King Saud University	College of Sciences	Department of Mathematics	
Second Examination	Math 228	1446(Semester 1)	Time:1H30

Question 1: (4 marks)

Let $f(x, y) = x^4 - 2x^2 + y^2 - 2$.

(a) Find $f_x(x, y)$ and $f_y(x, y)$. **(1 marks)**

(b) Find the local maximum and minimum values and saddle points of f. **(3 marks)**

Question 2: (14 marks)

1. Evaluate the integral $\iint_R x\cos(xy)\, dA$, where $R = [0, \pi] \times [1, 2]$. **(2 marks)**
2. Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy-plane, and inside the cylinder $x^2 + y^2 = 2y$. **(3 marks)**
3. evaluate $\iiint\limits_E x dV$, where E is the solid tetrahedron bounded by the four planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$. **(3 marks)**
4. Use cylindrical coordinates to evaluate $\iiint\limits_E \sqrt{x^2 + y^2} dV$, where E is the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$. **(3 marks)**
5. Use spherical coordinates to evaluate $\iiint\limits_B \sqrt{x^2 + y^2 + z^2} dV$, where B is the ball $B := \{(x, y, z) | x^2 + y^2 + z^2 \leq 9\}$. **(3 marks)**

Question 3: (7 marks)

1. Find the partial sum S_n of the arithmetic sequence that satisfies the given conditions, $a = 9$, $d = -2$ and $n = 22$.**(2 marks)**
2. Find the partial sum S_n of the geometric sequence that satisfies the given conditions, $a = 5$, $r = 2$ and $n = 8$. **(2 marks)**
3. Decide whether each of the following sequence converges or diverges. Justify you answer. **(3 marks)**

   (a) $a_n = 2 + \dfrac{\tan^{-1}(n)}{n^3}$.

   (b) $\{a_n\}$ is a geometric sequence with $a = -2$ and $r = \frac{1}{2}$.

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Q1) 4

1)

$f_x(x,y) = 4x^3 - 4x$ 1

$f_y(x,y) = 2y.$

2) $\begin{cases} 4x^3 - 4x = 0 \\ 2y = 0 \end{cases}$

$\Rightarrow \begin{cases} x(x^2-1) = 0 \\ y = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \text{ or } x = 1 \text{ or } x = -1 \\ y = 0. \end{cases}$ 1

$\Rightarrow (0,0), (1,0), (-1,0)$ are the critical points of f.

$f_{xx}(x,y) = 12x^2 - 4.$

$f_{yy}(x,y) = 2.$

$f_{xy}(x,y) = 0.$

$D(x,y) = 24x^2 - 8.$ 2

- $(0,0)$: $D(0,0) = -8 < 0 \Rightarrow f(0,0)$ is a saddle point
- $(1,0)$: $D(1,0) = 24 - 8 = 16 > 0$
  $f_{xx}(1,0) = 12 - 4 = 8 > 0$
  $\Rightarrow f(1,0)$ is a local minimum.
- $(-1,0)$: $D(-1,0) = 24 - 8 = 16 > 0.$
  $f_{xx}(1,0) = 12 - 4 = 8 > 0$
  $\Rightarrow f(-1,0)$ is a local minimum.

1)

Q2) 14

1. $\iint_R x\cos(xy)\,dA = \int_0^{\pi}\int_1^2 \overset{u'}{x}\cos(\overset{u}{xy})\,dy\,dx$

$= \int_0^{\pi}\left[\sin(xy)\right]_1^2 dx$

$= \int_0^{\pi}(\sin 2x - \sin x)\,dx$

2

$= \left[\frac{1}{2}\cos 2x + \cos x\right]_0^{\pi}$

$= \left(-\frac{1}{2}\cos 2\pi + \cos\pi\right) - \left(-\frac{1}{2}\cos 0 + \cos 0\right)$

$= \left(-\frac{1}{2}\times 1 + (-1)\right) - \left(-\frac{1}{2} + 1\right)$

$= \cancel{-\frac{1}{2}} - 1 \cancel{+\frac{1}{2}} - 1 = -2.$

2) $x^2+y^2 = 2y \Rightarrow r^2 = 2r\sin\theta$

$\Rightarrow r = 2\sin\theta \Rightarrow \sin\theta = 0$

$\Rightarrow \theta = 0$ or $\theta = \pi$

$V = \iint_E (x^2+y^2)\,dA = \int_0^{\pi}\int_0^{2\sin\theta} r^2 r\,dr\,d\theta$

$= \int_0^{\pi}\left[\frac{r^4}{4}\right]_0^{2\sin\theta} d\theta$

3

$= \int_0^{\pi}(4\sin^4\theta)\,d\theta = \int_0^{\pi} 4\left(\frac{1-\cos 2\theta}{2}\right)^2 d\theta$

$= \int_0^{\pi}\left(1 - 2\cos 2\theta + (\cos 2\theta)^2\right) d\theta$

$= \int_0^{\pi}\left(1 - 2\cos 2\theta + \frac{1+\cos 4\theta}{2}\right) d\theta$

$= \left[\theta - \sin 2\theta + \frac{1}{2}\theta + \frac{\sin 4\theta}{4}\right]_0^{\pi}$

$= \left(\pi - \sin 2\pi + \frac{1}{2}\pi + \frac{\sin 4\pi}{4}\right) - \left(\sin 0 + \frac{\sin 0}{4}\right)$

$= \frac{3\pi}{2} + 0 + 0 + 0 - 0 = \frac{3\pi}{2}$

2)

3) $\iiint_E x\,dV = \int_0^1 \int_0^{1-y} \int_0^{1-y-z} x \,dx\,dz\,dy$.

$$= \int_0^1 \int_0^{1-y} \left[\frac{x^2}{2}\right]_0^{1-y-z} dz\,dy$$

$$= \int_0^1 \int_0^{1-y} \frac{(1-y-z)^2}{2}\,dz\,dy$$

$$= -\frac{1}{2}\int_0^1 \left[\frac{(1-y-z)^3}{3}\right]_0^{1-y} dy$$

3

$$= \frac{1}{6}\int_0^1 (1-y)^3\,dy = \frac{1}{24}\left(-(1-x)^4\right]_0^1 = \frac{1}{24}$$

4) $x^2+y^2=4 \Rightarrow 0\le r\le 2;\quad 0\le\theta\le 2\pi$.

$r\le z\le 2$.

$$\int_0^{2\pi}\int_0^{2}\int_{r}^{2} r\,dz\,dr\,d\theta$$

3

$$= \int_0^{2\pi}\int_0^{2} r(2-r)\,dr\,d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^{2} (2r - r^2)\,dr$$

$$= (\theta)_0^{2\pi}\left(\frac{2r^3}{3} - \frac{r^5}{5}\right]_0^2 = 2\pi\left(\frac{64}{15}\right)$$

$$= \frac{128\pi}{15}$$

5) $\int_0^{\pi}\int_0^{2\pi}\int_0^{3} \rho\cdot\rho^2 \sin\phi \,d\rho\,d\theta\,d\phi$

3

$$= \int_0^{\pi} \sin\phi\,d\phi \int_0^{2\pi} d\theta \int_0^{3} \rho^3\,d\rho$$

$$= \left(-\cos\phi\right)_0^{\pi} (\theta)_0^{2\pi} \left[\frac{\rho^4}{4}\right]_0^3 = 4\pi\left(\frac{81}{4}\right) = 81\pi$$

3)

$Q_3)$ 7

1) $S_{22} = \frac{22}{2}\left[2 \times 9 + 21(-2)\right]$.

$= 11(18 - 42)$ 2

$= 11(-24) = -264$.

2) $S_8 = 5 \cdot \frac{1-2^8}{1-2} = -5(1-2^8) = 1275$ 2

3).

a).

$$-\frac{\pi}{2} \leq \tan^{-1}(n) \leq \frac{\pi}{2}$$

$$-\frac{\pi}{2n^3} \leq \frac{\tan^{-1}(n)}{n^3} \leq \frac{\pi}{2n^3}$$

$$2 - \frac{\pi}{2n^3} \leq 2 + \frac{\tan^{-1}(n)}{n^3} \leq 2 + \frac{\pi}{2n^3}$$

$$\lim_{n \to \infty} 2 - \frac{\pi}{2n^3} = 2$$

$$\lim_{n \to \infty} 2 + \frac{\pi}{2n^3} = 2$$ 1,5

$\Rightarrow \lim_{n \to \infty} a_n = 2 \Rightarrow \{a_n\}$ converges.

b) $\{a_n\}$ is a geometric sequences.

with $|r| = \frac{1}{2} < 1$.

$\Rightarrow \{a_n\}$ converges. 1,5

and $\lim_{n \to \infty} a_n = 0$.

4)