Recent Developments on Sliding Mode Control of Complex Dynamical Systems - Research Article

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Sliding mode control design for stabilization of underactuated mechanical systems

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Abstract

Stabilization of underactuated mechanical systems is one of the fundamental benchmark problem in the field of control theory. In this article, a hierarchical sliding mode control based on state-dependent switching gain is proposed for stabilization of underactuated mechanical systems. This controller is based on the so called first-level and second-level sliding surfaces. The asymptotic stability of these surfaces is proved by the Lyapunov stability theory. The proposed control technique is applied to two nonlinear underactuated mechanical systems and its feasibility is verified by numerical simulation. The proposed controller efficiently tackles the bounded external disturbance and shows robust performance. The convergence rate of the proposed controller is much faster as compared with the conventional decoupled sliding mode control and the integral sliding mode control.

Keywords

Sliding mode control, underactuated systems, state-dependent switching gain

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Introduction

The design of nonlinear controllers for mechanical systems has become an active field of research from the last few decades.[1] From a theoretical point of view, this attention can be attributed to their interesting dynamical behavior, which is well-suited for the validation and practical application of ideas emerging in control theory. However, recent technological advances have produced many real-world engineering applications that require the automatic control of mechanical systems.[2]

An underactuated mechanical system is a system that has a fewer number of control input actuators than the degrees of freedom of the system. These systems are widely used in spacecrafts, underwater vehicles, mobile robots, surface vessels, and many other systems. Due to numerous practical applications of these systems, many researchers have worked to find reliable mechanisms for the control of underactuated mechanical systems. Stabilization of these systems is a fundamental benchmark problem. These systems cannot be stabilized by smooth feedback because their dynamics are governed by differential equations in the presence of some non-integrable differential conditions.[1,3–5] The development of new control techniques and numerical tools have played a significant role in recognition and explaining the concept of stabilization. However, it is still an open and challenging task for researchers.

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Many techniques have been proposed to solve the stabilization problem for underactuated mechanical systems.[6–13] Most of the suggested control approaches undergo from either the oscillatory behavior, slow convergence rate, or lack of robustness against the external disturbances. The problem of robustness and slow convergence can be solved using sliding mode control (SMC). Due to inherent robustness properties of SMC, it has been widely used for stabilization of nonlinear systems. It has been used to design a controller for robotics,[14–16] under-actuated cranes,[17–20] unmanned aerial vehicle (UAV) quadrotors,[21] and under-actuated vessels.[22] SMC technique, which is a powerful approach to handle internal and external disturbances, was introduced in the literature.[23–25] The results developed by these techniques were very interesting. The control designed by these techniques had the ability to handle a class of uncertainties. However, these techniques require the use of virtual disturbances to satisfy some conditions. These controllers also require a lot of calculations that increase the computational burden.

Recently, many researchers have been showing their interest in combination of fuzzy logic control (FLC) and SMC which is referred to fuzzy SMC.[26–28] Fuzzy SMC has advantages of both FLC and SMC. However, there are some drawbacks of this technique. For higher order systems, the number of fuzzy sets and fuzzy rules becomes incredibly large, which compromises the applicability of this technique. An often remarked drawback of methods based on fuzzy logic is the lack of appropriate tools for analyzing the controller performance, such as stability, optimality, and so on. The SMC technique consists of two distinct phases. One is reaching phase and the other is sliding phase. In the reaching phase, the system states are forced toward the sliding surface while in the sliding phase, the system trajectories drive along the sliding surface.

A known limitation of SMC technique is chattering. To overcome this problem, Slotine proposed adopting thin boundary layer neighboring switching surfaces, by replacing the sign function with a saturation function.[29] The chattering phenomenon can be removed completely by utilizing observer. The main idea of using observers to prevent chattering is to generate an ideal sliding mode in the auxiliary loop including the observer.[30] Another way to reduce chattering without designing any asymptotic observer is to use state-dependent switching gain.[31] With the choice of state-dependent switching gain and saturation function, the chattering effect becomes negligible. The main goal of this article is to design nonlinear SMC for a class of mechanical systems based on state-dependent switching gain which is robust for bounded external disturbances, chattering free, and have low complexity and computational burden.

Using switching gain which are functions of the system states, the SMC guarantees the robustness and faster convergence. We design the proposed controller for stabilization of inverted pendulum (IP) and ball-beam systems, and simulate it using MATLAB. The simulation results show effectiveness of our proposed controller.

Problem statement

The general form of a class of underactuated mechanical systems is given by

$$\left.\begin{array}{lcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & g_1(x) + b_1(x)\tau + \Delta \\ \dot{x}_3 & = & x_4 \\ \dot{x}_4 & = & g_2(x) + b_2(x)\tau + \Delta \\ \vdots & \vdots & \vdots \\ \dot{x}_{2m-1} & = & x_{2m} \\ \dot{x}_{2m} & = & g_m(x) + b_m(x)\tau + \Delta \end{array}\right\} \tag{1}$$

where $x_1, x_2, x_3, \ldots, x_{2m}$ are state variables, $g_m(x)$ and $b_m(x)$ are defined as nonlinear functions of state variables, τ is control input, and Δ is bounded external disturbance.[32] Different nonlinear underactuated mechanical systems can be presented in the form of system (1) with different nonlinear functions $g_m(x)$ and $b_m(x)$. If $m = 2$, then system (1) represents overhead crane system, IP system, and so on. Similarly, for $m = 3$, system (1) represents the double rotary IP system, the double IP system, and so on. We can also write system (1) as follows

$$\left.\begin{array}{lcl} \dot{x}_{2k-1} & = & x_{2k} \\ \dot{x}_{2k} & = & g_k(x) + b_k(x)\tau + \Delta \end{array}\right\} \tag{2}$$

where $k = 1, 2, 3, \ldots, m$. The control objective is to design a τ such that as states of the system quickly converge to the equilibrium point in presence of external disturbances.

Control design

SMC due to simplicity of design has been successfully employed for solving nonlinear control problems.[33–36] It was first used in 1960s and its basic formulations are due to the work of Utkin.[37] Utkin provided the definition of sliding surface from which equivalent control is derived. In 1978, Utkin and Yang continued their work and derived the term nonlinear switching from linear state space derivation which ensures the robustness of SMC.[38] SMC is based on the concept of the sliding surface, and it switches the system trajectories on this sliding surface. The total control input is a combination of switching control and equivalent control, that is

$$\tau = \tau_{\text{eq}} + \tau_{\text{sw}} \tag{3}$$

where τ_{sw} is designed to switch system trajectories onto sliding surface and τ_{eq} is designed to keep system trajectories in closed neighborhood of sliding surface. Stabilization of underactuated mechanical systems has become a benchmark to verify the effectiveness of new control techniques. These systems are distinguished by the fact that they have fewer actuators. There are many examples of these systems such as manipulators, robots, and industrial equipment. The dynamics of these systems mostly contain feedback nonlinearities, couplings, nonholonomic constraints, and non-minimum phase zero dynamics, which makes it difficult to design controllers for these systems.

In this article, we are focusing on a class of nonlinear underactuated mechanical systems. SMC is a powerful and robust nonlinear feedback control method. However, for underactuated systems, designing a conventional single layer sliding surface is not appropriate because the parameters of the sliding mode surface can not be obtained directly according to the Hurwitz condition. The physical structure of the considered class also matters as it can be divided into several subsystems. Based on this structure, a simple way to design the sliding mode surface for this class was needed. To overcome this problem, Qian et al.[39] proposed a hierarchical sliding mode control (HSMC) for such a class of underactuated systems.

HSMC is a systematic and effective design procedure, which has both theoretical and practical significance. It fills the gap between SMC and its applications to underactuated mechanical systems.[40] It is composed of two types of sliding surfaces: the first-level sliding surfaces and a second-level sliding surface. The number of first-level sliding surfaces depends upon the number of subsystems and the second-level sliding surface is a linear combination of all first-level sliding surfaces. The schematic structure of the surfaces of HSMC is illustrated in Figure 1.

In order to reduce the response time of convergence and decrease chattering phenomenon, we proposed to design state-dependent switching gain instead of a constant switching gain. Both, first-level and second-level sliding surfaces, are asymptotically stable in the proposed control design scheme. Numerical simulations validate the effectiveness of the proposed HSMC. To design this controller, we start by defining the first-level sliding surfaces as follows

$$\sigma_k = \varrho_k x_{2k-1} + x_{2k} \tag{4}$$

where ϱ_k is a positive constant. Differentiating equation (4) with respect to t and using equations given in system (2), we get

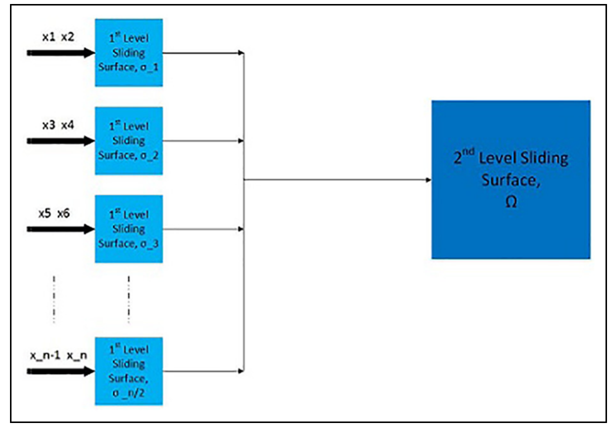

Figure 1. Schematic structure of sliding surfaces for HSMC.

$$\dot{\sigma}_k = \varrho_k \dot{x}_{2k-1} + \dot{x}_{2k} \tag{5}$$

$$\dot{\sigma}_k = \varrho_k x_{2k} + g_k + b_k\tau + \Delta \tag{6}$$

The equivalent control of the k^{th} subsystem can be obtained by assuming $\dot{\sigma}_k = 0$ and it is given by

$$\tau_{eqk} = -\frac{\varrho_k x_{2k} + g_k + \Delta}{b_k} \tag{7}$$

Now, we define the second-level sliding surface as follows

$$\Omega = \sum_{k=1}^{m} \lambda_k \sigma_k \tag{8}$$

where λ_k is a real constant. Differentiating equation (8) with respect to t, we have

$$\dot{\Omega} = \sum_{k=1}^{m} \lambda_k \dot{\sigma}_k \tag{9}$$

In order to derive a control law which drives the trajectories of the system to the sliding surface Ω, we define a Lyapunov function as follows

$$V = \frac{\Omega^2}{2} \tag{10}$$

$$\dot{V} = \Omega\dot{\Omega} \tag{11}$$

$$\dot{V} = \Omega\left[\sum_{k=1}^{m} \lambda_k \dot{\sigma}_k\right] \tag{12}$$

$$\dot{V} = \Omega\left[\sum_{k=1}^{m} \lambda_k(\varrho_k x_{2k} + g_k + b_k\tau + \Delta)\right] \tag{13}$$

$$\dot{V} = \Omega\left[\sum_{k=1}^{m} \lambda_k(\varrho_k x_{2k} + g_k + b_k(\tau_{eq} + \tau_{sw}) + \Delta)\right] \tag{14}$$

$$\dot{V} = \Omega\left[\sum_{k=1}^{m} \lambda_k \cdot b_k \cdot \tau_{sw}\right] \quad (15)$$

We design $\dot{\Omega}$ as

$$\dot{\Omega} = -\epsilon \cdot \text{sat}(\Omega) \quad (16)$$

where "sat" is the saturation function and ϵ is a switching gain. The saturation function in the form of sign function is defined as[41]

$$\text{sat}(\Omega) = \begin{cases} \text{sgn}(\Omega), & \text{if } |\Omega| \geq 1 \\ \Omega, & \text{if } |\Omega| < 1 \end{cases}$$

and

$$\text{sgn}(\Omega) = \begin{cases} +1, & \text{if } \Omega > 0 \\ -1, & \text{if } \Omega < 0 \end{cases}$$

The switching gain can be selected in two ways: one is the switching gain with a constant value ϵ and the other is the switching gain $\epsilon(x)$ which is a function of the state variables. If the switching gain is selected as a constant, then it must be a positive constant to fulfill the stability condition for second-level sliding surface. We proposed a state-dependent switching gain for fast convergence of the system states in presence of bounded external disturbances, that is

$$\epsilon(x) = \beta(\Omega^2 + \gamma) \quad (17)$$

where β and γ are positive constants. The switching control law obtained from equations (15) and (16) is given by

$$\tau_{\text{sw}} = \frac{-\epsilon(x) \cdot \text{sat}(\Omega)}{\sum_{k=1}^{m} \lambda_{k-1} \cdot b_k} \quad (18)$$

Theoretical stability

This section consists of two theorems which deal with the asymptotic stability of the second-level sliding surface and the first-level sliding surfaces. To prove these theorems, we use the Lyapunov stability theory and the Barbalat lemma.[29]

Theorem 1 *(Asymptotic stability of second-level sliding surface).* Consider the class of underactuated systems given in system (2). If we use the control law given in equation (3) along with sliding surfaces designed in equations (4) and (8), then the second-level sliding surface is asymptotically stable.

Proof. Consider the Lyapunov function defined in equation (10)

$$V = \frac{1}{2}\Omega^2$$

Differentiating both sides with respect to t, we get

$$\dot{V} = \Omega \cdot \dot{\Omega}$$

$$\dot{V} = -\epsilon(x) \cdot \Omega \cdot \text{sat}(\Omega)$$

Integrating both sides from 0 to t, we have

$$\int_0^t \dot{V} d\chi = \int_0^t -\epsilon(x) \cdot \Omega \cdot \text{sat}(\Omega) d\chi$$

$$V(t) - V(0) = \int_0^t -\epsilon(x) \cdot \Omega \cdot \text{sat}(\Omega) d\chi$$

$$V(0) = V(t) + \int_0^t \epsilon(x) \cdot \Omega \cdot \text{sat}(\Omega) d\chi$$

$$V(0) \geq \int_0^t \epsilon(x) \cdot \Omega \cdot \text{sat}(\Omega) d\chi$$

$$\Rightarrow \quad \lim_{t\to\infty} \int_0^t \epsilon(x) \cdot \Omega \cdot \text{sat}(\Omega) d\chi \leq V(0) < \infty \quad (19)$$

According to the Barbalat lemma[29]

$$\lim_{t\to\infty} \epsilon(x) \cdot \Omega \cdot \text{sat}(\Omega) = 0 \quad (20)$$

Therefore, $\lim_{t\to\infty} \Omega = 0$. Hence second-level sliding surface is asymptotically stable.

Theorem 2 *(Asymptotic stability of first-level sliding surfaces).* Consider the underactuated system given in system (2). If the control law is adopted as given in equation (3) with sliding surfaces designed in equations (4) and (8), then the first-level sliding surfaces are asymptotically stable.

Proof. The proof of this theorem is quite simple. From equation (15) and considering that $\dot{V} = \Omega\dot{\Omega} < \infty$, we can write

$$\Omega, \dot{\Omega} \in L^\infty$$

where L^∞ denotes the space of all bounded functions. It follows from equations (8) and (9) that $\sum_{k=1}^{m} \lambda_k \sigma_k$ and $\sum_{k=1}^{m} \lambda_k \dot{\sigma}_k$ both represent linear combinations of both bounded functions. It means that $\sigma_k, \dot{\sigma}_k \in L^\infty$. Therefore

$$\sup_{t\geq 0} |\sigma_k| = \|\sigma_k\|_\infty < \infty \Rightarrow \lim_{t\to\infty} \sigma_k = 0$$

Simulation results and discussion

The proposed controller is simulated in MATLAB and results are compared with the decoupled sliding mode controller (DSMC) and the integral sliding mode controller (ISMC). Consider a linear time-invariant (LTI) system of the form

$$\dot{x} = Ax + B\tau \quad (21)$$

where $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$. The integral surface is given by

$$\sigma = Cx - \int_0^t C(A - BK)x(\tau)d\tau \quad (22)$$

where $C \in \mathbb{R}^{m \times n}$ is selected to guarantee that CB is nonsingular and $K \in \mathbb{R}^{m \times n}$ is designed via pole placement such that all eigenvalues of the matrix $A - BK$ lie in the left half plane. The control input for ISMC is given by

$$\tau = \tau_{eq} + \tau_{sw} \quad (23)$$

where τ_{eq} and τ_{sw} are defined as

$$\tau_{eq} = -Kx$$
$$\tau_{sw} = -(CB)^{-1}[k\sigma + \eta \operatorname{sgn}(\sigma)]$$

where k and η are positive constants.

Application to IP

IP system is a strongly nonlinear underactuated mechanical system which is dynamically suitable to test new control strategies. The structure of IP system is shown in Figure 2. In case of IP model, system (1) can be represented as

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = g_1(x) + b_1(x)\tau + \Delta$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = g_2(x) + b_2(x)\tau + \Delta$$

The nonlinear functions $g_1(x)$, $g_2(x)$, $b_1(x)$, and $b_2(x)$ are defined as

$$g_1(x) = \frac{m_t g \sin x_1 - m_p L \sin x_1 \cos x_1 x_2^2}{L \cdot \left(\frac{4}{3}m_t - m_p \cos^2 x_1\right)}$$
$$g_2(x) = \frac{-\frac{4}{3}m_p L x_2^2 \sin x_1 + m_p g \sin x_1 \cos x_1}{\frac{4}{3}m_t - m_p \cos^2 x_1}$$
$$b_1(x) = \frac{\cos x_1}{L \cdot \left(\frac{4}{3}m_t - m_p \cos^2 x_1\right)}$$
$$b_2(x) = \frac{4}{3\left(\frac{4}{3}m_t - m_p \cos^2 x_1\right)}$$

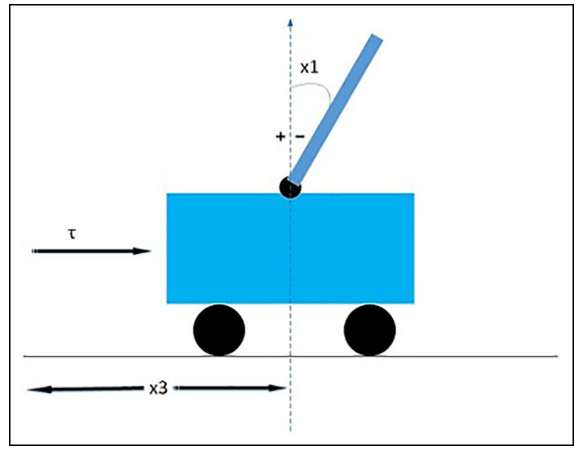

Figure 2. Structure of the inverted pendulum system.

Table 1. Description of variables and parameters used in inverted pendulum model.

Symbols	Description
x_1	Angle of the pole with vertical axis
x_2	Angular velocity of the pole
x_3	Position of the cart
x_4	Velocity of the cart
τ	Control input
Δ	External disturbances
L	Length of the pole
m_p	Mass of the pole
m_c	Mass of the cart

where $m_t = m_c + m_p$. The description of variables and parameters used in this model is given in Table 1. The first-level and second-level sliding surfaces for IP system are defined as

$$\sigma_1 = \varrho_1 x_1 + x_2 \quad (24)$$

$$\sigma_2 = \varrho_2 x_3 + x_4 \quad (25)$$

$$\Omega = \lambda_1\sigma_1 + \lambda_2\sigma_2 \quad (26)$$

The control input is given by

$$\tau = \tau_{eq1} + \tau_{eq2} + \tau_{sw} \quad (27)$$

where

$$\tau_{eq1} = \frac{-c_1x_2 - g_1 - \Delta}{b_1} \quad (28)$$

$$\tau_{eq2} = \frac{-c_2x_4 - g_2 - \Delta}{b_2} \quad (29)$$

$$\tau_{sw} = \frac{-\epsilon(x) \cdot \operatorname{sat}(\Omega)}{\lambda_1 b_1 + \lambda_2 b_2} \quad (30)$$

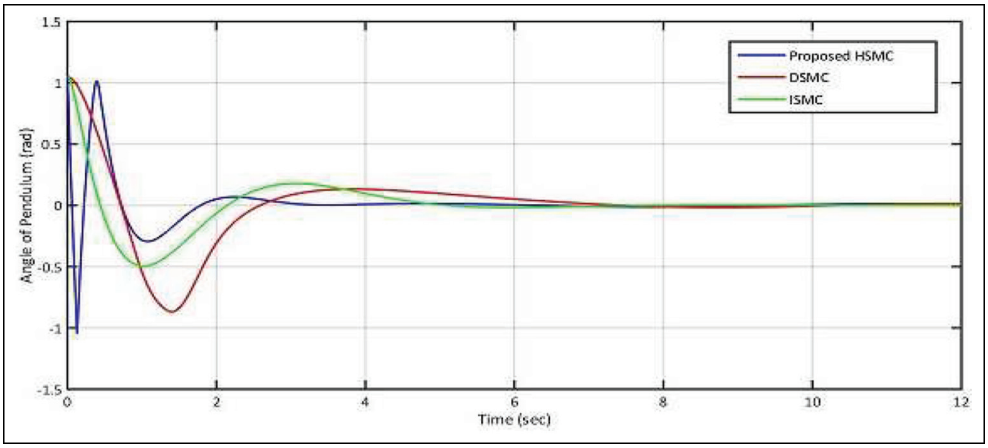

Figure 3. The angle of pendulum with time when system starts from $x_0 = [\frac{\pi}{3}\ 0\ 0\ 0]^T$.

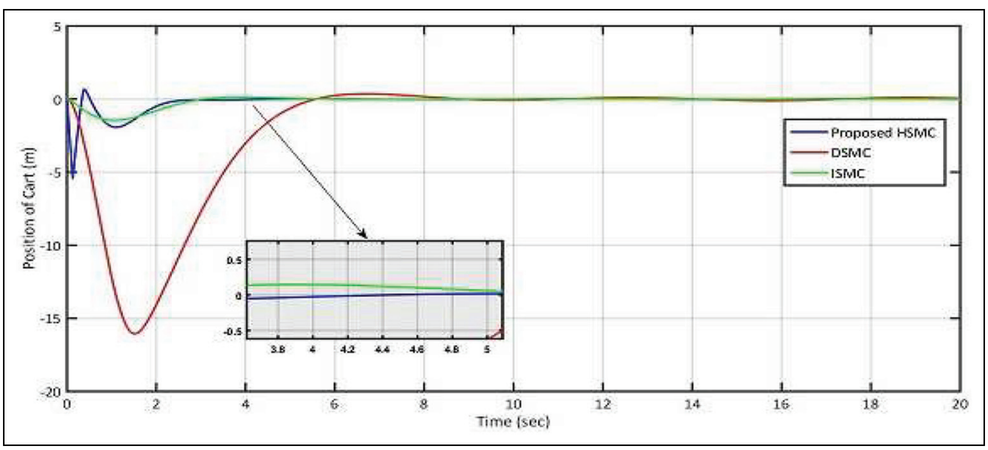

Figure 4. The position of cart with time when system starts from $x_0 = [\frac{\pi}{3}\ 0\ 0\ 0]^T$.

Table 2. Description of variables and parameters used in ball-beam model.

Symbols	Description
x_1	Angle of the beam with horizontal axis
x_2	Angular velocity of the beam
x_3	Position of the ball
x_4	Velocity of the ball
τ	Control input
Δ	External disturbances
m	Mass of the ball
r	Radius of the ball
j_b	Moment of inertia of the ball

and $\epsilon(x)$ is defined in equation (17). The values of parameters used for simulations are given in Table 3. We define a bounded external disturbance as $\Delta = \text{rand}().\sin(t)$, where MATLAB command "rand()" is used to describe the fluctuation of external disturbances. Stabilization of the angle of inverted pendulum is shown in Figure 3 and the position of the cart is shown in Figure 4. Both figures demonstrate that the proposed controller is robust and more efficient as compared with DSMC and ISMC.

Application to ball-beam system

Ball-beam (BB) system is another nonlinear underactuated system. Its structure is illustrated in Figure 5 and mathematical equations are given as

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \tau + \Delta \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{mr^2}{j_b + mr^2}(x_3 x_2^2 - g\sin x_1)
\end{aligned}$$

The description of different variables and parameters of ball-beam model is given in Table 2. The sliding surfaces are given by

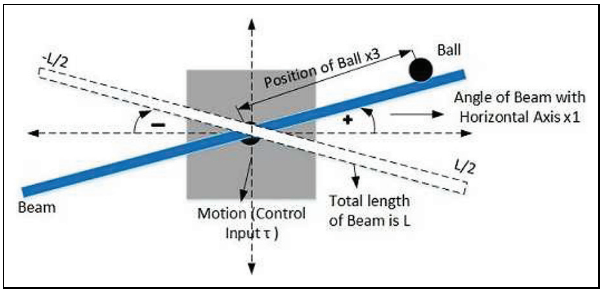

Figure 5. Structure of the ball-beam system.

$$\sigma_1 = \varrho_1 x_1 + x_2 \tag{31}$$

$$\sigma_2 = \varrho_2 x_3 + x_4 \tag{32}$$

$$\Omega = \lambda_1 \sigma_1 + \lambda_2 \sigma_2 \tag{33}$$

The control input is defined as

$$\tau = \tau_{eq} + \tau_{sw} \tag{34}$$

where

$$\tau_{eq} = -\varrho_1 x_2 - \Delta \tag{35}$$

$$\tau_{sw} = -\epsilon(x) \cdot \text{sat}(\Omega) - \lambda_1 \varrho_2 x_4 - \frac{\lambda_1 m r^2}{j_b + m r^2}(x_3 x_2^2 - g\sin x_1) \tag{36}$$

The values of parameters used in ball-beam model are given in Table 3. We use the same external disturbance as described in the inverted pendulum model. The change in the dynamics of angle of beam and position of the ball are shown in Figures 6 and 7. The simulation results show that the proposed controller has a faster convergence as compared with DSMC and ISMC in presence of bounded external disturbances.

Table 3. The values of parameters and constants used in the simulations of model.

Parameters	Values
L	0.5 m
m	0.5 kg
m_c	1 kg
m_p	0.05 kg
r	0.031 m
g	9.8 ms^{-2}
j_b	2×10^{-6}
ϱ_1	5
ϱ_2	0.5
λ_1	1
λ_2	–0.388
β	5
γ	4

We also compare HSMC with the state-dependent switching gain to HSMC with a constant switching gain. It can be seen that response time of the proposed HSMC is much smaller as compared with the conventional constant gain HSMC. Simulation results are shown in Figures 8 and 9. These results demonstrate the significance of proposed HSMC over existing HSMC, DSMC, and ISMC techniques. We also fluctuate the external disturbance to validate the robustness of proposed HSMC.

Conclusion

In this article, we discussed the general form of a class of underactuated mechanical systems and presented a nonlinear control strategy for stabilization of these systems. We focused on the potential uses of SMC methods for stabilization of underactuated mechanical systems. We proposed HSMC based on state-dependent switching gain. The proposed controller

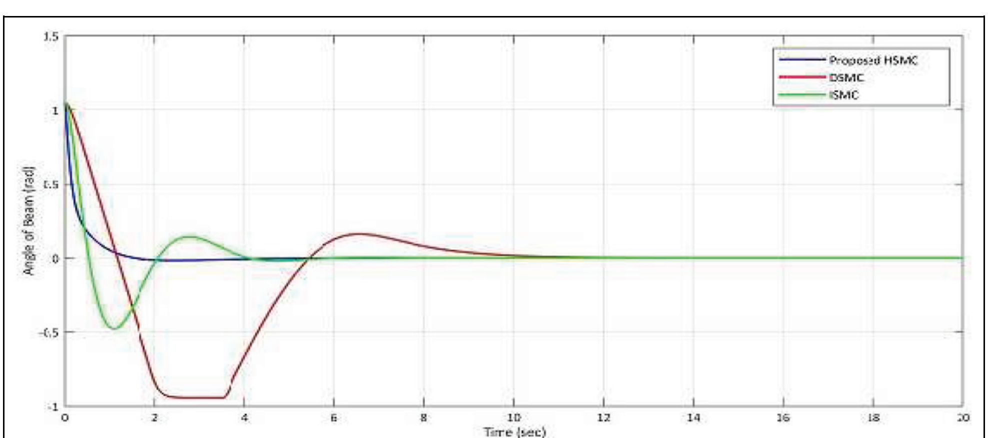

Figure 6. The angle of beam with time when system starts from $x_0 = [\frac{\pi}{3}\ 0\ 0.8\ 0]^T$.

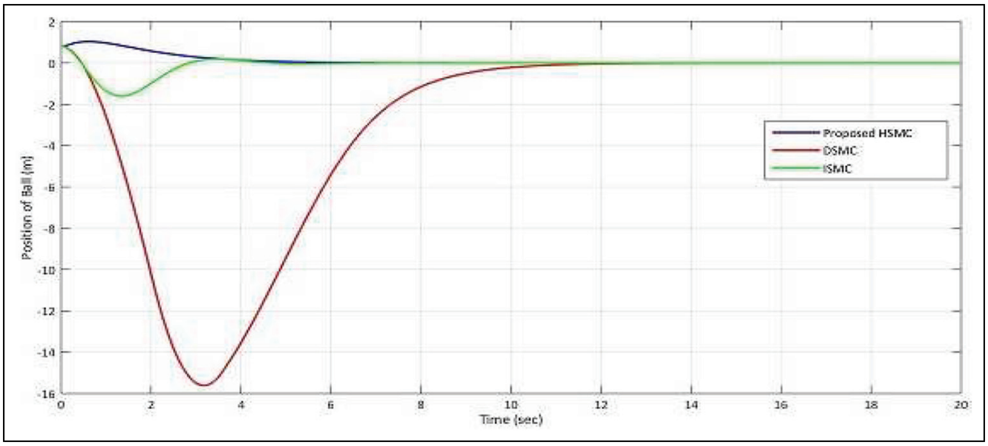

Figure 7. The position of ball with time when system starts from $x_0 = [\frac{\pi}{3}\ 0\ 0.8\ 0]^T$.

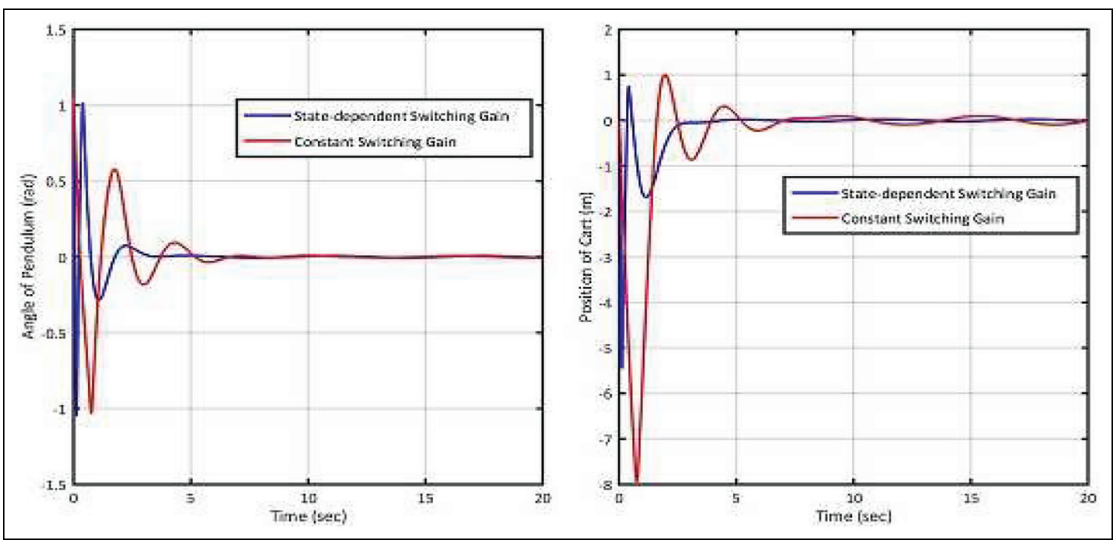

Figure 8. Comparison of proposed HSMC with existing HSMC in case of IP system.

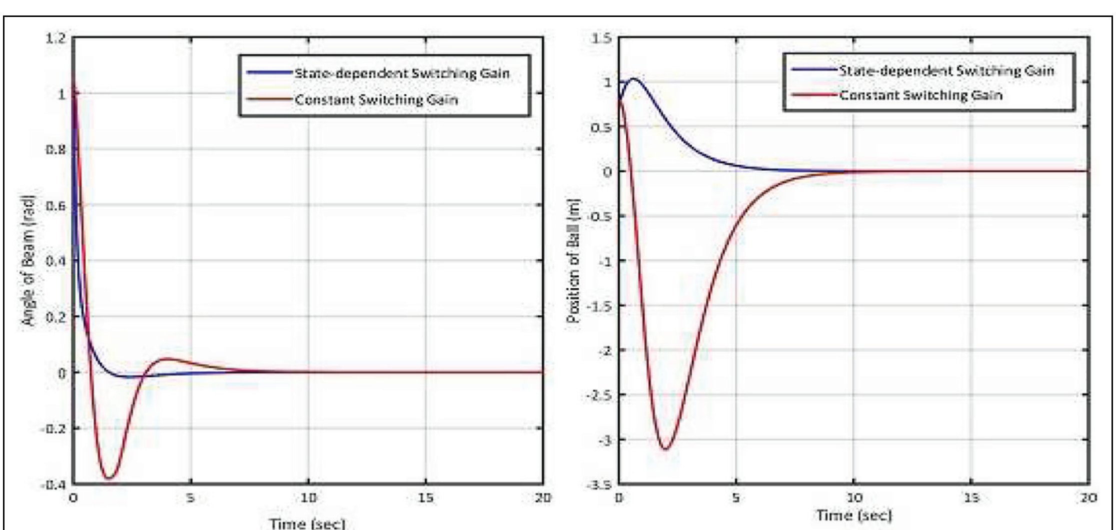

Figure 9. Comparison of proposed HSMC with existing HSMC in case of BB system.

consists of two first-level sliding surfaces and one second-level sliding surface which is essential to reduce chattering phenomenon.

The designed control technique is implemented on two nonlinear underactuated systems and simulated in MATLAB. The simulation results demonstrate that the proposed control technique is much efficient as compared with the existing SMC techniques such as HSMC, DSMC, and ISMC. The proposed control technique is robust and has a faster convergence as compared with these techniques. Furthermore, asymptotic stability of first-level and second-level sliding surfaces, used in the proposed technique, is also proved by the Lyapunov stability theory.

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References

1. Khan Q, Akmeliawati R, Bhatti AI, et al. Robust stabilization of underactuated nonlinear systems: a fast terminal sliding mode approach. *ISA Trans* 2017; 66: 241–248.
2. De Queiroz MS, Dawson DM, Nagarkatti SP, et al. Lyapunov-based control of mechanical systems. New York: Springer, 2012.
3. Berkemeier MD and Fearing RS. Tracking fast inverted trajectories of the underactuated Acrobot. *IEEE T Robot Autom* 1999; 15: 740–750.
4. Riachy S, Orlov Y, Floquet T, et al. Second-order sliding mode control of underactuated mechanical systems I: local stabilization with application to an inverted pendulum. *Int J Robust Nonlinear Control* 2008; 18: 529–543.
5. Zhang M and Tarn TJ. Hybrid control of the Pendubot. *IEEE/ASME T Mech* 2002; 7: 79–86.
6. Cheng J, Ahn CK, Karimi HR, et al. An event-based asynchronous approach to Markov jump systems with hidden mode detections and missing measurements. *IEEE T Syst Man Cyb* 2018; 99: 1–7.
7. He S, Ai Q, Ren C, et al. Finite-time resilient controller design of a class of uncertain nonlinear systems with time-delays under asynchronous switching. *IEEE T Syst Man Cyb* 2018; 99: 1–6.
8. Niu B, Wang D, Alotaibi ND, et al. Adaptive neural state-feedback tracking control of stochastic nonlinear switched systems: an average dwell-time method. *IEEE T Neural Netw Learn Syst* 2018; 99: 1–12.
9. Ortega R and Spong MW. Stabilization of underactuated mechanical systems via interconnection and damping assignment. *IFAC Proc Vol* 2000; 33: 69–74.
10. Ortega R, Spong MW, Gómez-Estern F, et al. Stabilization of a class of underactuated mechanical systems via interconnection and damping assignment. *IEEE T Automat Control* 2002; 47: 1218–1233.
11. Sarras I, Acosta JÁ, Ortega R, et al. Constructive immersion and invariance stabilization for a class of underactuated mechanical systems. *Automatica* 2013; 49: 1442–1448.
12. She J, Zhang A, Lai X, et al. Global stabilization of 2-dof underactuated mechanical systems—an equivalent-input-disturbance approach. *Nonlinear Dynam* 2012; 69: 495–509.
13. Shiriaev AS, Freidovich LB and Manchester IR. Can we make a robot ballerina perform a pirouette? Orbital stabilization of periodic motions of underactuated mechanical systems. *Ann Rev Control* 2008; 32: 200–211.
14. Qureshi MS, Swarnkar P and Gupta S. A supervisory on-line tuned fuzzy logic based sliding mode control for robotics: an application to surgical robots. *Robot Auton Syst* 2018; 109: 68–85.
15. Solanes JE, Gracia L, Muñoz-Benavent P, et al. Human-robot cooperation for robust surface treatment using non-conventional sliding mode control. *ISA Trans* 2018; 80: 528–541.
16. Zhao Y, Huang P and Zhang F. Dynamic modeling and super-twisting sliding mode control for tethered space robot. *Acta Astronaut* 2018; 143: 310–321.
17. Lu B, Fang Y and Sun N. Sliding mode control for underactuated overhead cranes suffering from both matched and unmatched disturbances. *Mechatronics* 2017; 47: 116–125.
18. Ngo QH, Nguyen NP, Nguyen CN, et al. Fuzzy sliding mode control of an offshore container crane. *Ocean Eng* 2017; 140: 125–134.
19. Tuan LA, Cuong HM, Lee SG, et al. Nonlinear feedback control of container crane mounted on elastic foundation with the flexibility of suspended cable. *J Vib Control* 2016; 22: 3067–3078.
20. Tuan LA, Lee SG, Nho LC, et al. Robust controls for ship-mounted container cranes with viscoelastic foundation and flexible hoisting cable. *Proc IMechE, Part I: J Systems and Control Engineering* 2015; 229: 662–674.
21. Antonio-Toledo ME, Sanchez EN, Alanis AY, et al. Real-time integral backstepping with sliding mode control for a quadrotor UAV. *IFAC-Papers Online* 2018; 51: 549–554.
22. Lin X, Nie J, Jiao Y, et al. Adaptive fuzzy output feedback stabilization control for the underactuated surface vessel. *Appl Ocean Res* 2018; 74: 40–48.
23. He S, Lyu W and Liu F. Robust H∞ sliding mode controller design of a class of time-delayed discrete conic-type nonlinear systems. *IEEE T Syst Man Cyb* 2018; 99: 1–8.

24. He S, Song J and Liu F. Robust finite-time bounded controller design of time-delay conic nonlinear systems using sliding mode control strategy. *IEEE T Syst Man Cyb* 2017: 99: 1–11.
25. Wang Y, Xia Y, Shen H, et al. SMC design for robust stabilization of nonlinear Markovian jump singular systems. *IEEE T Automat Control* 2018; 63: 219–224.
26. Jiang B, Karimi HR, Kao Y, et al. Adaptive control of nonlinear semi-Markovian jump T-S fuzzy systems with immeasurable premise variables via sliding mode observer. *IEEE T Cyberne* 2018; 99: 1–11.
27. Jiang B, Karimi HR, Kao Y, et al. A novel robust fuzzy integral sliding mode control for nonlinear semi-Markovian jump T-S fuzzy systems. *IEEE T Fuzzy Syst* 2018; 26: 3594–3604.
28. Wang Y, Shen H, Karimi HR, et al. Dissipativity-based fuzzy integral sliding mode control of continuous-time T-S fuzzy systems. *IEEE T Fuzzy Syst* 2018; 26: 1164–1176.
29. Slotine JJE and Li W. *Applied nonlinear control*. Upper Saddle River, NJ: Prentice Hall, 1991.
30. Lee H, Utkin VI and Malinin A. Chattering reduction using multiphase sliding mode control. *Int J Control* 2009; 82: 1720–1737.
31. Edwards C, Colet EF, Fridman L, et al. *Advances in variable structure and sliding mode control*. New York: Springer, 2006.
32. Muhammad S and Idrees M. Comparative study of hierarchical sliding mode control and decoupled sliding mode control. In: *Proceedings of the 12th IEEE conference on industrial electronics and applications (ICIEA), Siem Reap*, *Cambodia*, 2017, pp.818–824. New York: IEEE.
33. Jia Z, Yu J, Mei Y, et al. Integral backstepping sliding mode control for quadrotor helicopter under external uncertain disturbances. *Aerospace Sci Technol* 2017; 68: 299–307.
34. Kaynak O, Erbatur K and Ertugrul M. The fusion of computationally intelligent methodologies and sliding mode control—a survey. *IEEE T Ind Electron* 2001; 48: 4–17.
35. Kchaou A, Naamane A, Koubaa Y, et al. Second order sliding mode-based MPPT control for photovoltaic applications. *Solar Energ* 2017; 155: 758–769.
36. Liu X, Vargas AN, Yu X, et al. Stabilizing two-dimensional stochastic systems through sliding mode control. *J Franklin Inst* 2017; 354: 5813–5824.
37. Utkin VI and Korovin SK. Application of sliding mode to static optimization. *Automat Rem Control* 1972; 4: 570–579.
38. Yagn KD and Utkin VI. Methods for construction of discontinuity planes in multidimensional variable structure systems. *Automat Rem Control* 1978; 39: 72–77.
39. Qian D, Yi J, Zhao D, et al. Hierarchical sliding mode control for series double inverted pendulums system. In: *Proceeding of the international conference on intelligent robots and systems*, Beijing, China, 9–15 October 2006. New York: IEEE.
40. Qian D, Yi J and Zhao D. Hierarchical sliding mode control for a class of SIMO under-actuated systems. *Control Cyberne* 2008; 37: 159–175.
41. Lo JC and Kuo YH. Decoupled fuzzy sliding-mode control. *IEEE T Fuzzy Syst* 1998; 6: 426–435.