King Saud University
Department of Mathematics

Second Midterm Exam in Math 151, S2-1446H.

Calculators are not allowed

Q1. (a) Let R be the relation on $\mathbb{Z}$ defined by: $mRn \leftrightarrow m^2 > n$. Determine whether R is reflexive, symmetric, antisymmetric or transitive. (4)

(b) Let E be the relation on $\mathbb{R}$ defined by: $xEy \leftrightarrow x^2 - y^2$ is an **integer.**

(i) Show that E is an equivalence relation. (3)

(ii) Show that $[x] = [-x]$ for every real number x. (1)

(iii) Is $\sqrt{5}$ in $[5]$? (Justify your answer.) (1)

(c) Let $P = \{(a,a),(b,b),(c,a),(c,c),(d,a),(d,d),(e,b),(e,e)\}$ be a relation on $A = \{a,b,c,d,e\}$.

(i) Represent P with a digraph. (1)

(ii) Prove that P is a partial order. (3)

(iii) Is P a total order? (Justify your answer.) (1)

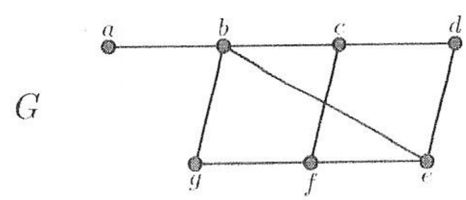
(iv) Represent P with a Hasse diagram. (1)

Q2. (a) If n is any positive integer, then show that a graph with $3n$ vertices and n edges cannot be regular. (2)

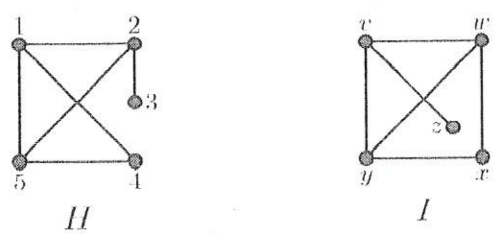
(b) Find the number of vertices of a complete graph $K = (V, E)$ with $|E| = 4|V|$. (2)

(c) Find all possible values for the number of edges of a complete bipartite graph $K_{m,n}$, which has 6 vertices. (2)

(d) Determine if the graph G below is bipartite. If so, find a bipartite representation. (2)



(e) Determine whether the graphs H and I below are isomorphic. (2)



Answer mid 2 (462)
math 151.

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Q1) a) 4

- $0 \in \mathbb{Z};\ 0^2 \not> 0 \Rightarrow 0 \not R 0 \Rightarrow$ R is not reflexive.
- $2, 1 \in \mathbb{Z};\ 2^2 = 4 > 1 \Rightarrow 2 R 1$ but $1^2 \not> 2 \Rightarrow 1 \not R 2$
  $\Rightarrow$ R is not symmetric.
- $-2, 1 \in \mathbb{Z};\ (-2)^2 = 4 > 1 \Rightarrow (-2) R 1$
  $1^2 = 1 > (-2) \Rightarrow 1 R (-2)$) $\Rightarrow$ R is not antisymmetric
- $(0) R (-1)$
  $(-1) R (0)$
  but $0 \not R 0$. $\Rightarrow$ R is not transitive.

b)

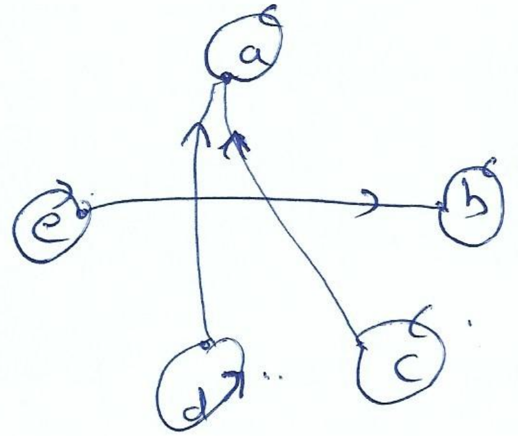
i) 3

- $x \in \mathbb{R};\ x^2 - x^2 = 0 \in \mathbb{Z} \Rightarrow x E x$
  $\Rightarrow$ E is reflexive.
- $x, y \in \mathbb{R};\ x E y \Rightarrow (x^2 - y^2) \in \mathbb{Z}$
  $\Rightarrow (y^2 - x^2) \in \mathbb{Z}$
  $\Rightarrow y E x$
  $\Rightarrow$ E is symmetric
- $x, y, z \in \mathbb{R};\ x E y$ and $y E z$
  $\Rightarrow (x^2 - y^2) \in \mathbb{Z}$ and $(y^2 - z^2) \in \mathbb{Z}$
  $\Rightarrow (x^2 - z^2) \in \mathbb{Z} \Rightarrow x E z$
  $\Rightarrow$ E is transitive
  $\Rightarrow$ E is an equivalence relation.

ii) 1 $x^2 - (-x)^2 = x^2 - x^2 = 0 \in \mathbb{Z} \Rightarrow x E (-x) \Rightarrow [x] = [-x]$

iii) $(\sqrt{5})^2 - (5)^2 = 5 - 25 = -20 \in \mathbb{Z} \Rightarrow \sqrt{5} E 5 \Rightarrow \sqrt{5} \in [5]$ 1 c)

i)



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ii) 3 • $(a,a) \in P$, $(b,b) \in P$, $(c,c) \in P$, $(d,d) \in P$, $(e,e) \in P$

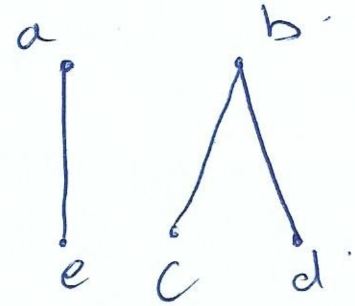
$\Rightarrow$ P is reflexive.

• $\left.\begin{array}{l} (c,a) \in P, (a,c) \notin P \\ (d,a) \in P, (a,d) \notin P \\ (e,b) \in P, (b,e) \notin P \end{array}\right) \Rightarrow$ P is antisymmetric.

• P is transitive $\Rightarrow$ P is a partial order.

iii) P is not a total order, a, b are incomparables. 1

iv)



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Q2) a) 2 if $G=(V,E)$ is an n-regular graph, then $|E| = \frac{|V| \cdot n}{2}$.

if $|E| = m$ and $|V| = 3m \Rightarrow m = \frac{3m \cdot n}{2}$.

$\Rightarrow \frac{3n}{2} = 1 \Rightarrow 3n = 2$

$\Rightarrow n = \frac{2}{3} \notin \mathbb{Z}^{+}$ contradiction

b)

$|E(K)| = \frac{|V| \cdot |V-1|}{2} \Rightarrow 4M = \frac{|V|\,|V-1|}{2}$ $\quad (|V| = m$

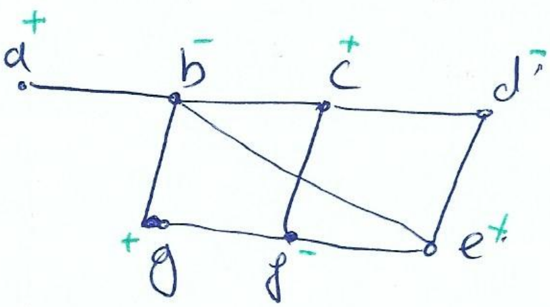
$\Rightarrow 8m = m(m-1)$

$\Rightarrow m^2 - m - 8m = 0 \Rightarrow m(m-9) = 0$ 2

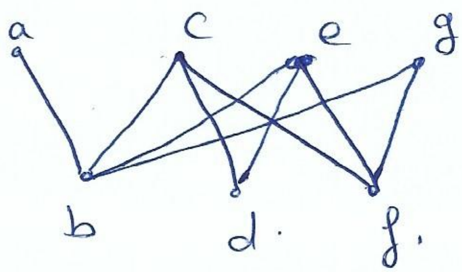
$\Rightarrow \boxed{m = 9}$ $(m \neq 0)$

c) 2 $m + n = 6 \Rightarrow |E| = 5$ or $|E| = 8$, or $|E| = 9$.

d) 2

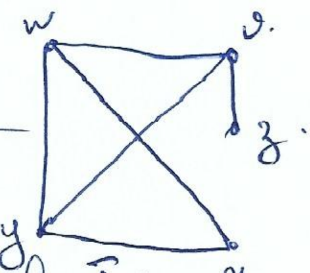


yes G is bipartite 2



e) 1) 2

a	3	1	2	4	5
f(a)	z	w	v	x	y



yes H and I are isomorphic.