King Saud University	College of Sciences		Department of Mathematics
Second Examination	Math 132	Semester 2 (1446)	Time:1H30

Question 1: (5 Marks)

1. Let X and Y be two sets such that: $X - Y = \{-1,0,1\}$, $Y - X = \{-3,-2,2,3\}$, and $X \cup Y = \{-3,-2,-1,0,1,2,3\}$. Find X and Y. (2 marks)
2. Let $A = \{a,b,c\}$ and $B = \{b,c,d,e\}$ be two sets.
   (a) Find $B - A$. (1 mark)
   (b) Find $(B - A) \times B$. (1 mark)
   (c) Find $\emptyset \times A$. (1 mark)

Question 2: (12 Marks)

1. Let R be the relation from $C = \{0,1,2,3\}$ to $D = \{3,4,5\}$ defined by:
$$\forall a \in C \text{ and } b \in D,\ aTb \iff a^2 + b \leq 6.$$
   (a) List all ordered pairs of R. (2 marks)
   (b) Find the domain and the image of R. (1 mark)
   (c) Represent R with a matrix. (1 mark)
   (d) Let $S = R \circ R^{-1}$. List all ordered pairs of S. (2 marks)
   (e) Find $\overline{S}$ and S^2. (2 marks)
2. Let T be the relation on the set $\mathbb{Z}$ of integers defined by:
$$\forall a,b \in \mathbb{Z},\ aTb \iff 3|(5a+b).$$
   Determine whether T is reflexive, symmetric, antisymmetric, or transitive. (4 marks)

Question 3: (8 Marks)

1. Let E be the relation on the set $\mathbb{Z} - \{0\}$ defined by:
$$\forall a,b \in \mathbb{Z} - \{0\},\ aEb \iff \frac{a}{b} > 0.$$
   (a) Show that E is an equivalence relation. (3 marks)
   (b) Find $[1]$ and $[-1]$. (2 marks)
2. Let P be the equivalence relation on the set $\{-1,0,1,2,3,4\}$ with equivalence classes $\{-1,4\}$, $\{0,1,2\}$, and $\{3\}$.
   (a) Draw the digraph of P. (1 mark)
   (b) List all ordered pairs of P. (2 marks)

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Q1) 5

1) $X = \{-1, 0, 1\}$; $Y = \{-3, -2, 2, 3\}$. 2

2)
a) $B - A = \{d, e\}$. 1

b) $(B - A) \times B = \{(d,b), (d,c), (d,d), (d,e), (e,b), (e,c), (e,d), (e,e)\}$. 1

c) $\phi \times A = \phi$. 1

Q2) 12

1) a) $R = \{(0,3), (0,4), (0,5), (1,3), (1,4), (1,5)\}$. 2

b). $\text{domain}(R) = \{0, 1\}$; $\text{image}(R) = \{3, 4, 5\}$. 1

c)

$$M_R = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$ 1.

d) $R : C \longrightarrow D$; $R^{-1} : D \longrightarrow C$.

$R^{-1} = \{(3,0), (3,1), (4,0), (4,1), (5,0), (5,1)\}$.

$S = R \circ R^{-1} : D \longrightarrow D$. 2

$S = \{(3,3), (3,4), (3,5), (4,3), (4,4), (4,5), (5,3), (5,4), (5,5)\}$.

e) $\overline{S} = \phi$. 2.

$S^2 = \{(3,3), (3,4), (3,5), (4,3), (4,4), (4,5), (5,3), (5,4), (5,5)\}$.

1). $a \in \mathbb{Z}$; $5a + a = 6a$, $3 | 6a \Rightarrow 3 | (5a+a) \Rightarrow a T a$.
$\Rightarrow$ T is reflexive.

- $a, b \in \mathbb{Z}$; $aTb \Rightarrow 3 | (5a+b) \Rightarrow 5a + b = 3q$, $q \in \mathbb{Z}$.

$\Rightarrow b = 3q - 5a$.

$\Rightarrow 5b + a = 15q - 25a + a$.

$= 15q - 24a$

$= 3(\underbrace{5q - 12a}_{t \in \mathbb{Z}})$

$= 3t$, $t \in \mathbb{Z}$.

$\Rightarrow 3 | (5b + a)$

$\Rightarrow bTa \Rightarrow$ T is symmetric.

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- $0, 3 \in \mathbb{Z}$; $0 T 3$
$3 T 0$ $\Rightarrow$ T is not antisymmetric.
but $0 \neq 3$

- $a, b, c \in \mathbb{Z}$; aTb and bTc.

$\Rightarrow 3 | (5a+b)$ and $3 | (5b + c)$

$\Rightarrow 5a + b = 3q_1$ and $5b + c = 3q_2$.

$\Rightarrow 5a + 6b + c = 3q_1 + 3q_2$.

$\Rightarrow 5a + c = 3(\underbrace{q_1 + q_2 - 2b}_{\in \mathbb{Z}})$. $\Rightarrow 3 | (5a + c)$

$\Rightarrow aTc \Rightarrow$ T is transitive.

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Q3) 8

1) a) • $a \in \mathbb{Z}-\{0\}$; $\frac{a}{a}=1>0 \Rightarrow aEa \Rightarrow E$ is reflexive.

• $a,b \in \mathbb{Z}-\{0\}$; $aEb \Rightarrow \frac{a}{b}>0 \Rightarrow a>0$ and $b>0$ or $a<0$ and $b<0$

$\Rightarrow \frac{b}{a}>0 \Rightarrow bEa$

$\Rightarrow E$ is symmetric.

• $a,b,c \in \mathbb{Z}-\{0\}$; aEb and bEc

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$\Rightarrow \frac{a}{b}>0$ and $\frac{b}{c}>0$

$\Rightarrow \frac{a}{\cancel{b}}\cdot\frac{\cancel{b}}{c}>0 \Rightarrow \frac{a}{c}>0 \Rightarrow aEc$

$\Rightarrow E$ is transitive

$\Rightarrow E$ is an equivalence relation.

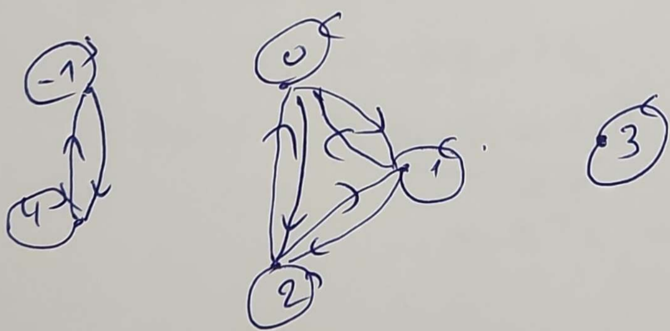
b)

$[1] = \{a \in \mathbb{Z}-\{0\};\ aE1\}$

$= \{a \in \mathbb{Z}-\{0\};\ a>0\} = \mathbb{Z}^+$. 1

$[-1] = \{a \in \mathbb{Z}-\{0\};\ aE(-1)\} = \{a \in \mathbb{Z}-\{0\},\ -a>0\} = \mathbb{Z}^-$. 1

2) a)



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b) $P = \{(-1,-1), (-1,4), (4,-1), (4,4), (0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,1), (2,2), (3,3)\}$. 2