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Solution of Quiz 1 February 12, 2020 ACTU 464

Question (**5 marks**)

1. Find θ such that $\mathrm{P}\left(Z \leq \Pi_{\mathrm{SL}}\left(\theta\right)\right) = \alpha$ if you are given the following: $Z = 0.7X$, and $X \hookrightarrow \mathcal{E}xp(\lambda = 2)$ and $\alpha = 0.05$.
2. Calculate $\Pi_{\mathrm{SL}}\left(\theta\right)$.

Solution

1. We would like to find θ such that

$$\mathrm{P}\left(\frac{Z - \mathrm{E}\left[Z\right]}{\sigma_Z} \leq \theta\frac{\mathrm{E}\left[Z\right]}{\sigma_Z}\right) = 1 - 0.05 = 0.95. \tag{1}$$

Remark first that $\mathrm{E}\left[Z\right] = 0.7\mathrm{E}\left[X\right] = 0.7 \times 0.5$ and $\sigma_Z = 0.7\sigma_X = 0.7 \times 0.5$, hence $\frac{\mathrm{E}[Z]}{\sigma_Z} = \frac{1}{2}$. Now, the equation (1) becomes

$$\begin{aligned} 0.95 &= \mathrm{P}\left(\frac{0.7X - 0.5)}{0.5} \leq \theta\right) = \mathrm{P}\left(0.7X - 0.5 \leq \frac{\theta}{2}\right) \\ &= \mathrm{P}\left(X \leq \left(\frac{\theta}{2} + \frac{1}{2}\right)\frac{10}{7}\right) = \mathrm{P}\left(X \leq \left(\theta + 1\right)\frac{5}{7}\right) \\ &= F_X\left(\left(\theta + 1\right)\frac{5}{7}\right) \end{aligned}$$

Since X is exponentially distributed with parameter $\lambda = 2$ its c.d.f. is $F_X(x) = 1 - e^{-2x}$, then;

$$F_X\left(\left(\theta + 1\right)\frac{5}{7}\right) = 1 - \exp\left(-2\left(\theta + 1\right)\frac{5}{7}\right) = 0.95$$

Solving for θ :

$$\begin{aligned} 1 - \exp\left(-\left(\theta + 1\right)\frac{10}{7}\right) = \frac{95}{100} &\iff \exp\left(-\left(\theta + 1\right)\frac{10}{7}\right) = \frac{1}{20} \\ &\iff -\left(\theta + 1\right)\frac{10}{7} = \ln\left(\frac{1}{20}\right) = -\ln\left(20\right) \\ &\iff \theta + 1 = \frac{7}{10}\ln\left(20\right) \\ &\iff \theta = \frac{7}{10}\ln\left(20\right) - 1 = 1.097. \end{aligned}$$

2. Consequently the 1.097–safety loading premium for the proportional loss $Z = 0.7X$ is given by

$$\Pi_{\mathrm{SL}}\left(1.097\right) = (1 + 1.097)\mathrm{E}\left[Z\right] = 2.097 \times 0.7\mathrm{E}\left[X\right] = 2.097 \times 0.7 \times 0.5 = 0.73395.$$