King Saud University
Department of Mathematics

First Midterm Exam in Math 151, S2-1446H.
Calculators are not allowed

Q1. (a) Construct the truth table for $A = (\neg p \wedge q) \vee (r \rightarrow \neg q)$. [3]
(b) Without using truth tables, show that $p \rightarrow [\neg(p \wedge \neg q) \rightarrow (p \wedge q)]$ is a tautology. [3]
(c) Let m and n be integers. Use a direct proof to show that if both $m-1$ and $n+2$ are divisible by 3, then $mn + 2m - n + 7$ is divisible by 9. [3]

Q2. (a) Use induction to show that $5n^2 - 3n$ is even for all $n \geq 0$. [4]
(b) Let $\{a_n\}$ be a sequence defined by:

$$a_1 = 1, a_2 = 3, \text{ and } a_{n+1} = (a_n)^2 - (a_{n-1})^2 - 3a_{n-1} \text{ for } n \geq 2.$$

Show that $a_n = 2n - 1$ for all $n \geq 1$. [4]

Q3. (a) Let R be the relation from $A = \{2, 3, 4, 5\}$ to $B = \{0, 1, 2, 3, 4\}$ defined by:

$$aRb \Leftrightarrow 2a \leq b^2.$$

(i) List all ordered pairs of R. [2]
(ii) Represent R with a matrix. [1]
(iii) Find the domain and image (range) of R. [1]

(b) Let $S = \{(v, w), (w, x), (x, y), (y, z), (z, v)\}$ be a relation on $C = \{v, w, x, y, z\}$.
(i) Represent S with a digraph. [1]
(ii) Find $S \cap S^{-1}$. [1]
(iii) Find S^2. [1]
(iv) Find S^3. [1]

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Q1 a) 9

p	q	r	$\neg p$	$\neg p\wedge q$	$\neg q$	$r\to\neg q$	R
T	T	T	F	F	F	F	F
T	T	F	F	F	F	T	T
T	F	T	F	F	T	T	T
T	F	F	F	F	T	T	T
F	T	T	T	T	F	F	T
F	T	F	T	T	F	T	T
F	F	T	T	F	T	T	T
F	F	F	T	F	T	T	T

3

b)

$$p\to[\neg(p\wedge\neg q)\to(p\wedge q)] \equiv p\to[(p\wedge\neg q)\vee(p\wedge q)]$$

$$\equiv p\to[p\wedge(\neg q\vee q)]$$

3

$$\equiv p\to(p\wedge T)\equiv p\to p\equiv T.$$

c) Let $m,n\in\mathbb{Z}$. Assume that $3\mid(m-1)$ and $3\mid(n+2)$

$$\Rightarrow m-1=3q_1 \text{ and } n+2=3q_2,\ q_1,q_2\in\mathbb{Z}.$$

$$\Rightarrow m=3q_1+1 \text{ and } n=3q_2-2.$$

3

$$\Rightarrow mn+2m-n+7=(3q_1+1)(3q_2-2)+6q_1+2-3q_2+2+7$$

$$=9q_1q_2\cancel{-6q_1}+\cancel{3q_2}\cancel{-2}\cancel{+6q_1}\cancel{+2}\cancel{-3q_2}+2+7$$

$$=9\underbrace{(q_1q_2+1)}_{\in\mathbb{Z}}\Rightarrow 9\mid(mn+2m-n+7)$$

P1

2) a) $\frac{8}{9}$

$P(n)$: $5n^2-3n$ is even $\forall n \geq 0$.

B.S: $P(0)$: $5\times 0-3\times 0=0 \Rightarrow 2|0 \Rightarrow 5\times 0-3\times 0$ is even.

I.S: let $k\geq 0$, we prove that $P(k) \rightarrow P(k+1)$

we assume that $P(k)$ true $\Rightarrow 5k^2-3k$ is even.

$\Rightarrow 5k^2-3k=2a; a\in\mathbb{Z}$.

and we prove that $P(k+1)$ is true $5(k+1)^2-3(k+1)$ is even.

4

$$5(k+1)^2-3(k+1)=5k^2+10k+5-3k-3$$
$$=5k^2-3k+10k+2$$
$$=2a+10k+2$$
$$=2(\underbrace{a+5k+1}_{\in\mathbb{Z}})$$

$\Rightarrow 5(k+1)^2-3(k+1)$ is even.

$\Rightarrow P(k+1)$ is true.

b)

$P(n)$: $a_n=2n-1$; $\forall n\geq 1$.

B.S: ~~we prove that P(k) → P~~

$P(1)$: $a_1=2-1=1$ True.

$P(2)$: $a_2=4-1=3$ True.

I.S: let $k\geq 2$, we assume that $P(1), P(2), \dots P(k)$ are true

and we prove that $P(k+1)$ is true $\left(a_{k+1}=2(k+1)-1 \atop =2k+1\right)$.

4

$a_{k+1}=(a_k)^2-(a_{k-1})^2-3a_{k-1}$,

$P(k)$ true $\Rightarrow a_k=2k-1$.

$P(k-1)$ true $\Rightarrow a_{k-1}=2(k-1)-1=2k-3$.

$\Rightarrow a_{k+1}=(\cancel{4k^2}-4k+1)-(\cancel{4k^2}-12k+\cancel{9})-6k+\cancel{9}$.

$=2k+1$.

$\Rightarrow P(k+1)$ is true.

Q3 (8)

a) i) $R = \{(2,2),(2,3),(2,4),(3,3),(3,4),(4,3),(4,4),(5,4)\}$ (2)

ii)

$$M_R = \begin{array}{c|c} & \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$ (1)

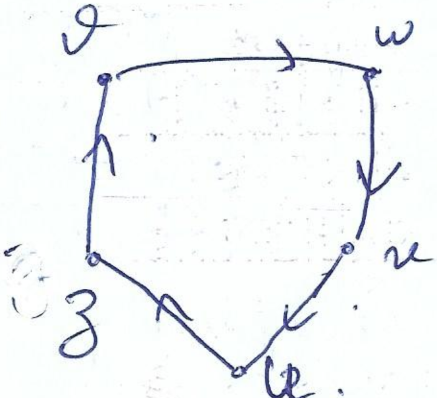
iii)

domain$(R) = \{2,3,4,5\}$. (1)

image$(R) = \{2,3,4\}$.

b)

i)



(1)

ii) $S^{-1} = \{(v,z),(w,v),(x,w),(y,x),(z,y)\}$.

$S \cap S^{-1} = \emptyset$ (1)

iii) $S^2 = \{(v,x),(w,y),(x,z),(y,v),(z,w)\}$ (1)

iv) $S^3 = \{(v,y),(w,z),(x,v),(y,w),(z,x)\}$. (1)