0$, $\mathfrak{v}$, and $\mathfrak{A}$, $\{\mathfrak{u} : \mathfrak{u} \leq_\alpha \mathfrak{v}\}$ forms a submodel of $\beta\mathfrak{A}$.*

If R, S are binary relations on $\beta\omega$, let us say that $\mathfrak{u}$ is (R, S)-*minimal* iff $\mathfrak{v}\, R\, \mathfrak{u}$ implies $\mathfrak{u}\, S\, \mathfrak{v}$, for all $\mathfrak{v}$. So (R_1, R_1)- and $(R_{<\omega_1}, R_{<\omega_1})$-minimal are just $\leq_{\mathrm{RK}}$- and $\leq_{\mathrm{C}}$-minimal, respectively.

Proposition 3. *Let $\mathfrak{u}$ be non-principal.*

(i) *If $\mathfrak{v}$ is $\leq_{\mathrm{RK}}$-minimal and $\mathfrak{u}\, R_\alpha\, \mathfrak{v}$, then $\mathfrak{u}$ is $(R_{<\omega_1}, R_\alpha)$-minimal.*

(ii) *$\mathfrak{u}$ is $\leq_{\mathrm{RK}}$-minimal iff it is a $(R_{<\alpha}, R_{<\beta})$-minimal weak p-point, for every $\alpha, \beta \leq \omega_1$.*

(iii) *$\mathfrak{u}$ is a weak p-point iff for any $g : \omega^2 \to \omega$ and $\mathfrak{v}, \mathfrak{w} \in \beta\omega$ with $\mathfrak{u} = \widetilde{g}(\mathfrak{w}, \mathfrak{v})$ there exists $n \in \omega$ such that $\widetilde{g}(n, \mathfrak{v})$ is either $\mathfrak{u}$ or principal.*

Many natural questions on minimality seem hard to answer; we are not aware, e.g., whether each (R_2, R_2)-minimal is (R_3, R_3)-minimal or at least $(R_3, R_{<\omega_1})$-minimal.

Ultrafilter extensions of ω-ary operations can be used to state Ramsey-type results. Let $f[X]$ be the image of X under f, and let $I = \{x \in \omega^\omega : x$ is increasing$\}$. If $X \subseteq \omega$ and $f : \omega^\omega \to Y$, we say that f is *constant upward on X* iff $|f[X^\omega \cap I]| = 1$, and *quasi-invertible upward on X* iff there exists $g : Y \to \omega$ such that for any infinite $A \subseteq X$ we have $g[f[A^\omega \cap I]] \subseteq A$ and $|A \setminus g[f[A^\omega \cap I]]| < \omega$. The following refines the well-known characterization of Ramsey ultrafilters as selective ones (see [1, 7]).

Proposition 4. *A non-principal $\mathfrak{u} \in \beta\omega$ is $\leq_{\mathrm{RK}}$-minimal iff any continuous $f : \omega^\omega \to \omega$ is either constant upward or quasi-invertible upward on some $X \in \mathfrak{u}$.*

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On algebraic K-theories parametrised by polyhedra for associative rings

Abstract. In a series of papers, Bruns and Gubeladze defined an algebraic K-functor for associative rings, which uses a convex polyhedron satisfying certain properties as the second argument (the so-called polytopal K-theory). They found all polygons suitable for these properties and proved that the corresponding K-theories decompose into a direct sum of Quillen's K-theories.

Thus, they made a conjecture that this fact is true for any polyhedron suitable for their conditions. The report will be devoted to the description of the Brans-Gubeladze construction and the proof of their conjecture in the general case.

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Generalization of Arc Shift for Twisted Knots

Mathematics Subject Classification (MSC): 57K10, 57K14

Abstract. M. O. Bourgoin [1] introduced the twisted knot theory as a generalization of classical knot theory, virtual knot theory, and projective knot theory. Twisted knots are stable equivalence classes of oriented knots in orientable three-manifolds that are orientation I-bundles over closed but not necessarily orientable surfaces. Diagrammatic representations of twisted knots are twisted knot diagrams. Twisted knot diagrams are knot diagrams on $\mathbb{R}^2$ possibly with some crossings called virtual crossings and bars which are short arcs intersecting the arcs of the diagrams. Unlike virtual knots,

only a few invariants are known for twisted knots. Hence, finding invariants for twisted knots is quite an interesting problem. Arc shift move is an unknotting operation for virtual knots [2].

In this talk, I will define arc shift move for twisted knots and establish that it is an unknotting operation for twisted knots. Further, we establish that this arc shift number is an invariant for twisted knots and discuss some properties of arc shift move.

This is a joint work with my P.hD. student Ms. Komal Negi.

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Independence complexes of wedge of graphs

Mathematics Subject Classification (MSC): 05C69, 55P15, 05C10

Abstract. We provide detailed computations for the homotopy type of the independence complexes of a wedge of path and cycle graphs. In particular, we show that these complexes are either contractible or wedges of spheres. In most cases, we determine the dimensions of these spheres.

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Pseudocompact algebraic structures

Mathematics Subject Classification (MSC): 54H99, 54D30, 54D35, 08B05

Abstract. An *algebra* (or *algebraic structure*) $\mathbf{X} = (X, \{f_\alpha : \alpha \in A\})$ is a set X together with a collection $\{f_\alpha : \alpha \in A\}$ of operations on X, where $f_\alpha : X^{n_\alpha} \to X$ is n_α-ary *operation.* If X is a topological space and the operations f_α are (separately) continuous, then $\mathbf{X}$ will be called a *topological (semitopological) algebra.* We say that $\beta\,\mathbf{X}$ is a topological (semitopological) algebra if each operation f_α extends to a (separately) continuous operation $\hat{f}_\alpha : (\beta\,X)^n \to \beta\,X$.

Theorem 1. *If $\mathbf{X}$ is a pseudocompact topological algebra, then (1) $\beta\,\mathbf{X}$ is a semitopological algebra; (2) $\beta\,\mathbf{X}$ is a topological algebra if X^{n_α} is pseudocompact for all α.*

Theorem 2. *If $\mathbf{X}$ is a countably compact semitopological algebra, then $\beta\,\mathbf{X}$ is a semitopological algebra.*

Theorem 3. *Let $\mathbf{X}$ be a compact topological algebra and $|A| \le \omega$. Then $\mathbf{X}$ embeds in a product of metrizable topological algebras.*

An operation $M : X^3 \to X$ is called a *Mal'cev operation* if $M(x, y, y) = M(y, y, x) = x$ for all $x, y \in X$.

Theorem 4. *Let $\mathbf{X}$ be a compact semitopological algebra and $|A| \le \omega$. Then (1) if X has caliber ω_1, then $\mathbf{X}$ embeds in a product of metrizable semitopological algebras; (2) if there is a Mal'cev operation among f_α operations, then X is a Dugundji compact space and $\mathbf{X}$ embeds in a product of metrizable semitopological algebras.*

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Kuratowski Convergence of Nets of Sets in Approach Spaces

Mathematics Subject Classification (MSC): 54B20, 54B30

Abstract. lt is well known that, in a Hausdorff topological space (X,τ), the Kuratowski convergence of nets of sets with respect to τ can also be characterized by means of the Fell topology defined on the closed subsets of X [3]. We give an analogue definition for Kuratowski convergence of nets of sets in an Hausdorff approach space in [2]. In addition, we obtain a relation with this new convergence and the recently defined Fell approach structure in [1].

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Uniform selection principles and its applications

Mathematics Subject Classification (MSC): 54E15

Abstract. In recent times, the theory of the selection principles in uniform spaces has been developing intensively.

L.D.R. Kočinac introduced and characterized uniform versions of classical topological notions of the Menger, Hurewicz and Rothberger properties, also uniform γ-sets.

This work study explores certain important properties of the uniform selection principle, such as the uniform Menger, uniform Hurewicz, uniform Rothberger properties, and uniform γ-sets.

Throughout the work uniform spaces are assumed to be Hausdorff, and mappings are uniformly continuous. The uniform structure is defined in terms of covers. The terms of cover has an advantage because many important concepts such as compactness, paracompactness, Lindelöf space and so on are defined through the concept of covers [1], [3].

As known, to each selection principle for topological spaces it is naturally associated the corresponding game and often selection principles can be characterized game-theoretically. In uniform case to each uniform selection principle one can assign also the corresponding game. For example, the game UG associated to the uniform Hurewicz property is defined in the following way. Two players, ONE and TWO, play a round for each positive integer. In the n-th round ONE chooses a uniform cover $\alpha_n \in U$, and TWO responds choosing a finite subfamily β_n. TWO wins a play $\alpha_1, \beta_1; \alpha_2, \beta_2, ...$ if, for each $x \in X$, $x \in \bigcup\limits_{n \in N} \beta_n$, for all but finitely many n; and otherwise ONE wins [2].

The uniform space (X, U) is uniform Hurewicz space iff ONE does not have a winning strategy in UG. TWO has a winning strategy in UG iff the uniform space (X, U) is σ-precompact.

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Unknottability, unlinkability and splittability of tangles

Mathematics Subject Classification (MSC): 57M25, 57M27

Abstract. A *tangle* $\mathcal{T}$ is a pair (B, σ) formed by a ball B and a collection of properly embedded disjoint arcs σ in B. If σ has n components we say that $\mathcal{T}$ is a n-string tangle.

Let K be a link in S^3, and B a ball in S^3 with exterior B'. If $\mathcal{T} = (B, B \cap K)$ and $\mathcal{T}' = (B', B' \cap K)$ are tangles, then we say that $\mathcal{T} \cup \mathcal{T}'$ is a *tangle decomposition* of K and that K is a *closure* of $\mathcal{T}$ (and of $\mathcal{T}'$). In case there is a tangle decomposition of K with $\mathcal{T}$ one of the tangle components, we also say that $\mathcal{T}$ embeds into the pair (S^3, K), or, for abbreviation, that it embeds into K.

If a tangle $\mathcal{T}$ embeds into the unknot, an unlink or a split link, we say that $\mathcal{T}$ is *unknottable*, *unlinkable* or *splittable*.

In this talk, we discuss obstructions to these properties through geometric characterizations, tangle sums and colorings. These obstructions allow us to determine when several 2-string tangles are unknottable, unlinkable or splittable, including all prime 2-string tangles with up to seven crossings.

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A characterization of homology d-manifolds with $g_2 \leq 3$

Mathematics Subject Classification (MSC): Primary 57Q25; Secondary 05E45, 55U10, 57Q05, 57Q15

Abstract. The g-vector of a simplicial complex contains a lot of information about the combinatorial and topological structure. Several classification results on the structure of normal pseudomanifolds and homology manifolds have been given concerning the value g_2. It is known that for $g_2 = 0$, all the normal pseudomanifolds of dimensions at least three are stacked spheres. In the case of $g_2 = 1$ and 2, all the prime homology manifolds are the polytopal spheres and are obtained by some sort of retriangulation or join operation from the previous one. In this talk, we shall present a combinatorial characterization of the homology d-manifolds, $d \geq 3$, with $g_2 = 3$. These are spheres and are obtained by operations such as join, some retriangulations, and connected sums from spheres with $g_2 \leq 2$. Further, we will see a structural result on some prime normal d-pseudomanifolds with $g_2 = 3$. Our results, together with some previous work, classify (combinatorially) all the normal 3-pseudomanifolds with $g_2 = 3$.

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On reduction and separation of projective sets in Tychonoff spaces

Abstract. Below $\mathscr{F}$, $\mathscr{G}$, $\mathscr{K}$, $\mathscr{Z}$ denote the classes of closed, open, compact, and zero sets (pre-images of $0 \in [0,1] \subseteq \mathbb{R}$ under continuous maps), resp.; $\mathscr{S}$ denotes an unspecified class. The classes are treated as operators so $\mathscr{F}(X)$ consists of all closed sets in X, etc.; $\mathscr{S}(X) = \mathscr{S} \cap \mathscr{P}(X)$. Let $\mathscr{S}(Y) \restriction X = \{S \cap X : S \in \mathscr{S}(Y)\}$ for $F : X \to Y$ let FA and $F^{-1}A$ denote the image and pre-image of A. $\mathbf{\Phi}$ is an ω-ary *Hausdorff* (or δs-)*operation* iff there is $S \subseteq \omega^\omega$ (the *base* of $\mathbf{\Phi}$) such that $\mathbf{\Phi}(A_s)_{s\in\omega^{<\omega}} = \bigcup_{f\in S}\bigcap_{n\in\omega} A_{f\restriction n}$ for all A_s, $s \in \omega^{<\omega}$ ([6], [5], [1]; for κ-Suslin sets, see [9], [8]). E.g., if $S = \omega^\omega$, $\mathbf{\Phi}$ is the A-operation. A $\mathbf{\Phi}$-*set* is a set obtained by $\mathbf{\Phi}$. Let $\mathbf{\Phi}(\mathscr{S}, X)$ denote the class of $\mathbf{\Phi}$-sets generated by sets in $\mathscr{S}(X)$ and $\mathbf{\Phi}(\mathscr{S})$ the union of $\mathbf{\Phi}(\mathscr{S}, X)$ for all X.

The *Borel hierarchy* generated by $\mathscr{S}(X)$ is defined by alternating countable unions and complements; $\mathbf{\Sigma}^0_\alpha(\mathscr{S}, X)$ and $\mathbf{\Pi}^0_\alpha(\mathscr{S}, X)$ are its αth additive and multiplicative classes. E.g., $\mathbf{\Sigma}^0_2(\mathscr{F}, X)$ is $\mathscr{F}_\sigma(X)$ and $\mathbf{\Pi}^0_2(\mathscr{F}, X)$ is $\mathscr{G}_\delta(X)$. By induction on α, each Borel class is of form $\mathbf{\Phi}(\mathscr{S}, X)$ for some $\mathbf{\Phi}$. The *projective hierarchy* generated by $\mathscr{S}(X)$, for Polish spaces X, is defined by alternating projections of subsets of $X \times \omega^\omega$ onto X and complements; $\mathbf{\Sigma}^1_n(\mathscr{S}, X)$ and $\mathbf{\Pi}^1_n(\mathscr{S}, X)$ are its nth additive and multiplicative classes. E.g., $\mathbf{\Sigma}^1_1(\mathscr{F}, \mathbb{R})$ and $\mathbf{\Pi}^1_1(\mathscr{F}, \mathbb{R})$ consist of A-sets and CA-sets of reals. By the Fundamental Theorem on Projections ([6], p. 264), if X is Polish, the class of projections of sets in $\mathbf{\Phi}(\mathscr{F}, X \times \omega^\omega)$ onto X is of form $\mathbf{\Psi}(\mathscr{F}, X)$ for $\mathbf{\Psi}$ with a base in $\mathbf{\Phi}(\mathscr{F}_\sigma, \omega^\omega)$; so by induction on n, each projective class is of form $\mathbf{\Phi}(\mathscr{F}, X)$ for some $\mathbf{\Phi}$. For arbitrary X, we define projective classes as $\mathbf{\Phi}(\mathscr{S}, X)$ for $\mathbf{\Phi}$ such that the corresponding projective class in $\mathbb{R}$ is $\mathbf{\Phi}(\mathscr{S}, \mathbb{R})$. This approach clearly extends to σ-projective sets ([2], [7]) for any X.

$\mathscr{S}(X)$ has the *reduction* property iff for any $A, B \in \mathscr{S}(X)$ there are $C, D \in \mathscr{S}(X)$ such that $C \subseteq A$, $D \subseteq B$, $C \cap D = \emptyset$, and $C \cup D = A \cup B$; the *separation* property iff for any disjoint $A, B \in \mathscr{S}(X)$ there is

$C \in \mathscr{S}(X) \cap \{X \setminus S : S \in \mathscr{S}(X)\}$ such that $A \subseteq C$ and $B \cap C = \emptyset$. If $\mathscr{S}(X)$ has reduction then the dual class $\{X \setminus S : S \in \mathscr{S}(X)\}$ has separation. The classes $\mathbf{\Sigma}^0_\alpha(\mathscr{F}, \mathbb{R})$, $\alpha > 1$, $\mathbf{\Pi}^1_1(\mathscr{F}, \mathbb{R})$, $\mathbf{\Sigma}^1_2(\mathscr{F}, \mathbb{R})$ have reduction and the stronger pre-well-ordering property (we do not formulate it here); $V = L$ implies reduction in $\mathbf{\Sigma}^1_n(\mathscr{F}, \mathbb{R})$ for all $n \geq 2$; and under PD (the Projective Determinacy), $\mathbf{\Sigma}^1_{2n}(\mathscr{F}, \mathbb{R})$ and $\mathbf{\Pi}^1_{2n+1}(\mathscr{F}, \mathbb{R})$ have pre-well-ordering (the fact known as the First Periodicity Theorem) and so reduction ([7], [9], [4], [8]). If $\mathscr{S}(Y)$ has reduction (separation) then $\mathscr{S}(Y) \restriction X$ has the same property; $\mathbf{\Phi}(\mathscr{S}(Y) \restriction X) = \mathbf{\Phi}(\mathscr{S}, Y) \restriction X$ for all $\mathbf{\Phi}$; whence we get:

Lemma 1. *Let $X \subseteq Y$ and $\mathscr{S}(X) = \mathscr{S}(Y) \restriction X$. Then $\mathbf{\Phi}(\mathscr{S}, X) = \mathbf{\Phi}(\mathscr{S}, Y) \restriction X$ and if $\mathbf{\Phi}(\mathscr{S}, Y)$ has reduction (separation) then $\mathbf{\Phi}(\mathscr{S}, X)$ has the same property.*

E.g., $\mathscr{S}(X) = \mathscr{S}(Y) \restriction X$ holds if $\mathscr{S}$ is $\mathscr{F}$ or $\mathscr{G}$, and also if $\mathscr{S}$ is $\mathscr{Z}$ for Tychonoff X, Y (Lemma 4). Given $F : X \to Y$, F *preserves* $\mathscr{S}$ iff $A \in \mathscr{S}(X)$ implies $FA \in \mathscr{S}(Y)$, and F^{-1} *preserves* $\mathscr{S}$ iff $B \in \mathscr{S}(Y)$ implies $F^{-1}B \in \mathscr{S}(X)$. E.g., F is closed iff F preserves $\mathscr{F}$, continuous iff F^{-1} preserves $\mathscr{F}$ (or $\mathscr{G}$), compact iff F^{-1} preserves $\mathscr{K}$, and perfect iff it is closed, continuous, and compact. Since $F^{-1}\mathbf{\Phi}(A_s)_{s\in\omega^{<\omega}} = \mathbf{\Phi}(F^{-1}A_s)_{s\in\omega^{<\omega}}$ for all $\mathbf{\Phi}$, F, $(A_s)_{s\in\omega^{<\omega}}$, we get:

Lemma 2. *If F^{-1} preserves $\mathscr{S}$ then F^{-1} preserves $\mathbf{\Phi}(\mathscr{S})$.*

E.g., if F is continuous then F^{-1} preserves each of $\mathbf{\Phi}(\mathscr{F})$, $\mathbf{\Phi}(\mathscr{G})$, $\mathbf{\Phi}(\mathscr{Z})$, and if F is compact then F^{-1} preserves $\mathbf{\Phi}(\mathscr{K})$. Given $F : X \to Y$, define its *kernel* $\ker F = \{F^{-1}\{y\} : y \in Y\}$ and *algebra of pre-images* $\operatorname{alg} F = \{F^{-1}B : B \subseteq Y\}$. Clearly, $\operatorname{alg} F = \{A \subseteq X : F^{-1}FA = A\}$, $\operatorname{alg} F$ is the complete subalgebra of $\mathscr{P}(X)$ generated by $\ker F$, so it is closed under all $\mathbf{\Phi}$. Using the diagonal product of maps witnessing that A_s are zero sets, we get:

Proposition 1. *If $(A_s)_{s\in\omega^{<\omega}}$ is in $\mathscr{Z}(X)$, then there is a continuous $F : X \to [0,1]^\omega$ such that $A_s \in \operatorname{alg} F$ for all $s \in \omega^{<\omega}$ and so $\mathbf{\Phi}(A_s)_{s\in\omega^{<\omega}} \in \operatorname{alg} F$ for all $\mathbf{\Phi}$.*

Given $(I, \leq)$, a family $(A_i)_{i\in I}$ is *decreasing* iff $A_i \supseteq A_j$ for all $i \leq j$. A map $F : X \to Y$ is *closed-to-one* iff $\ker F \subseteq \mathscr{F}(X)$. It can be shown that for such F, $F\bigcap_{i\in I} A_i = \bigcap_{i\in I} FA_i$ for all directed $(I,\leq)$ and decreasing $(A_i)_{i\in I}$ in $(\mathscr{F}\cap\mathscr{K})(X)$, and so $F\mathbf{\Phi}(A_s)_{s\in\omega^{<\omega}} = \mathbf{\Phi}(FA_s)_{s\in\omega^{<\omega}}$ for all decreasing $(A_s)_{s\in\omega^{<\omega}}$ in $(\mathscr{F}\cap\mathscr{K})(X)$ and all $\mathbf{\Phi}$, whence we get:

Lemma 3. *If $\mathscr{S}(X) \subseteq (\mathscr{F}\cap\mathscr{K})(X)$ is closed under finite intersections and $F : X \to Y$ is closed-to-one and preserves $\mathscr{S}$, then F preserves $\mathbf{\Phi}(\mathscr{S})$.*

E.g., for Hausdorff X, Y and continuous $F : X \to Y$, if X is compact then F preserves $\mathbf{\Phi}(\mathscr{F})$; if moreover Y is perfectly normal then F preserves also $\mathbf{\Phi}(\mathscr{Z})$. Lemmas 2 and 3 allow to transfer reduction (separation) to the pre-image direction:

Proposition 2. *Let $\mathscr{S}(X) \subseteq (\mathscr{F}\cap\mathscr{K})(X)$ be closed under finite intersections and for any $(A_s)_{s\in\omega^{<\omega}}$ in $\mathscr{S}(X)$ there exist Y and a closed-to-one $F : X \to Y$ such that F and F^{-1} preserve $\mathscr{S}$, $(A_s)_{s\in\omega^{<\omega}}$ is in* alg F, *and $\mathbf{\Phi}(\mathscr{S}, Y)$ has the reduction (separation) property. Then $\mathbf{\Phi}(\mathscr{S}, X)$ has the same property.*

Lemma 4. *If $X \subseteq Y$ are Tychonoff, then $\mathscr{Z}(X) = \mathscr{Z}(Y) \restriction X$, $\mathbf{\Phi}(\mathscr{Z}, X) = \mathbf{\Phi}(\mathscr{Z}, Y) \restriction X$ and if $\mathbf{\Phi}(\mathscr{Z}, Y)$ has reduction (separation) then $\mathbf{\Phi}(\mathscr{Z}, X)$ has the same property.*

For $\mathscr{Z}(X) \subseteq \mathscr{Z}(Y) \restriction X$, note that all $F : X \to [0,1]$ continuously extend to βX, the Čech–Stone compactification of X, and then to $[0,1]^\kappa$ with a suitable κ (see [3]). The main result of this note is:

Theorem 1. *Let X be a Tychonoff space and $\mathbf{\Phi}$ a Hausdorff operation. If $\mathbf{\Phi}(\mathscr{F}, \mathbb{R})$ has the reduction (separation) property, then $\mathbf{\Phi}(\mathscr{Z}, X)$ has the same property.*

By Lemma 4, it suffices to handle $X = [0,1]^\kappa$; using Proposition 1, verify the assumptions of Proposition 2 with $\mathscr{S} = \mathscr{Z}$ and $Y = [0,1]^\omega$ common for all $(A_s)_{s\in\omega^{<\omega}}$ in $\mathscr{Z}([0,1]^\kappa)$.

Corollary 1. *If X is Tychonoff, for all $\alpha < \omega_1$, $\alpha > 1$, $\mathbf{\Sigma}^0_\alpha(\mathscr{Z}, X)$, $\mathbf{\Pi}^1_1(\mathscr{Z}, X)$, $\mathbf{\Sigma}^1_2(\mathscr{Z}, X)$ have reduction; under* PD, *for all $n < \omega$, $n > 0$, $\mathbf{\Sigma}^1_{2n}(\mathscr{Z}, X)$, $\mathbf{\Pi}^1_{2n+1}(\mathscr{Z}, X)$ have reduction.*

Under σ-PD, Corollary 1 extends to the σ-projective classes generated by $\mathscr{Z}(X)$.

The results were announced in [10].

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Examples in dimension theory of topological groups

Mathematics Subject Classification (MSC): 22A05, 54H11, 54F45

Abstract. Several examples concerning the covering dimension $\dim_0$ in the sense of Katětov of topological groups are presented. Recall that, given a topological space X, $\dim_0 X$ is the least integer $n \geq -1$ such that any finite cozero cover of X has a finite cozero refinement of order n, provided that such an integer exists. If it does not exist, then $\dim_0 X = \infty$. For normal spaces, this dimension coincides with the covering dimension dim in the sense of Čech, whose definition coincides with that of $\dim_0$ in which "cozero" is everywhere replaced by "open." In all of the examples "$\dim_0$" can be replaced by dim. A space X with $\dim_0(X) = 0$ is said to be *strongly zero-dimensional*.

All examples are based on the following one.

Universal example. There exist spaces C_1 and C_2 with the following properties:

(1) $\dim_0 C_i = 0$ for $i = 1, 2$;

(2) $\dim_0(C_1 \times C_2) > 0$;

(3) C_1^n is Lindelöf for each $n \in \mathbb{N}$;

(4) the underlying set of the space C_1 is the Cantor set C, its topology is finer than that of C, and it has a base consisting of sets closed in C;

(5) C_2 is second-countable; in fact, C_2 is a subspace of the Cantor set C.

Both spaces C_1 and C_2 are retracts of certain Abelian topological groups G_1 and G_2, respectively. Therefore, $C_1 \times C_2$ is a retract of (and hence C-embedded in) $G_1 \times G_2$, which implies that $\dim_0(G_1 \times G_2) > 0$. The groups G_1 and G_2 have the following properties:

(1) G_1^n is Lindelöf for every $n \in \mathbb{N}$;

(2) G_1 is topologically isomorphic to a closed subgroup of a group M_1 being a product of zero-dimensional second-countable topological groups;

(3) G_2 is second-countable;

(4) $\dim_0(G_1) = \dim_0(G_2) = 0$;

(5) $\dim_0(G_1 \times G_2) > 0$.

This gives the following examples, which answer old questions of Arkhangel'skii, Shakhmatov, Tkachenko, and Zambakhidze.

Theorem 1. *There exist two strongly zero-dimensional Abelian topological groups whose product has positive dimension.*

Theorem 2. *There exists a strongly zero-dimensional Abelian topological group containing a closed subgroup of positive dimension.*

Recall that a topological group G is said to be $\mathbb{R}$-*factorizable* if, for every continuous function $f\colon G \to \mathbb{R}$, there exists a continuous epimorphism $h\colon G \to H$ onto a second-countable topological group H and a continuous function $g\colon H \to \mathbb{R}$ such that $f = g \circ h$. It is known that any Lindelöf group is $\mathbb{R}$-factorizable and that $\dim_0 H \leq \dim_0 G$ for any topological group G and any $\mathbb{R}$-factorizable subgroup H of G. This implies the following result.

Theorem 3. *There exists an $\mathbb{R}$-factorizable topological group G and a second-countable topological group H such that the product $G \times H$ is not $\mathbb{R}$-factorizable.*

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Roelcke compactifications of ultra-transitive groups

Mathematics Subject Classification (MSC): 22F30, 22F50, 57S05

Abstract. A group G of homeomorphisms of (a linearly ordered) space X is ultra-transitive if for any pairwise disjoint points $x_1, \dots, x_n$ ($x_1 < \dots < x_n$) and $y_1, \dots, y_n$ ($y_1 < \dots < y_n$) there exists $g \in G$ such that $g(x_i) = y_i$, $i = 1, \dots, n$, $n \in \mathbb{N}$.

The least admissible group topology on G is:

the permutation topology τ_∂ if X is discrete;

the topology of pointwise convergence τ_p if X is a linearly ordered space (the permutation topology $\tau_\partial \geq \tau_p$ is also an admissible group topology on G).

The usage of Ellis's compactification of a transformation group allows to construct the Roelcke compactification of the group (G, τ_∂) and formulate sufficient condition for the group (G, τ_p) to be Roelcke-precompact [1].

1. The group of homeomorphisms of a metrizable CDH compactum for which the complement to a finite set is connected is ultra-transitive and Roelcke-precompact in the permutation topology. But the groups of homeomorphisms of the spheres $S^n, n \geq 2$ and the Hilbert cube Q are not Roelcke-precompact in the compact-open topology [2].

2. For a simple chain X its o-primitive automorphism groups $\mathrm{Aut}(X)$ is Roelcke precompact in the topology of pointwise convergence iff it is Roelcke precompact in the permutation topology.

3. The permutation topology is the least admissible group topology on ultra-transitive automorphism group of a homogeneous GO-space which is not a linearly ordered space. In particular, automorphism groups of the Sorgenfrey line and the Michael line are Roelcke precompact. Their Roelcke compactifications can be described.

4. The usage of the structure of an automorphism group of the lexicographically ordered product of linearly ordered homogeneous spaces (if $X = X_1 \bigcirc X_2$, X_1 is a regular interval, then $\mathrm{Aut}(X) \cong \mathrm{Aut}(X_2)^{X_1} \ltimes \mathrm{Aut}(X_1)$, where $\ltimes$ denotes a semidirect product) and the results of T. Tsankov [3] allows to find sufficient condition for the Roelcke precompactness of automorphism groups of non-simple homogeneous chains. If $\mathrm{Aut}(X_1)$, $\mathrm{Aut}(X_2)$ are Roelcke precompact in the permutation topology, then $\mathrm{Aut}(X)$ is Roelcke precompact in the permutation topology. In particular, this approach allows to verify the Roelcke precompactness of the automorphism group of the lexicographically ordered square in the permutation topology and give description of its Roelcke compactification.

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On the connectedness of Stone-Čech remainders of locally compact spaces

Mathematics Subject Classification (MSC): 54D05, 54D35, 54D40, 54D45

Abstract. A compactum bX is said to be a compactification of a Tychonoff space X if there exists a homeomorphic embedding $b : X \to bX$ such that $\overline{b(X)} = bX$. The subspace $bX \setminus b(X)$ is called a remainder of space X in compactification bX. The following theorem is a generalization of the result in [1].

Theorem 1. Let X be a noncompact locally compact Hausdorff space. If for every compactum $K \subset X$ there exists a compactum $C(K)$ such that $K \subset C(K) \subset X$ and the subspace $X \setminus C(K)$ is connected, then $\beta X \setminus X$ is connected.

Theorem 2. For a noncompact locally compact Hausdorff space X the following properties are equivalent:

1. $\beta X \setminus X$ is connected;
2. X has no compactification bX such that the remainder $bX \setminus b(X)$ is a two point set.

From Theorem 2 we obtain the following corollary.

Corollary. Let X be a noncompact locally compact Hausdorff space. Connectedness of $\beta X \setminus X$ is equivalent to the connectedness of all metrizable remainders of X.

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A fuzzy extension of *L*-topologies

Mathematics Subject Classification (MSC): 54A40

Abstract. Given a crisp, i.e. ordinary, topological space (X, τ) and its subset $A \subseteq X$, one can definitely say say whether this set is open or not. On the other hand in the framework of fuzzy mathematical structures question like *how much this set is open ?* is quite justified. Namely, with the help of fuzzy logic rules, one can develop a model that allows one to measure the degree to which a given property is satisfied for a given object of study. In this talk we present a scheme that allows us to extend an L-topology τ on a set X to an L-fuzzy topology $\mathcal{T} : L^X \to L$ on this set allowing to associate with each L-subset A of X its degree of openness $\mathcal{T}(A) \in L^X$. We discuss some properties of the L-fuzzy topology $\mathcal{T}$, in particular, show that $\mathcal{T}(A) = \mathbf{1}$ for every $A \in \tau$ and, under some additional assumptions on L, show relations between the degrees of openness and closeness et al. For the convenience of the listeners, we recall the basic concepts that will be used in the talk.

- Let $(L, \leq \wedge, \vee)$ be a complete lattice with top $\mathbf{1}$ and bottom $\mathbf{0}$ elements [1]. Given a set X its L-subset is a mapping $A : X \to L$. Note that in case $L = \{0, 1\}$ an L-subset of a set X is just its subset and in case $L = [0, 1]$ an L-subset is just a fuzzy set as it is defined in the fundamental work by Zadeh [9].
- An L-topology on a set X is a family $\tau \subseteq L^X$ satisfying the analogues of topological axioms, i.e. (1) $\mathbf{0}_X, \mathbf{1}_X \in \tau$, (2) τ is closed under finite intersection, i.e. $A_1, \ldots A_n \in \tau \Longrightarrow \bigwedge_{i=1}^n A_i \in \tau$ (3) τ is closed under arbitrary unions, i.e. $\{A_i \mid i \in I\} \subseteq \tau \Longrightarrow \bigvee_{i \in I} A_i \in \tau$ [2], [3].

- An L-fuzzy topology on a set X is a mapping $\mathcal{T} : L^X \to L$ satisfying fuzzy analogues of a topology, i.e. (1) $\mathcal{T}(\mathbf{1}_X) = \mathcal{T}(\mathbf{0}_X) = \mathbf{1}$, (2) $\mathcal{T}(\bigwedge_{i=1}^n A_i) \geq \bigwedge_{i=1}^n \mathcal{T}(A_i)$ for any $A_1, \ldots A_n \in L^X$ and (3) $\mathcal{T}(\bigvee_{i\in I} A_i) \geq \bigwedge_{i\in I} \mathcal{T}(A_i)$ for any $\{A_i \mid i \in I\} \subseteq L^X$ [5], [7], [8].
- A residuated lattice is a complete lattice enriched with a pair of binary operations: $* : L \times L \to L$ and $\mapsto: L \times L \to L$ related by Galois connection i.e. $a * b \leq c \iff a \leq b \mapsto c \ \forall a, b, c \in L$ [6]. Important pairs of such operation in case $L = [0, 1]$ are the following three $a * b = a \wedge b$ (minimum t-norm), $a * b = a \cdot b$ (product t-norm) and $a * b = \max\{a + b - 1, 0\}$ (Łukasiewicz t-norm) and the corresponding residua $\mapsto$, see e.g. [4]. Operation $*$ in fuzzy logic is interpreted as conjunction and $\mapsto$ as implication. In case $L = \{0, 1\}$ they indeed reduce to ordinary logical conjunction & and implication $\Rightarrow$.

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Game-theoretic results in regards to cardinal functions

Mathematics Subject Classification (MSC): 54A25, 91A44, 54D10

Abstract. In 1970, Arhangel'skii asked whether for every compact space X, $wL(X_\delta) \leq 2^{\aleph_0}$; and Bell, Ginsburg and Woods posed the following question in 1978: is there a regular space such that $|X| \leq 2^{\chi(X).wL(X)}$? Since then, a negative answer has been given to the first question as well as some partial answers to the second one. We provide a few results and examples related to these questions and their answers through the use of topological games.

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Ideal version of the Fréchet–Urysohn property

Mathematics Subject Classification (MSC): 40A35, 54G15, 26A03

Abstract. We consider the ideal version of the Fréchet–Urysohn property of a space of continuous functions. We show its role in a solution [1] to the problem posed by J. Gerlits and Zs. Nagy [2] and to two problems by M. Sakai [3, 4].

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Totally imperfect Menger sets

Mathematics Subject Classification (MSC): 54D20, 03E17

Abstract. A set of reals X is *Menger* if for every sequence of open covers $\mathcal{U}_0, \mathcal{U}_1, \dots$ there are finite families $\mathcal{F}_0 \subseteq \mathcal{U}_0, \mathcal{F}_2 \subseteq \mathcal{U}_2, \dots$ such that the family $\bigcup_{n\in\omega} \mathcal{F}_n$ is a cover of X. Any set of reals of cardinality smaller than the dominating number $\mathfrak{d}$ is Menger and there is a non-Menger set of cardinality $\mathfrak{d}$. By the result of Bartoszyński and Tsaban, in ZFC, there is a totally imperfect (with no copy of the Cantor set inside) Menger set of cardinality $\mathfrak{d}$. We discuss a problem, whether in ZFC there is such a set of cardinality continuum.

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On the Ellis–Numakura Lemma, Free Idempotent Ultrafilters on ω and Choice

Mathematics Subject Classification (MSC): Primary 03E25; Secondary 03E35, 54D30, 54D80, 54H11, 54H20

Abstract. I will discuss, in set theory without the Axiom of Choice (AC), the open problem of the deductive strength of the following two statements:

(1) The *Ellis–Numakura Lemma* (ENL): "Every compact Hausdorff right topological semigroup has an idempotent element";

(2) "There exists a free idempotent ultrafilter on ω".

The chief motivation for investigating this intriguing open problem stems from the fact that the above two consequences of AC are (famously known to be) strongly related to *Hindman's Theorem*: "For any finite colouring of ω, there exists an infinite set $H \subseteq \omega$ such that the set $\mathrm{FS}(H) = \{\sum_{x\in F} x : F \in [H]^{<\omega} \setminus \{\emptyset\}\}$ is monochromatic", which is a cornerstone of the Ramsey theory of numbers and, as shown by W.W. Comfort [2], it is also provable without invoking any choice principle.

Typical results that will be presented are:

(a) ENL for well-orderable semigroups is provable in ZF (i.e. Zermelo–Fraenkel set theory without AC).

(b) ENL for Loeb semigroups is provable in ZF.

(c) The Boolean Prime Ideal Theorem (BPI) implies ENL, and thus ENL is *not* equivalent to AC in ZF.

(d) In ZFA (i.e. ZF with atoms), the Axiom of Multiple Choice (MC) implies ENL restricted to abelian semigroups, or to linearly orderable semigroups.

(e) ENL does not imply MC in ZF (or in ZFA).

(f) "The Cantor cube $2^{\mathbb{R}}$ is compact and a Loeb space" (which, in ZF, is strictly weaker than BPI restricted to $\mathbb{R}$) implies (2) and the implication is not reversible in ZF.

The presentation will be based on results from Tachtsis [11].

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Hartman–Mycielski construction and minimally almost periodic groups

Mathematics Subject Classification (MSC): 54H10, 54H11

Abstract. A topological group G is *minimally almost periodic* if every continuous homomorphism of G to a compact Hausdorff topological group is trivial. According to [NW], there exist countable discrete minimally almost periodic groups. These groups are necessarily infinite and non-Abelian.

We present another source of minimally almost periodic topological (Abelian) groups. In 1958, S. Hartman and J. Mycielski constructed a functorial embedding of any topological group G into a connected, locally connected topological group, $G^\bullet$. Elements of the group $G^\bullet$ are the so-called step-functions from the half-open interval $[0, 1)$ to G. Endowed with an appropriate topology, $G^\bullet$ becomes a topological group containing G as a closed topological subgroup.

Theorem. *For every topological group G, the group $G^\bullet$ is minimally almost periodic.*

Therefore, if G is a compact topological (Abelian) group with $|G| > 1$, then $G^\bullet$ is a connected, locally connected minimally almost periodic topological (Abelian) group which is algebraically generated by a compact connected subset.

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On the degree of connectedness of compact sets

Mathematics Subject Classification (MSC): 54D30, 54F45

Abstract. We consider compact metric spaces. The *n-dimensional diameter* of a space X is the infimum of all $\delta > 0$ such that X admits a δ-map onto a space of dimension n (here, by a dimension, we mean we the covering dimension dim). Recall that if P, Q are closed subsets of X, then a *partition of X between P and Q* is a closed set $C \subset X$ such that $X \setminus C = U_P \cup U_Q$, where U_P and U_Q are disjoint open subsets of X containing P and Q, respectively.

The following notion was introduced by Alexandroff: Two closed disjoint subsets P and Q of X are said to be *V^n-connected* if there exists $\varepsilon > 0$ such that the $(n-2)$-dimensional diameter of any partition C between P and Q is $\geq \varepsilon$.

For any open cover ω of a space X, let N_ω be the nerve of ω and $\pi_\omega : X \to N_\omega$ be the natural map. In this talk we provide the following result:

Let $\dim X = n$ *and* P, Q *be closed disjoint sets in* X *with non-empty interiors. Then* P *are* Q *are* V^n*-connected in* X *if and only if there is an open cover* ω *of* X *such that the sets* $\pi_\omega(P)$ *and* $\pi_\omega(Q)$ *are* V^n*-connected in* N_ω*.*
Some corollaries will be also discussed.

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Asymptotic dimension of hyperbolic, geodesic, proper, quasi-cobounded spaces

Mathematics Subject Classification (MSC): 51F30, 54F45

Abstract. There is a well-known theorem utilizing asymptotic dimension in geometric group theory, proven by S. Buyalo and N. Lebedeva, which states that for a hyperbolic group G, the equality asdim $G = \dim(\partial G) + 1$ holds. More generally, Buyalo and Lebedeva show that the same equality is true for metric spaces which are hyperbolic, geodesic, proper and cobounded.

We will show that Buyalo–Lebedeva's theorem can be further generalized to hyperbolic, geodesic, proper and *quasi-cobounded* spaces. Quasi-coboundedness of a metric space X means that there is a constant $R > 0$ and a uniform collection $\mathcal{A}$ of quasi-isometries of X such that for any chosen base point $o \in X$, and for any $x \in X$, there is a $g \in \mathcal{A}$ so that $g(x) \in B(o, R)$. As a consequence, the equality mentioned above is also true for hyperbolic approximate groups.

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Persistent homology: an application to functional brain networks

Mathematics Subject Classification (MSC): 55N31

Abstract. Persistent homology is a powerful tool from algebraic topology that enables the computation of topological features while keeping track of them along different scales. It has been widely applied to data analysis, including point cloud data, complex networks, images, etc.

In this talk, I will give an overview of persistent homology, describe a pipeline of how this tool is commonly used to analyze data and present a specific application in the analysis of functional brain networks. This application is based in a work where we have focused on the study of individuals with an inhalant substance abuse disorder using resting-state functional Magnetic Resonance Imaging (rs-fMRI).

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Finitely supported functionals vs linear and general ones

Mathematics Subject Classification (MSC): 54C35

Abstract. For given a Tykhonoff space X we call *a functional* any continuous function $f : C_p(X) \to \mathbb{R}$ which maps the zero-function 0^X to 0. We denote the subspace of all functionals in $C_pC_p(X)$ by $C_p^0C_p(X)$.

We call a functional *finitely supported* or a functional *with a finite support* if there exists a finite subset $K \subset X$, such that the following two conditions hold:

(fs 1) For any positive ε, any $\varphi \in C_p(X)$, there exists a positive δ such that if $\psi \in C_p(X)$ and $|\varphi(x)-\psi(x)| < \delta$ for all $x \in K$, then $|f(\varphi)-f(\psi)| < \varepsilon$;

(fs 2) There exists a positive ε, such that for each $x \in K$, each its neighborhood U one can find two functions φ^x, $\psi^x \in C_p(X)$ which coincide out of U, but $|f(\varphi^x) - f(\psi^x)| > \varepsilon$.

Let us denote by $\hat{L}_p(X)$ the subspace of those finitely supported functionals f which satisfy

(I) If $f(\varphi) \neq 0$ then there exists $n_0 \in \mathbb{N}$ such that $|f(n \cdot \varphi)| > 1$ for all $n > n_0$;

(II) If $|f(n \cdot \varphi)| > 1$ for some $n \in \mathbb{N}$ then $f(\varphi) \neq 0$.

It is known [1] that if the spaces $C_p(X)$, $C_p(Y)$ are linearly homeomorphic then Lindelöf numbers of X, Y are equal: $l(X) = l(Y)$.

The linearity of a homeomorphism $h : C_p(X) \to C_p(Y)$ is equivalent to the inclusions $h^*(Y) \subset L_p(X)$ and $(h^{-1})^*(X) \subset L_p(Y)$. Replacing in this sentence L_p by $\hat{L}_p$ we obtain the definition of some new class H of homeomorphisms of function spaces.

Theorem 1. [2] If $h : C_p(X) \to C_p(Y)$, $h \in H$ then $l(X) = l(Y)$.

The theorem just formulated inspires the question about comparison of the spaces $L_p(X)$, $\hat{L}_p(X)$, $FS(X)$, $C_p^0C_p(X)$. Here $FS(X)$ is the subspace of all finitely supported functionals.

It is evidently from the definitions that the chain written above is increasing and the first two inclusions are strong. Moreover, it is well known that $L_p(X)$ is closed in $C_p^0C_p(X)$. Now we claim

Theorem 2. (a) [3] $FS(X) \neq C_p^0C_p(X)$;

(b) $FS(X)$ is dense in $C_p^0C_p(X)$;

(c) $L_p(X)$ is nowhere dense in $C_p^0C_p(X)$;

(d) $\hat{L}_p(X)$ is not nowhere dense in $C_p^0C_p(X)$.

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Local structure of homogeneous *ANR*-spaces

Mathematics Subject Classification (MSC): 54C55, 55M15

Abstract. The importance of finite-dimensional homogeneous ANRs is based on the well known Bing-Borsuk conjecture stating that every homogeneous ANR compactum of dimension n is an n-manifold. It seems that this conjecture is still open. Because of that it is interesting to investigate to what extend finite-dimensional homogeneous locally compact ANR-spaces have common properties with Euclidean manifolds. In the present talk

the local structure of homogeneous ANR-spaces is described. Using that description, we provide a positive solution of the problem whether every finite-dimensional homogeneous metric ANR-compactum X is dimensionally full-valued, i.e. $\dim X \times Y = \dim X + \dim Y$ for any metric compactum Y.

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Perfectly meager sets in the transitive sense and the Hurewicz property

Mathematics Subject Classification (MSC): 54D20, 03E17

Abstract. We work in the Cantor space with the usual group operation $+$. A set X is *perfectly meager in the transitive sense* if for any perfect set P there is an F_σ-set F containing X such that for every point t the intersection $F \cap (t+P)$ is meager in the relative topology of $t+P$. A set X is *Hurewicz* if for any sequence $\mathcal{U}_0, \mathcal{U}_1, \dots$ of open covers of X, there are finite families $\mathcal{F}_0 \subseteq \mathcal{U}_0, \mathcal{F}_1 \subseteq \mathcal{U}_1, \dots$ such that the family $\{\bigcup \mathcal{F}_n : n \in \omega\}$ is a γ-cover of X, i.e., the sets $\{n : x \notin \bigcup \mathcal{F}_n\}$ are finite for all points $x \in X$. Nowik proved that each Hurewicz set which cannot be mapped continuously onto the Cantor set is perfectly meager in the transitive sense. We present results related to the question, whether the same assertion holds for each Hurewicz set with no copy of the Cantor set inside.

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An Introduction to Topological Data Analysis with an Example in Social Networks

Mathematics Subject Classification (MSC): 62R40, 55N31

Abstract. As artificial intelligence improves, the detection of non-human users in social networks becomes a more difficult task. While not all non-human social network users are harmful, many are programmed to spread disinformation or cause civil unrest. Thus, determining whether a social network user is human is an important task. In this talk, we provide an introduction to topological data analysis and provide preliminary results on how these methods could be used to detect non-human users in Twitter.

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On the lower-semi-continuity of the fundamental group

Abstract. The Gromov-Hausdorff distance on the class of compact metric spaces quantifies how far two metric spaces are from being isometric. The only topological feature that presents good behavior with respect to this distance is the fundamental group; it is in some sense lower-semi-continuous. I will present the problem of generalizing this property to non-compact Gromov-Hausdorff convergence and how with certain forms of symmetry, such lower-semi-continuity can be recovered.

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On topology of spaces of persistence diagrams

Mathematics Subject Classification (MSC): 54E35, 55N31

Abstract. Persistence homology and persistence diagrams are important tool in the Topological Data Analysis. Metric spaces of persistence diagrams are objects of consideration in numerous publications (see, e.g., [1]-[6]). In particular, it is shown in [6] that methods of infinite-dimensional topology are useful for description of topology of spaces of persistence diagrams. The talk is devoted to extensions and generalizations of results from [6].

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On $\omega_{\mathcal{N}_T}$-limit sets of discrete dynamical systems

Mathematics Subject Classification (MSC): 37B02, 37B20, 37E05, 37H99

Abstract. In the year 2016 in [1] Wen Huang, Danylo Khilko, Sergii Kolyada and Guohua Zhang introduced the concept of $\omega_{\mathcal{N}_T}$-limit sets and transitive compactness. In this talk I focus on the properties of $\omega_{\mathcal{N}_T}$ -limit sets, their similarities and differences to standart ω-limit sets and show that if we restrict ourselves to interval mappings, then transitive compactness and weakly mixing are equivalent.

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Commutators on generalized power series spaces over non-Archimedean fields

Mathematics Subject Classification (MSC): 26S10, 47S10, 46A35

Abstract. By a non-Archimedean field we mean a non-trivially valued field $\mathbb{K}$ which is complete under the metric induced by the valuation $|\cdot| : \mathbb{K} \to [0, \infty)$ with the *strong triangle inequality:* $|\alpha + \beta| \leq \max\{|\alpha|, |\beta|\}$ for all scalars $\alpha, \beta \in \mathbb{K}$. The generalized power series spaces $D_f(a, r)$ over non-Archimedean fields are the most known and important examples of non-Archimedean nuclear Fréchet spaces. The *commutator* of a pair of operators A and B on a locally convex space E is given by $[A, B] := AB - BA$. An operator T on E is said to be a commutator if T can be expressed in the form $T = [A, B]$ for some operators A and B on E.

We prove among other things the following:

(I) Every operator on $D_f(a, r)$ is a commutator, if (1) $r \in \{0, \infty\}$ and $\sup_n[a_{2n}/a_n] < \infty$ or (2) $r \in (-\infty, 0) \cup (0, \infty)$, $\lim_n[a_{2n}/a_n] = 1$ and f is rapidly increasing.

(II) If $\lim_n[a_{n+1}/a_n] = \infty$ and an operator T on $D_f(a, r)$ is a commutator, then T is bounded. In particular, the identity operator on $D_f(a, r)$ is not a commutator, if $\lim_n[a_{n+1}/a_n] = \infty$.

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Completeness-type properties of hyperspace topologies

Mathematics Subject Classification (MSC): Primary 54B20; Secondary 46A17, 54E50, 54E52, 91A44

Abstract. The purpose of the talk is to present some new completeness-type results concerning the hyperspace $CL(X)$ of the nonempty closed subsets of a metric space (X, d) endowed with functional type hypertopologies such as the Hausdorff metric topology, the Attouch-Wets topology and, in general, the topology of bornological convergence. The properties considered include Čech-completeness, α-favorability, Baireness.

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The gluing problem in General relativity

Abstract. In this talk will discuss the gluing problem for hyperbolic equations along characteristic hypersurfaces. We will cover both the linear case of the wave question and the nonlinear case of the Einstein equations.

This is joint work with Stefan Czimek (Leipzig) and Igor Rodnianski (Princeton).

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Remarks on a dicotomy for the remainder of a topological group

Abstract. Following Arhangel'skiĭ, we discuss a possible dicotomy for the remainder in the compactification of a topological group involving the Menger property.

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Quasi alpha (quasi-α) convergence

Abstract. We introduce and study the notion of quasi-α convergence for sequences of functions. We give some examples which distinguishe the quasi-α convergence from Semi-α, pointwise and α convergence. Using the notion of quasi exhaustiveness we can describe the relation between pointwise convergence and quasi-α convergence for a sequence of functions. Finally we give a characterization of a pseudocompact space using quasi-α convergence.

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Some Notes on Connectedness in point-free topology

Abstract. We look at the notion of connectedness in sigma-frames and its extensions to uniformity and metrizability. We provide a construction of the uniformly locally connected reflection of a locally connected metric sigma-frame.

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A universal hyperspace for posets

Mathematics Subject Classification (MSC): 54B20, 54C25, 54D10, 54D35

Abstract. Finite topological spaces have recently gained interest due to the development of computational and applied topology in recent years. Techniques such as persistent homology allow for improved detection of structures in noisy datasets compared to other methods. In this talk, we will introduce a space (or rather, a hyperspace) as the universal topological space for all finite (and Alexandroff) spaces, and study its properties. In particular, this space allows for comparison of finite spaces and their sequences, resulting in a universal space for certain algebraic topological properties of compact metric spaces.

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Cohomology with fragmented finite supports

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