King Saud University | College of Sciences | Department of Mathematics

Final Examination | Math 151 | Semester I(1442) | Time: 3H

**Calculators are not allowed**

***Exercise 1:*** (10 pts)

(1) Without using truth tables, show that: $(p \wedge q) \vee (p \wedge \neg q) \vee \neg p$ is a tautology. (2 pts)

(2) Show that:

$$\frac{1}{1(2)} + \frac{1}{2(3)} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

for all integers $n \geq 1$. (4 pts)

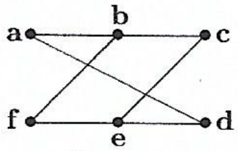
(3) Let $R$ be the relation on $\mathbb{Z}$ defined by: $mRn \iff m + n = 4$. Determine whether $R$ is reflexive, symmetric, antisymmetric, or transitive. (4 pts)

***Exercise 2:*** (6 pts)

(1) Let $G$ be an $r$-regular graph with $4r$ vertices. Find $r$ if $G$ has exactly 18 edges. (2 pts)

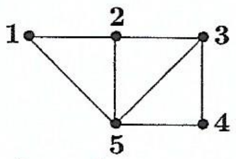
(2) Find the number of edges in a complete graph $K$, whose vertices all have degree 9. (1 pts)

(3) Determine whether the following graph is bipartite, and if so, give a bipartite representation. (1 pts)

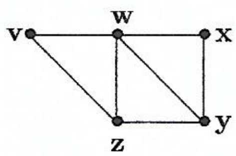



(4) Determine whether the following graphs are isomorphic. (2 pts)

$G_1$




$G_2$




***Exercise 3:*** (11 pts)

(1) Using alphabetical order, form a binary search tree for the words: soccer, handball, volleyball, basketball, tennis, badminton, squash. (2pts)

(2) Let $E$ be the arithmetic expression $(3 * ((7 - x) \uparrow 2)) + ((y/5) - x)$.

(a) Represent $E$ by an ordered rooted tree. (2 pts)

(b) Write $E$ in prefix notation. (1.5 pts)

(c) Write $E$ in postfix notation. (1.5 pts)

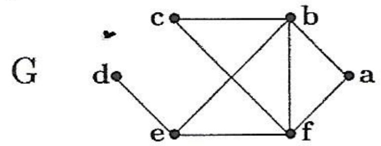
(3) (a) Find the value of the prefix expression: $* + -532/93$. (1 pts)

(b) Find the value of the postfix expression: $53 - 2 + 93/*$. (1 pts)

(4) For the graph $G$ below, find a spanning tree with root $a$,

(a) using depth-first search; (1 pts)

(b) using breadth-first search. (1 pts)




***Exercise 4:*** (13 pts)

(1) Let $f(x,y,z) = (x + y\bar{z})(\bar{y} + z)$ be a Boolean function.

(a) Draw the K-map (Karnaugh map) for $f$. (2 pts)

(b) Write $f$ in **CSP** form. (2 pts)

(2) Write the Boolean function $g(x,y,z) = (x + \bar{z})(\bar{y} + z)$ in **CPS** form. (2 pts)

(3) Let $h$ be the Boolean function represented by the K-map below.

(a) Write $h$ in **MSP** form. (2 pts)

(b) Write $h$ in **MPS** form. (2 pts)

(c) Construct a minimal "**AND-OR**" circuit for $h$. (1 pts)

(d) Construct a circuit for $h$ using **NAND** gates only. (1 pts)

(e) Construct a circuit for $h$ using **NOR** gates only. (1 pts)

| | $zw$ | $z\bar{w}$ | $\bar{z}\bar{w}$ | $\bar{z}w$ |
|---|---|---|---|---|
| $xy$ | 1 | 1 | 1 | |
| $x\bar{y}$ | | 1 | 1 | |
| $\bar{x}\bar{y}$ | | 1 | 1 | |
| $\bar{x}y$ | 1 | 1 | | |

## Exercice 1 : (10pts)

1) (2pts)

$(p \wedge q) \vee (p \wedge \neg q) \vee \neg p$

$\equiv [p \wedge (q \vee \neg q)] \vee \neg p$.

$\equiv [p \wedge T] \vee \neg p \equiv p \vee \neg p \equiv T$.

2) (4pts)

$P(n) : \frac{1}{1(2)} + \frac{1}{2(3)} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$.

$n \geq 1$.

B.S. : $P(1)$: $\frac{1}{1(2)} = \frac{1}{1+1}$ true.

I.S. : we assume that $P(k)$ is true ($k \geq 1$)

and we prove that $P(k+1)$ is true.

$P(k)$ true $\Rightarrow \frac{1}{1(2)} + \frac{1}{2(3)} + \cdots + \frac{1}{k(k+1)} \underset{I.H.}{=} \frac{k}{k+1}$.

Prove $P(k+1)$: $\frac{1}{1(2)} + \frac{1}{2(3)} + \cdots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$.

$\underbrace{\frac{1}{1(2)} + \frac{1}{2(3)} + \cdots + \frac{1}{k(k+1)}}_{IH} + \frac{1}{(k+1)(k+2)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$

$= \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{(k+1)^2}{(k+1)(k+2)} = \frac{k+1}{k+2}$.

$\Rightarrow P(k+1)$ is true $\Rightarrow P(n)$ is true $\forall n \geq 1$

3) (4pts)

- $3+3 = 6 \neq 4 \Rightarrow 3 \not R 3 \Rightarrow R$ not reflexive.
- $m, n \in \mathbb{Z}$; $mRn \Rightarrow m+n = 4 \Rightarrow n+m = 4$

$\Rightarrow nRm \Rightarrow R$ symmetric

- $1R3$, $3R1$ but $1 \neq 3$.

$\Rightarrow R$ not antisymmetric.

- $m, n, p \in \mathbb{Z}$; $mRn$; $nRp$

$\Rightarrow m+n = 4 \Rightarrow n = 4-m$

$n+p = 4 \Rightarrow p = 4-n$

$\Rightarrow m = p$

$1R3$; $3R1$.

but $1 \not R 1 \Rightarrow R$ not reflexive.

## Exercice 2 : (6pts)

1) (2pts)

number of edges of $C_n$ is $\frac{4n \times n}{2} = \frac{4n^2}{2}$.

$\Rightarrow \frac{4n^2}{2} = 18 \Rightarrow 2n^2 = 18 \Rightarrow n^2 = 9$

$\Rightarrow n = 3$.

2) (1pts) K have 10 vertices.

so the number of edges is

$\frac{10 \times 9}{2} = 45$.

3) (1pts) no is not bipartite;

because it contain a 5-cycle

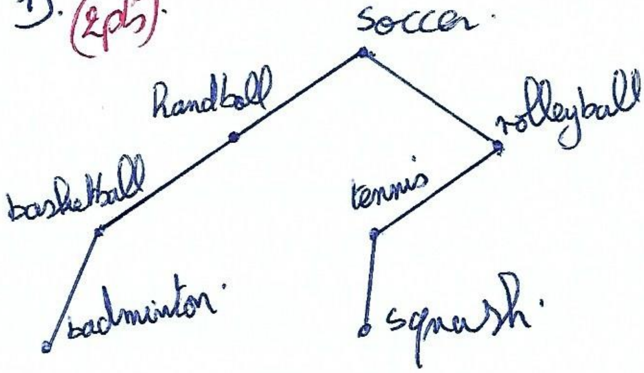
in the vertices; a, b, c, e, d.

4) (2pts) Yes isomorphic.

| | | | | | |
|---|---|---|---|---|---|
| a | 5 | 1 | 2 | 3 | 4 |
| f(a) | w | x | y | z | v |

## Exercice 3 : (11pts)

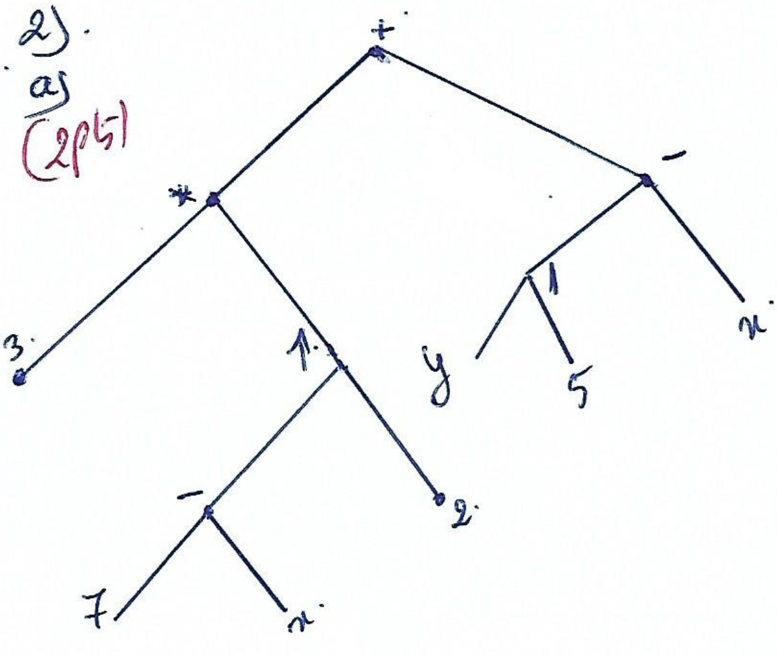
1). (2pts).




2).

a)

(2pts)



b) (1,5pts) + * 3 ↑ _ 7 x 2 _ 1 y 5 x .

c) (1,5pts) 3 7 x − 2 ↑ * y 5 / x − + .

3)

a) (1pts)

$* + - 5\ 3\ 2\ \underbrace{1\ 9\ 3}_{9/3=3}$

$* + \underbrace{-\ 5\ 3}_{5-3=2}\ 2\ 3$

$* \underbrace{+\ 2\ 2}_{2+2=4}\ 3$

$\underbrace{*\ 4\ 3}$

$12.\quad 4\times 3=12$

b) (1pts)

$\underbrace{5\ 3\ -}_{5-3=2}\ 2 + 9\ 3\ /\ *$

$\underbrace{2\ 2\ +}_{2+2=4}\ 9\ 3\ /\ *$

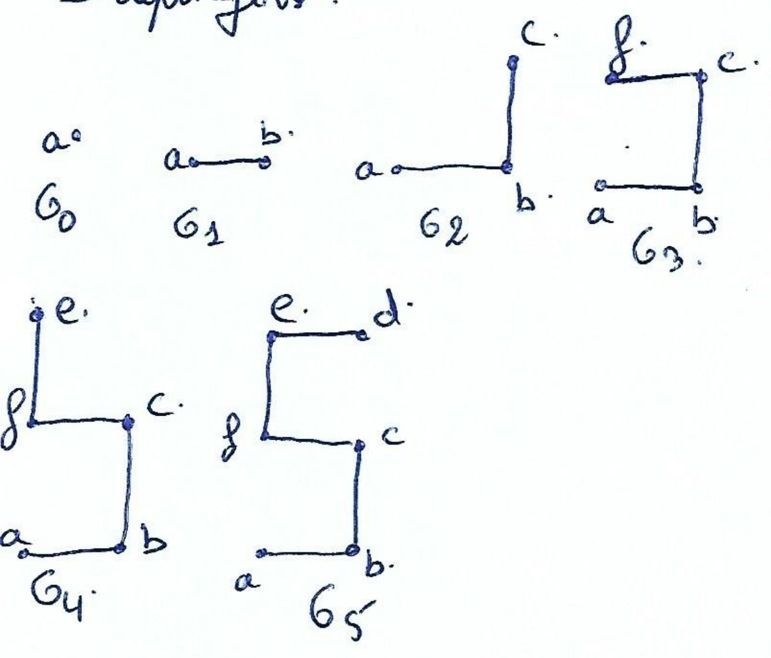
$4\ \underbrace{9\ 3\ /}_{9/3=3}\ *$

$\underbrace{4\ 3\ *}_{4\times 3=12}$

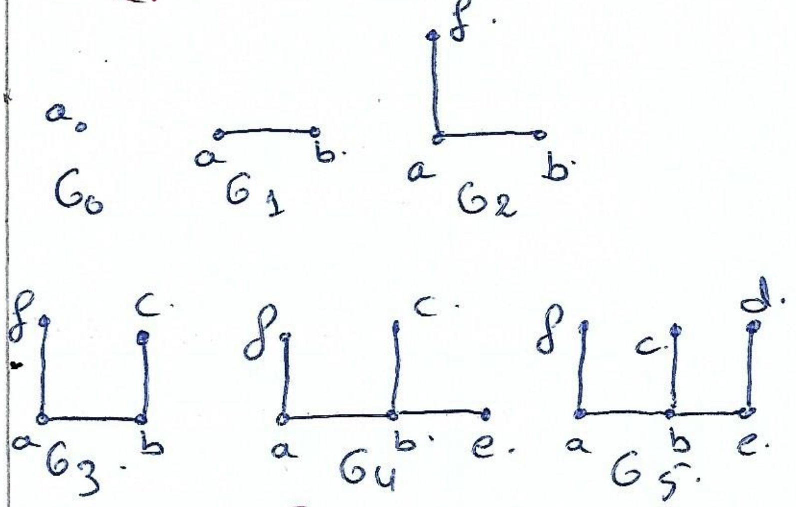
12.

4) (1pts)

a) depth first:



b) (1pts)



Exercice 4 : (13pts)

1)

$f(x,y,z) = (x + y\bar{z})(\bar{y} + z)$

$= x\bar{y} + xz$

a) (2pts)

| | $yz$ | $y\bar{z}$ | $\bar{y}\bar{z}$ | $\bar{y}z$ |
|---|---|---|---|---|
| $x$ | 1 | 0 | 1 | 1 |
| $\bar{x}$ | 0 | 0 | 0 | 0 |

b) (2pts) $f(x,y,z) = x\bar{y}\bar{z} + x\bar{y}z + xyz$

2) (2pts)

$g(x,y,z) = (x+\bar{z})(\bar{y}+z)$

$= x\bar{y} + xz + \bar{y}\bar{z}$

$= x\bar{y}z + x\bar{y}\bar{z} + xyz + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}\bar{y}\bar{z}$

$CSP(g) = x\bar{y}z + x\bar{y}\bar{z} + xyz + \bar{x}\bar{y}\bar{z}$

$\Rightarrow CSP(\bar{g}) = xy\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}z$

$\Rightarrow CPS(g) = (\bar{x}+\bar{y}+z)(x+\bar{y}+\bar{z})(x+\bar{y}+z)(x+y+\bar{z}$

3)

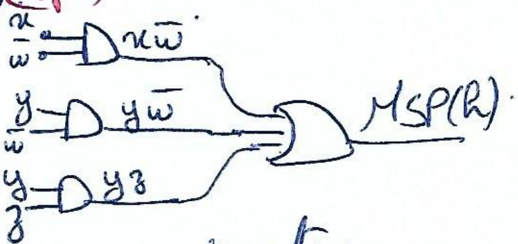
| | $z w$ | $z\bar{w}$ | $\bar{z}\bar{w}$ | $\bar{z} w$ |
|---|---|---|---|---|
| $xy$ | 1 | 1 | 1 | 0 |
| $x\bar{y}$ | 0 | 1 | 1 | 0 |
| $\bar{x}\bar{y}$ | 0 | 1 | 1 | 0 |
| $\bar{x}y$ | 1 | 1 | 0 | 0 |

a) (2pts) $MSP(h) = x\bar{w} + y\bar{w} + yz$

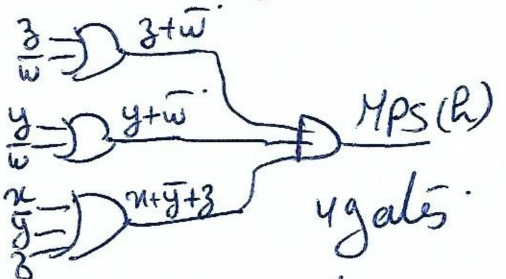
b) (2pts) $MSP(\bar{h}) = \bar{z}w + \bar{y}w + \bar{x}y\bar{z}$

$\Rightarrow MPS(h) = (z+\bar{w})(y+\bar{w})(x+\bar{y}+z)$

c) (1pts)




4 gates



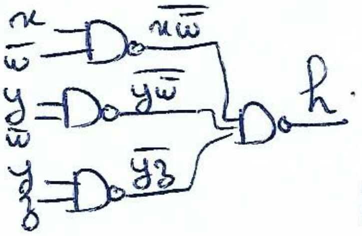

4 gates

the two circuits are minimal.

d) (1pts)

$$h(x,y,z,w) = \overline{\overline{x\bar{w} + y\bar{w} + yz}}$$

$$= \overline{\overline{x\bar{w}} \cdot \overline{y\bar{w}} \cdot \overline{yz}}$$

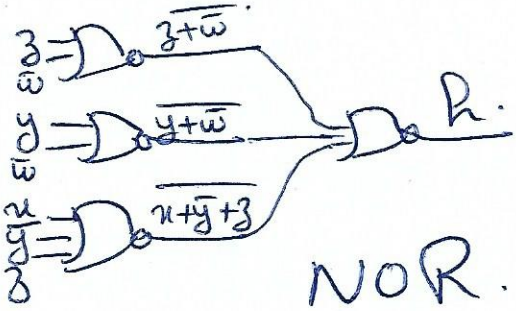



NAND.

e) (1pts)

$$h(x,y,z,w) = \overline{\overline{(z+\bar{w})(y+\bar{w})(x+\bar{y}+z)}}$$

$$= \overline{\overline{(z+\bar{w})} + \overline{(y+\bar{w})} + \overline{(x+\bar{y}+z)}}$$




NOR.

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