King Saud University	College of Sciences		Department of Mathematics
Second Examination	Math 132	Semester 1 (1447)	Time:1H30

Question 1: (7 marks)

1. Consider the two sets $A := \{1, 2, 3, 4, \{1\}, \{2\}, \{1,2\}, \{1, \{1\}\}, \{2, \{2\}\}\}$ and $B := \{\emptyset, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$. Determine whether each of the following seven statements is true or false. (Justify your answer). **(4 marks)**
   (i) S_1: "$\{1,2\} \in A$". **(ii)** S_2: "$\{1,2\} \subseteq A$". **(iii)** S_3: "$\{1, \{2\}\} \subseteq A$".
   (iv) S_4: "$A \cap \{1, 2, \{\{1\}, \{2\}\}\} = \{1, 2\}$". **(v)** S_5: "$\emptyset \in B$".
   (vi) S_6: "$\{\{\emptyset\}\} \subseteq B$". **(vii)** S_7: "$(\{\emptyset, \{\emptyset\}\} \cap \{\emptyset, \{\emptyset, \{\emptyset\}\}\}) \subseteq B$".

2. Consider the following three sets $C := \{a, b, c\}, D := \{1, 2, a\}$, and $E := \{(a, 1), (1, a), (b, b), (2, 2), (c, 2), (c, a)\}$. Find the following sets: **(3 marks)**
   (i) $(C \cap D) \times C$. **(ii)** $E \setminus (C \times D)$. **(iii)** $\{\emptyset\} \times E$.

Question 2: (11 marks)

1. Let R be the relation from the set $A := \{1, 3, 5, 7\}$ to the set $B := \{0, 1, 2, 3, 4\}$ defined as follows: for $a \in A$ and $b \in B$, $[(aRb) \Leftrightarrow (a \leq b)]$.
   (a) List all the ordered pairs in the relation R. **(2 marks)**
   (b) Represent the relation R with a matrix. **(1 mark)**
   (c) Let S be the relation from B to A defined by $S := \{(0, 1), (1, 1), (2, 1), (3, 5)\}$.
      i. Find the following relations: $S^{-1} \cap R$, $\overline{R}$ and $S \circ R$. **(3 marks)**
      ii. Draw the digraph of the relation $S \circ R$. **(1 mark)**

2. Let T be the relation on $\mathbb{Z}$ defined as follows:
$$\text{for } m, n \in \mathbb{Z}, \quad (mTn) \Longleftrightarrow 5|(3m + 2n).$$
   Determine whether T is reflexive, symmetric, antisymmetric or transitive. (Justify your answers) **(4 marks)**

Question 3: (7 marks)

1. Let E be the relation on the set $\mathbb{Z}$ defined by:
$$\text{for } a, b \in \mathbb{Z}, \quad (aEb) \Longleftrightarrow (3a - b) \text{ is even.}$$
   (a) Show that the relation E is an equivalence relation on $\mathbb{Z}$. **(3 marks)**
   (b) Decide whether $-3 \in [6]$, justify your answer.. **(1 mark)**

2. Given $A_1 = \{-2, 2\}$, $A_2 = \{-1, 0, 1\}$, and $A_3 = \{3\}$ be the equivalence classes of the relation P on the set $\{-2, -1, 0, 1, 2, 3\}$.
   (a) Draw the digraph of P.**(1 mark)**
   (b) List all ordered pairs of P. **(2 marks)**

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Q1) 7

1)

i) S_1: True: $\{1,2\} \in A$.

ii) S_2: True: $1 \in A$; $2 \in A$.

iii) S_3: True: $1 \in A$; $\{2\} \in A$.

iv) S_4: True because $\{\{1\},\{2\}\} \notin A$.

v) S_5: True; $\phi \in B$. 4

vi) S_6: False; $\{\phi\} \notin B$.

vii) S_7: True.

2)

i) $C \cap B = \{a\}$;

$(C \cap D) \times C = \{(a,a), (a,b), (a,c)\}$. 1

ii) $C \times D = \{(a,1), (a,2), (a,a), (b,1), (b,2), (b,a), (c,1), (c,2), (c,a)\}$.

$E \setminus (C \times D) = \{(1,a), (b,b), (2,2)\}$. 1

iii)

$\{\phi\} \times E = \{(\phi,(a,1)), (\phi,(1,a)), (\phi,(b,b)), (\phi,(2,2)), (\phi,(c,2)), (\phi$

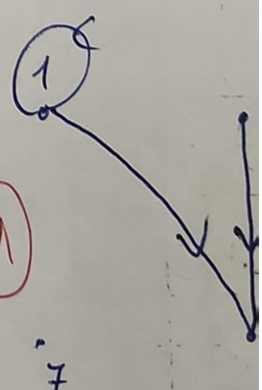
Q2)

a) $R = \{(1,1), (1,2), (1,3), (1,4), (3,3), (3,4)\}$ 2

b)

$$M_R = \begin{array}{c} \\ 1 \\ 3 \\ 5 \\ 7 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$ 1

c) ii)



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c) i) $S^{-1} = \{(1,0), (1,1), (1,2), (5,3)\}$.

$S^{-1} \cap R = \{(1,1), (1,2)\}$. 1

1 $\bar{R} = \{(1,0), (3,0), (3,1), (3,2), (5,0), (5,1), (5,2), (5,3), (5,4), (7,0), (7,1), (7,$

1 $S \circ R = \{(1,1), (1,5), (3,5)\}$.

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2) $\cdot\ m \in \mathbb{Z};\ 3m + 2m = 5m \Rightarrow 5 \mid 5m \Rightarrow m T m$

$\Rightarrow T$ is reflexive.

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$\cdot\ m, n \in \mathbb{Z},\ m T n \Rightarrow 5 \mid (3m+2n) \Rightarrow 3m + 2n = 5q.\quad q \in \mathbb{Z}.$

1 $3m + 2n + 3n + 2m = 5q + 3n + 2m \Rightarrow \underbrace{5m + 5n}_{} = 5q + 3n + 2m$

$5 \mid (5m+5n) \Rightarrow 5 \mid (3n+2m)$

$\Rightarrow n T m$

$\Rightarrow T$ symmetric.

$\cdot$ Not antisymmetric. $2 T 7$ and $7 T 2$

1

$\cdot\ m, n, p \in \mathbb{Z};\ m T n$ and $n T p \Rightarrow 5 \mid (3m + 2n)$ and $5 \mid (3n + 2p)$

$\Rightarrow 3m + 2n = 5q_1,\ 3n + 2p = 5q_2.\quad q_1, q_2 \in \mathbb{Z}.$

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$\Rightarrow 3m + 5n + 2p = 5q_1 + 5q_2.$

$\Rightarrow 3m + 2p = 5\underbrace{(q_1 + q_2 - n)}_{\in \mathbb{Z}}$

$\Rightarrow 5 \mid (3m + 2p) \Rightarrow m T p.$

$\Rightarrow T$ is transitive.

Q3) 1)

a) $a \in \mathbb{Z};\ 3a - a = 2a \Rightarrow$ even; $\Rightarrow a E a \Rightarrow E$ is reflexive

1 $\cdot\ a, b \in \mathbb{Z};\ a E b \Rightarrow 3a - b$ is even $\Rightarrow 3a - b = 2k,\ k \in \mathbb{Z}.$

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$\Rightarrow -9a + 3b = -2k.$

$\Rightarrow 3b - a = -2k + 8a = 2\underbrace{(4a - k)}_{\in \mathbb{Z}} \Rightarrow (3b - a)$ even

$\Rightarrow b E a \Rightarrow E$ is symmetric.

$\cdot\ a, b, c \in \mathbb{Z};\ a E b$ and $b E c \Rightarrow 3a - b$ is even and $3b - c$ is even.

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$\Rightarrow 3a - b + 3b - c$ is even

$\Rightarrow 3a - c + 2b$ is even $\Rightarrow 3a - c$ is even

$\Rightarrow a E c \Rightarrow E$ is transitive

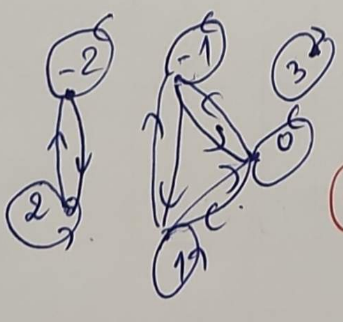
$\Rightarrow E$ is an equivalence relation.

b)

$3(-3) - 6 = -9 - 6 = -15$ odd $\Rightarrow -3 \notin [6].$ 1

2)

a)



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b)

2

$P = \{(-2,-2), (-2,2), (2,-2), (2,2), (-1,-1), (-1,0), (-1,1)$

$(0,-1), (0,0), (0,1), (1,-1), (1,0), (1,1), (3,3)\}.$

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