King Saud University
Department of Mathematics

First Midterm Exam in Math 151, S1-1446H.
Calculators are not allowed

Q1. (a) Construct the truth table of $A = \neg(p \wedge \neg q) \rightarrow (q \vee \neg r)$. [3]
(b) Without using truth tables, show that

$$(\neg p \rightarrow q) \rightarrow p \equiv \neg(\neg p \wedge q). \text{ [3]}$$

(c) Let x, y and z be real numbers. Use contraposition to show that if $2x = y - z + 1$, then $x > 5$ or $y < 9$ or $z > -2$. [3]

Q2. (a) Use induction to show that $2+8+14+\ldots+(6n-10) = 3n^2-7n+4$ for all $n \geq 2$. [4]
(b) Let $\{u_n\}$ be a sequence defined by:

$$u_1 = 0, u_2 = 3, u_3 = 8 \text{ and } u_{n+1} = u_n + u_{n-1} - u_{n-2} + 4 \text{ for } n \geq 3.$$

Show that $u_n = n^2 - 1$ for all $n \geq 1$. [4]

Q3. (a) Let R be the relation from $A = \{-1, 0, 1, 2, 3\}$ to $B = \{1, 2, 3, 4, 5\}$ defined by:

$$aRb \Leftrightarrow a^2 = b - 1.$$

(i) List all ordered pairs of R. [2]
(ii) Represent R with a matrix. [1]
(iii) Find the domain and image (range) of R. [1]

(b) Let $T = \{(a,c), (b,a), (b,d), (d,a), (d,b), (d,d)\}$ be a relation on $C = \{a, b, c, d\}$.
(i) Find $\overline{T} \cap T^{-1}$. [2]
(ii) Find $T \circ T$. [2]

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Q1) 9

a)

p	q	r	$\neg q$	$p\wedge\neg q$	$\neg(p\wedge\neg q)$	$\neg r$	$q\vee\neg r$	A
T	T	T	F	F	T	F	T	T
T	T	F	F	F	T	T	T	T
T	F	T	T	T	F	F	F	T
T	F	F	T	T	F	T	T	T
F	T	T	F	F	T	F	T	T
F	T	F	F	F	T	T	T	T
F	F	T	T	F	T	F	F	F
F	F	F	T	F	T	T	T	T

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b)

$\cancel{(p \to q) \to p \equiv (\neg p \vee q) \to p}$

$(\neg p \to q) \to p \equiv (\neg(\neg p) \vee q) \to p$

$\equiv \neg(p \vee q) \vee p$

$\equiv (\neg p \wedge \neg q) \vee p$

$\equiv (\neg p \vee p) \wedge (\neg q \vee p)$ 3

$\equiv \neg q \vee p \equiv \neg(\neg p \wedge q)$.

c) By contraposition we prove that:
if $x \leq 5$ and $y \geq 9$ and $z \leq -2$ then $2x \neq y - z + 1$.

$x \leq 5 \Rightarrow 2x \leq 10$.

$y \geq 9$, $-z \geq 2 \Rightarrow y - z + 1 \geq 12$. $\Big\} \Rightarrow 2x \neq y - z + 1$.

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Q2) 8

a) $P(n): 2+8+14+\dots+(6n-10) = 3n^2-7n+4, \forall n \geq 2$.

B.S: $P(2)$: L.H.S $= 2$.

R.H.S $= 3\times4-14+4 = 2$.

$\Rightarrow P(2)$ is true.

I.S: Let $k \geq 2$, we assume that $P(k)$ is true and we prove that $P(k+1)$ is true.

$P(k): 2+8+14+\dots+(6k-10) = 3k^2-7k+4$.

$P(k+1): 2+8+\dots+(6k-4) = 3(k+1)^2-7(k+1)+4$.

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$2+8+\dots+(6k-10)+(6k-4) = 3k^2-7k+4+6k-4$
$= 3k^2-k = $ L.H.S.

RHS $= 3(k+1)^2-7(k+1)+4$
$= 3k^2+6k+\cancel{3}-7k\cancel{-7}\cancel{+4}$.
$= 3k^2-k = $ L.H.S.

$\Rightarrow P(k+1)$ is true $\Rightarrow \forall n \geq 2$ $P(n)$ true.

b) $P(n): u_n = n^2-1 \quad \forall n \geq 1$.

B.S: $P(1): u_1 = 0$ True.
$P(2): u_2 = 4-1 = 3$ True.
$P(3): u_3 = 9-1 = 8$ True.

I.S: Let $k \geq 3$, we assume that $P(1), \dots, P(k)$ are true and we prove that $P(k+1)$ is true.

$P(k+1): u_{k+1} = (k+1)^2-1 = k^2+2k$.

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$u_{k+1} = u_k + u_{k-1} - u_{k-2} + 4$.
$P(k)$ true $\Rightarrow u_k = k^2-1$.
$P(k-1)$ true $\Rightarrow u_{k-1} = (k-1)^2-1 = k^2-2k$
$P(k-2)$ true $\Rightarrow u_{k-2} = (k-2)^2-1 = k^2-4k+3$.
$u_{k+1} = k^2-1+k^2-2k-k^2+4k-3+4$
$= k^2+2k \Rightarrow P(k+1)$ true
$\Rightarrow \forall n \geq 1$ $P(n)$ true.

Q3) 8

a)

i) $R = \{(-1,2),(0,1),(1,2),(2,5)\}$ 2

ii)

$$M_R = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 & 0 & 0 \end{array}$$ 1

iii) $domain(R) = \{-1, 0, 1, 2\}$.

$image(R) = \{1, 2, 5\}$. 1

b)

i) $\overline{T} = \{(a,a),(a,b),(a,d),(b,b),(b,c),(c,a)(c,b)(c,c),(c,d),(d,c)\}$.

$T^{-1} = \{(a,b),(a,d),(b,d),(c,a),(d,b)(d,d)\}$.

$\overline{T} \cap T^{-1} = \{(a,b),(a,d),(c,a)\}$. 2

ii)

$T \circ T = \{(b,c),(b,a)(b,b),(b,d),(d,c)(d,a),(d,b),(d,d)\}$.

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3)