King Saud University
Department of Mathematics

Second Midterm Exam in Math 151, S1-1446H.
Calculators are not allowed

Q1. (a) Let R be the relation on $\mathbb{R}$ defined by: $xRy \Leftrightarrow x + y = 40$. Determine whether R is reflexive, symmetric, antisymmetric or transitive. (4)

(b) Let E be the relation on the set A of **even** integers defined by:

$$mEn \Leftrightarrow 6 \mid (2m + n).$$

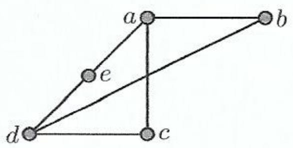
(i) Show that E is an equivalence relation. (3)
(ii) Find $[0]$, and show that $2 \notin [-2]$. (2)

(c) Let $P = \{(a,a),(a,c),(b,a),(b,b),(b,c),(c,c),(d,a),(d,c),(d,d)\}$ be a relation on $B = \{a,b,c,d\}$.
(i) Represent P with a digraph. (1)
(ii) Show that P is a partial order. (3)
(iii) Is P a total order? (Justify your answer.) (1)
(iv) Represent P with a Hasse diagram. (2)

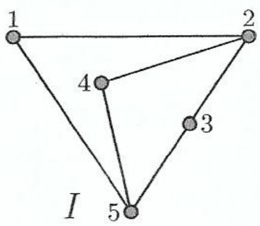
Q2. (a) If a graph G has degree sequence $x, x, 2x, 2x, 2x, 3x, 3x$, then find the value(s) of x, knowing that G has 14 edges. (2)

(b) Find the number of edges of a complete bipartite graph K_{n,n^2} that has 12 vertices. (3)

(c) Determine whether the following graphs H and I are isomorphic. (2)



H



I

(d) Determine whether the graph H in **(c)** is bipartite. If so, then give a bipartite representation. (2)

Q1) 16

a) 4 . R is not reflexive: $1 \in \mathbb{R}$; $1+1=2 \neq 40 \Rightarrow 1 \not R 1$.

. R is symmetric: $x, y \in \mathbb{R}$; $xRy \Rightarrow x+y=40$
$\Rightarrow y+x=40$
$\Rightarrow yRx \Rightarrow$ R is symmetric.

. R is not antisymmetric: $10, 30 \in \mathbb{R}$; $10R30$ and $30R10$
but $10 \neq 30$

. R is not transitive: $30, 10 \in \mathbb{R}$; $30R10$ and $10R30$
but $30 \not R 30$.

b) 5 i) 3 . Let $m \in A \Rightarrow m = 2k$; $k \in \mathbb{Z}$.
$\Rightarrow 2m+m = 2(2k)+2k = 6k \Rightarrow 6 | 6k$.
$\Rightarrow 6 | (2m+m)$
$\Rightarrow mEm \Rightarrow$ E is reflexive

. Let $m, n \in A$, $mEn \Rightarrow 6 | (2m+n)$ $(6 | (2n+m))$
$\Rightarrow 2m+n = 6q$. $q \in \mathbb{Z}$.
$\Rightarrow 4m+2n = 12q$.
$\Rightarrow 2n+m = 12q - 3m$
as $m \in A \Rightarrow m = 2k$; $k \in \mathbb{Z}$.
$\Rightarrow 2n+m = 12q - 6k$
$= 6\underbrace{(2q-k)}_{\in \mathbb{Z}} \Rightarrow 6 | (2n+m)$
$\Rightarrow nEm \Rightarrow$ E is symmetric.

. Let $m, n, p \in A$; mEn and nEp
$\Rightarrow 6 | (2m+n)$ and $6 | (2n+p)$
$\Rightarrow 2m+n = 6q_1$ and $2n+p = 6q_2$, $q_1, q_2 \in \mathbb{Z}$.
$\Rightarrow 2m+p = 6q_1 + 6q_2 - 3n$ $\quad n \in A \Rightarrow n = 2k$, $k \in \mathbb{Z}$.
$= 6q_1 + 6q_2 - 6k$
$= 6\underbrace{(q_1+q_2-k)}_{\in \mathbb{Z}} \Rightarrow 6 | (2m+p)$
$\Rightarrow mEp \Rightarrow$ E is transitive
$\Rightarrow$ E is an equivalence relation

MA.

ii) $[0] = \{m \in A,\ m \mathcal{E} 0\} = \{m \in A,\ 6 \mid 2m\} = \{m \in A,\ 6 \mid m\}$.

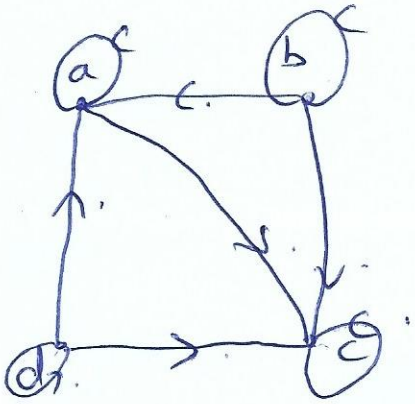
1

$2\times 2 - 2 = 2$, $6 \nmid 2 \Rightarrow 6 \nmid ((2\times 2) - 2) \Rightarrow 2 \not\mathcal{E} (-2)$

$\Rightarrow 2 \notin [-2]$. 1

c) 7

i) 1



ii)

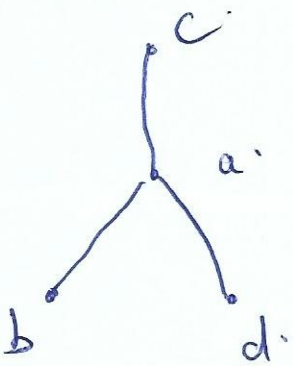
- $(a,a), (b,b), (c,c), (d,d) \in P \Rightarrow P$ is reflexive.
- $(a,c) \in P; (c,a) \notin P; (b,a) \in P; (a,b) \notin P; (b,c) \in P; (c,b) \notin P$

3 $(d,a) \in P; (a,d) \notin P; (d,c) \in P, (c,d) \notin P \Rightarrow P$ is antisymmetric

- $(b,a) \in P; (a,c) \in P \Rightarrow (b,c) \in P$, $(d,a) \in P; (a,c) \in P \Rightarrow (d,c) \in P$.

iii) No; $(b,d) \notin P, (d,b) \notin P \Rightarrow$ b, d are incomparable.
1

iv)



2

2)

Q2) 9

a)

$14x = 28 \Rightarrow x = 2.$ 2

b)

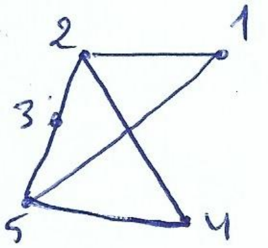
$n + n^2 = 12 \Rightarrow n(n+1) = 12 \Rightarrow n = 3.$

3 $\Rightarrow |E(K_{3,9})| = 9 \times 3 = 27.$

c).

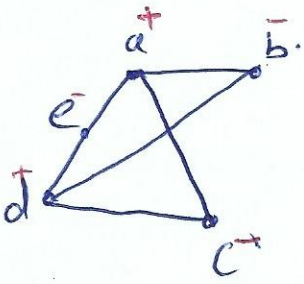
x	b	a	c	d	e
$f(x)$	1	2	4	5	3.

1	2	3	4	5
c	d	b	e	a.

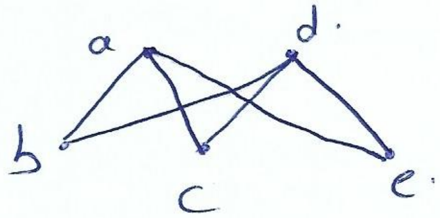


2

d)



yes bipartite



2

3